Journal of Project Management 10 (2025) 613–628

Contents lists available at GrowingScience

Journal of Project Management

homepage: www.GrowingScience.com/jpm

API-based dynamic programming model and optimization of vehicle routing: Cases of fluctuations in demand, traffic, capacity, and availability

Osamah Abdulhameed[a*] and Naveed Ahmed[a]

[a]*Department of Industrial Engineering, College of Engineering, Al Yamamah University, Riyadh 11512, Saudi Arabia*

C H R O N I C L E

Article history:
Received April 22, 2025
Received in revised format June 25, 2025
Accepted August 28 2025
Available online
August 28 2025

Keywords:
Vehicle routing
Vehicle capacity
Network
Customer nodes
Demand quantities
Dynamic programming model
API distance matrix

A B S T R A C T

The primary challenge in the supply chain is minimizing travel distance and time between hubs and customers. Inappropriate assignment of vehicle routing results in travel distances longer than required, causing delays in achieving timely deliveries, and ultimately negatively affect the customer expectations routes. In this study, the selecting optimal vehicle routing has been addressed. This involves calculating the shortest possible that meets the demand effectively while adhering to various logistical constraints like warehouse fixed positions, demand variety, demand quantity, and destination locations. Dynamic programming has been developed where numerous time period-based fluctuations can be accommodated such as fluctuations in traffic, alternative routes availability, and changes in travel distances. The weighted demand cost matrix has been introduced to prioritize and cluster the group of customer nodes for the assignment of certain vehicles. Moreover, API google distance matrix (latitudes and longitudes) has been integrated into the model to extract live locations of source-and-demand nodes which are a function of different time periods of a day. The dynamic has optimized the vehicle routes and results in 30.9% reduction comparing the existing case. The validation was done through four more cases where different possibilities such as business expansion, network growth, demand fluctuations, and vehicle capacities.

© 2025 by the authors; licensee Growing Science, Canada.

1. Introduction

The integration of location, inventory, and routing problems is a significant area of focus. It is noted that solving these components together rather than separately can lead to substantial improvements in overall efficiency and cost reduction in logistics networks. This approach addresses the complexities of modern logistics demands by considering multiple facets such as transportation costs, storage capabilities, and distribution efficiency simultaneously. Research presented in (Almalki & Alkahtani, 2022) has categorized hub location models into multi-criteria and single-criterion based on the number of decision criteria they consider. These models factor in various aspects like transport costs, hub functionality, investment needs, supply and demand dynamics, market proximity, policy implications, and environmental impacts. Various case studies are presented in (Vieira & Luna, 2016) highlighting the practical applications of these models in diverse geographic and industrial contexts. For example, studies have focused on optimizing network designs for better cargo flow management, minimizing environmental impacts of traffic and congestion, and enhancing the service levels through strategic hub placements. Recent advances in methodologies include the use of Geographic Information Systems (GIS), heuristic and metaheuristic algorithms, and various optimization models to tackle complex logistics problems as presented in a review (Almalki & Alkahtani, 2022). These tools help in better visualization, decision-making, and implementation of logistics strategies that are robust and adaptable to changing market and environmental conditions.

* Corresponding author Tel.: +966580915397
E-mail address o_abdulhameed@yu.edu.sa (O. Abdulhameed)

ISSN 2371-8374 (Online) - ISSN 2371-8366 (Print)
© 2025 by the authors; licensee Growing Science, Canada
doi: 10.5267/j.jpm.2025.8.007

Several researches related to hub location and its optimization are found in literature. Different methods are applied such as mathematical programming, heuristic techniques, fuzzy logic-based formulations, multi-criteria decision making (MCDM) and multi-criteria decision analysis (MCDA). For example, the use of MCDM is in widespread disciplines such as environmental sciences, computer science, materials sciences, and social sciences. Among the areas of MCDM application, its use in engineering field is the most common (Taherdoost & Madanchian, 2023) Based on these techniques, mathematical models are developed and validated using assumed numerical data despite of validating with the data pertaining to real case-studies. For instance, (Sarker, Wu, & Paudel, 2018) developed mixed integer non-linear model for multi-hub location allocation problem. The model was verified through the data of numerical examples. Similarly, in research presented in (Gruca & Klemz, 2003) genetic algorithm-based procedure has been developed to address the problem of optimal positioning of hubs for new products to be launched. The results were compared through simulation and empirical study instead of applying the developed algorithm to actual case study. The research (Dye & Hsieh, 2024) analyzed a joint dynamic pricing, investment, and advertisement model with an objective of maximizing the profit. The application of the model is found to be generating theoretical results. Thus, it leads to the point that the outcome of such research is the development and/or the improvement in the given solution techniques.

While making decisions on optimization of locations and transportations several factors of consideration should be involved such as capacity and number of facilities, network design, and the products' flow etc. (Campbell & O'Kelly, 2012). Hub-location problem (HLP) models seem to be more applicable in the interest of hub location and flow application throughout the network. However, the limitation of HLP is not to be the most suitable choice in case of logistics hubs having large-scale networks. Polynomial functions are involved in many of the types of networks especially considering the time-dependency and different time-periods. For this reason, non-linear mathematical models are expected to be developed for such problems. Meta-heuristic techniques and algorithms are used to solve non-linear problems and models such as NSGA, NSGA, NRGA, and RESA-II etc. It is also stated that the facility location allocation (FLA) decisions fall under the category of NP-hard optimization problems (Fahmy, Zaki, & Gaber, 2023). In the area of network design with rapid changes and developments, the attention should be paid to consider the strategic decisions. Dynamic changes include, for instance, the environmental changes, road traffic, movability, and mobility of transportation in different periods etc. Moreover, if the source or destination hubs deal with perishable products, for example fresh produce, having different rates and dates of spoiling then the dynamic problems attain a further higher level of complexity. Perishable products are dependent over the transportation time, distances between origins and the destination nodes, services provided at the transportation hubs, and other conditions. Hence, accommodating the changes in the resources and developments in the distribution units, the mathematical models should be dynamic in nature so that the effect of changes on decision making can also be ensured. In this connection, bi-objective model is proposed in (Eydi & Saghez, 2024). aiming to reduce the travel time and network costs.

Shared mobility is another concept adapted in hub locations wherein the use of transport in the given network is shared by cars, trucks, mopeds, and motor bikes etc. It improves the connectivity and minimizes transportation trips. The amount of requirement at demand nodes is also a time-dependent function. Hub capacity can also be modular and each module associated with a certain number of servers serving a hub. In this context the hub network can be termed a multi-period network. (Xanthopoulos et al., 2024) proposed a model addressing the location and capacity issues of multimodal hubs. Non-linear mathematical model presented in (Khaleghi, Eydi, & Research, 2023) considered the time-dependent function of demand nodes. It helps in minimizing the cost of networks and travel distances. Including the dynamic and stochastic nature of network designing in capacity planning is the basis of dynamic mathematical models. (Liu, Li, Dahan, Montreuil, & Review, 2025) applied combinatorial Benders Decomposition (CBD) meta-heuristic algorithm to develop a dynamic model to control the hub costs. The applicability of the model was tested on a case study selected from the automotive delivery sector. The proposed model resulted in a significant reduction (22.96%) in hub costs for the automotive delivery case. Optimal-transport measure has been used by (Hu, Liu, Chen, Wang, & Research, 2024) as a criterion for minimizing the setup and shipping cost of selected hubs. In this way the capacitation problem of hub location was solved considering the uncertainty in demand and cost.

(Fahmy et al., 2023) developed a mixed integer LP model considering dynamic changes in transportation time, spoilage rate of products, and capacity based hub costs. Due to the complexity involved in the problem because of spoilage rate of product, the researchers proposed a hybrid algorithm combining meta-heuristic with perishability transportation algorithms such as particle swarm optimization and simulated annealing. Similarly, maintaining the freshness of products is also required in the area of agriculture, therefore minimizing transportation time and hub location distances is also another area of interest. (Wang, Cheng, Huang, Cheng, & Applications, 2021) have modeled the transportation problem related to agricultural products while setting the objective functions to be the timely delivery along with minimum network cost. They verified their model through the transportation data of different states of the United States. Their model resulted in a 16.73% reduction in cost as compared to the direct routing transport method. Another model presented in (Zhu, Schneider, & Oude Lansink, 2023) has been developed to measure the dynamic nature of economic, environmental, and inefficiency of Dutch dairy farms. (Shi, Yu, & Dong, 2021) applied dynamic programming techniques to address optimal space allocation decisions. They have tested the effectiveness of their heuristic through applying the model on a real dataset provided by Red-Star Macalline (China-based furniture retail). Finally, the literature suggests that future research could further explore the integration of real-time data

analytics, the impact of evolving global trade policies, and sustainability considerations in hub location optimization (Vieira & Luna, 2016). Emphasizing these areas could lead to more dynamic and responsive logistics networks that are capable of addressing both immediate and long-term logistical challenges. Vehicle routing problems are common in operations research and supply chain logistics. The solvers propose a solution to provide deliveries at desired locations in a timely manner (Praveen, Keerthika, Sivapriya, Sarankumar, & Bhasker, 2022). The vehicle routing problem for an FMCG has been addressed in (Jayarathna, Lanel, Juman, & Trade, 2022) where the researchers have proposed a gravity model to optimize the truck route plan, number of trucks allocation, and the transportation cost.

Vehicle routing problem was applied in several research studies without considering the balance between customer's distance and its demand. The novelty of this research is to give priority to those having high demand and low distance and least priority for those having low demand and high amount of distance. Also, this method can be applied for emergency situations to fulfil the demand in emergency. Weighted cost matrix based on customer demand and distance has been introduced in the model. Moreover, API google distance matrix is integrated with the model to accommodate the dynamic fluctuations. One real-case has been taken to apply the model and compare results with the existing routing and travel cost. The proposed dynamic programming model offered optimized vehicle routing and customer clustering in such a way that 30% reduction in travel distance has been experienced. The model is further validated through four different cases to testify that the optimized vehicle routing follows the vehicle capacity and other set constraints.

2. Methodology

A dynamic programming model that incorporates constraints and objectives related to the distribution network. In order to get deep insight into the problem and develop the solution, data pertaining to the different aspects of the problem was collected. It includes the current location of the existing customers, warehouse location, quantity of orders per customer per location, list of the products, products demanded by customers, availability of the number of transport means (in specific the trucks availability), and the capacity of each transportation means.

In the phase of developing dynamic programming the locations (warehouse and customer) were linked with the google coordinates by considering both the latitudes and longitudes of each location i.e. the node points of the model. In this way, an API distance matrix was developed as shown in Table 1. The delivery or demand cost mainly depends on the length of route travel by a truck from the warehouse to the customer locations. Thus, the demand cost was computed for each combination of the truck-warehouse-customer. For example, the demand cost for a certain customer, for instance i^{th} customer, was computed through the distance covered by the truck, for instance truck *j,* starting its travel from the warehouse to the i^{th} customer. Moreover, the capacity of each of the trucks was also incorporated in the programming model. The decision criterion for assigning the right allocation for customers was established by considering the set of constraints. The methodology flowchart is shown in Fig. 1 depicting the programming stages along with the selection criterion.

Table 1
API distance matrix based on latitudes and longitudes of customer nodes

ID	Latitude	Longitude	Demand (unit boxes)
Warehouse	21.43385	39.21683	0
C1	21.56520833	39.19109833	5
C2	21.55959887	39.24909585	6
C3	21.5959682	39.1674466	3
C4	21.51347202	39.20027886	9
C5	21.40930802	39.25782021	12
C6	21.59208767	39.21878148	2
C7	21.43965667	39.20573667	20
C8	21.62124689	39.17101239	4
C9	21.58334166	39.12929166	50
C10	21.43381696	39.20308027	6
C11	21.45884577	39.26184909	3
C12	21.61951666	39.19491833	9
C13	21.51479333	39.16489833	12
C14	21.41975666	39.29488833	10
C15	21.55039	39.205445	6
C16	21.43623019	39.19652984	5
C17	21.43699533	39.19666294	12
C18	21.48383112	39.3092635	23

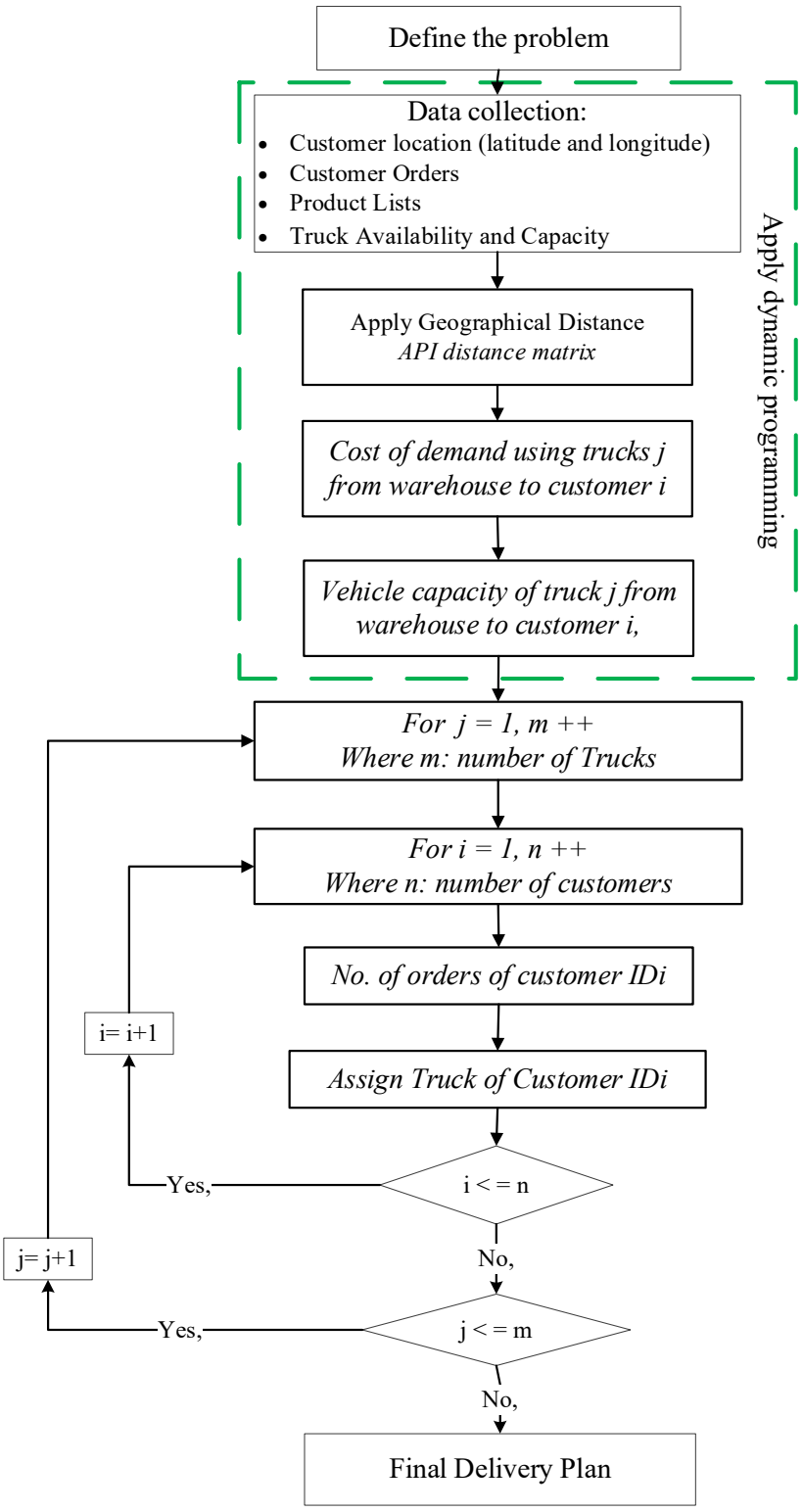


Fig. 1. Flowchart of the research methodology

3. Model Formulation

A standard syntax of mathematical model formulation was applied such as identifying the decision variables, setting the objective function, formulation of set of constraints, and other parameters associated with the distribution network. The model uses a distance matrix as the primary input, which represents the cost or distance from potential hubs to various demand points.

3.1 Identifying the decision variable(s)

In this model a compound decision variable is defined based on the different parameters. The first parameter is the interrelated distances from warehouse node to the customer node and from each individual customer node to the other remaining customer nodes in the network. A sample inter-related distance matrix (From-To-Distance) is shown in Table 2. This parameter is denoted as $d_{i,j}$.

$$d_{i,j} = APi\ distance\ matrix\ of\ trucks\ j\ from\ warehouse\ to\ customer\ i\ (km)$$

The second parameter is the quantity of orders requested by the customers from each location in the network. Since the traveling of a certain vehicle is to cover the demand of several node points, therefore the vehicle capacity has related to the demand nodes. Thus, the quantity of orders is defined as $Q_{i,j}$.

$$Q_{i,j} = Vehicle\ capacity\ of\ the\ truck\ j\ from\ warehouse\ to\ customer\ i$$

The third component of the compound decision variable is the demand cost per customer node. A sample demand cost is shown in Table 3. This component is denoted as $C_{i,j}$.

$$C_{i,j} = Cost\ of\ providing\ the\ demand\ using\ truck\ j\ from\ warehouse\ to\ customer\ i$$

Thus, the compound decision variable is set as the weighted travel cost. The weighted travel cost is a function of the network's inter-related distances, the quantity of orders, and the demand cost matrix as shown in Eq. 1.

$$wt_{i,j} = \left\{ {Q_{i,j}}/{(d_{i,j}C_{i,j})} \right\} \tag{1}$$

Table 2
A sample inter-related distance matrix of the network.

Customer nodes		WH	1	2	3	4	5
		Distance (km)					
WH	Distance (km)	0	20.4	23	24.5	14.2	8.6
1		18.1	0	8.5	6.4	7.7	24.9
2		24.2	9.8	0	14.4	13.6	22.1
3		22.2	5.5	12.7	0	11.7	29.7
4		13.9	7.5	12.7	12	0	22.6
5		8.6	29	23.7	31.9	21.7	0

Table 3
A sample demand cost matric of the network.

Customer nodes		WH	1	2	3	4	5
		Demand Cost (SAR)					
WH	Demand Cost (SAR)	0.000	1.007	1.072	0.503	2.605	5.734
1		1.135	0.000	2.901	1.926	4.803	1.981
2		1.019	2.516	0.000	0.856	2.720	2.231
3		0.555	2.242	0.971	0.000	3.161	1.660
4		2.138	4.932	2.912	3.082	0.000	2.182
5		5.734	1.701	2.081	1.546	2.273	0.000

3.2 Defining the Objective Function

Instead of using the direct cost as a measure of objective, it is set to be the minimization of the total weighted travel cost pertaining to the hub and customer network. The objective function is presented in Eq. (2).

$$\max \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K} wt_i\, \lambda_{ij}^{k}$$

$$\begin{bmatrix} wt_{1,1} & \dots & wt_{1,j} \\ wt_{2,1} & \dots & wt_{2,j} \\ \dots & \dots & \dots \\ wt_{i,1} & \dots & wt_{i,j} \end{bmatrix} \begin{bmatrix} \lambda_{1,1} & \dots & \lambda_{1,j} \\ \lambda_{2,1} & \dots & \lambda_{2,j} \\ \dots & \dots & \dots \\ \lambda_{i,1} & \dots & \lambda_{i,j} \end{bmatrix} \tag{2}$$

where

$$wt_{i,j} = \left\{ {Q_{i,j}}/{(d_{i,j}C_{i,j})} \right\}$$

$wt_{i,j}$ *is the wighted travel distance* (*priority wieght*) *of trucks* j *from the warehouse hub to customer node* i (km)

$d_{i,j} = APi$ *distance matrix of trucks* j *from warehouse to customer* i (km)

$C_{i,j} = Cost$ *of providing the demand using truck* j *from warehouse to customer* i

$Q_{i,j} = Customer\ demand$

$\lambda_{i,j}$ *is a binary variable; value* 1 *means the* $i - hub$ *serves to* $j - customer$, *and value* 0 *is otherwise*

$$\sum_{j=0}^{n}\sum_{k=1}^{K}\lambda_{ij}^{k} = \begin{cases}1, \\ 0,\end{cases} \qquad (1, if\ truck\ k\ is\ assigned\ to\ travel\ from\ customer\ i\ to\ customer\ j, 0, otherwise)$$

n *is the number of customers to be served in one full routing*, $i = \{1,2,\dots,j,\dots,n\}$

K *is the number of availability of trucks to be used in one full routing*, $k = \{1,2,\dots,K\}$

3.3 Constraints and Parameters

The model includes sets of constraints. These constraints are defined to meet the supply chain requirement of the hub-customer network and to have error free results upon the model execution. Hence, the constraints ($C_1, C_2, C_3, C_4, and\ C_5$) and their short description are listed in Eq. (3),

$$C_1: \quad \sum_{j=0}^{n}\sum_{k=1}^{K}\lambda_{ij}^{k} = 1\,, \qquad \forall i,j \in \{1,\dots,n\} \tag{3}$$

The constraint (C_1) ensures that each customer must be visited exactly once by one vehicle.

$$C_2: \quad \sum_{j=1}^{n}\lambda_{0j}^{k} = 1, \sum_{i=1}^{n}\lambda_{i0}^{k} = 1, \qquad \forall k \in \{1,\dots,K\} \tag{4}$$

This constraint (C_2) pertains to the fact that each vehicle must start and end its route at the warehouse.

$$C_3: \quad \sum_{j=0}^{n}\lambda_{ij}^{k} = \sum_{j=0}^{n}\lambda_{ji}^{k}, \qquad \forall i,j \in \{1,\dots,n\}, \qquad \mathrm{k} \in \{1,\dots,K\} \tag{5}$$

This constraint (C_3) defines that if a vehicle arrives at a j[th] customer, it must leave from that customer. It is called as flow conservation constraint.

$$C_4: \quad \sum_{i=1}^{n} Q_i\lambda_{ji}^{k} \le T_K, \qquad \forall i,j \in \{1,\dots,n\}, \qquad \mathrm{k} \in \{1,\dots,K\} \tag{6}$$

The constraint (C_4) belongs to the capacity of the vehicles.

$$C_5: \quad u_i - u_j + T_K\lambda_{ji}^{k} \le T_K - u_j \qquad \forall i,j \in \{1,\dots,n\}, i \ne j, \mathrm{k} \in \{1,\dots,K\} \tag{7}$$

The constraint (C_5) belongs to the capacity constraints which is stated here, making sure that the sum of the demands of the customers visited in a route is less than or equal to the capacity of the vehicle performing the service, where u_i represents the cumulative demand along the route.

4. Computational Implementation

Using Microsoft Excel Solver and VBA scripting, and python, the optimization process was automated. The steps involved are as follows:

1. ***Data Preparation:*** Assemble the distance matrix and define all relevant parameters and constraints in the Excel environment.
2. ***Model Setup:*** Implement the dynamic programming model using the Excel Solver, setting up the objective function to minimize total travel costs.
3. ***Optimization Execution:*** Run the Solver to find the optimal solution, adjusting parameters and constraints iteratively to explore different scenarios and refine the results.

4.1 The Current Case

The first case was considered as the current existing case of the company wherein there are 18 customer nodes, 4 trucks with a capacity of 60 units each, and varying demand of each customer node. The current map of the customer network is shown in Fig. 2.

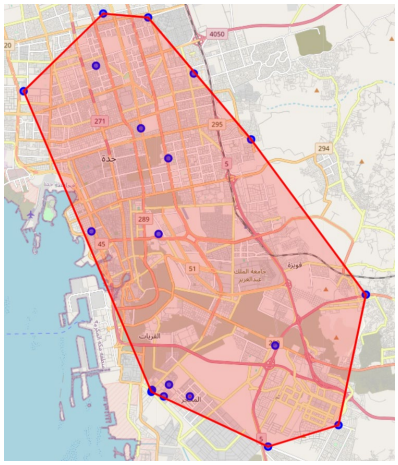


Fig. 2. The existing map locations of the network of 18 customers

The company has been adopting the First-Order-First-Delivery approach without checking their locations. As per this approach, the demand, travel distance and vehicle routing are presented in Table 4. With 4 vehicles the demand is being fulfilled by covering a total travel distance of 177.37 km. The current has also a lacking in the appropriate assignment of customer nodes in the network. By the implementation of the proposed dynamic mathematical model, a significant improvement has been found in the demand fulfillment, the vehicle routing, and the total travel distance. Prior to running the model, the input data was arranged in the form of different required tables and formats. Pertaining to this input data, the detailed inter-related (From-To) distance matrix is shown in Table 5, the quantity order matrix is shown in Table 6, and the weighted demand cost matrix is tabulated in Table 7.

Table 4
Network demand, travel distance and vehicle routing (the existing case of First-Order-First-Delivery)

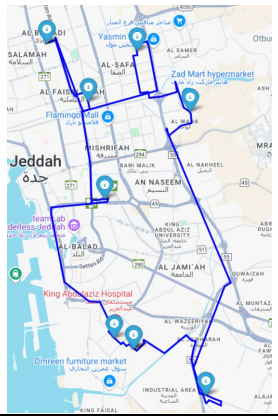
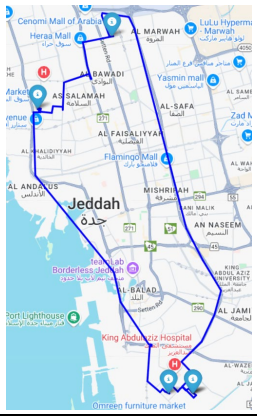
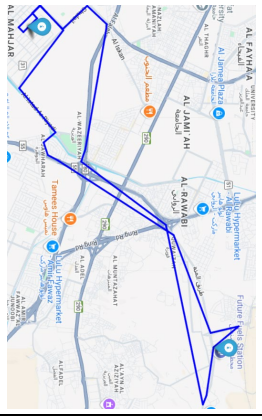
Vehicle 1	**Vehicle 2**	**Vehicle 3**	**Vehicle 4**
Demand: 57 units Distance: 51.33 km	Demand: 60 units Distance: 46.97 km	Demand: 57 units Distance: 56.70 km	Demand: 23 units Distance: 22.13 km
Total Demand: 197 units Total Distance: 177.13 km			
			

Table 5

Detailed inter-related (From-To) distance matrix of the network of 18 customer node points

Customer Nodes		WH	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		Distance (km)																		
WH	Distance (km)	0	20.4	23	24.5	14.2	8.6	25.3	2.1	26.5	25.5	3.6	7.3	25	16.6	15.2	16.8	3.5	3.7	19.7
1		18.1	0	8.5	6.4	7.7	24.9	6.4	16.5	8.4	12.2	17.5	16.8	9.3	9.4	24.7	6.1	19.6	19.6	25.2
2		24.2	9.8	0	14.4	13.6	22.1	8.1	22.9	16.4	19.7	23.9	17.4	13.8	14	21.9	10.4	25.5	23.6	22.4
3		22.2	5.5	12.7	0	11.7	29.7	10.2	20.6	3.9	8.3	21.6	20.9	5.8	12.7	29.5	9.6	23.7	23.7	29.9
4		13.9	7.5	12.7	12	0	22.6	15.4	14.4	14	15.7	15.4	11.4	15.1	7.3	19.4	6.9	14.8	14.8	19.8
5		8.6	29	23.7	31.9	21.7	0	27.3	10.2	33.9	33.8	11.7	10.3	32.6	24.8	7.6	25.1	12.5	12.7	18.1
6		26.3	10.2	8.3	14.8	16	24.1	0	25	9.6	20.1	25.9	19.5	7	17.1	24	9.1	27.5	25.6	24.4
7		1.6	19.8	22.5	23.9	13.7	12.7	24.7	0	25.9	24.9	1.8	9.6	24.5	16	14.6	16.2	2.6	2.7	19.1
8		28.3	8.1	17.6	4.6	14.4	33.5	13	23.2	0	12	24.2	23.5	3.6	15	33.3	13.7	26.3	26.3	33.8
9		27.1	9.6	16.6	6.5	17.8	32.4	12.9	24.1	9.9	0	25.1	29.2	12.9	13.4	34.3	13	24.5	24.5	38.8
10		3	19.6	22.2	24.3	14.1	12.5	24.5	1.9	26.4	22.5	0	9.3	24.2	13.6	14.4	16	1	1	18.9
11		10.4	18.9	17.6	23	12.8	9.2	21.2	9.7	25	29.1	10.7	0	23.5	20.1	7.5	15.3	11.6	10.4	10.7
12		24.6	7.8	14.6	9	16	30.5	7.2	23.3	3.5	13.5	24.3	22.2	0	17	30.4	10	25.8	23.9	30.8
13		14.2	10.8	17.2	15.2	4.9	19.5	18.9	11.2	17.3	12.6	12.2	16.3	18.6	0	21.4	10.1	11.6	11.6	25.8
14		12.9	26.8	21.5	30	19.8	7.7	25.1	17.3	32	33.5	18.3	10.1	30.5	24.6	0	22.9	16.8	18	16.1
15		20.8	4.3	7.4	8.8	12.2	23.7	9.2	19.5	10.1	14.1	20.5	18.4	8.9	11.1	23.6	0	22.1	20.2	24
16		3.1	18.8	21.8	23.3	13	12.1	24.1	1.5	25.3	21.5	1	8.9	23.8	12.6	14	15.5	0	0.1	18.4
17		3	19	21.7	21.6	11.3	11.9	23.9	1.4	23.6	22.4	1	8.8	23.7	13.5	13.8	15.4	0.4	0	18.3
18		16.8	22.2	18.4	28.1	18.7	13.6	22	17.8	30.1	34	18.8	9	27.3	25	13.7	18.6	18	18.4	0

Table 6

The quantity order matrix of 18 customer nodes

Customer	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Quantity	5	6	3	9	12	2	20	4	50	6	3	9	12	10	6	5	12	23

Table 7

The detailed weighted demand cost matrix of the 18 customer nodes network

Customer		**WH**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
		Demand Cost (SAR)*																		
WH	Demand Cost (SAR)*	0.000	1.007	1.072	0.503	2.605	5.734	0.325	39.139	0.620	8.058	6.849	1.689	1.479	2.971	2.704	1.468	5.871	13.328	4.798
1		1.135	0.000	2.901	1.926	4.803	1.981	1.284	4.981	1.957	16.843	1.409	0.734	3.977	5.246	1.664	4.042	1.048	2.516	3.751
2		1.019	2.516	0.000	0.856	2.720	2.231	1.015	3.589	1.002	10.430	1.032	0.709	2.680	3.523	1.877	2.371	0.806	2.090	4.220
3		0.555	2.242	0.971	0.000	3.161	1.660	0.806	3.990	4.215	24.757	1.142	0.590	6.377	3.883	1.393	2.568	0.867	2.081	3.161
4		2.138	4.932	2.912	3.082	0.000	2.182	0.534	5.708	1.174	13.088	1.601	1.081	2.449	6.755	2.118	3.574	1.388	3.332	4.774
5		5.734	1.701	2.081	1.546	2.273	0.000	0.301	8.058	0.485	6.079	2.107	1.197	1.135	1.989	5.407	0.982	1.644	3.883	5.222
6		0.301	0.806	0.990	0.555	0.514	0.341	0.000	3.288	1.712	10.223	0.952	0.632	5.284	2.884	1.712	2.710	0.747	1.926	3.874
7		51.370	4.151	3.653	3.439	5.999	6.472	3.328	0.000	0.635	8.252	13.699	1.284	1.510	3.082	2.815	1.522	7.903	18.265	4.949
8		0.660	2.029	0.934	3.574	1.142	0.491	1.264	0.709	0.000	17.123	1.019	0.525	10.274	3.288	1.234	1.800	0.781	1.875	2.796
9		7.582	21.404	12.378	31.612	11.544	6.342	15.929	8.526	20.756	0.000	0.491	1.267	2.867	3.680	1.198	1.897	0.839	2.013	2.436
10		8.219	1.258	1.111	1.015	1.749	1.973	1.006	12.978	0.934	0.548	0.000	1.326	1.528	3.626	2.854	1.541	20.548	49.315	5.001
11		1.185	0.652	0.700	0.536	0.963	1.340	0.582	1.271	0.493	1.271	1.054	0.000	1.574	2.453	5.479	1.612	1.771	4.179	8.834
12		1.504	4.742	2.533	4.110	2.312	1.213	5.137	1.587	10.568	3.653	1.522	1.666	0.000	2.901	1.352	2.466	0.796	2.063	3.069
13		3.473	4.566	2.867	3.244	10.064	2.529	2.609	4.403	2.851	3.262	4.042	3.025	2.651	0.000	1.920	2.441	1.771	4.251	3.664
14		3.186	1.533	1.911	1.370	2.076	5.337	1.637	2.375	1.284	0.736	2.246	4.069	1.347	1.671	0.000	1.077	1.223	2.740	5.871
15		1.185	5.734	3.332	2.802	2.021	1.040	2.680	1.264	2.441	1.457	1.203	1.340	2.771	2.221	1.045	0.000	0.930	2.441	3.938
16		6.628	1.093	0.943	0.882	1.581	1.698	0.853	13.699	0.812	2.294	20.548	2.309	0.863	1.631	1.468	1.326	0.000	493.151	5.137
17		16.438	2.596	2.273	2.283	4.364	4.144	2.063	35.225	2.090	0.183	49.315	5.604	2.081	3.653	3.574	3.202	123.288	0.000	5.165
18		5.626	4.258	5.137	3.364	5.055	6.950	4.296	5.310	3.140	2.780	5.028	10.502	3.462	3.781	6.899	5.082	5.251	5.137	0.000

* Demand cost is computed as follows: Weighted travel distance matrix: $wt_{i,j} = \left(\frac{9}{d_{i,j}*2.18}\right) * Q_{i,j}$ where: cost of fuel/liter = 2.18 Saudi Riyal Fuel consumption = 9 km/liter

4.1.1. Model Search for visiting a node

Starting at the warehouse, which is node 0, the truck needs to visit all nine customers from nodes 1 to 9 exactly one time and then return to the warehouse. This is a problem where the goal is to minimize the total cost of visiting all nodes. The inputs are a from-to-distance matrix that was found by Google Maps distance matrix API using the warehouse and customer's latitudes and longitudes. Next, the cost per unit distance was found since it costs approximately 2.18 SAR for every 9km. These two matrices were multiplied together to find the total fuel cost matrix where C_{ij} is the fuel cost of traveling from node *i* to node *j*. For example, $C_{0,1}$ =237.30, which means that the cost of traveling from the warehouse to Customer 1. Next, dynamic programming DP is incorporated into the solution to find the minimum cost. A 1024 × 10 table is created from 2^{10} =1024 so there are 1024 possible subsets of 10 nodes. DP [mask][i] is the minimum cost to visit all the nodes in the subset mask, ending at node *i*. The subset mask is a binary number representing the nodes that have been visited so for example if mask = 0000000011, this means that only customers 1 and 2 have been visited. The last node *i* represents the last node visited in the subset. First, we start at the warehouse which is Node 0 with no customers visited which is DP [1][0] = 0. This means the cost of starting at the warehouse and visiting no nodes is 0.

To find the cost of visiting a new node j from a subset mask, the formula is DP[mask][j] = min (DP[prev_mask}[i] + C[i][j]) where prev_mask =mask\{j} which is the subset of nodes excluding j and C[i][j] is the cost of traveling from node i to j. After visiting all nodes, the cost of returning to the warehouse should be calculated using this formula:

MinCost =i=1min9 (DP[all_visited][i]+C[i][0])

To find the optimal route using the backward approach, the model starts from the final state and finds the last node that gives the minimum cost. We used the parent table to trace backward through the DP to find which node led to the current state and we continued till we reached the warehouse (Node 0).

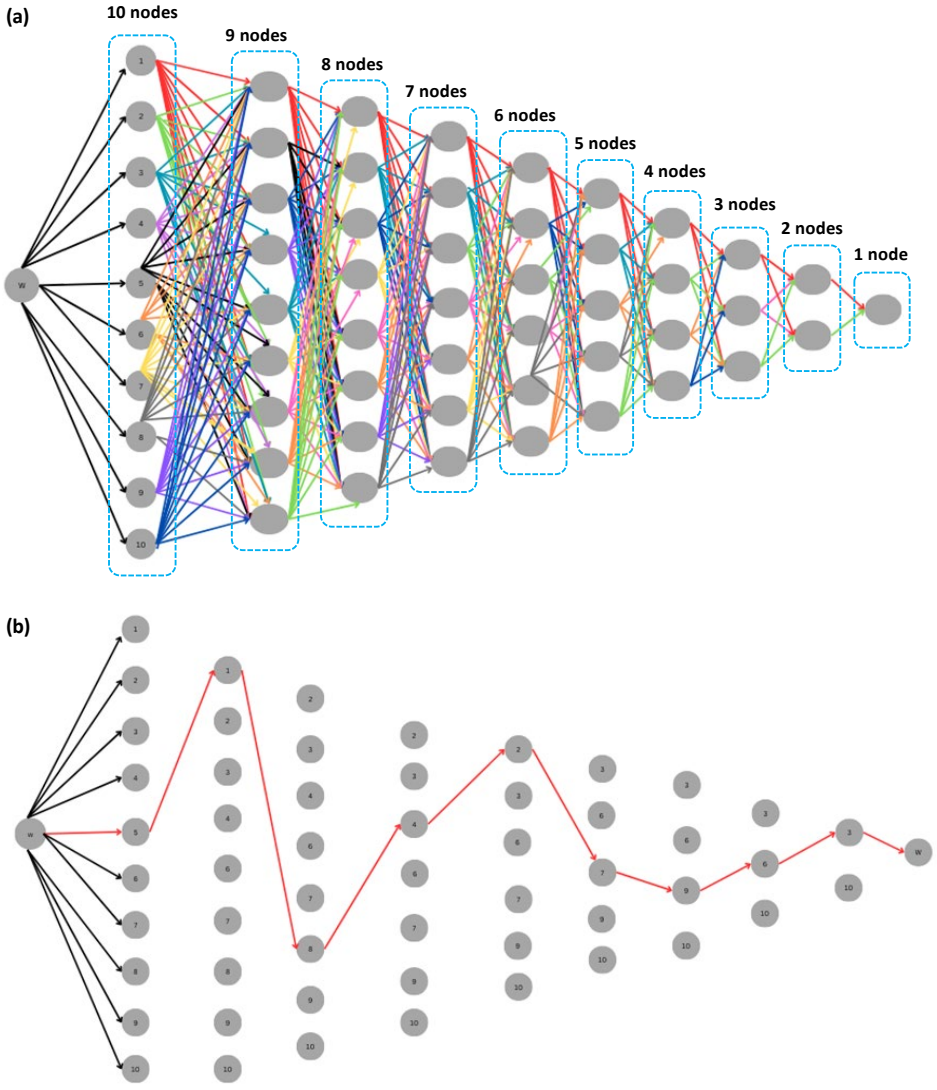


Fig. 3. Model search through iterative process (nodes selection and elimination)

A sample graphical search output is shown in Fig. 3 wherein the search begins from warehouse (W). At a first search point there are 10 nodes available to visit from the warehouse. For the second iteration out of 10 nodes one node is selected (for instance node 5) and the remaining 9 nodes are eliminated. For the third iteration there are 9 nodes available to make selection of one node (for instance node 1) and eliminating the other remaining. The model goes for the fourth iteration and repeats the searching process until the last iteration where finally the last node is selected based on the set criteria. Hence the search follows a funneling approach and the iterative process continues until the selection of a complete route for the vehicle. The search operation is shown in Fig. 3 where how the iteration process proceeds with the selection and elimination of customer nodes. For the case shown in Fig. 3, the selected route is [Warehouse→ Customer 5→ 1→ 8→ 4→ 2→ 7→ 9→ 6→ 3→ Warehouse]. This is called a vehicle routing problem and algorithm/model based search for customers fitting with the capacity of vehicle 1. The DP table in Excel that solves TSP. Each of its rows corresponds to some state, which is a mask showing the subset of the nodes visited so far, including the warehouse and customers. Each of its columns corresponds to the last node visited in the state. The values are minimum costs to reach a certain node while visiting a subset of nodes given by a state.

4.1.2. Improvement in the current state

The model was run for the current state of the company considering the same number of vehicles (4 trucks) and their equal capacities (60 units per truck). Based on the model generated weighted distance matrix the allocation of trucks was executed and trucks were assigned to the customers. Prioritizing customers depending on the balance between their orders quantity and their location distancing. Therefore, by the end of delivery day all customers will receive their orders but customers with high weight will be first delivered. For instance, the vehicle 1 is assigned to customer 9 (C9) based on the highest value of weighted matrix or priority with a quantity order of 50 units. Through this allocation, the distance covered by vehicle 1 was reduced from 51.33 km to 37.75 km and the route is set as [WH, 9, WH]. Similarly, the allocation of vehicle 2 has been executed. The clustering of customers for vehicle 2 includes a group of nine customer nodes (C13, C1, C3, C8, C12, 6, C2, C15, C4) each having varying demand quantity. The sub-total collective demand of this group of nine customers is 56 units and the corresponding route assigned to vehicle 2 is [WH, 13, 1, 3, 8, 12, 6, 2, 15, 4, WH]. In a similar pattern, the customer clustering and the vehicle routing is obtained for the other vehicles. The detail on each cluster of customer nodes, demand per customer, demand per vehicle, total travel distance and the vehicle routing is provided in Table 8. The total travel distance covered by all the four vehicles is 122.24 km which shows an improvement of more than 30% as compared to the current case of the company where their total travel distance is 177.13 km. The API-based maps after the model-generated improvements for each of the four vehicle routings are presented in Table 9. A summary of the vehicle routing (the current state and the improvement through dynamic modelling) has been presented in Table 10. The comparative analysis of the demand quantities and the travel distances associated with the before applying the model and after applying the model is shown in Fig. 4. Before applying the model, there were 57 quantities assigned to vehicle 1 and the vehicle needed to travel a distance of 51.33 km. With the help of the model, 50 quantities were assigned to vehicle 1 and travel distance reached 37.75 km with an improvement of 26.45% in the travel distance. The most significant improvement in travel distance is found in the assignment of vehicle 4. Before applying the model 23 shipment units were assigned to vehicle 4 and it needs to travel 22.13 km, whereas the model-based assignment consists of 43 shipping units for vehicle 4 and the corresponding travel distance was just 4.54 km. It leads to an improvement of 79.48% in the traveling cost or distance of vehicle 4. The collective improvement in the vehicle routing is found to be 30% with the help of model generated assignment and allocation of customers to the vehicles.

Table 8
Vehicle routing and the assignment of customers with their demand (improvement in the existing case after model application)

<table>
<tr><th>Vehicle 1</th><th>Vehicle 2</th><th>Vehicle 3</th><th>Vehicle 4</th></tr>
<tr><td>Vehicle 1 route:
[WH, 9, WH]</td><td>Vehicle 2 route:
[WH, 13, 1, 3, 8, 12, 6, 2, 15, 4, WH]</td><td>Vehicle 3 route:
[WH, 5, 14, 18, 11, WH]</td><td>Vehicle 4 route:
[WH, 10, 16, 17, 7, WH]</td></tr>
<tr><td>Customer ID:
C9</td><td>Customer ID:
C13, C1, C3, C8, C12, 6, C2, C15, C4</td><td>Customer ID:
C5, C14, C18, C11</td><td>Customer ID:
C10, C16, C17, C7</td></tr>
<tr><td>Individual Customer ID and Demand
<table><tr><td>ID</td><td>C9</td></tr><tr><td>Q</td><td>50</td></tr></table></td><td>Individual Customer ID and Demand
<table><tr><td>ID</td><td>C13</td><td>C1</td><td>C3</td><td>C8</td><td>C12</td><td>C6</td><td>C2</td><td>C15</td><td>C4</td></tr><tr><td>Q</td><td>12</td><td>5</td><td>3</td><td>4</td><td>9</td><td>2</td><td>6</td><td>6</td><td>9</td></tr></table></td><td>Individual Customer ID and Demand
<table><tr><td>ID</td><td>C5</td><td>C14</td><td>C18</td><td>C11</td></tr><tr><td>Q</td><td>12</td><td>10</td><td>23</td><td>3</td></tr></table></td><td>Individual Customer ID and Demand
<table><tr><td>ID</td><td>C10</td><td>C16</td><td>C17</td><td>C7</td></tr><tr><td>Q</td><td>6</td><td>5</td><td>12</td><td>20</td></tr></table></td></tr>
<tr><td>Sub-Demand for Vehicle 1: 50 units</td><td>Sub-Demand for Vehicle 2: 56 units</td><td>Sub-Demand for Vehicle 3: 48 units</td><td>Sub-Demand for Vehicle 4: 43 units</td></tr>
<tr><td>Distance: 37.75 km</td><td>Distance: 52.59 km</td><td>Distance: 27.36 km</td><td>Distance: 4.54 km</td></tr>
<tr><td colspan="4">Total Demand: 197 units
Total Distance: 122.24 km</td></tr>
</table>

Table 9
Vehicle routing along with API matrix based maps (improvement in the existing case after model application)

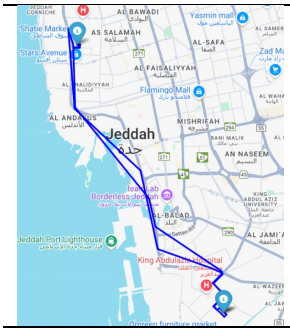
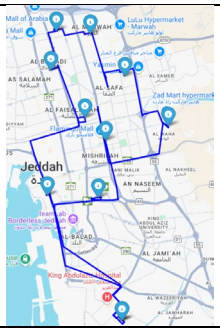
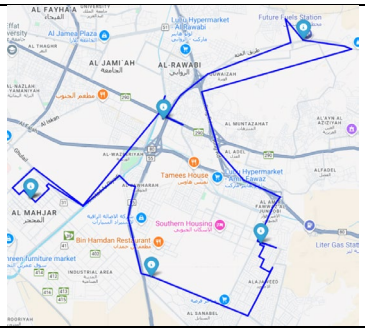
Vehicle 1	Vehicle 2	Vehicle 3	Vehicle 4
			
Vehicle 1 route: [WH, 9, WH]	Vehicle 2 route: [WH, 13, 1, 3, 8, 12, 6, 2, 15, 4, WH]	Vehicle 3 route: [WH, 5, 14, 18, 11, WH]	Vehicle 4 route: [WH, 10, 16, 17, 7, WH]

Table 10
Summary of the vehicle routing (before and after applying the model)

Vehicles	Capacity (units)	Demand (Q units)	Distance (km)	Capacity (units)	Demand (Q units)	Distance (km)	% Improvement
	Before Applying the Model			After Applying the Model			
Vehicle 1	60	57	51.33	60	50	37.75	26.45
Vehicle 2	60	60	46.97	60	56	52.59	-11.96
Vehicle 3	60	57	56.7	60	48	27.36	51.74
Vehicle 4	60	23	22.13	60	43	4.54	79.48
SUM	60	197	177.13	60	197	122.24	30.98

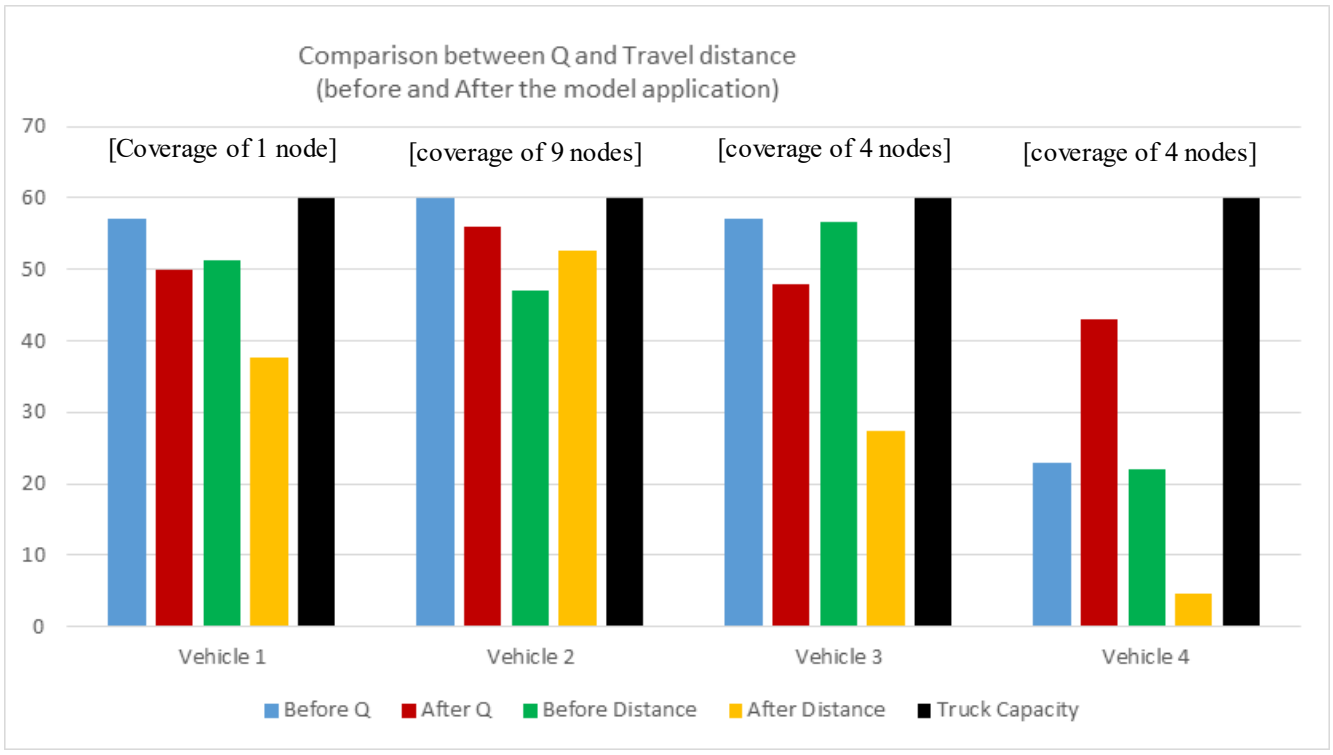


Fig. 4. Comparative analysis of the demand quantities and the travel distances (before and after the model application)

4.2 Model Validation

The validation of successful implementation of the model was performed in different cases. These cases are explained hereunder.

4.2.1 A case of the current demand and varying vehicle capacity

In this case the demand quantity was considered as the current demand data of the company. It consists of demand units to be supplied (Q), the location of each customer node (fixed latitudes and altitudes), number of vehicles, and vehicle capacity. The company has four vehicles with a fixed capacity of 60 units per vehicle. In this case of validation all the parameters were kept the same as of current state except the vehicle capacity. This case is built assuming that if the company made some changes in the vehicles and the capacity is changed then whether the model has a capability to accommodate those changes or not. Therefore, the model was run based on the new set of vehicle capacities such as all the four vehicles have their capacities as 70, 50, 80, and 50 units. The new vehicle routing was generated by the model and the assignment is shown in Table 11 and Fig. 5.

Table 11
Model validation for the case of variable vehicle capacity and the corresponding vehicle routing

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Quantity	5	6	3	9	12	2	20	4	50	6	3	9	12	10	6	5	12	23

	Truck Capacity (units)	**Cumulative demand; Q (units)**	**Distance (km)**	**Vehicle Route**
Vehicle 1	70	62	37.77	[WH, 13, 9, WH]
Vehicle 2	50	44	47.83	[WH, 2, 6, 12, 8, 3, 1, 15, 4, WH]
Vehicle 3	80	48	27.37	[WH, 5, 14, 18, 11, WH]
Vehicle 4	50	43	4.54	[WH, 10, 16, 17, 7, WH]
SUM	250	197	117.51	

4.2.2 A case of the variable demand and variable vehicle capacity

The model was tested and validated through this second case in which the demand is varied as compared to the current demand. Moreover, the number of vehicles is the same as of the current company state (4 vehicles) but each vehicle differs from others in terms of its capacity. The purpose of this testing was to ensure the robustness of the proposed model against the future possibilistic as well as real-case expected situation. For instance, if the customer demand is changed for the next period, then whether the model still has the potential to accommodate the change and produce the optimal routing for the vehicles or not. Thus, the model was run with the following primary set of data inputs as shown in Table 12.

Table 12
Model validation for the case of variable demand, variable vehicle capacity, and the corresponding vehicle routing

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Quantity	25	15	16	15	12	23	15	16	13	19	12	20	7	4	50	6	3	9

	Truck Capacity (units)	**Cumulative demand; Q (units)**	**Distance (km)**	**Vehicle Route**
Vehicle 1	70	62	30.08	[WH, 15, 11, WH]
Vehicle 2	50	43	4.54	[WH, 10, 16, 17, 7, WH]
Vehicle 3	80	80	49.11	[WH, 9, 3, 8, 12, 4, WH]
Vehicle 4	100	95	52.33	[WH, 13, 1, 6, 2, 18, 14, 5, WH]
SUM	300	280	136.06	

4.2.3 A case of futuristic business expansion-1

This case is built based on the possibility and futuristic expansion of business. It considered the assumption that what if there is an increase in the number of customers, number of demand units, and a greater number of vehicles etc. The model was tested for such future expected cases. Therefore, the case was built by setting 53 customers, each customer having variable demand, 12 number of vehicles, and each vehicle having different capacity. Therefore, prior to running the model the API matrix was developed for 53 customers using locations (latitudes and longitudes). The sample table of selected values of latitudes and altitudes is shown in Table 13.

Table 13
Selected data of latitudes and longitudes from a range of 53 customer nodes

Customer ID	Customer No.	Latitude	Longitude
Warehouse	0	21.4339	39.2168
102215	1	21.501795	39.24419833
104085	2	21.74588167	39.19407
103678	3	21.58945167	39.21912167
101773	-	21.5414813	39.2886656
102264	-	21.42749167	39.19056333
101747	-	21.43649657	39.19649943
101894	21	21.27643333	39.276445
101906	22	21.51312667	39.25092833
100096	23	21.4607257	39.2468377
103947	-	21.78748459	39.1762178
100388	-	21.576593	39.24061
100364	-	21.75840166	39.189875
101730	51	21.43623019	39.19652984
101728	52	21.43699533	39.19666294
102854	53	21.48383112	39.3092635

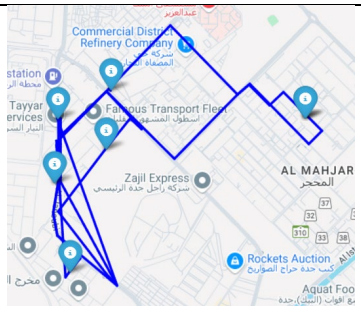
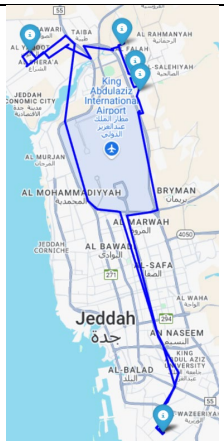
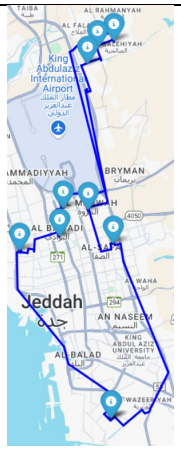
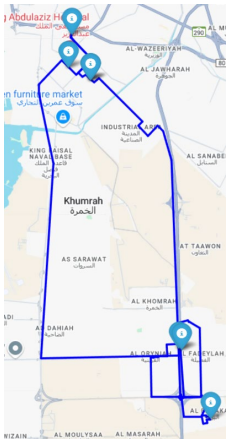
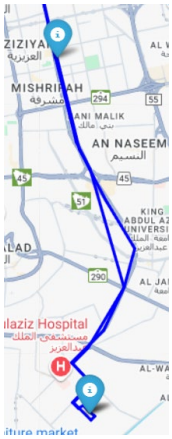
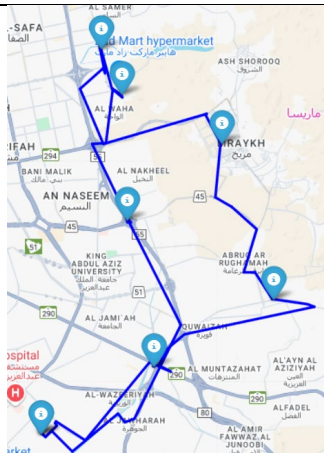
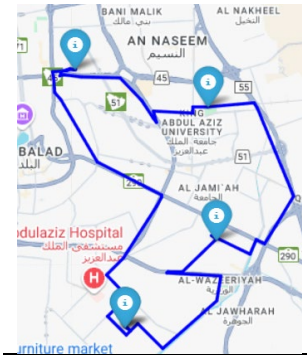
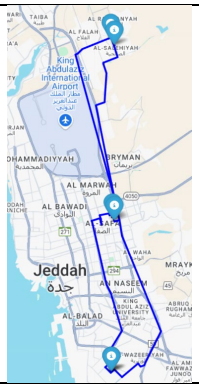
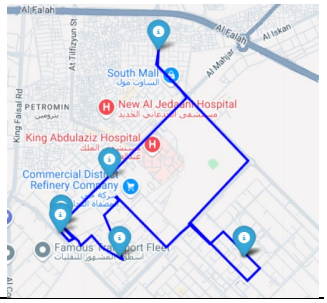
The weighted demand cost matrix (53×53 matrix) for all the 53 customers network was developed. Based on this weighted demand cost and the API matrices, number of vehicles, vehicle capacities and other related input data, the model was executed. The output is shown in Table 14. The allocation of customer nodes from the overall network of 53 customers has been distributed among all the 12 vehicles. None of the available vehicles exceeds its pre-defined capacity (variable capacity of different vehicles). The clustering of the customers is based on the weighted demand cost. For instance, five nodes are grouped together to form one cluster for vehicle 1 having 70 units' capacity to fulfill the cumulative demand of 70 units. The vehicle 1 travels 101.57 km distance among its customer node points located at different but neighboring locations. The assigned route for vehicle 1 is [WH, 18, 11, 49, 50, 9, WH] and the travel distance for vehicle 1 is 101.57 km. On the other hand, five different customers (27, 7, 48, 32, & 8) have cumulative demand of 44 units, therefore, the model has grouped these customers in one cluster and assigned to vehicle 2 which has a capacity of 50 units. Thus, the routing for vehicle 2 is [WH, 27, 7, 48, 32, 8, WH] with a total travel distance of only 8.08 km. Similar is the case of other customers and remaining vehicles. Hence, in order to meet the total demand of 879 units for a network of 53 nodes, the total travel distance covered by 12 vehicles will be 557.46 km. The google maps and the vehicle routings for each of the 12 vehicles are separately presented in individual cells of Table 15.

Table 14
Model validation for the case of futuristic business expansion-1 and vehicle routing

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Quantity (units)	25	15	16	15	12	23	15	16	13	19	12	20	7	4	50	6	15	5
Customer	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Quantity (units)	8	5	10	30	35	14	7	45	3	9	25	15	16	15	12	23	15	16
Customer	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	
Quantity (units)	13	19	20	20	7	4	50	6	15	5	8	5	10	30	35	14	7	

	Truck Capacity (units)	**Cumulative demand; Q (units)**	**Distance (km)**	**Vehicle Route**
Vehicle 1	70	70	101.57	[WH, 18, 11, 49, 50, 9, WH]
Vehicle 2	50	44	8.08	[WH, 27, 7, 48, 32, 8, WH]
Vehicle 3	80	80	89.86	[WH, 2, 26, 24, 44, WH]
Vehicle 4	100	100	82.28	[WH, 37, 17, 36, 19, 45, 53, 40, 3, WH]
Vehicle 5	70	69	42.24	[WH, 10, 21, 12, 35, 4, WH]
Vehicle 6	50	50	25.92	[WH, 43, WH]
Vehicle 7	80	79	37.81	[WH, 41, 47, 28, 15, 46, WH]
Vehicle 8	100	98	40.8	[WH, 39, 56, 6, 25, 16, 22, WH]
Vehicle 9	70	65	22.57	[WH, 10, 21, 12, 35, 4, WH]
Vehicle 10	50	47	22.86	[WH, 30, 31, 42, 33, WH]
Vehicle 11	80	77	76.25	[WH, 41, 47, 28, 15, 46, WH]
Vehicle 12	100	100	7.22	[WH, 38, 55, 54, 52, 14, 29, WH]
SUM	900	879	557.46	

Table 15
Vehicle routing along with API matrix based maps (for the case of futuristic business expansion-1)

Vehicle 1	Vehicle 2	Vehicle 3	Vehicle 4
			
Customers [WH, 18, 11, 49, 50, 9, WH] Total Demand: 70 Total Distance: 101.57 km	Customers [WH, 27, 7, 48, 32, 8, WH] Total Demand: 44 Total Distance: 8.08 km	Customers [WH, 2, 26, 24, 44, WH] Total Demand: 80 Total Distance: 89.86 km	Customers [WH, 37, 17, 36, 19, 45, 53, 40, 3, WH] Total Demand: 100 Total Distance: 82.28 km
Vehicle 5	**Vehicle 6**	**Vehicle 7**	**Vehicle 8**
			
Customers [WH, 10, 21, 12, 35, 4, WH] Total Demand: 69 Total Distance: 42.24 km	Customers [WH, 43, WH] Total Demand: 50 Total Distance: 25.92 km	Customers [WH, 41, 47, 28, 15, 46, WH] Total Demand: 79 Total Distance: 37.81 km	Customers [WH, 39, 56, 6, 25, 16, 22, WH] Total Demand: 98 Total Distance: 40.80 km
Vehicle 9	**Vehicle 10**	**Vehicle 11**	**Vehicle 12**
			
Customers [WH, 20, 1, 23, WH] Total Demand: 65 Total Distance: 22.57 km	Customers [WH, 30, 31, 42, 33, WH] Total Demand: 47 Total Distance: 22.86 km	Customers [WH, 34, 5, 51, 13, WH] Total Demand: 77 Total Distance: 76.25 km	Customers [WH, 38, 55, 54, 52, 14, 29, WH] Total Demand: 100 Total Distance: 7.22 km

4.2.4 A case of futuristic business expansion-2

This case is the extension of the previous case, i.e. the customer demand is the same. However, if the company reduces the number of vehicles and at the same time expands the vehicle capacity to fulfill the demand of each network node then the model response is evaluated. The vehicle capacity is considered as accommodating 100 boxes of shipment per vehicle with a total of 9 vehicles as transportation means. The model output is summarized in Table 16 where it can be seen that the customers are grouped (based on the model's calculated weighted demand quantity) into 9 clusters. For each group of customers, the assigned number of shipments to each vehicle is either less than or equal to the vehicle capacity. None of the vehicle assignments exceeds the capacity of 100 units. Vehicle routing for each group of customer network nodes is presented in the last column of Table 16. Comparing the previous case of futuristic expansion-1 with this case, the total travel distance has been found to be 458.53 km. The total travel distance of the previous case was 557. Hence, it can be inferred that there is a 100km travel distance saving which directly affects the travel time, fueling cost, and other associated costs per route. If the capacity expansion is preferred over the increase in number of vehicles then there is a significant reduction in travel distance, time, fueling and other related cost elements.

Table 16
Model validation for the case of futuristic business expansion-2 and vehicle routing

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Quantity	25	15	16	15	12	23	15	16	13	19	12	20	7	4	50	6	15	5
Customer	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Quantity	8	5	10	30	35	14	7	45	3	9	25	15	16	15	12	23	15	16
Customer	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	
Quantity	13	19	20	20	7	4	50	6	15	5	8	5	10	30	35	14	7	

	Truck Capacity (units)	**Cumulative demand; Q (units)**	**Distance (km)**	**Vehicle Route**
Vehicle 1	100	100	103.92	[WH, 44, 18, 11, 49, 50, 9, 24, 45, 20, WH]
Vehicle 2	100	99	78.44	[WH, 26, 53, 51, 5, WH]
Vehicle 3	100	99	85.79	[WH, 10, 21, 12, 3, 36, 17, 37, WH]
Vehicle 4	100	100	29.96	[WH, 43, 15, WH]
Vehicle 5	100	97	74.32	[WH, 25, 34, 40, 2, 19, 28, 47, 41, WH]
Vehicle 6	100	96	41.16	[WH, 1, 22, 6, 16, 46, 13, WH]
Vehicle 7	100	99	28.08	[WH, 31, 23, 39, 56, 42, 33, WH]
Vehicle 8	100	89	7.36	[WH, 38, 35, 29, 4, 30, WH]
Vehicle 9	100	100	9.5	[WH, 27, 7, 48, 55, 54, 52, 32, 8, 14, WH]
SUM	900	879	458.53	

5. Conclusion

This study highlights the importance of strategic decision making in optimizing distribution networks. A dynamic programming mathematical model is developed and validated to address the optimization in vehicles' routing for transportation of shipment units to customers located at different locations with different demand quantities. The model, through optimized grouping of customer nodes using K-means clustering and vehicle routings, has shown a significant reduction in travel distances for individual vehicles as well as the reduction in overall total distance covered by a group of vehicles to fulfill the demand quantity of each node in the customer oriented network. Google API matrix of customer nodes has been utilized to accommodate the dynamic nature of the problem. Since, at different time periods the road traffic fluctuates, therefore the model accommodates this dynamic change and performs the optimized customer clustering and consequently the vehicle routing. Through the model based optimization, a reduction of 30% travel distance has been found in the existing vehicle routing for the selected case of the company. The robustness of the model is further validated by applying at four different scenarios as follows:

- What if the demand remains the same but the vehicle's capacity doesn't remain fixed? In this case, the vehicle capacity was varied and the model has proven the appropriate allocation of customer clustering and vehicle routing without exceeding the vehicle capacity.
- What if both the demand and vehicle capacity is varied? In this case the demand was changed for each customer and the vehicle capacity was also changed simultaneously. The model resulted in optimized vehicle routing with satisfying the constraints limits.
- What if the business expands in the future and there is expansion in the customer network and their demand quantities? In this case 53 customers located at different network locations each having varying demand quantities were selected. The optimization was performed for 12 vehicles each having varying capacities. The model produced 12 clusters of different

customer nodes (based on weighted cost matrix) and resulted in optimized routing for each vehicle without exceeding the capacity.

- What if the business expands, more customers, different demand quantities, and the limited vehicles with large capacity? This fourth scenario was executed to verify the model's robustness for 53 customers for 9 vehicles each having a capacity of 100 units. All the set constraints and limits were completely satisfied and the optimized vehicle routing was achieved similar to previous cases.

Acknowledgements

Author would like to acknowledge Al Yamamah University in supporting this research.

References

Almalki, M., & Alkahtani, M. Practical Cases of Locating Regional Logistics Hubs: A Systematic Literature Review.

Campbell, J. F., & O'Kelly, M. E. J. T. S. (2012). Twenty-five years of hub location research. *46*(2), 153-169.

Dye, C.-Y., & Hsieh, T.-P. J. E. J. o. O. R. (2024). A sustainable dynamic optimization model of pricing and advertising in the presence of green innovation investment. *315*(2), 654-667.

Eydi, A. R., & Saghez, S. S. J. A. S. E. J. (2024). A p-center mobile hub location in a dynamic environment with capacity constraints and multiple allocation. *15*(6), 102712.

Fahmy, S. A., Zaki, A. M., & Gaber, Y. H. J. S.-E. P. S. (2023). Optimal locations and flow allocations for aggregation hubs in supply chain networks of perishable products. *86*, 101500.

Gruca, T. S., & Klemz, B. R. J. E. J. o. O. R. (2003). Optimal new product positioning: A genetic algorithm approach. *146*(3), 621-633.

Hu, J., Liu, T., Chen, Z., Wang, S. J. C., & Research, O. (2024). Optimal-transport satisficing with applications to capacitated hub location. *165*, 106566.

Jayarathna, D., Lanel, G., Juman, Z. J. J. o. S., & Trade. (2022). Industrial vehicle routing problem: a case study. *7*(1), 6.

Khaleghi, A., Eydi, A. J. C., & Research, O. (2023). Multi-period hub location problem considering polynomial time-dependent demand. *159*, 106357.

Liu, X., Li, J., Dahan, M., Montreuil, B. J. T. R. P. E. L., & Review, T. (2025). Dynamic hub capacity planning in hyperconnected relay transportation networks under uncertainty. *194*, 103940.

Praveen, V., Keerthika, P., Sivapriya, G., Sarankumar, A., & Bhasker, B. J. M. T. P. (2022). Vehicle routing optimization problem: a study on capacitated vehicle routing problem. *64*, 670-674.

Sarker, B. R., Wu, B., & Paudel, K. P. J. A. M. M. (2018). Optimal number and location of storage hubs and biogas production reactors in farmlands with allocation of multiple feedstocks. *55*, 447-465.

Shi, Y., Yu, Y., & Dong, Y. J. E. J. o. O. R. (2021). Warehousing platform's revenue management: A dynamic model of coordinating space allocation for self-use and rent. *293*(1), 167-176.

Taherdoost, H., & Madanchian, M. J. E. (2023). Multi-criteria decision making (MCDM) methods and concepts. *3*(1), 77-87.

Vieira, C. L. d. S., & Luna, M. M. M. J. P. O. (2016). Models and methods for logistics hub location: a review towards transportation networks design. *36*, 375-397.

Wang, M., Cheng, Q., Huang, J., Cheng, G. J. P. A. S. M., & Applications, i. (2021). Research on optimal hub location of agricultural product transportation network based on hierarchical hub-and-spoke network model. *566*, 125412.

Xanthopoulos, S., van der Tuin, M., Azadeh, S. S., de Almeida Correia, G. H., van Oort, N., Snelder, M. J. T. R. P. A. P., & Practice. (2024). Optimization of the location and capacity of shared multimodal mobility hubs to maximize travel utility in urban areas. *179*, 103934.

Zhu, L., Schneider, K., & Oude Lansink, A. J. E. J. o. O. R. (2023). Economic, environmental, and social inefficiency assessment of Dutch dairy farms based on the dynamic by-production model. *311*(3), 1134-1145.

© 2025 by the authors; licensee Growing Science, Canada. This is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (http://creativecommons.org/licenses/by/4.0/).