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A mixed-integer linear programming formulation for the periodic vehicle routing problem with application to pathological waste collection

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A B S T R A C T

Pathological waste generated in healthcare facilities poses a significant risk of infectious disease transmission and therefore requires specialized collection, transportation, and disposal systems. Efficient logistical planning is essential for companies responsible for managing this waste in order to reduce operational costs while ensuring adequate health and environmental standards. In this context, this study develops an optimization strategy based on the Periodic Vehicle Routing Problem (PVRP) to support the planning of pathological waste collection and transportation activities. A mixed-integer linear programming model (MILP) is proposed to determine the visit schedule for healthcare centers, assign vehicles to service days, and design vehicle routes that satisfy demand and service frequency requirements over a weekly planning horizon while minimizing total operational costs. The model is validated through a real case study from Argentina, demonstrating its applicability to real-world waste management systems. Additionally, a benchmark case from the literature is solved to evaluate the impact and performance of the proposed approach under a deterministic demand scenario derived from historical data. Finally, the performance of the proposed model is compared with alternative PVRP formulations using several benchmark instances, showing the reliability and consistency of the solutions obtained.

1. Introduction

The collection and transportation of pathological waste represent a critical public health and environmental concern. This type of waste, which includes contaminated materials such as cotton, gauze, used bandages, and other disposable medical supplies, is mainly generated in hospitals, clinics, laboratories, and nursing homes and carries a high risk of transmitting infectious diseases. Due to this risk, such waste cannot be disposed of directly in landfills and must undergo specialized treatment and handling procedures. Given the strict precautions required for its collection, treatment and disposal, specialized companies are responsible for managing these operations. Ensuring an efficient and safe collection process is essential to minimize both potential health hazards and operational costs. However, these companies face significant logistical challenges when meeting the collection needs of healthcare facilities, including limited vehicle capacity, regulatory restrictions, waste storage limitations, and service frequency requirements. Consequently, the design of optimal collection routes and schedules becomes a complex task that has attracted increasing attention from both practitioners and researchers, particularly after the COVID-19 pandemic caused by SARS-CoV-2 intensified waste management challenges worldwide.

To address these complexities, different optimization approaches have been applied to pathological waste collection problems. Among them, the *Periodic Vehicle Routing Problem* (PVRP) has become the most widely adopted framework, as it allows the planning of vehicle routes over multiple periods while satisfying customer-specific service frequency requirements. In fact, Beltrami & Bodin (1974) were the first to apply the PVRP to this type of problem, establishing a foundation for subsequent research and demonstrating its suitability for waste collection operations. Since then, the PVRP has been widely used to model and optimize multi-period routing problems in this context, and numerous extensions have been developed to capture the operational and environmental characteristics of medical waste logistics.

Formally, the PVRP extends the classical Vehicle Routing Problem (VRP), one of the most widely studied problems in combinatorial optimization (Irnich et al., 2014b). According to Francis et al. (2008), the PVRP generalizes the VRP by incorporating periodicity into the routing process. The goal is to determine optimal routes for a fleet of capacitated vehicles over a defined planning horizon, typically consisting of multiple days, while minimizing transportation costs and respecting

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the visit frequency required by each customer. These authors provide a literature review on the evolution of PVRP, discussing various extensions of this problem and its solution methods. Later, Campbell & Wilson (2013) and Irnich et al. (2014a) introduce additional variants. Although these studies indicate that heuristics and metaheuristics remain the predominant solution approaches, significant progress toward exact methods has been achieved in recent years through the combination of new powerful technologies and innovative mathematical programming techniques.

The literature on waste collection and transportation is extensive. Recent review papers of Hess et al. (2024) and Li et al. (2025) provide comprehensible overviews of the research in this area, covering aspects such as waste types, model characteristics, objective functions, solution methods, benchmark datasets and case studies based on real-world applications. In particular, Li et al. (2025) point out that most studies address municipal solid waste management, whereas other categories, such as medical waste, have received comparatively less attention. Regarding this type of waste, most existing studies focus on the location of facilities for storage, treatment and disposal, as well as on determining their capacities and the size and capacity of vehicle fleets These approaches also determine the amount of waste transported from medical centers to treatment plants over the planning horizon (Mantzaras & Voudrias, 2017; Mete & Serin, 2019; Kargar et al., 2020; Suksee & Sindhuchao, 2021; Tirkolaee et al., 2021; Xu et al., 2023; Palomino-Perez et al., 2024). Some studies address a daily VRP to manage collection and transport processes efficiently, where all medical waste generation centers must be visited (Simon et al., 2012; Mohamed Faizal et al., 2021; Daoud et al., 2020; Wang et al., 2023). In contrast, others works consider the periodic variant of the VRP, which provides a more realistic representation of the typical collection scenario for this type of waste.

From this last perspective, Alshraideh and Qdais (2016) present a study based on a real case where two trucks are used to collect waste from 19 hospitals located at the northern region of Jordan. They develop a mathematical model to minimize the total travel distance of collection vehicles while guaranteeing an acceptable service level for medical centers. Waste accumulation at hospitals is modeled using a Log-Normal distribution, and each hospital must be visited at least once every three consecutive days. The resulting stochastic model is solved using a Genetic Algorithm.

A related PVRP is addressed by Alves et al. (2019), who consider the logistics of home healthcare services delivered from a health unit. In their formulation, a homogeneous fleet of vehicles is available and used by healthcare staff to travel to patients' locations. The visit frequency required by each patient over a multi-day planning horizon is known, as well as the minimum and/or maximum time intervals between successive visits. The objective is to determine both the scheduling of patients visits and the routes of the vehicles in order to minimize the total travel costs. The model is applied a real case study of a Bragança Health Unit with a planning horizon of five days.

Taslimi et al. (2020) propose a decomposition-based heuristic approach to solve a load-dependent capacitated PVRP related to medical waste. The problem addresses the design of a weekly inventory routing schedule that minimizes both the transport risk and the risk associated with the temporary storage of waste at healthcare facilities. Partial pickups are not allowed, meaning that a vehicle can visit a medical center only if it has enough capacity to collect all the waste stored there. Although it is not necessary to visit each customer every day, all facilities must be served by the end of the planning horizon. The authors also consider different objective functions related to transportation risk, occupational risk, and transportation cost. The proposed approach is tested using a case study based on medical waste management in Dolj, Romania. Zhang et al. (2022) consider a PVRP with Time Windows (PVRPTW) for the transportation of medical waste. They propose a Mixed Integer Linear Programming (MILP) model to minimize the transportation cost, which is solved using a neighborhood search algorithm. The problem is applied to a case study of a treatment plant located in China, where the collection policy requires visiting each medical institution every two days. More recently, Mufarrege et al. (2024) develop a MILP model based on the concept of light robustness to address regular pathological waste collection and transportation problems. In addition to the formulation, the authors present a Branch-and-Bound algorithm designed to improve the efficiency of the solution process. The performance of the proposed formulation and solution algorithm is evaluated through computational experiments and compared with a previously proposed approach.

Finally, Anityasari et al. (2025) propose a PVRPTW model to address the complexities of medical waste management, incorporating time-sensitive constraints and regulatory requirements, in a region of Indonesia that requires the standards strict compliance. To resolve this, the authors propose a problem decomposition that combines a heuristic method with an exact method.

Despite these advances, most of the previously mentioned studies assume a homogeneous fleet, which allows the omission of explicit vehicle-indexed decision variables. Under this assumption and considering that the collection operation at a customer on a specific day is carried out by a single vehicle, the PVRP can be represented using formulations that rely on sequencing variables for each pair of customers and commodity flow variables to track the accumulated load at each served customer. However, homogeneous fleets are not typical in real-world applications, where transportation units often differ in terms of operating, service, and depreciation costs. In addition, many formulations rely on predefined visit patterns for each customer based on service frequency within the planning horizon. Although this approach reduces the size of the search space by eliminating alternatives, it may also exclude solutions that could potentially be globally optimal. These simplifying assumptions may significantly reduce the realism of the resulting models, particularly in practical waste collection systems where vehicles often differ in capacity, operating costs, and service capabilities.

In a recent work, Basir et al. (2024) present a comparative study of three basic formulations for the PVRP and their extensions to address the PVRPTW. However, the key assumptions in this study include a homogeneous fleet of vehicles and a predefined set of possible visit schedules. The authors conduct a comprehensive set of computational experiments to evaluate the different formulations, both with and without considering valid inequalities and breaking constraints, and provide insights into their performance across instances of different sizes. Finally, they emphasize the importance of revising the existing formulations so that they can be easily adaptable to new variants of the PVRP and provide high-quality solutions within reasonable computation times.

In this context, this work proposes a MILP formulation to address the weekly planning of pathological waste collection and transportation services. The model simultaneously determines visit schedules for healthcare facilities and the daily routes of a fleet of vehicles, ensuring that service frequency requirements and waste demand are satisfied while minimizing total transportation costs. The main contributions of this study are threefold. First, we develop a flexible MILP formulation for the periodic routing of pathological waste collection vehicles that explicitly considers vehicle assignment decisions and heterogeneous operational characteristics, allowing a more realistic representation of real-world collection systems. Second, the proposed model is applied to a real case study of a pathological waste collection company operating in Argentina, using updated operational data and demonstrating its practical applicability for decision support in healthcare waste management. Third, the performance of the formulation is evaluated through computational experiments on benchmark instances and compared with alternative PVRP formulations, showing the robustness and consistency of the solutions obtained. Therefore, this work contributes both methodologically, by proposing an alternative formulation for periodic waste collection problems, and practically, by providing a decision-support tool capable of improving the planning of pathological waste collection systems.

The remainder of this paper is organized as follows. Section 2 provides a detailed description of the problem. Section 3 presents the mathematical model and the key assumptions. Section 4 describes the real-world case studies based on pathological waste collection and reports several numerical tests to illustrate the applicability and flexibility of the proposed model. Finally, Section 5 presents the conclusions and outlines directions for future research.

2. Problem description

The problem addressed in this paper involves a company responsible for the collection, transportation, and treatment of pathological waste generated across various healthcare facilities. Since the waste generated by these institutions poses significant risks to human health and the environment, government regulatory frameworks establish standards or technical guidelines for its safe management, within which waste pickup activities are generally planned and executed on a weekly basis. Thus, the company enters into an agreement with each health center, also referred to as customer from now on and denoted by $i \in I_c$, where the weekly frequency of visits F_i and the estimated amount of waste to be collected, Dem_{id}, are established, being $I_c = \{i_1, i_2, \ldots, i_n\}$ and $D = \{d_1, \ldots, d_6\}$ the sets of customers and working days of the week, respectively. It is assumed that, for the estimated demand, historical data on collections from each customer are available, and based on the frequency of visits, the amount of waste to be collected can be determined. While this parameter is set for each day of the week, it is assumed that the demand will only be collected on the corresponding visit days according to the weekly frequency and the policy established by the company. For example, if a customer has historically contracted the company for three visits per week, say Monday, Wednesday, and Friday, and cannot be visited on two consecutive days, the demand for Monday and Tuesday is set to the historical average for Mondays, the demand for Wednesday and Thursday is set to the historical average for Wednesdays, and the demand for Friday and Saturday is set to the historical average for Fridays. Finally, the model will determine whether the customer is visited on Monday, Wednesday, and Friday; or Tuesday, Thursday, and Saturday; or Monday, Thursday, and Saturday; or any other combination that meets the contract requirements.

To fulfill the collection tasks, a heterogeneous fleet of vehicles $k \in K$ is available. Each vehicle has a maximum load capacity, Cap_k, and it can only perform one trip per day, starting and ending its route at the treatment plant, denoted as i_0. Additionally, for each vehicle k, a daily working time limit WT_{kd} is defined, representing the duration of a time interval during which the vehicle can operate on day d. Let $I = \{i_0\} \cup I_c$ be the set of all the locations (plant and customers). Based on the required visit frequency, customers are divided into pairwise disjoint sets I_f, $I_c = \bigcup_f I_f$, with each set corresponding to customers visited f times during the considered period, where $1 \leq f \leq max_i\{F_i\}$. When customer i is such that its visit frequency satisfies $F_i = |D|$, then at least one vehicle must depart from the treatment plant each day to visit a set of customers, without exceeding the daily working time limit and its maximum load capacity. A vehicle can only visit a customer if its entire demand can be collected, i.e. split collection is not allowed. Furthermore, the time required for the pickup operation at customer i on day d is directly related to the amount of waste to be collected that day and is denoted by ST_{id}. Given this context, and considering the distances (D_{ij}) and travel times (TT_{ij}) between all locations, as well as the fixed and variable vehicle usage costs, denoted by FC_k and VC_k, respectively, the problem consists of determining, for each day of the week:

(a) The assignment of vehicles to be used on that day.
(b) The set of customers to be visited by each selected vehicle.
(c) The route for each vehicle, i.e. the sequence of customers visited by each vehicle.
(d) The amount of waste transported between consecutive customers.
(e) The total working time for each used vehicle.

3. Mathematical model

In this section, the mathematical formulation of the simultaneous customer–visit day–vehicle assignment and routing problem is presented. The nomenclature section provides the definitions of the sets, parameters, and variables used throughout the formulation. Next, the various constraints are described and grouped according to the type of condition they address.

3.1. Assignment constraints

Let X_{kd} be the binary variable that takes value 1 if vehicle *k* is used on day *d*, and 0 otherwise. When daily waste collection is required for some customer(s), at least one vehicle must depart from the plant every workday. This is represented by the following constraint:

$$\sum_{k \in K} X_{kd} \geq 1 \qquad \forall\, d \in D \tag{1}$$

Since the day(s) on which each customer is visited and the vehicle assigned for each visit are decision variables, the binary variable Y_{ikd} is defined. This variable takes value 1 if customer *i* is visited by vehicle *k* on day *d*, and 0 otherwise. Eq. (2) guarantees that each customer is visited according to the required frequency (F_i) along the week by any vehicle:

$$\sum_{k \in K} \sum_{d \in \mathrm{D}} Y_{ikd} = F_i \qquad \forall\, i \in I_c \tag{2}$$

Additionally, the following expression ensures that each customer is visited by at most one vehicle per day:

$$\sum_{k \in K} Y_{ikd} \leq 1 \qquad \forall\, i \in I_{\mathrm{c}}, d \in D \tag{3}$$

Eq. (4) states that if vehicle *k* is not used on day *d*, no customer can be visited by that vehicle on that day:

$$Y_{ikd} \leq \mathrm{X}_{kd} \qquad \forall\, i \in I_c, k \in K, d \in \mathrm{D} \tag{4}$$

In order to ensure a more uniform distribution of visits throughout the week for customers who are not visited daily, some constraints are imposed by the companies to achieve a relatively even distribution of the total waste collected across the weekly visits. For example, considering a time horizon of 6 days, composed of {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}, for customers with a visit frequency of two, denoted by $i \in I_2$, it is common to set the maximum number of days between two consecutive visits at two. Thus, the no allowable visit days combinations are: {Monday, Friday}, {Monday, Saturday}, {Tuesday, Saturday}, which can be avoided by Eq. (5) and Eq. (6). Also, a usual requirement for customers with frequency two or three, that is $i \in I_2$ or I_3, is that visits do not occur on consecutive days. Eq. (7) ensures this last condition. It is worth clarifying that any other conditions set by the company can be incorporated through this type of constraints.

$$\sum_{k \in K} Y_{ikd} + \sum_{k \in K} Y_{ik,d+4} \leq 1 \qquad \forall\, i \in I_2, d \in \{d_1, d_2\} \tag{5}$$

$$\sum_{k \in K} Y_{ikd_1} + \sum_{k \in K} Y_{ikd_6} \leq 1 \qquad \forall\, i \in I_2 \tag{6}$$

$$\sum_{k \in K} Y_{ikd} + \sum_{k \in K} Y_{ikd+1} \leq 1 \qquad \forall\, i \in I_2 \cup I_3, d \in D - \{d_6\}$$

3.2. Routing constraints

To generate the daily route for each used vehicle, the variable Z_{ijkd} is introduced, which takes value 1 if vehicle *k* travels from customer *i* to customer *j* on day *d*, and 0 otherwise. If vehicle *k* is assigned to day *d*, then that vehicle must depart from the plant i_0 exactly once:

$$\sum_{j \in I_c} Z_{i_0 jkd} = X_{kd} \qquad \forall\, k \in K, d \in D \tag{8}$$

Eq. (9) indicates that if customer *i* is assigned to vehicle *k* on day *d*, it must have a successor in the route, where node *j* can be a new health facility or the return to the treatment plant.

$$Y_{ikd} = \sum_{\substack{j \in I \\ j \neq i}} Z_{ijkd} \qquad \forall\, i \in I_c, k \in K, d \in D \tag{9}$$

Similarly, Eq. (10) states that if customer *j* is visited by vehicle *k* on day *d*, it must have a predecessor in the route.

$$Y_{jkd} = \sum_{\substack{i \in I \\ i \neq j}} Z_{ijkd} \qquad \forall\, j \in I_c, k \in K, d \in D \tag{10}$$

Constraint (11) represents the flow conservation constraint, which guarantees the continuity of the route performed by each vehicle on a given day. Specifically, it states that if vehicle *k* travels from customer *i* to customer *u* on day d, then the same vehicle must also travel from customer *u* to another customer *j* (either another healthcare facility or the treatment plant). This constraint prevents disconnected routes and guarantees the logical consistency of each vehicle tour.

$$\sum_{\substack{i \in I \\ i \neq \mathrm{u}}} Z_{iukd} = \sum_{\substack{j \in I \\ \mathrm{u} \neq j}} Z_{ujkd} \qquad \forall\, u \in I_c, k \in K, d \in D \tag{11}$$

3.3. Capacity constraints

The total demand collected by any vehicle must not exceed its capacity, as indicated in Eq. (12):

$$\sum_{i \in I_c} Dem_{id}\, Y_{ikd} \leq Cap_k\, X_{kd} \qquad \forall\, k \in K, d \in \mathrm{D} \tag{12}$$

To describe the cumulative waste collected by a vehicle up to a certain customer on a specific day, the non-negative variable Q_{ijkd} is defined. This variable represents the load of vehicle *k* after visiting customer *i* on day *d*, i.e. it denotes the vehicle load upon arrival at customer *j*. In particular, when vehicle *k* departs from the plant, its load is zero, which corresponds to the base case. This condition is stated by Eq. (13).

$$\sum_{j \in I_c} Q_{i_0 jkd} = 0 \qquad \forall\, k \in K, d \in D \tag{13}$$

Eq. (14) states that the amount of waste transported from a customer to customer *j* by vehicle *k* on day *d*, plus the waste collected at customer *j*, must be equal to the amount transported after visiting customer *j*.

$$\sum_{\substack{i \in I \\ i \neq j}} Q_{ijkd} + \sum_{\substack{i \in I \\ i \neq j}} Dem_{jd}\, Z_{ijkd} = \sum_{\substack{i \in I \\ i \neq j}} Q_{jikd} \quad \forall\, j \in I_c, k \in K, d \in D \tag{14}$$

Eq. (15) and Eq. (16) state the lower and the upper bounds for the amount of transported waste between two consecutively served customers by vehicle *k* on day *d*. On the one hand, Constraint (15) ensures that the vehicle load after visiting customer *i* must be at least equal to the demand of that customer. On the other hand, Eq. (16) states that the amount picked up at customer *j* must not exceed the remaining capacity of the vehicle.

$$Dem_{id} Z_{ijkd} \leq Q_{ijkd} \qquad \forall\, i, j \in I\ (i \neq j), k \in K, d \in D \tag{15}$$

$$Q_{ijkd} \leq \left(Cap_k - Dem_{jd}\right) Z_{ijkd} \qquad \forall\, i, j \in I\ (i \neq j), k \in K, d \in D \tag{16}$$

3.4. Time routing constraint

The total travel and service time incurred by a vehicle must not exceed its daily working time limit, as indicated in Eq. (17):

$$\sum_{j\in I}\sum_{\substack{i\in I\\ i\neq j}} TT_{ij}Z_{ijkd} + \sum_{i\in I_c} ST_{id}Y_{ikd} \leq WT_{kd}X_{kd} \qquad \forall\ k\in K, d\in D \tag{17}$$

It is worth mentioning that this formulation does not consider waiting times or time windows for customers, as the healthcare facilities are open during the operating hours of the collection company.

3.5. Valid inequalities

With the aim of improving the solution performance, the following constraints are proposed. These constraints reduce the search space in the solution process, thereby decreasing computation time. For each day, Eq. (18) imposes an order on the use of vehicles with the same capacity, in such a way that symmetries are avoided. Moreover, if more than one vehicle of the same capacity is selected, Constraint (19) sorts the allocation, assigning vehicles with lower indices to larger routes:

$$X_{kd} \geq X_{pd} \qquad \forall\ k,p\in K{:}\ Cap_k = Cap_p \wedge p > k, d\ \in D \tag{18}$$

$$\sum_{j\in I}\sum_{\substack{i\in I\\ i\neq j}} D_{ij}Z_{ijkd} \geq \sum_{j\in I}\sum_{\substack{i\in I\\ i\neq j}} D_{ij}Z_{ijpd} \quad \forall\ k,p\in K{:}\ Cap_k = Cap_p \wedge p > k, d\in D \tag{19}$$

Eq. (20) prevents subtours between two customers. This constraint is redundant given that subtour elimination is represented by Eq. (14), however its inclusion improves the performance of the model.

$$Z_{ijkd} + Z_{jikd} \leq 1 \qquad \forall\ i,j\in I_c(i\neq j), k\in K\ , d\in D \tag{20}$$

The relationship between binary variables Z_{ijkd} and X_{kd} established in Eq. (21) can be derived from Eq. (4), Eq. (9), and Eq. (10), but the model's performance improves when it is considered on the formulation:

$$Z_{ijkd} \leq X_{kd} \qquad \forall\ i,j\in I\ (i\neq j), k\in K, d\in D \tag{21}$$

3.6. Objective function

The problem goal is to minimize the total cost, as presented in Eq. (22). The first term represents the fixed cost associated with the use of each vehicle, and the second term corresponds to the variable cost, which is directly related to the distance traveled by each vehicle during each workday:

$$min \sum_{k\in K}\sum_{d\in D}\left(FC_kX_{kd} + \sum_{j\in I}\sum_{\substack{i\in I\\ i\neq j}} VC_kD_{ij}Z_{ijkd} \right) \tag{22}$$

Therefore, the MILP model for the simultaneous customer, visit day, vehicle allocation and routing problem considers the objective function, as expressed in Eq. (22), subject to constraints (1) – (21).

4. Cases studies and Computational experiments

To show the validity of the model and its applicability to real-world contexts, two case studies are analyzed. In addition, the performance of the proposed model is compared against alternative PVRP formulations on several benchmark small, medium and large size instances. All models were coded and implemented in Python 3.7.9 and solved using the solver Gurobi v.10.0.0 on a PC with an Intel Core i7-11700 processor, 2.50GHz and 16 GB of RAM.

4.1. Motivating scenario: a pathological waste collection company in Argentina

To test the proposed formulation, the model is implemented and solved for a real-world case study based on a pathological waste collection company located in a province in central-eastern region of Argentina. In this scenario, the company, denoted as i_0, employs two vehicles, k_1 and k_2, with capacities of 1500 kg and 1800 kg, respectively, and it is required to visit 41 customers (i_1, i_2, …, i_{41}) over a weekly planning horizon consisting of six working days, from Monday to Saturday, denoted by $D = \{d_1, d_2, \ldots, d_6\}$. The spatial distribution is illustrated in Fig. 1, where the blue marker represents the plant, green markers indicate customers with a frequency of 1, red markers are the customers with a frequency of 2, pink markers represent customers with a frequency of 3, and orange markers are customers visited daily. The picture also highlights that the company is responsible for collecting pathological waste across five geographical regions (R_1, …, R_5), with a notably high customer density in region R_4.

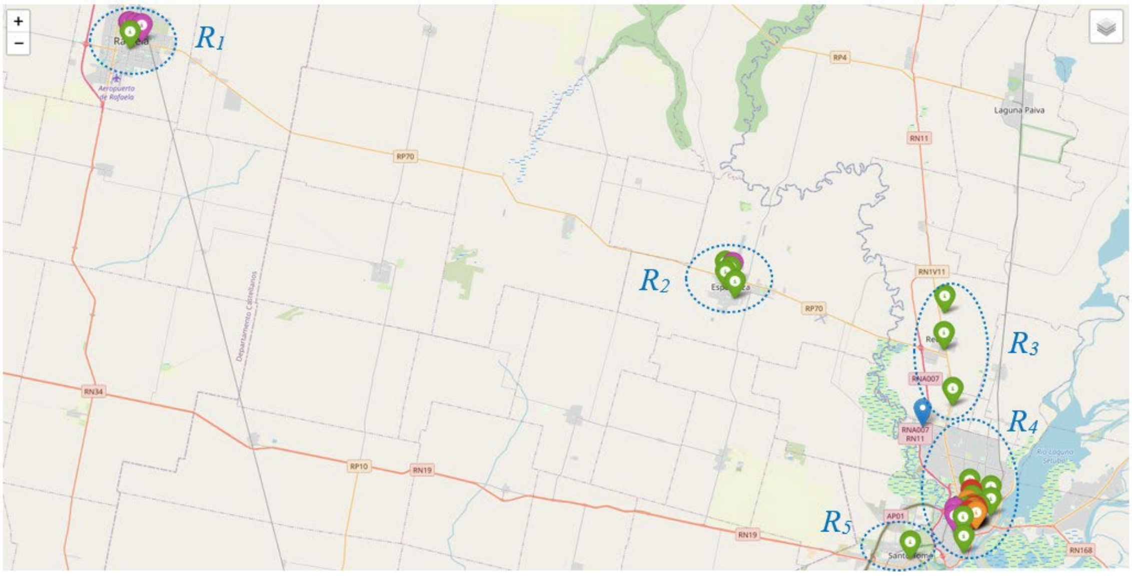

Fig. 1. Geographical distribution of company and healthcare facilities.

Table 1 displays the number of customers in each region, sorted by their visit frequency. Parameter Dem_{id} is estimated based on historical data from the company and is set as described in Section 2, while the service time is directly related to the demand. The variable and fixed costs associated with each vehicle are also estimated considering that it is more cost-effective to use the vehicle with the lower capacity. Besides, parameters D_{ij} and TT_{ij} are obtained through the Application Programming Interface (API) of Project *Open Source Routing Machine* (OSRM, https://project-osrm.org), which uses data from OpenStreetMap. Therefore, both parameters capture the real distance and travel time, respectively, required for a vehicle to travel between locations, leading to asymmetric matrices. The vehicles' time windows are set from 8:00 AM to 4:00 PM, which the daily operating time of each vehicle is 8 hours. It is important to highlight that, for confidentiality reasons, the information and obtained results cannot be disclosed in their entirety. Thus, only the most representative results are reported.

Table 1
Number of customers for each region and frequency

	Visit Frequency				
	$f = 1$	$f = 2$	$f = 3$	$f = 6$	**Total**
R_1	1		4		5
R_2	4		1		5
R_3	3				3
R_4	11	8	2	6	27
R_5	1				1
Total	20	8	7	6	**41**

The model formulation considers the objective function, expressed in Eq. (22), subject to constraints (1) – (21). For solving this problem, the time limit is set to two hours. The model involves 20691 continuous and 21168 binary variables, and 85071 constraints. It is worth mentioning that, despite the combinatorial nature of this problem, the model achieves a good solution after two hours of computation. Moreover, the solver only took 89 seconds to find a feasible solution with an optimality gap of 38.3%. After 3182 seconds, a 5% optimality gap was achieved. This shows a good solution performance of the model. At the time limit, the objective function reached is equal to 319601.21, with 3.03% of optimality gap, which indicates that the solution is very close to the global optimal.

In Table 2 the main results are displayed. For each day and each used vehicle, the route, total load percentage, total traveled distance (in kilometers), and duration of the route (in hours) are presented. It can be observed that, on most days, one of the vehicles operates with a load close to its full capacity. An exception is on d_1, where the limiting factor for vehicle k_1 is the time constraint, which is close to its daily operating time limit (approximately 7 hours and 50 minutes).

Table 2
Routes, load percentage, travel distance and time of each vehicle on each day

		Vehicles	
		k_1	k_2
d_1	Route	i_0 - i_{40} - i_{37} - i_{38} - i_{39} - i_{35} - i_{32} - i_{36} - i_{33} - i_3 - i_{17} -i_{18} - i_{21} - i_{10} - i_{19} - i_{20} - i_{12} - i_{22} - i_0	i_0 - i_1 - i_6 - i_5 - i_7 - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{28} - i_{15} - i_0
	Capacity utilization (%)	60.27	78.56
	Distance (km)	223.1	25.29
	Time (h)	7.82	4.37
d_2	Route	i_0 - i_8 - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{22} - i_0	
	Capacity utilization (%)	99.85	
	Distance (km)	28.01	
	Time (h)	4.13	
d_3	Route	i_0 - i_{35} - i_{40} - i_{37} - i_{38} - i_{39} - i_0	i_0 - i_{17} - i_{18} - i_{16} - i_2 - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{22} - i_0
	Capacity utilization (%)	32.73	92.63
	Distance (km)	173.64	29.84
	Time (h)	4.43	4.81
d_4	Route		i_0 - i_1 - i_9 - i_6 - i_5 - i_7 - i_{20} - i_{19} - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{22} - i_0
	Capacity utilization (%)		90.32
	Distance (km)		27.94
	Time(h)		4.99
d_5	Route	i_0 - i_{29} - i_{31} - i_{30} - i_{35} - i_{34} - i_{40} - i_{37} - i_{38} - i_{39} - i_{41} - i_0	i_0 - i_{17} - i_{18} - i_{11} - i_{16} - i_2 - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{22} - i_0
	Capacity utilization (%)	44.57	99.51
	Distance (km)	186.79	30.46
	Time (h)	5.51	5.17
d_6	Route	i_0 - i_{13} - i_{14} - i_4 - i_{26} - i_{24} - i_{23} - i_{25} - i_{27} - i_{22} - i_0	
	Capacity utilization (%)	94.84	
	Distance (km)	30.79	
	Time(h)	4.28	

Additionally, on d_1, the total demand to be collected is higher because the previous day is not a working day, leading to an accumulation of waste. As a result, finding the optimal route for this day is more complex. On d_1, d_3 and d_5, both vehicles are deployed to meet the frequency and demand of customers, demonstrating efficient vehicle utilization. This is an advantage of the proposed approach, which simultaneously solves the vehicle allocation to customers and vehicle routing, thereby reducing transportation costs.

Furthermore, for each day, the detailed sequence of visits for each vehicle is obtained. As an example, Fig. 2 shows the routes on d_1 and Fig. 3 the routes on d_3. In both cases, the geographic area of region R_4 is shown with a zoom-in, as it is the one with the highest concentration of health facilities. The route of vehicle k_1 is represented in red and of vehicle k_2 in blue. It is worth mentioning that the shortest path to cross between region 4 and 5 is a bridge, as shown in Fig. 2, and it is represented in purple. However, this road is currently under repair, making it inaccessible. The information retrieved from the consulted API is updated to reflect this restriction. This is the reason why vehicle k_1 takes a longer route to travel from region 5 to region 4. This reflects the high level of integration and connection with the real-world characteristics, which enhance the approach and make it a useful and powerful tool for obtaining the detailed weekly allocation and routing of the company.

In contrast, Fig. 3 shows the routes for d_3, where vehicle k_1 is responsible for collection in regions R_1, R_2 and R_3, while vehicle k_2 handles the collection exclusively in region R_4. As can be noted, the health facilities located in R_1, R_2, and R_3 have visit frequencies of 1 and 3. Therefore, these customers are visited on days d_1, d_3, and d_5, when two vehicles are needed to carry out the collection tasks.

Finally, because in the proposed real case there are several small customers with a visit frequency of 1 (once a week) and low demands, there are alternative solutions that achieve the same cost and accommodate the visit of these customers on different days. This is one of the reasons why the optimality gap decreases very slowly, and it is difficult to find the global optimum.

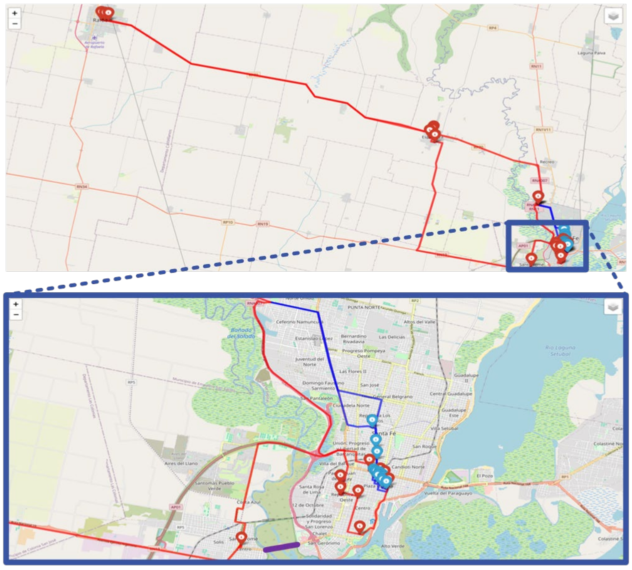

Fig. 2. Routes for each used vehicle on day d_1.

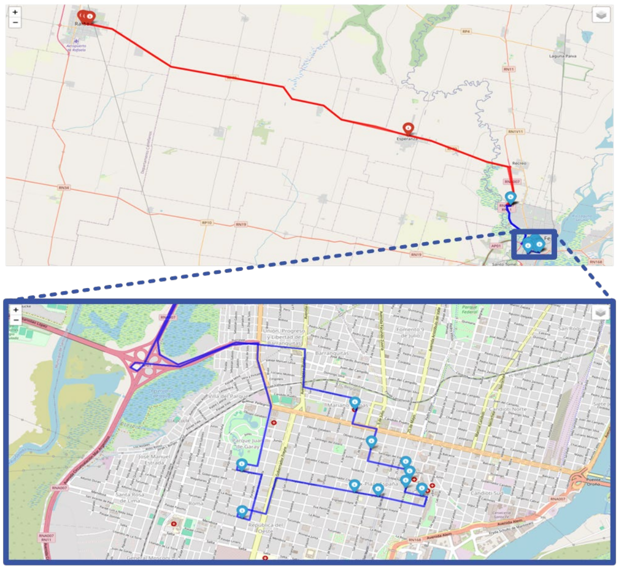

Fig. 3. Routes for each used vehicle on day d_3.

4.2. Application to a real-world benchmark case

The proposed model is applied to a real case study taken from the literature, which involves a company (i_0) that collects and treats pathological wastes from nineteen hospitals (H1 to H19) located in the northern region of Jordan (Alshraideh & Qdais, 2016). Two trucks, each with a capacity of 1500 kg of collected waste, are used to visit the healthcare facilities, and each can make a maximum of one trip per day. Historical data of generated and collected medical waste from these centers, as well as parameters of travel distance from the company to each of the hospitals along with the distance between hospitals are reported by the authors. In addition, owing to local environmental regulations and considering a weekly planning horizon, each healthcare facility must be visited at least once every three days, with Sundays considered non-working days. Thus, the possible visit frequencies within this context are three, four, or five times per week, or daily. The authors present a stochastic model for determining the weekly routing scheduling that minimizes the total travel distance of vehicles while ensuring a high level of service through a probability constraint. Following a Genetic Algorithm rutine implemented in Matlab, a less costly collection schedule is obtained compared to the ad-hoc procedure previously used by the company. However, no model computational statistics or solution times are reported for the authors.

More recently, based on all these data, Mufarrege et al. (2024) address the mentioned case study considering the incorporation of other factors, in order to analyze their influence on the solutions obtained. The analysis is based on the incorporation of two key components: a fixed service time of 20 minutes at each visited hospital and a daily working time limit of 8 hours per vehicle, together with an average vehicle speed set at 35 km/h. Furthermore, based on the historical data provided in Alshraideh & Qdais (2016), the authors present a table with estimates of the average amount of waste generated per week at each hospital. These estimates provide the fundamental basis for a deterministic formulation of the problem, enabling the derivation of more efficient solutions for each scenario considered. Under these conditions and assuming that, for each hospital, the daily waste amount is estimated as the average weekly amount distributed uniformly over the working days, a model MILP and solution strategy are presented to address a classical pathological waste collection and transportation problem. The obtained solution leads to a reduction in the total weekly travel distance, with the consequent economic and environmental benefits.

In this context, the applicability of the model presented in Section 3 of this manuscript is tested, and a comparison with the approaches proposed in Alshraideh and Qdais (2016) and Mufarrege et al. (2024) is carried out. To comply with the requirement imposed by the case study, namely, that each hospital must be visited at least once within any three consecutive days, the following constraints are introduced:

$$\sum_{k\in K} Y_{ikd} + \sum_{k\in K} Y_{ik,d+1} + \sum_{k\in K} Y_{ik,d+2} \geq 1 \qquad \forall\, i \in I_3, d \in \{d_1, d_2, d_3, d_4\} \tag{23}$$

$$\sum_{k\in K} Y_{ikd} + \sum_{k\in K} Y_{ik,d+1} \geq 1 \qquad \forall\, i \in I_3, d = d_5 \tag{24}$$

while constraints (5) – (7) are omitted. Table 3 shows that the solution obtained with the proposed model improves upon the solutions achieved by the other two approaches, considering a computational time limit of 3600 seconds for the deterministic models. As shown in Table 4, six weekly trips are scheduled: no trips are performed on Wednesdays and Fridays, a single trip is carried out on Mondays and Tuesdays, and on Thursdays and Saturdays, both trucks perform one trip each. This solution is more efficient than the one reported in Mufarrege et al. (2024) in terms of vehicle utilization, vehicle availability for other tasks, total distance traveled, and associated costs, yielding a reduction of 37 km in the total weekly distance with respect to the objective function considered.

Table 3
Comparison of solutions

	Total travel distance (km)	Number of routes	Total route duration (h)	Collected waste (kg)	Weekly visit frequency	Optimality gap (%)
Alshraideh & Qdais (2016)	1185	11	54.18	8647.23	6 (H1) 4 (H11) 3 (all others)	----
Mufarrege et al. (2024)	991	8	49.98	8812.16	3 for all hospitals	12.81
Our approach	954	6	46.26	8812.16	3 for all hospitals	2.01

Table 4
Solution obtained using the model proposed in Section 3

Day	Trip	Route	Time route (h)	Collected waste (kg)
1	1	i_0-H1-H5-H7-H18-H19-H13-H10-H17-i_0	7.55	1497.44
2	1	i_0-H2-H15-H3-H4-H11-H6-H12-H14-H9-H16-H8-i_0	7.87	1439.95
3				
4	1	i_0-H1-H5-H6-H18-H19-H13-H10-H17-i_0	7.55	1494.83
	2	i_0-H2-H3-H15-H4-H11-H7-H12-H14-H9-H16-H8-i_0	7.87	1442.56
5				
6	1	i_0-H1-H17-H10-H13-H19-H18-H7-H5-i_0	7.55	1497.44
	2	i_0-H2-H15-H3-H4-H11-H6-H12-H14-H9-H16-H8-i_0	7.87	1439.95

4.3. Numerical tests

To evaluate the computational performance of the model, a set of instances taken from the literature is used. These instances were presented by Basir et al. (2024) on six alternative mathematical models. They suggest a total of 80 different instances, combining the number of customers n, $n \in \{10, 20, 30\ 40, 50, 60, 70\}$, the planning horizon of t days, $t \in \{2, 3, 4, 5\}$, the number of vehicles p, $p \in \{2, 3, 4\}$ and their capacities. Given that some instances are infeasible due to capacity constraints, Basir et al. (2024) adjust the vehicle capacity to ensure feasibility. This data set was originally proposed by Rodriguez-Martín et al. (2019), who also considered different options of visit schedules for customers with known visit frequency. Basir et al. (2024) select one of the visit patterns proposed by Rodriguez-Martín et al. (2019) and denoted the data set with "*S5*". From the 80 instances, 60 were selected to evaluate the proposed approach.

It is worth mentioning that Basir et al. (2024) fix the possible days for customer visits without exploring all possible options. They evaluate each instance across the six proposed mathematical formulations. In this paper, in order to carry out a comparison, from the point of view of performance, with the formulations presented in Basir et al. (2024), the best solution among the six is selected for each instance. Additionally, given that these authors address the PVRP considering periodic delivery operations, and that the present work develops an approach to PVRP based on periodic collection activities, the corresponding constraints reflecting the available vehicle capacities and the flow direction (Equations (13) to (16)) were reformulated to ensure a consistent and rigorous comparison. Also, the formulation presented in Section 3 is modified by relaxing some constraints that are not necessary for the comparison: Eq. (1), as some instances do not meet its condition, given that there may be days in the time horizon in which no demands are recorded; Eq. (17), due to the lack of information on service and travel times; and Eq. (6), since the planning horizon is not long enough to consider this constraint.

Two scenarios are performed using the data set S5. The scenario named "*SC1*" represents the formulation proposed in Section 3, where the constraints related to the visit distribution policy (Equations (5) and (7)) are progressively added as the time horizon increases. When the length of the planning horizon is 2 days, these equations are unnecessary. When t is 3 or 4, only constraint (7) is activated for customers visited twice, i.e. those belonging to set I_2. When the time horizon is extended to 5 days, constraint (7) is activated for customers in sets I_2 and I_3, along with constraint (5). The second scenario "*SC2*" allows any possible visits schedule, therefore constraints (5) to (7) are relaxed from the formulation presented in Section 3. This results in a larger solution search space, increasing the combinatorial complexity but also enhancing the probability of finding a schedule that reduces costs. In other words, the main difference among the three cases is that Basir et al. (2024) consider managing the visit schedules using a very limited number of patterns for each customer without including all possible options, *SC1* treats the assignment of visit days to customers as a decision variable, while respecting predefined visit frequencies and specific conditions regarding the minimum and maximum number of days required between two consecutive visits. and finally, *SC2* considers all possible combinations of customer-visit day allocations, guaranteeing compliance with the prescribed visit frequency for each customer. In order to compare the results, the objective function considered is the minimization of the total distance. Additionally, since all analyzed instances consider symmetric distance matrixes, a new constraint is added to break the symmetry between identical paths with different directions. Equation (25) imposes a direction on the route, ensuring that each vehicle k used on a day d must returns to the plant from a customer with a higher index than that of the first visited customer:

$$Y_{i_0ukd} \leq \sum_{\substack{j\in I_c \\ j>u}} Y_{ji_0kd} \qquad \forall\, u \in I_c, k \in K, d \in D \tag{25}$$

In Table 5, the results of the computational experiments are presented. The first column shows the instance name, taken from Basir et al. (2024), where the sequence of digits that composes it indicates: the number of locations (plant and customers), the number of days of the time horizon, the number of vehicles, the set of schedule patterns given by case "a" from Rodriguez-Martín et al. (2019), and the vehicle capacity that belong to the homogeneous fleet considered, respectively. The following block presents the best value of the objective function (*BestOFV*) obtained among the six alternative formulations proposed by Basir et al. (2024), and *BestTime* reports the computational time if the optimal solution was reached. The subsequent blocks present the results for scenarios *SC1* and *SC2*, respectively. The column *OFV* reports the objective function value obtained within the given time limit, and the computational time (*Time*) required to reach that solution. If the global solution is not reached before the time limit, the optimality gap (*gap*) and the time of the first appearance of the objective function value (*FirstApp*) are shown. In all cases, the computational time limit is set to two hours (7200 seconds). For each instance, the best solution is highlighted in bold.

From the 60 instances, in 4 cases the same solution is obtained by Basir et al. (2024) and at one least of the new approaches, in 34 cases superior solutions are obtained through formulations *SC1* and/or *SC2*, and in the remaining instances, Basir et al. (2024) obtain better solutions. Additionally, in four instances, only one of the scenarios is able to reach a feasible solution within the computation time limit.

Generally, Basir et al. (2024) computing time is shorter because fewer possible visit schedules are considered. Moreover, it is evident that as the size of the instance increases, *SC1* and *SC2* involve more combinations, making the problem more difficult to solve due to its combinatorial nature. For example, in instances with 4 working days, various customers with visit frequency of 2 have only one possible visit combination according to the dataset used by Basir et al. (2024). However, *SC1* approach always considers 3 schedules (d_1 and d_3, d_1 and d_4, d_2 and d_4), while approach *SC2* involves 6 possible schedules (d_1 and d_2, d_1 and d_3, d_1 and d_4, d_2 and d_3, d_2 and d_4, d_3 and d_4). Nevertheless, while the problem size is critical for the solution performance, it is also dependent on the specific characteristics of the dataset.

From Table 5, other conclusions can be highlighted:

(a) When Basir et al. (2024) or *SC1* achieve the same optimal solution as *SC2*, it implies that the corresponding case involves the optimal patterns. For instance, *test11-3-2-a-Q51* demonstrates that the original instance includes the optimal patterns, while *test11-4-2-a-Q51* shows that only *SC1* contains them.

(b) In cases like *test21-3-2-a-Q71*, when the optimal solution is not the same as *SC2*, it shows that neither the original instance used by Basir et al. (2024) nor *SC1* includes the optimal patterns.

(c) For instances like *test31-2-3-a-Q64* and *test31-2-4-a-Q64* where the same objective function value is reached, the addition of one vehicle does not improve the solution.

(d) In smaller instances, it can be observed that the first appearance of the objective function value (*FirstApp*) generally occurs within the first hour of computation. The remaining time is then spent trying to close the optimality gap, which is likely difficult to achieve due to the presence of multiple alternative solutions. A similar situation occurs as described in Section 4.1. However, in these instances, the combinatorial complexity is further increased because the demands of each client are the same for all days and the fleet is homogeneous. For example, in instance *test41-4-2-a-Q180*, *SC1* successfully closed the optimality gap whereas *SC2* did not, with the first appearance occurring at 1071 seconds. Different in *test31-4-2-a-Q80,* where both scenarios achieve a good solution within the first 10 minutes of computation, but neither was able to close the optimality gap within the time limit.

Table 5
Results of different approaches for the dataset S5 taken from Basir et al. (2024)

Instance	Basir et al. (2024)		*SC1*				*SC2*			
	BestOFV	BestTime	OFV	Time	gap	FirstApp	OFV	Time	gap	FirstApp
test11-2-2-a-Q51	**620,48**	0,81	**620,48**	2,2			**620,48**	2,44		
test11-3-2-a-Q51	**834,96**	1,26	**834,96**	3,02			**834,96**	3,47		
test11-4-2-a-Q51	1043,91	0,93	**1027,52**	10,24			**1027,52**	15,85		
test11-5-2-a-Q51	1083,97	7,36	**1064,49**	21,29			**1064,49**	81,94		
test21-2-2-a-Q71	828,98	1,92	**827,88**	20,27			**827,88**	31,94		
test21-3-2-a-Q71	1088,36	4,21	1063,53	80,69			**1042,44**	348,48		
test21-4-2-a-Q71	1253,47	76,29	1219,16	1003,17			**1212,97**	704,73		
test21-5-2-a-Q71	**1729**	1293,83	1768,06		2,12	1624	**1729**	1197,66		
test31-2-2-a-Q80	841,63	15,72	**812,84**	127,23			**812,84**	205,03		
test31-2-3-a-Q64	917,28	1757,67	**880,99**	2959,89			**880,99**	2730,49		
test31-2-4-a-Q64	911,89	3423,2	**880,99**	3609,6			**880,99**	5599,08		
test31-3-2-a-Q80	1190,97	70,13	1190,41	101,08			**1176,27**	1152,83		
test31-3-3-a-Q64	1285,54		1284,91	6432,47			**1248,99**		2,43	2409
test31-3-4-a-Q64	1287,1		1294,91		0,73	1858	**1268,94**		5,44	6913
test31-4-2-a-Q80	1625,85	846,62	**1624,87**		0,68	239	**1624,87**		0,46	562
test31-4-3-a-Q64	1742,31		**1738,72**		3,59	5596	1745,98		3,73	6400
test31-4-4-a-Q64	**1741,92**		2233,4		25,98	7129	1764,15		6,01	1977
test31-5-2-a-Q80	1673,06		1666,41		1,55	3010	**1654,47**		1,46	3764
test31-5-3-a-Q64	**1773,1**		1810,75		6,04	4454	1787,69		6,69	6874
test31-5-4-a-Q64	**1771,75**		1867,47		10,03	5809	1796,66		7,43	1471
test41-2-2-a-Q180	871,32	114,72	**837,69**	88,42			**837,69**	64,83		
test41-2-3-a-Q150	889,32	70,19	**846,1**	1026,15			**846,1**	482,7		
test41-2-4-a-Q60	**1073,75**		1095,29		9,27	7162	1075,13		7,14	4588
test41-3-2-a-Q180	1185,19	272,55	1175,43	248,97			**1155,56**	2280,48		
est41-3-3-a-Q150	1214,05	754,25	1212,09	803,92			**1177,04**		1,03	2348
test41-3-4-a-Q60	**1489,58**		1575,21		11,04	7079	1822,67		25,37	7200
test41-4-2-a-Q180	1460,46	962,98	**1440,75**	3974,14			**1440,75**		0,21	1071
test41-4-3-a-Q150	1476,33	1804,66	1482,83		3,16	6165	**1475,07**		2,45	5567
test41-4-4-a-Q60	**1750,77**		1938,19		17,43	5664	-			
test41-5-2-a-Q190	1921,38	1396,48	1881,84		1,26	6077	**1848,07**		0,55	4337
test41-5-3-a-Q160	1952,17	4425,48	**1926,03**		2,71	3908	1945,96		4,95	6849
test51-2-2-a-Q190	1085,59	730,91	**1053,61**	1922,39			**1053,61**	3700,4		
test51-2-3-a-Q110	1217,06		1215,16		6,94	7195	**1175,28**		3,51	3903
test51-2-4-a-Q100	1248,6		1249,48		6,98	6821	**1220,01**		5,1	5508
test51-3-2-a-Q160	1677,65		1633,3		2,99	3997	**1608,94**		2,52	6233
test51-3-3-a-Q110	**1804,88**		2019,73		15,6	7188	2024,42		16,49	7117
test51-3-4-a-Q100	**1888,78**		2166,75		19,79	6783	2266,61		24,03	7062
test51-4-2-a-Q160	1936,78		1888,23		4,68	7116	**1857,8**		3,04	6546
test51-4-3-a-Q120	**2020,04**		2140,34		14,15	5884	2305,28		20,15	6818
test51-4-4-a-Q100	**2128,49**		-				2478,59		22,78	7070
test51-5-2-a-Q200	2401,49		2394,75		4,11	4080	**2354,9**		2,9	4382
test51-5-3-a-Q190	2425,05		**2403,97**		4,67	6102	-			
test61-2-2-a-Q180	1168,95	5037,86	**1142,15**		0,92	6707	1149,36		1,31	6414
test61-2-3-a-Q120	**1267,71**		1291,3		8,6	6467	1312,67		10,1	7156
test61-2-4-a-Q97	**1323,03**		1552,71		20,66	6714	1794,99		31,5	6254
test61-3-2-a-Q190	1741,95		1785,99		2,86	5632	**1730,45**		2,39	5394
test61-3-3-a-Q150	**1790,93**		1815,68		3,3	6574	1970,72		12,13	6202
test61-3-4-a-Q97	**1999,44**		2498,67		22,97	6847	2845,29		34,06	7125
test61-4-2-a-Q180	2200,08		2316,74		9,71	6447	**2152,8**		2,79	7200
test61-5-2-a-Q180	2450,39		2462,89		4,29	6746	**2418,45**		2,4	5738
test61-5-3-a-Q150	**2513,36**		2754,78		12,87	6958	2672,85		10,78	7027
test71-2-2-a-Q200	1243,98	128,48	**1210,93**	2221,95			**1210,93**	1201,8		
test71-2-3-a-Q150	**1326,69**	6163,58	1392,94		11,01	6369	1417,55		12,79	6932
test71-2-4-a-Q120	**1395,07**		1613,65		20,3	7082	1472,29		12,67	6290
test71-3-2-a-Q200	1641,94	1428,56	1642,8		2,19	6473	**1597,65**		2,24	6920
test71-3-3-a-Q150	**1733,11**		1802,49		8,49	6910	1951,34		17,76	6164
test71-3-4-a-Q120	**1790,09**		2227,17		23,29	6695	1944,14		14,81	14,9
test71-4-2-a-Q200	**2167,52**		2298,96		11,58	6879	2301,39		11,6	7200
test71-4-4-a-Q120	**2322,54**		-				3262,66		33,66	7030
test71-5-2-a-Q200	**3009,01**		3053,4		9,38	7200	3025,99		9,22	5611

5. Conclusions

This work presents a new MILP formulation for the PVRP applied to pathological waste collection and transportation. The proposed model simultaneously determines the assignments of vehicles to customers and service days, as well as the daily routing of vehicle over a weekly planning horizon. The objective is to minimize total transportation costs while fulfilling the service frequency requirements for each healthcare facility and explicitly considering a heterogeneous fleet of vehicles. The formulation incorporates several characteristics commonly observed in real-world waste collection systems, including heterogeneous vehicle and flexible visit schedules. In addition, the integration of geographic information through a GIS database allows the model to use updated distances and travel times between locations, contributing to a more realistic representation of the operational environment. The proposed model was tested using real-world scenarios from a pathological waste collection company, generating detailed logistical plans for daily operations. For each day of the planning horizon, the model determines the healthcare facilities to be visited, the vehicles to be deployed, the assignment of vehicles to customers, and the corresponding collection routes. Additional operational information, such as route duration, collected waste quantities, traveled distances, and visit schedules, is also obtained. The computational results show that the proposed approach is capable of generating high-quality solutions for real instances within reasonable computational times, supporting its potential as a decision-support tool for pathological waste collection planning. Furthermore, the proposed formulation was compared with alternative PVRP formulations from the literature. The results indicate improvements in total costs, mainly due to the generation of more efficient routes enabled by the consideration of additional visit schedules alternatives. However, as the size of the problem increases, solving the model to optimality becomes more challenging due to its combinatorial complexity.

Future research will focus on the development of advanced solution methodologies, such as decomposition techniques and hybrid metaheuristic approaches, to address larger instances and further improve computational efficiency.

Nomenclature

Sets and elements	
$D = \{d_1, d_2, d_3, d_4, d_5, d_6\}$	Set of working days
$I = \{i_0, i_1, \ldots, i_n\}$	Set of nodes, being i_0 the treatment plant and *n* the number of customers
$K = \{k_1, k_2, \ldots, k_p\}$	Set of vehicles, being *p* the number of vehicles
$I_c = I_1 \cup I_2 \cup I_3 \cup I_6$	Subsets of customers
I_f	Set of customers visited f times in the considered period
Parameters	
F_i	Weekly visit frequency of customer *i*
Cap_k	Capacity of vehicle *k*
Dem_{id}	Estimated demand of customer *i* on day *d*
D_{ij}	Distance from node *i* to node *j*
TT_{ij}	Travel time from node *i* to node *j*
ST_{id}	Service time in customer *i* on day *d*
VC_k	Variable transport cost per unit of traveled distance for vehicle *k*
FC_k	Fixed transport cost for using vehicle *k*
WT_{kd}	daily working time limit of vehicle *k* on day *d*
Variables	
Binaries	
X_{kd}	Takes value 1 if vehicle *k* is used on day *d*, and 0 otherwise.
Y_{ikd}	Takes value 1 if customer *i* is visited by vehicle *k* on day *d*, and 0 otherwise.
Z_{ijkd}	Takes value 1 if node *i* is visited immediately before node *j* by vehicle *k* on day *d*, and 0 otherwise.
Continuous	
Q_{ijkd}	Amount of waste in vehicle *k*, travelling from node *i* to node *j* on day *d*.

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