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Research on the game strategy of manufacturer channel encroachment and AI empowerment in the remanufacturing outsourcing supply chain

Xiaoying Cai[a*] and Bing Jiang[b]

[a]*School of Business, Zibo Polytechnic University, Zibo, China*
[b]*Business School, Shandong University of Technology, Zibo, China*

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A B S T R A C T

The continuous advancement of artificial intelligence technology is impacting the operational mechanisms and market competition structures of closed-loop supply chains. How to integrate AI empowerment with channel strategies for coordinated optimization has become a core decision-making issue for manufacturers. By establishing three progressive game models: the benchmark model (M0), the encroachment model (M1), and the AI empowerment model (M2), this study investigates the strategic interplay arising from manufacturers' channel encroachment decisions and AI technology deployment in the context of remanufacturing outsourcing, along with the resulting economic implications. The study finds that when channel differentiation is significant, the manufacturer's encroachment can enhance its profits through market expansion effects. On this basis, by introducing AI technology, its enabling effect exhibits a non-linear characteristic. Only when the channel substitution degree is low and the price sensitivity is moderate, can AI achieve the synergistic benefits of cost savings and market expansion. In terms of revenue distribution, the AI dividend has extensive penetration among supply chain members. Retailers only suffer losses in extreme scenarios where the channels are highly homogeneous and consumers are not price-sensitive. In most market conditions, they can share the efficiency improvement brought about by technology empowerment. This study theoretically reveals the interaction mechanism between channel encroachment and AI empowerment, expands the analytical boundaries of closed-loop supply chain management, and provides strategic guidance for manufacturing enterprises on how to formulate collaborative strategies in the wave of digital transformation.

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1. Introduction

Under the dual impetus of global resource constraints and the wave of circular economy, remanufacturing, as an important means to achieve sustainable development, has increasingly attracted the attention of both the academic community and the industry. By restoring the efficacy of end-of-life products to a state comparable to that of new units, remanufacturing not only significantly reduces resource consumption and environmental impact, but also creates considerable economic value for enterprises (Mondal & Giri, 2020; J. Shen et al., 2025). In this context, remanufacturing, as the crucial part of the closed-loop supply chain, is receiving increasing attention from more and more enterprises. However, the choice of operational model for remanufacturing, whether implemented independently by the original equipment manufacturer (OEM) or authorized or outsourced to a professional third party remanufacturer (TPR), profoundly affects the structure and performance of the supply chain (Guo et al., 2024). Among them, the remanufacturing outsourcing model is favored because it allows OEMs to focus on core design and manufacturing while leveraging the professional capabilities and scale economy of TPRs (Cheng and Pei, 2025; L. Wang et al., 2017), but this model also brings new challenges in supply chain coordination and profit distribution.

Meanwhile, the channel strategies and competitive landscape of manufacturing enterprises have undergone profound changes due to the rapid rise of e-commerce platforms. Currently, many OEMs have expanded from traditional retail to self-operated direct sales channels in order to achieve direct coverage of end-users (Arya et al., 2007; Huang et al., 2021; J. Wang et al., 2026). This channel encroachment behavior, although helping manufacturers enhance brand control and achieve higher profits, inevitably leads to direct competition with long-term cooperative retailers, triggering channel conflicts (Ha et al.,

* Corresponding author
E-mail xiaoying0223@163.com (X. Cai)
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2016; Z. Zhang et al., 2021). Existing research on channel encroachment has predominantly centered on the market for new products (Huang et al., 2026; Z. Li et al., 2014), however, when the remanufacturing business adopts an outsourcing model, the strategic effects brought about by the OEM introducing direct sales channels will become more complex. It not only affects the competitive dynamics of new products across both channels but may also indirectly affect the remanufacturing market through mechanisms such as wholesale prices and product substitutability, causing systemic impacts on the closed-loop supply chain and its overall efficiency.

The new generation of information technologies, represented by artificial intelligence (AI), is also rapidly penetrating various industries, providing new impetus for the transformation and upgrading of supply chains (Masoomi et al., 2026; L. Yang and Xie, 2026). AI can optimize product design, production, and quality control through intelligent detection and machine learning, improving accuracy and reducing waste (Khoshgoftar Manesh & Shahin, 2026; L. Zhang et al., 2024). At the same time, by leveraging demand forecasting and intelligent operation technology, it can precisely grasp market fluctuations, optimize inventory and achieve dynamic monitoring throughout the entire life cycle, thereby reducing capital occupation and operational costs (Chaudhary et al., 2026; Y.-G. Zhao et al., 2025). However, the introduction of AI also comes with high fixed investment costs and uncertain returns, requiring enterprises to make careful considerations when making decisions. In the context of channel encroachment and outsourcing of remanufacturing, how the enabling effect of AI is transmitted and distributed among different entities, whether it will alleviate or exacerbate channel competition, and its consequences for customer uptake of new and remanufactured items, the cost framework, and the overall balance of the supply chain, are key issues that manufacturers face when promoting the digital transformation of supply chains.

Although existing literature has conducted extensive discussions on the decision of remanufacturing models, channel encroachment tendency, and the application of AI technologies, few studies have examined the interactive effects of these three elements within a unified framework. Firstly, scholars have mostly focused on whether manufacturers should outsource remanufacturing and the design of outsourcing contracts, but there has been relatively insufficient attention paid to the structural impact of manufacturers extending their channels to the downstream under the given outsourcing premise. Secondly, the existing results mainly explore the impact of manufacturers establishing direct sales channels on retailers' pricing, services, and channel coordination, but generally ignore how the existence of remanufacturing business changes this game pattern. Finally, regarding the application of AI technology in the supply chain, existing research mostly focuses on its demand forecasting or cost optimization functions, and rarely embeds it in the complex context of remanufacturing outsourcing and channel encroachment to analyze its effect on the competitive and cooperative relationship of manufacturers with dual identities.

Based on this, this article intends to explore the following issues:

1. How do cost variations for new and remanufactured products influence the optimal decisions within the supply chain? Does the change in profits always align with the direction of cost changes?

2. Does a manufacturer opening a direct sales channel necessarily increase its own profits? How do the degree of channel substitution and price sensitivity affect the encroachment motivation?

3. Can the introduction of AI technology enhance the interests of OEMs and retailers? How does its effect depend on the market structure and channel characteristics?

To address these issues, this paper constructs a three-level closed-loop supply chain consisting of an original equipment manufacturer (OEM), a third party remanufacturer (TPR), and a retailer (R). Based on this, by introducing different strategic dimensions, three game models are constructed: the benchmark model (M0), the encroachment model (M1), and the AI-enabled model (M2), and the applicable boundaries and internal operation mechanisms of each model are revealed. When the product market structure is good, the channel differentiation degree is moderate, and the consumer price sensitivity is reasonable, the manufacturer can capture additional profits through encroachment behavior without overly impacting the retail channel. At this time, the encroachment model becomes a feasible market expansion strategy. If the channel differentiation is limited or the market competition is intense, the benchmark model without encroachment remains the preferred choice for maintaining channel stability and avoiding destructive conflicts. When AI technology is introduced and has sufficient capabilities to reduce costs and expand market potential, and the channel differentiation is significant enough to prevent intense price competition, the AI-enabled encroachment model can ensure that both parties are able to participate in the value creation process brought about by technological upgrades.

This study makes three primary theoretical contributions, which are outlined below. First, an analytical framework integrating remanufacturing outsourcing and manufacturer channel encroachment was constructed, revealing how remanufacturing business alters the channel game pattern and the strategic value of retailers as the sole sales channel for remanufactured products, advancing the analytical framework for remanufacturing supply chain governance. Second, AI was introduced into the combined context of remanufacturing outsourcing and channel encroachment, considering the dual impacts of AI on cost structure and market potential, analyzing its transmission mechanism in the competitive relationship between manufacturers and retailers, and expanding the boundaries of research on closed-loop supply chain coordination under technological empowerment. Third, through a progressive model design, the structural influence of remanufacturing outsourcing, channel

encroachment, and AI empowerment on supply chain equilibrium was systematically depicted, providing testable theoretical propositions for subsequent empirical research and offering methodological references for exploring supply chain governance issues in complex scenarios.

The subsequent structure of this article is as follows: Section 2 summarizes the relevant literature. Section 3 elaborates on the model formulation; Section 4 separately solves the three models and compares and analyzes the equilibrium results; Section 5 summarizes the main research findings and their implications for management.

2. Literature Review

Based on a review of the relevant literature, this study primarily relates to three research streams, namely remanufacturing, manufacturer encroachment, and AI and supply chain. The following content is a review of the existing research on these topics.

2.1 Remanufacturing

With the implementation of green policies such as carbon taxes, many scholars have conducted research on the factors influencing the environmental effects and economic value of the remanufacturing in closed-loop supply chains, including online platform signals (Shi and Zhu, 2024), product quality (Pazouki et al., 2025), inventory and pricing strategies (Das et al., 2026), and government subsidies (B. Liu et al., 2026). Remanufacturing encompasses three primary modes: self-operation, licensing, and outsourcing. For manufacturers operating independently, Z. Liu et al. (2018) argued that monopolistic manufacturers can maximize their profits by independently pricing new products and remanufactured products, using price discrimination strategies. J. Zhao et al. (2026) further pointed out that although serviceization would reduce the applicability of this strategy， it could promote enterprises to engage in remanufacturing by blurring product differences and using mixed sales strategies to expand the market. For licensed remanufacturing, Lv et al. (2023) analyzed how consumer brand loyalty affects the formulation of the optimal remanufacturing licensing strategy by OEMs. Lin et al. (2026) found that the delayed network effect would raise the OEM's licensing threshold, and it is worth noting that the economic rationality of the OEM does not always align with its environmental strategy. For outsourced remanufacturing, W. Li et al. (2024) discovered that the internal architecture choice (centralized and decentralized) of the OEM and the outsourcing decision are mutually influential. Xia et al. (2026) compared the impacts of two carbon quota allocation methods, historical method and benchmark method, on the decisions of OEMs and outsourcing remanufacturers in the supply chain, and pointed out that the centralized decision-making of both parties performed better than the decentralized decision-making. Additionally, Q. Zhou et al. (2023) found that the outsourcing remanufacturing strategy of contract manufacturers would affect the choice of remanufacturing mode by OEMs. F. Zhou et al. (2026) compared the profit performance of the three remanufacturing modes under the trade-in scheme, and found that the autonomous and outsourcing modes performed better than the licensing mode.

2.2 Manufacturer's Encroachment

With the widespread adoption of e-commerce and the flattening of distribution channels, the issue of manufacturer encroachment has become a hot topic. Existing research has conducted multi-angle discussions on the impact of manufacturer encroachment, mainly including information asymmetry (Z. Li et al., 2014), changes in retailer profits (S. Zheng and Yu, 2021), and the decision-making of supply chain members (S. Chang et al., 2023). Y. Zhao and Hou (2024) found that under the carbon emission rights trading policy, the encroachment behavior of manufacturers can be incentivized to invest in carbon reduction through lowering wholesale prices. S. Zhao et al. (2025) analyzed how manufacturers in a closed-loop supply chain can address information asymmetry by designing nonlinear contracts and selecting invasive channels. L. Li et al. (2025) revealed that in response to manufacturer encroachment, e-commerce platforms tend to strengthen blockchain deployment to mitigate cybersecurity risks. Duan et al. (2026) examined how manufacturers can strategically choose between direct sales and agency sales channels when implementing data-driven marketing on the platform. From the retailer's perspective, Kittaka and Pan (2023) pointed out that retailers may actively introduce external competitors to secure more favorable purchasing conditions or avoid being squeezed out of the market by manufacturer encroachment. L. Chang et al. (2024) believed that platforms can prevent manufacturer encroachment through information sharing or sales model selection, while manufacturers can avoid the platform's anti-encroachment measures by shifting to agency sales. L. Chang et al. (2025) found that market encroachment through the agent channel by manufacturers can alleviate channel competition to a certain extent. Meanwhile, if retailers adopt a quality differentiation strategy, they can effectively prevent encroachment. J. Zhang et al. (2020) explored how manufacturers and retailers should optimize advertising placement patterns to cope with the competitive challenges brought by encroachment. Additionally, H. Li et al. (2025) found that C2M platforms can guide manufacturers to choose cooperation or give up market encroachment through third-party platforms through strategic investment.

2.3 Application of AI in Supply Chain

Considering AI's technological pervasiveness and data-driven capabilities, researchers have discussed the value generated by AI in the supply chain domain, including demand forecasting (Ramos et al., 2026), pricing optimization (Salehi et al., 2026), production process control (Massaro et al., 2025), and inventory management (Chaudhary et al., 2026). W. Zhou et al., (2025)

argued that when retail platforms adopt AI-driven pricing, manufacturers' channel encroachment behavior can only effectively increase their profits if supported by AI pricing. Song et al., (2026) revealed that the intervention of cost-based pricing on manufacturers' quality decisions stems from the coexistence of two opposing effects: "demand segmentation" and "profit compression". Regarding supply chain resilience, Boone et al. (2025) pointed out that GenAI transforms the supply chain from a passive response system to an active foresight and self-healing resilient system by generating unknown risk scenarios, dynamically optimizing decisions at all levels of the supply chain, and simulating adaptive strategies. Lu et al. (2026) examined how different combinations of AI-enabled capabilities and supply chain integration affect the three stages of supply chain resilience. Regarding supply chain sustainability, Aziz et al. (2021) found that AI can significantly improve the efficiency, accuracy, and sustainability of the remanufacturing process by optimizing the design of components for additive manufacturing repair. A thorough literature review is conducted by L. Zheng et al. (2025) to deeply explore how AI drives the carbon neutrality transformation in supply chain operations. Park (2026) discovered that GenAI adopts measures that can significantly reduce the carbon emission intensity of enterprises in the short term and effectively promote the implementation of circular supply chains. Additionally, Z.-J. M. Shen and Lin (2026) proposed a “three-chain four-intelligence” framework, explaining how AI enhances situational awareness capabilities, reconfigures the logistics chain, information chain, and value chain of the supply chain, and promotes its evolution from digital connection to an adaptive intelligent system.

3. Model

This section systematically studies the strategic interactions among remanufacturing outsourcing, manufacturer channel encroachment, and the application of AI through the construction of a game model. We consider a three-level closed-loop supply chain consisting of original equipment manufacturers (OEM), third-party remanufacturers (TPR), and retailers (R). Based on this, by introducing different strategic dimensions, three game models are constructed: the benchmark model (M0), the encroachment model (M1), and the AI-enabled model (M2).

3.1 Model Setting

We examine a single-cycle closed-loop supply chain for a single product, consisting of three main participants: (1) Original Equipment Manufacturer (OEM): designs and manufactures new products while retaining brand ownership. The OEM fully outsourced remanufacturing activities and does not engage directly in collection or reprocessing. (2) Third Party Remanufacturer (TPR): a professional remanufacturing service provider. TPR is responsible for recycling used products (cores) from the consumer end, conducting inspections, disassembly, and remanufacturing, and delivering qualified remanufactured products to the OEM. (3) Retailer (R): collaborates with the OEM to distribute both new and remanufactured products to consumers via traditional retail channels. At the same time, we consider two expansion strategies: Firstly, in the channel strategy, the OEM can choose whether to open its own direct sales channel (such as the official mall) to sell new products directly to consumers, thereby competing with the retailer in a "encroachment" manner; Secondly, in terms of technology, the OEM and the retailer can choose whether to introduce AI technology. AI can play a role in the back-end (such as the optimization of the OEM's remanufacturing process and cost control) and the front-end (such as the dynamic pricing and personalized recommendations of the retailer). Based on this, we propose the following three structures, and the specific structure diagram is shown in Fig.1:

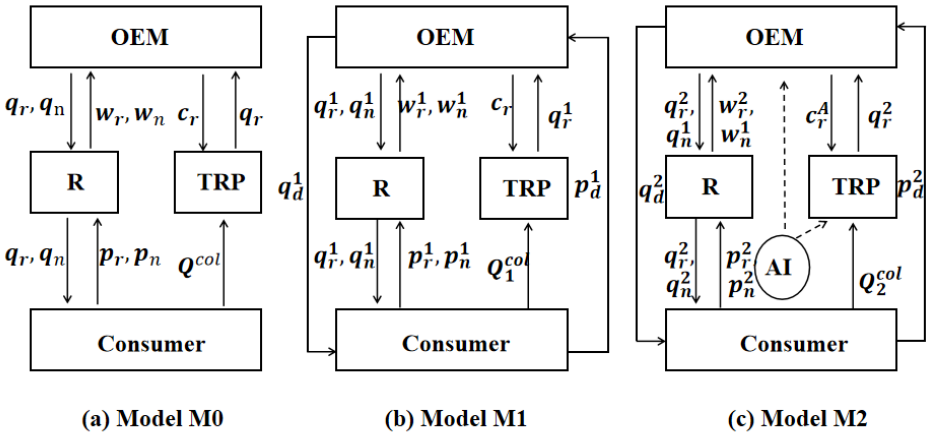

Fig. 1. Supply Chain Structure

Model M0: In the scenario of outsourcing for remanufacturing, manufacturers do not adopt nor introduce AI technologies (neither do retailers). The OEM independently manufactures new products and procures remanufactured units from the TPR. The retailer purchases both product types from the OEM and sells them to end consumers. Consumers perceive

remanufactured products as functionally equivalent to new ones, though potentially lower in perceived quality, while still valuing their environmental benefits.

Model M1: In the scenario of outsourcing remanufacturing, OEM directly encroaches on the market (without AI). Based on M0, OEM additionally establishes its own direct sales channels (such as its own online stores), "infiltrating" the downstream market of retailers R in the new product market, while the remanufacturing business is still outsourced to TPR.

Model M2: With the empowerment of AI, remanufacturing outsourcing and OEM directly encroach on the market. In the M1 structure, OEM and TPR introduce AI technology in the production - remanufacturing - service process, thereby changing the cost parameters and the effective market size.

3.2 Basic Assumptions

Assumption 1. The market comprises both new products (denoted by n) and remanufactured products (denoted by r). In all model settings, p_n and p_r denote the corresponding retail prices. Consumer demand for new and remanufactured products is characterized by the following linear functions:

$$\begin{cases} q_n = a_n - bp_n + sp_r \\ q_r = a_r - bp_r + sp_n \end{cases} \tag{1}$$

Among them, q_n and q_r represent the market demands for new products and remanufactured products respectively; $a_n \geq a_r > 0$ indicates the potential market size under zero price and no environmental efforts; b represents the own-price sensitivity ($b > 0$); $s \in (0, b)$ represents the cross-price effect, ensuring that the two products are "substitutable rather than identical", and it can also represent the degree of substitution between different sales channels in models M1 and M2, with a higher s value indicating a stronger substitutability between sales channels, and thus indicating a more intense competition among the channels (Feng & Lu, 2013; Huang et al., 2018).

Assumption 2. The unit production cost of the new product is $c_n > 0$, and the total cost is $C_n(q_n) = c_n\, q_n$; while the unit processing cost (including processing service fees) of the remanufactured product is $c_r > 0$, usually it is assumed that $c_r < c_n$ (Y. Zhang et al., 2025), corresponding to the total cost $C_r(q_r) = c_r\, q_r$. The AI mainly functions by reducing the unit production and remanufacturing costs and improving the recycling efficiency, that is: $c_n^A = c_n - \lambda_o \tau_o, c_r^A = c_r - \lambda_c \tau_c$; κ is the unit cost of AI investment, and the fixed investment cost of AI is $\frac{1}{2} k_A \kappa^2$ (Cohen et al., 2016; Y. Yang et al., 2025).

Decision-making process: Firstly, OEM needs to make strategic decisions on a long-term basis, including whether to establish direct sales channels and whether to adopt AI technology. Secondly, after determining the strategic decisions, OEM decides the wholesale prices of new products and remanufactured products for retailers, and also decides the retail prices of new products in the direct sales channels (in the M1 and M2 models). After observing the upstream decisions, retailer R (and in the encroachment model, OEM's direct sales channel) selects the retail prices of new products and remanufactured products, and the market demand is realized, and the profit function is determined. Table 1 lists all the symbols used in this article.

Table 1
Notations

Notations	Definitions
q_n^i	Quantity of new products sold through the retailer channel under scenario i
q_r^i	Quantity of remanufactured products sold through the retailer channel under scenario i
q_d^i	Quantity of new products sold through the direct channel under scenario i
p_n^i	Retail price of new products in scenario i
p_r^i	Retail price of remanufactured products in scenario i
w_n^i	Wholesale price of new products in scenario i
w_r^i	Wholesale price of remanufactured products in scenario i
τ_o, τ_c	Individually representing the AI innovation levels of OEM and TRP
φ_o, φ_c	The marginal enhancement coefficient of AI on the market potential of OEM and TRP sectors
a_n	The potential market size of the new product
a_r	The potential market size of remanufactured products
b	The price sensitivity of the product itself
s	Cross-price effect
c_n	Unit production cost of new products
c_r	Processing cost of remanufactured products (including processing service fees)
κ	The unit cost of AI investment

4. Equilibrium Analysis

4.1 Model M0: Baseline Scenario

In the baseline scenario M0, we considered a green supply chain model based on remanufacturing, in which the original equipment manufacturer (OEM) outsources all remanufacturing activities to an independent contract remanufacturer (TRP).

This TRP only provides processing services, and the OEM sells the products through retailer R, without directly entering the market (no infringement behavior). Moreover, neither the OEM nor the retailer R has deployed artificial intelligence technology; all demand information and operational decisions are made through conventional (non-artificial intelligence) processes. Based on the above, the profit expressions for the OEM and the retailer in the baseline model can be formulated as follows:

$$\pi_O = (w_n - c_n)\, q_n + (w_r - c_r)\, q_r \tag{2}$$

$$\pi_R = (p_n - w_n)\, q_n + (p_r - w_r)\, q_r \tag{3}$$

Theorem 1. Based on the reverse deduction, the optimal decisions and maximum profits of the manufacturer and retailer in model M0 are as follows:

$$q_n = \frac{1}{4}(a_n - bc_n + sc_r) \tag{4}$$

$$q_r = \frac{1}{4}(a_r + sc_n - bc_r) \tag{5}$$

$$\Pi_O = \frac{ba_n^2 + ba_r^2 + 2a_n(sa_r + (-b+s)(b+s)c_n) + 2(-b^2+s^2)a_r c_r + (b-s)(b+s)(bc_n^2 - 2sc_n c_r + bc_r^2)}{8(b-s)(b+s)} \tag{6}$$

$$\Pi_R = \frac{ba_n^2 + ba_r^2 + 2a_n(sa_r + (-b+s)(b+s)c_n) + 2(-b^2+s^2)a_r c_r + (b-s)(b+s)(bc_n^2 - 2sc_n c_r + bc_r^2)}{8(b-s)(b+s)} \tag{7}$$

$$w_r = \frac{1}{2}\left(\frac{sa_n + ba_r}{b^2 - s^2} + c_r\right) \tag{8}$$

$$w_n = -\frac{ba_n + sa_r + b^2 c_n - s^2 c_n}{2(-b^2 + s^2)} \tag{9}$$

$$p_n = \frac{1}{4}\left(\frac{3(ba_n + sa_r)}{b^2 - s^2} + c_n\right) \tag{10}$$

$$p_r = \frac{1}{4}\left(\frac{3(sa_n + ba_r)}{b^2 - s^2} + c_r\right) \tag{11}$$

Proposition 1. The impact of new product unit cost c_n on the optimal decision-making of the supply chain:

$\frac{\partial p_n}{\partial c_n} > 0$; $\frac{\partial p_r}{\partial c_n} = 0$; $\frac{\partial q_n}{\partial c_n} < 0$; $\frac{\partial q_r}{\partial c_n} > 0$; $\frac{\partial w_r}{\partial c_n} = 0$; $\frac{\partial w_n}{\partial c_n} > 0$; when $0 < a_n < bc_n - sc_r$, $\frac{\partial \pi_O}{\partial c_n} > 0$ and $\frac{\partial \pi_R}{\partial c_n} > 0$ hold true; conversely, $\frac{\partial \pi_O}{\partial c_n} < 0$ and $\frac{\partial \pi_R}{\partial c_n} < 0$.

Proposition 1 reveals the impact of changes in the unit cost c_n of new products on various decision variables in the supply chain. Its core conclusion is as follows: In response to rising costs for new products, their demand declines in favor of remanufactured ones. This is accompanied by an increase in both wholesale and retail prices for new products, while prices for remanufactured products remain stable. Interestingly, the changes in the profits of OEMs and retailers are not always negative; instead, they depend on the size of the new product market a_n: when a_n is smaller than the critical value $bc_n - sc_r$, profits increase with the increase in c_n, and vice versa. This seemingly counterintuitive conclusion stems from the product substitution effect within the supply chain and the transfer of profit structure: The increase in the cost of the new product leads to its price increase, and some consumers shift to the remanufactured product. Since the remanufactured product has a cost advantage ($c_r < c_n$), its unit profit is higher. Therefore, when the new product market is not large (a_n is small), the profit center of OEMs will naturally lean more towards the remanufactured product. An uptick in new product expenses, through the "crowding-out effect", pushes more consumers towards the high-profit remanufactured product, thereby increasing the total profit. Conversely, if the new product market is the main source of profits (a_n is large), then the increase in cost will directly harm the core business, leading to a decrease in profits. Taking Apple Inc. as an example, when the unit cost of its new products (such as iPhone) increases, Apple usually raises the wholesale price and retail price, causing some price-sensitive consumers to switch to purchasing refurbished remanufactured iPhones. These refurbished phones are manufactured by third-party authorized by Apple and have lower costs and higher profits. If the growth rate of the new product market slows down (i.e., a_n is relatively small), Apple may gain more profits from the growth in the sales of remanufactured products, forming the phenomenon of "cost increase leading to increased profits". Therefore, we find that the final effect of cost increase

depends on the profit center of the enterprise. When remanufactured product serves as the primary profit driver, rising costs for new products can paradoxically fuel overall profitability.

Proposition 2. The impact of remanufactured product unit cost c_r on the optimal decision-making of the supply chain:

$\frac{\partial p_n}{\partial c_r} = 0$; $\frac{\partial p_r}{\partial c_r} > 0$; $\frac{\partial q_n}{\partial c_r} > 0$; $\frac{\partial q_r}{\partial c_r} < 0$; $\frac{\partial w_r}{\partial c_r} > 0$; $\frac{\partial w_n}{\partial c_r} = 0$; when $0 < s < \frac{bc_r}{c_n}$ and $0 < a_r < -sc_n + bc_r$, $\frac{\partial \pi_O}{\partial c_r} > 0$ and $\frac{\partial \pi_R}{\partial c_r} > 0$ hold true; conversely, $\frac{\partial \pi_O}{\partial c_r} < 0$ and $\frac{\partial \pi_R}{\partial c_r} < 0$.

This proposition examines how variations in the unit cost of remanufactured products (c_r) influence key decision variables within the supply chain. The main findings are as follows: An increase in remanufacturing costs triggers a demand shift from remanufactured to new products. This is accompanied by higher wholesale and retail prices for remanufactured items, while prices for new products stay constant. Consistent with Proposition 1, the profit implications for the OEM and the retailer are not uniformly negative; rather, they depend on the market size of remanufactured products (a_r). When a_r is less than the critical value $-sc_n + bc_r$ and the substitution degree s satisfies $0 < s < \frac{bc_r}{c_n}$, profits increase instead as c_r rises, and decrease otherwise. This conclusion is also derived from the product substitution effect within the supply chain and the transfer of profit structure: The increase in remanufacturing cost leads to an increase in its price, and some consumers switch to new products. Since new products typically have higher absolute profits (although unit profits may be lower than those of remanufactured products, but the unit price is high and the market size is large), when the remanufactured product market itself is not large (a_r is small), the profit center of OEMs is originally on new products, and the increase in remanufacturing cost instead "crowds out" more consumers to high-value new products, thereby increasing total profits. Conversely, if the remanufactured product is the main source of profit (a_r is large), then an increase in cost will directly harm the core business, leading to a decrease in profits. Taking Tesla as an example, its remanufactured battery business is undertaken by a third-party partner. When the remanufacturing cost of recycled batteries increases due to raw material or process requirements, Tesla raises the wholesale and retail prices of remanufactured batteries, and some consumers switch to new cars (especially the entry-level Model 3 or Model Y), because new cars have higher profits and a strong market growth rate (that is, a_r is relatively small, and remanufactured batteries are not the main source of profit), Tesla's total profits may increase instead due to the increase in remanufacturing costs, so when the remanufactured product market is not large, its cost increase may instead "crowd out" consumers to higher-value new products, thereby increasing total profits.

4.2 Model M1: The Scenario of OEM Infringement under Outsourced Remanufacturing (Without AI)

Based on the base case M0, this section considers that the OEM maintains the outsourcing of remanufacturing unchanged and instead, through opening its own direct sales channels, "infiltrates" the retailers. Meanwhile, the retailers sell both new products and remanufactured products via traditional retail channels, pitting them against manufacturers' direct channels in the new product space. Consumers have the option to choose among the three scenarios: purchasing new products through retailers, purchasing new products through the manufacturer's direct sales channels, or purchasing remanufactured products through retailers. To identify the marginal effect of channel encroachment, we introduce the market segmentation assumption: the demand for remanufactured products is exogenous to the fluctuations in direct sales prices, while new products form horizontal competition between the retail and direct sales channels. Here, p_n^1 refers to the retail channel price for new products, while p_d^1 refers to the price in the manufacturer's direct sales channel. The demand functions are then specified as follows:

$$q_n^1 = a_n - bp_n^1 + sp_d^1 \tag{12}$$

$$q_d^1 = a_n - bp_d^1 + sp_n^1 \tag{13}$$

The demand for remanufactured products is given by:

$$q_r^1 = a_r - bp_r^1 + sp_n^1 \tag{14}$$

Under this setting, the profit functions for the OEM and the retailer are:

$$\pi_O^1 = (w_n^1 - c_n)q_n^1 + (p_d^1 - c_n)q_d^1 + (w_r^1 - c_r)q_r^1 \tag{15}$$

$$\pi_R^1 = (p_n^1 - w_n^1)q_n^1 + (p_r^1 - w_r^1)q_r^1 \tag{16}$$

Theorem 2. Through reverse deduction, the partial optimal decisions and maximum profits of the manufacturer and retailer in Model M1 are as follows. Due to the lengthy explicit expressions, the details can be found in the appendix:

$$q_d^1 = -\frac{(2b+s)(-a_n + (b-s)c_n)}{4b} \tag{17}$$

$$q_r^1 = -\frac{b^2(b+s)\left(-sa_n + (b-s)(-2a_r - sc_n + 2bc_r)\right)}{8b^4 - 10b^2s^2 + s^4} \tag{18}$$

$$w_n^1 = \frac{(2b-s)(b+s)(2b+s)a_n + s^3a_r + (4b^4 - 5b^2s^2 + s^4)c_n - bs^3c_r}{8b^4 - 10b^2s^2 + s^4} \tag{19}$$

$$p_d^1 = \frac{(8b^3 + 8b^2s - 2bs^2 - s^3)a_n + 2bs^2a_r + (8b^4 - 10b^2s^2 + bs^3 + s^4)c_n - 2b^2s^2c_r}{2(8b^4 - 10b^2s^2 + s^4)} \tag{20}$$

In the equilibrium solution of the above model, the market size parameters a_n and a_r may vary depending on the product type. However, to focus on the impact of cost structure and market substitutability on profit comparisons, this section considers the special case of market size symmetry, that is, assuming that the market demand scales for the two types of products are equal, and is standardized as $a_n = a_r = 1$(Niu et al., 2025).

Proposition 3. Compared with model M0, when the following range is satisfied, in the encroachment scenario, the new product price in the retail channel will increase ($p_n^1 - p_n > 0$): (1) $0 < s < s_1, b_1 < b < b_2$; (2) $s_1 < s < s_2, b_1 < b < b_2$; (3) $s_2 \le s < s_3, b_1 < b < 1$.

A comparison between the new product prices in the retail channel with encroachment (p_n^1) and without encroachment (p_n) is presented in Proposition 3. The results show that manufacturer encroachment leads to an increase in the retail price of new products. Furthermore, the extent of this price increase is contingent upon the cross-price effect s and the price sensitivity parameter b. From Proposition 1, we can observe that when $0 < s < s_1, b_1 < b < b_2$, s is small and b is moderate, the channel differences are obvious and the consumer price sensitivity is moderate. The retailer has sufficient pricing space, and the price increase is the most stable. For example, for Apple, there is a significant service differentiation between its retail channels (such as JD.com and Suning) and its direct sales channels (official website and Apple Store): The official website provides exclusive services such as engraving, trade-in, and 24-month interest-free installment, while authorized stores offer offline experience and immediate availability. This channel difference results in a small cross-price effect s, and consumers will not significantly switch to the official website due to a small retail price increase. At the same time, Apple users are relatively insensitive to prices (b is moderate), and a small price increase will not lead to a significant decline in demand. When $s_1 < s < s_2, b_1 < b < b_2$, s increases to a medium level, the channels have some similarity but still have differences. The price increase still holds, for example, for Nike (in the US market), the official website provides customized services, but there is still customer overlap with Foot Locker and other stores. The retail price can increase but the range is limited; however, when $s_2 \le s < s_3, b_1 < b < 1$, s is relatively large but not completely identical, and at this time, consumers have a high sensitivity to prices. We find that the price in the retail channel still increases. In practice, it often cannot be realized due to demand contraction or profit squeeze, ultimately manifesting as "publicly rising but secretly falling" or price stability. For example, in the case of Dell's later entry into Best Buy, the official website and store products are highly similar, and the retail price was initially consistent with the official website but soon reduced implicitly through "time-limited discounts" and "full deduction" methods, and the final transaction price was often lower than the official website. The retailer cannot maintain a long-term price increase.

Proposition 4. When $0 < s < s_1$ and $b_1 < b < 1$, the manufacturer's OEM profit will increase ($\Pi_O^1 - \Pi_O > 0$), and the OEM will choose to encroach on the market.

Proposition 4 reveals that under the encroachment condition, the increase in manufacturer profits holds true within the range of $0 < s < s_1, b_1 < b < 1$: at this time, s is relatively small, the substitutability of the channels is low, the differences between the retail and direct sales channels are obvious, consumers will not easily switch from retail to direct sales due to a price increase, and at the same time, the price sensitivity b is within an appropriate range ($b_1 < b < 1$), ensuring that a price increase will not lead to a collapse in demand. The manufacturer can attract new customer groups through market expansion effects, thus, in this scenario, OEMs will choose to encroach on the market, thereby achieving profit growth. For example, Apple, whose official website and authorized stores have significant differences in services, customization, and trade-in programs, consumers are relatively insensitive to prices. After encroachment, the manufacturer not only does not damage the retail channel but also expands the overall market through differentiated services, and profits continue to rise. From this, we can observe that manufacturers' profit gains from encroachment are not inevitable. Only when the channel differentiation is significant, and the market expansion effect dominates, can manufacturers attract new customer groups and achieve profit growth; once s is too large or b exceeds the moderate range, the profit enhancement effect weakens or disappears.

A comparison of the manufacturer's profits in the encroachment and non-encroachment scenarios is shown in Fig. 2 In the blue area ($\Pi_O^1 - \Pi_O > 0$), the OEM's profit under encroachment exceeds that in the non-encroachment case, making encroachment the preferred strategy. Conversely, the green area ($\Pi_O^1 - \Pi_O < 0$) reflects situations where the manufacturer's profit is higher without encroachment. At this time, the OEM will not choose to occupy the market. Fig. 2 further demonstrates the conclusion of Proposition 4 and provides an intuitive decision boundary for managers to identify the applicable conditions of the occupation strategy.

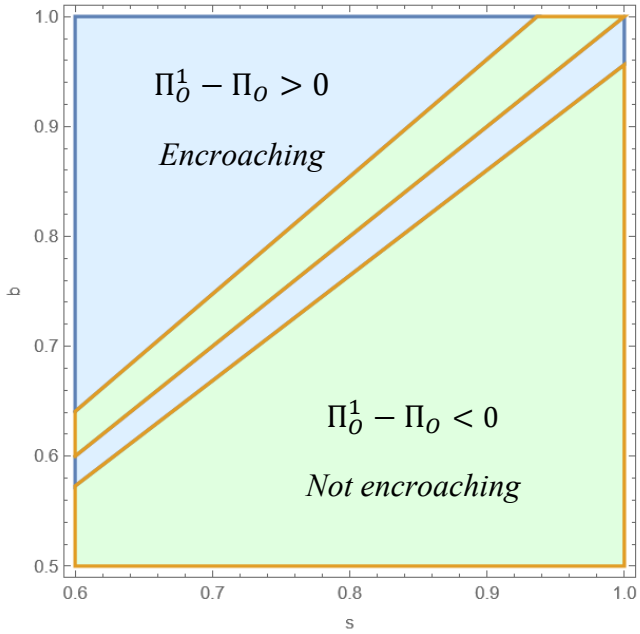

Fig. 2. Diagram of OEM encroachment

4.3 Model M2: Outsourcing of Remanufacturing Enabled by AI and OEM Encroachment

Based on Model M1, this section introduces AI technology to construct the AI-enabled remanufacturing outsourcing and manufacturer encroachment model M2. We deploy AI technology in both OEM and TRP. The AI deployment in OEM can be used for predictive maintenance and intelligent quality inspection, thereby reducing unit manufacturing costs; while the TRP employs AI technology to enhance recycling volumes and streamline remanufacturing operations, thereby reducing unit costs. Therefore, the new product market demand is as follows:

$$q_n^2 = a_n + \varphi_o \tau_o - b p_n^2 + s p_d^2 \tag{21}$$

$$q_d^2 = a_n + \varphi_o \tau_o - b p_d^2 + s p_n^2 \tag{22}$$

For remanufactured products, the demand function is:

$$q_r^2 = a_r + \varphi_c \tau_c - b p_r^2 + s p_n^2 \tag{23}$$

The profits of the OEM and retailer R in this case can be expressed as:

$$\pi_O^2 = (w_n^2 - c_n^A) q_n^2 + (p_d^2 - c_n^A) q_d^2 + (w_r^2 - c_r^A) q_r^2 - \frac{1}{2} k_A \kappa^2 \tag{24}$$

$$\pi_R^2 = (p_n^2 - w_n^2) q_n^2 + (p_r^2 - w_r^2) q_r^2 \tag{25}$$

Theorem 3. Through backward induction, the optimal strategies and resulting profits for the manufacturer and retailer in Model M2 can be characterized as follows. Due to the lengthy explicit expressions, the details can be found in the appendix:

$$q_d^2 = -\frac{(2b+s)(-a_n + (b-s)(c_n - \lambda_o \tau_o) - \tau_o \varphi_o)}{4b} \tag{26}$$

$$q_r^2 = -\frac{b^2(b+s)\big(-s a_n + (b-s)\big(-2a_r - s c_n + 2b c_r + s\lambda_o \tau_o - 2\tau_c(b\lambda_c + \varphi_c)\big) - s\tau_o \varphi_o}{8b^4 - 10b^2 s^2 + s^4} \tag{27}$$

Proposition 5. The impact of OEM's AI innovation level τ_o on supply chain demands:

(1) when $s_1 \le s < 1, 0 < \varphi_o < \varphi_{o1}, \lambda_{o1} < \lambda_o < 1$, then $\frac{\partial q_n^2}{\partial \tau_o} < 0$;

(2) when $s_1 \le s < 1, 0 < \varphi_o < \varphi_{o2}, 0 < \lambda_o < \lambda_{o2}$ or $s_1 \le s < 1, \varphi_{o2} \le \varphi_o < 1$, then $\frac{\partial q_r^2}{\partial \tau_o} < 0$;

(3) $\frac{\partial q_d^2}{\partial \tau_o} < 0$ always holds.

Proposition 5 reveals that the demand for new products in the direct sales channel (q_d^2) is consistently negatively correlated with the OEM's AI innovation level. This is because although AI empowerment reduces production costs, manufacturers tend to maintain long-term cooperative relationships through retail channels rather than over-expanding the direct sales channels. Taking Xiaomi as an example, although it significantly reduced the production cost of mobile phones through AI technology (λ_0 is large), in its channel strategy, Xiaomi always uses offline retail partners (such as Xiaomi Home Authorized Stores) as the main sales channels, while the official direct sales channel mainly serves as a showcase and limited release function, and did not expand the scale of direct sales due to the cost reduction. Instead, it maintained the price system of the retail channels by controlling the supply volume of the direct sales channels. Secondly, the demand for new products in the retail channel (q_n^2) will also decrease under certain conditions due to OEM's AI innovation. This mainly occurs in scenarios where the channel substitutability is strong (s is large) and AI has a limited effect on market expansion (φ_0 is small). Taking Dell as an example, its products on the official website are highly homogeneous with those on retail channels (such as JD.com and Suning), when Dell optimizes the supply chain through AI to reduce costs, it often prioritizes launching limited-time discounts on the official website, significantly squeezing out the new product market in the retail channels, thus creating a competitive situation of "one growing while the other declines". Finally, the OEM's AI innovation may also suppress q_r^2. Taking Apple's official refurbished iPhones as an example, when Apple optimizes the new product production line (such as iPhone 15) and launches more attractive entry-level models, the attention of consumers to the official refurbished phones (such as the iPhone 13 re-release version) significantly decreases. Even though the refurbished phones have lower prices, they cannot resist the appeal of the new product's AI functions (such as computational photography, intelligent battery management), and as a result, the market share of remanufactured products is gradually diminished.

Proposition 6. The impact of TRP's AI innovation level τ_c on supply chain demands:

(1) when $s_1 \le s < 1, \varphi_c < \varphi_{c1}$, then $\frac{\partial q_n^2}{\partial \tau_c} < 0$;

(2) when $s_1 \le s < 1, \varphi_c > \varphi_{c1}$, then $\frac{\partial q_r^2}{\partial \tau_c} < 0$;

(3) $\frac{\partial q_d^2}{\partial \tau_c} = 0$.

From Proposition 6, we can observe that the elasticity of the q_d^2 through the direct sales channel with respect to the AI innovation level of TRP is zero. This is because the technological upgrade of TRP only affects the remanufacturing process and has no direct correlation with the new product sales through the manufacturer's direct sales channel. Taking Tesla's remanufactured battery business as an example, its third-party partners (such as Redwood Materials) enhance the efficiency of battery recycling and remanufacturing through AI technology, but this does not affect the demand for new cars on Tesla's official website. Consumers mainly focus on core performance such as range and autonomous driving when purchasing new cars, rather than changes in the cost of remanufactured batteries. Secondly, the q_n^2 may decline when the AI innovation level of TRP increases. This occurs when the expansion effect of AI on the remanufactured product market is limited (φ_c is small). Take HP's remanufactured printer as an example, when third-party remanufacturers use AI to reduce the cost of remanufacturing, the price of remanufactured printers decreases, and some budget-sensitive enterprise customers switch to purchasing remanufactured products through retailers (such as JD Enterprise Purchase). This has led to a diversion of demand for new printers, resulting in pressure on the sales of new products in the retail channels. Finally, the demand for remanufactured products (q_r^2) may decline instead when the AI innovation level of TRP increases. This seemingly counterintuitive phenomenon occurs when the expansion effect of AI on the market is significant (φ_c is large). Take Samsung's officially authorized third-party remanufactured refurbished mobile phones as an example, when TRP significantly improves the remanufacturing efficiency and recovery scale through AI technology, the market supply of remanufactured products significantly increases. To maintain its upscale brand identity and profit structure, Samsung may proactively adjust its wholesale strategy at this time, such as lowering new product prices or enhancing trade-in incentives, with the aim of limiting the market penetration of remanufactured products. This ultimately leads to a situation where the demand for remanufactured products actually decreases instead of increasing, creating a "gameplay dilemma where technology empowerment is counteracted by channel strategies".

$$\Delta\Pi_O = \Pi_O^2 - \Pi_O^1 = \frac{A}{8b(8b^4 - 10b^2s^2 + s^4)} \tag{28}$$

$$\Delta\Pi_R = \Pi_R^2 - \Pi_R^1 = \frac{B}{16b(8b^4 - 10b^2s^2 + s^4)^2} \tag{29}$$

Among them, the appendix contains the complete expressions for A and B.

Proposition 7. Based on the manufacturer's encroachment, when $\Delta\Pi_O > 0$, OEM prefers to introduce AI technology; when $\Delta\Pi_O < 0$, OEM prefers not to introduce AI technology.

Proposition 7 describes the impact of introducing AI technology on the profits of manufacturers (OEMs) when they have already established direct sales channels. When $\Delta\Pi_O > 0$, it indicates that introducing AI can enhance the profits of OEMs, while when $\Delta\Pi_O < 0$, it shows that introducing AI harms the profits of OEMs. This is illustrated in Fig. 3. Additionally, based on Fig. 3, we can observe that OEMs' preference for AI exhibits distinct regional characteristics. When the s is small and the price sensitivity is moderate, OEMs tend to introduce AI. At this time, channel differentiation is significant, and the cost savings and market expansion effects brought by AI can be captured by OEMs without triggering intense price wars, thereby increasing profits. However, when the s is large and the price sensitivity is either too high or too low, OEMs tend not to introduce AI because the channels are highly homogeneous. The cost savings caused by AI instead intensify price competition, resulting in the erosion of profits. Price sensitivity (b) also plays a crucial role. When b is too high, the cost savings must be fully transferred to consumers; when b is too low, the market expansion effect cannot be realized; only when b is moderate can the AI investment achieve the optimal return. Therefore, we find that OEMs' AI investment decisions have a deep dependence on the market structure. AI is not always profitable, and its value realization requires a moderate degree of channel differentiation and consumer price response.

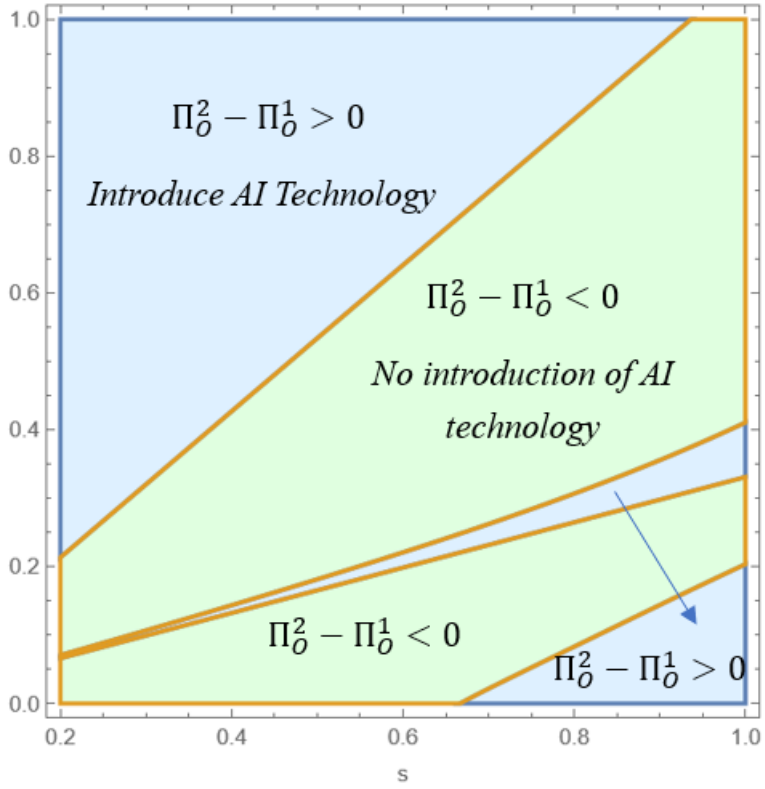

Fig. 3. OEM's preference regarding the adoption of AI technology

Proposition 8. Based on the manufacturer's exploitation, when $\Delta\Pi_R > 0$时, retailer R prefers to introduce AI technology; when $\Delta\Pi_R < 0$, retailer R prefers not to introduce AI technology.

Through this proposition, we find that when the s is large and the consumer price sensitivity is low (b is small), the retailer's profits are affected negatively ($\Delta\Pi_R < 0$): On the one hand, the high degree of channel homogeneity gives OEM a strong motivation to lower prices in the direct sales channel to capture the market, directly eroding the retailer's customer base; on the other hand, the consumers' low price sensitivity means that the retailer cannot effectively compete for customers by lowering prices and can only passively bear the loss of declining sales. At this time, the cost savings brought by AI are completely monopolized by OEM, and even turn into competitive pressure on the retailer, significantly eroding the profits of retailers. Conversely, when the degree of channel substitution is low (s is small) or when the degree of channel substitution is high but the price sensitivity is also high (s is large, b is large), the retailer can benefit from the introduction of AI ($\Delta\Pi_R > 0$): On the one hand, when the channel differentiation is significant, OEM and the retailer have a complementary relationship. The cost savings brought by AI are transmitted to the retailer through the wholesale price adjustment, and the new customer groups attracted by the market expansion effect benefit the retail channel, enabling both parties to share the benefits; on the other hand, when the degree of channel substitution is high but the price sensitivity is also high, the cost savings brought by AI give OEM the space to lower the wholesale price, and the retailer can attract more price-sensitive consumers by lowering prices, achieving "low profit but high sales" profit growth. The price transmission mechanism benefits the retailer at this time. As shown in Fig. 4, we find that retailers only suffer losses in the case of highly homogeneous channels and low price sensitivity of consumers, while in most market scenarios, retailers can share the efficiency improvement brought by AI.

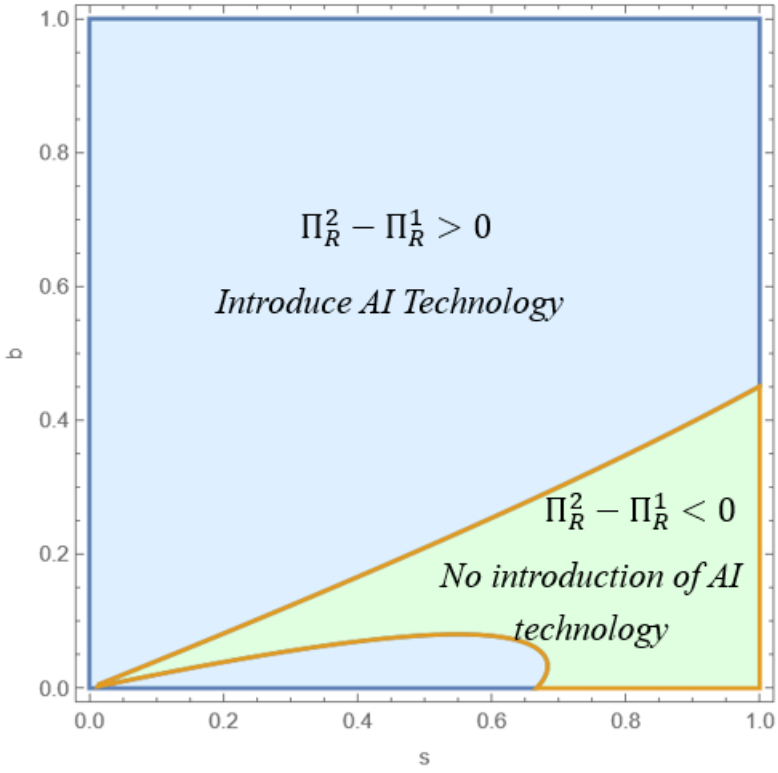

Fig. 4. Retailer R's preference regarding the introduction of AI technology

5. Conclusion

5.1 Main Findings

This study focuses on a three-level closed-loop supply chain. A manufacturer-led Stackelberg game model was constructed to study the three typical scenarios: the benchmark model (M0), the encroachment model (M1), and the AI-enabled model (M2). The study systematically examined the strategies of AI technology application in outsourcing re-manufacturing and channel encroachment situations. It explored how the outsourcing strategies of remanufacturing, channel encroachment decisions, and AI technology investment of supply chain members jointly affect product pricing, production allocation, and channel profit distribution. We found that:

First, in the benchmark remanufacturing outsourcing model without infringement and without AI, the impact of product cost fluctuations on supply chain profits is asymmetric. The direction of profit changes depends on the market structure. The research finds that an increase in the unit cost of new products does not necessarily harm the profits of manufacturers and retailers. When the market size of new products is small, the cost increase will, through substitution effects, push consumers towards remanufactured products with higher profits, thereby increasing overall profits; similarly, an increase in remanufacturing costs may bring positive returns when new products are the profit center.

Secondly, in the scenario where remanufacturing outsourcing is combined with the channel encroachment by manufacturers, the conditions for manufacturers to benefit from the encroachment have strict boundary constraints. The key lies in the degree of differentiation between the channels. Research shows that only when the degree of channel substitution is moderate and the price sensitivity is within a reasonable range can manufacturers achieve profit growth by opening direct sales channels; if the channel homogeneity is too high or the price sensitivity is too low, the encroachment behavior will trigger intense channel conflicts, which will instead harm the manufacturers' own interests.

Thirdly, in the combined context of remanufacturing outsourcing enabled by AI technology and channel encroachment, the impact of AI on the interests of each member of the supply chain shows significant differentiation. Its value realization is highly dependent on the matching of market structure and technology deployment. The research reveals that manufacturers' preference for AI is concentrated in regions with significant channel differentiation and moderate price sensitivity. At this time, the cost savings and market expansion effects brought by AI can be effectively captured. Retailers only suffer when the channels are highly homogeneous and consumers are not sensitive to prices. In the vast majority of market scenarios, they can share the efficiency improvement brought by AI. Further, the technical empowerment of AI by the remanufacturing

business may be counteracted by the channel strategies of manufacturers, resulting in a game situation where there is both technological gain and strategy loss.

5.2 Managerial Implications

First, enterprises should manage cost fluctuations dynamically rather than simply controlling costs. When the potential market for new products is limited, an increase in the cost of new products may, through substitution effects, push consumers towards more profitable remanufactured products, thereby enhancing overall profits. Enterprises should view cost fluctuations from the perspective of product mix. If remanufactured products have become the profit pillar, they should prioritize controlling the cost risks of these products; if the growth of new products is weak, they can moderately tolerate the increase in costs and take advantage of this to drive consumers to shift towards more profitable remanufactured products, achieving an optimization of the profit structure.

Secondly, channel encroachment should be based on differentiation and avoid blind expansion. When channel homogeneity is too high, encroachment will trigger intense price competition, ultimately harming the manufacturer's own profits. Enterprises should assess whether the direct sales channels can provide differentiated value, the price sensitivity of the target customer group, and the strategic value of retailers as the sole channel for re-manufacturing products. Only when the channel differentiation is significant and can attract new customers, can the encroachment strategy be cautiously implemented, and the incentive consistency with retailers can be maintained through wholesale price contracts.

Thirdly, the realization of the value of AI investment highly depends on the matching of market structure. OEMs should introduce AI under conditions where channel differentiation is significant and price sensitivity is moderate, in order to capture the dual benefits of cost savings and market expansion. Retailers can share the efficiency improvements brought by AI in most situations, but they need to be cautious about profit losses only in extreme scenarios where channels are highly homogeneous and consumers are not sensitive to prices. When there are differences in interests between the two parties, OEMs should coordinate channel relationships through wholesale price adjustments or revenue sharing mechanisms to ensure that technological upgrades and channel cooperation work together.

5.3 Limitations and Future Research

The primary constraint of this study lies in the relatively idealized nature of the model construction. Future research could consider adopting a more general demand function or incorporating consumer heterogeneity preferences (such as environmental awareness, brand loyalty) into the model to enhance the realistic explanatory power of the conclusions. Moreover, this paper assumes that the introduction of AI technology is a binary decision (introduction or not), without considering the continuity and dynamic adjustment process of AI investment. Subsequent research could focus on optimizing strategies for the intensity of AI investment, and further investigate the interaction and coordination mechanisms of technological deployment among supply chain members. Additionally, this paper focuses on the Stackelberg game framework dominated by manufacturers. Future research could expand to consider demand uncertainty and information sharing strategies, or extend the model to multi-period dynamic scenarios to explore the structural reshaping effect of AI technology on the remanufacturing closed-loop supply chain in the long term.

Data disclosure statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclosure statement

The authors report there are no competing interests to declare.

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Appendix

Proof of Theorem 1

First, we substitute the demand functions $q_r = a_r - bp_r + sp_n$ and $q_n = a_n - bp_n + sp_r$ into the profit functions π_R and π_O. Then, in the profit function π_R, we take the first-order derivative with p_n and p_r to obtain $p_n = -\frac{-ba_n - sa_r - b^2w_n + s^2w_n}{2(b^2-s^2)}; p_r = -\frac{-sa_n - ba_r - b^2w_r + s^2w_r}{2(b^2-s^2)}$; Then, substitute p_n and p_r into the profit function of OEM and differentiate w_r and w_n to obtain $w_r = \frac{1}{2}\left(\frac{sa_n + ba_r}{b^2-s^2} + c_r\right); w_n = -\frac{ba_n + sa_r + b^2c_n - s^2c_n}{2(-b^2+s^2)}$, thereby obtain the decision equilibrium solution:

$$q_n = \frac{1}{4}(a_n - bc_n + sc_r); q_r = \frac{1}{4}(a_r + sc_n - bc_r);$$

$$\Pi_O = \frac{ba_n^2 + ba_r^2 + 2a_n(sa_r + (-b+s)(b+s)c_n) + 2(-b^2+s^2)a_rc_r + (b-s)(b+s)(bc_n^2 - 2sc_nc_r + bc_r^2)}{8(b-s)(b+s)};$$

$$\Pi_R = \frac{ba_n^2 + ba_r^2 + 2a_n(sa_r + (-b+s)(b+s)c_n) + 2(-b^2+s^2)a_rc_r + (b-s)(b+s)(bc_n^2 - 2sc_nc_r + bc_r^2)}{8(b-s)(b+s)};$$

$$w_r = \frac{1}{2}\left(\frac{sa_n + ba_r}{b^2-s^2} + c_r\right);\ w_n = -\frac{ba_n + sa_r + b^2c_n - s^2c_n}{2(-b^2+s^2)};\ p_n = \frac{1}{4}\left(\frac{3(ba_n + sa_r)}{b^2-s^2} + c_n\right);\ p_r = \frac{1}{4}\left(\frac{3(sa_n + ba_r)}{b^2-s^2} + c_r\right).$$

Proof of Proposition 1

By separately calculating the first-order derivatives of the production quantity, wholesale price, retail price, and enterprise profit, we have: $\frac{\partial q_n}{\partial c_n} = -\frac{b}{4} < 0; \frac{\partial q_r}{\partial c_n} = \frac{s}{4} > 0; \frac{\partial w_r}{\partial c_n} = 0; \frac{\partial w_n}{\partial c_n} = -\frac{b^2-s^2}{2(-b^2+s^2)} > 0; \frac{\partial p_n}{\partial c_n} = \frac{1}{4} > 0; \frac{\partial p_r}{\partial c_n} = 0$; when $0 < a_n < bc_n - sc_r$, $\frac{\partial \pi_O}{\partial c_n} = \frac{2(-b+s)(b+s)a_n + (b-s)(b+s)(2bc_n - 2sc_r)}{8(b-s)(b+s)} > 0$ and $\frac{\partial \pi_R}{\partial c_n} = \frac{2(-b+s)(b+s)a_n + (b-s)(b+s)(2bc_n - 2sc_r)}{8(b-s)(b+s)} > 0$.

Proof of Proposition 2

By separately calculating the first-order derivatives of the production quantity, wholesale price, retail price, and enterprise profit. We have: $\frac{\partial q_n}{\partial c_r} = \frac{s}{4} > 0; \frac{\partial q_r}{\partial c_r} = -\frac{b}{4} < 0; \frac{\partial w_r}{\partial c_r} = \frac{1}{2} > 0; \frac{\partial w_n}{\partial c_r} = 0; \frac{\partial p_n}{\partial c_r} = 0; \frac{\partial p_r}{\partial c_r} = \frac{1}{4} > 0$; when $0 < s < \frac{bc_r}{c_n}$ and $0 < a_r < -sc_n + bc_r$, $\frac{\partial \pi_O}{\partial c_r} = \frac{2(-b^2+s^2)a_r + (b-s)(b+s)(-2sc_n + 2bc_r)}{8(b-s)(b+s)} > 0$ and $\frac{\partial \pi_R}{\partial c_r} = \frac{2(-b^2+s^2)a_r + (b-s)(b+s)(-2sc_n + 2bc_r)}{8(b-s)(b+s)} > 0$.

Proof of Theorem 2

First, we substitute the demand functions $q_r = a_r - bp_r + sp_n$ and $q_n = a_n - bp_n + sp_r$ into the profit functions π_R^1 and π_O^1. Then, in the profit function π_R^1, we take the first-order derivative with respect to p_n^1 and p_r^1 to obtain $p_n^1 = -\frac{-2ba_n - sa_r - 2bsp_d^1 - 2b^2w_n^1 + bsw_r^1}{4b^2-s^2}; p_r^1 = -\frac{-sa_n - 2ba_r - s^2p_d^1 - bsw_n^1 - 2b^2w_r^1 + s^2w_r^1}{4b^2-s^2}$; Then substitute p_n^1 and p_r^1 into the function of the OEM and differentiate p_d^1 to obtain $p_d^1 = \frac{1}{8b^3-6bs^2}\left((4b^2 + 2bs - s^2)a_n + 4b^3c_n + s(-2b^2c_n + s(a_r + (-3b+s)c_n - bc_r) + (4b^2 - s^2)w_{n1})\right)$, and then substitute it into π_O^1 and differentiate w_n^1 and w_r^1 with respect to the first order. The results are: $w_n^1 = \frac{(2b-s)(b+s)(2b+s)a_n + s^3a_r + (4b^4 - 5b^2s^2 + s^4)c_n - bs^3c_r}{8b^4 - 10b^2s^2 + s^4}; w_r^1 =$

$\frac{s(16b^4+8b^3s-12b^2s^2-2bs^3+s^4)a_n+4b(4b^4-5b^2s^2+s^4)a_r+s^2(8b^4-10b^2s^2+bs^3+s^4)c_n+4b^4(4b^2-5s^2)c_r}{4b^2(8b^4-10b^2s^2+s^4)}$. Thues, the decision equilibrium solution can be obtained:

$q_d^1 = -\frac{(2b+s)(-a_n+(b-s)c_n)}{4b}$; $q_r^1 = -\frac{b^2(b+s)(-sa_n+(b-s)(-2a_r-sc_n+2bc_r))}{8b^4-10b^2s^2+s^4}$;

$q_n^1 = \frac{(8b^4-12b^2s^2-2bs^3+s^4)a_n+(b-s)(-4bs(b+s)a_r-(8b^4-8b^2s^2+2bs^3+s^4)c_n+4b^2s(b+s)c_r)}{4(8b^4-10b^2s^2+s^4)}$;

$w_n^1 = \frac{(2b-s)(b+s)(2b+s)a_n+s^3a_r+(4b^4-5b^2s^2+s^4)c_n-bs^3c_r}{8b^4-10b^2s^2+s^4}$;

$w_r^1 = \frac{s(16b^4+8b^3s-12b^2s^2-2bs^3+s^4)a_n+4b(4b^4-5b^2s^2+s^4)a_r+s^2(8b^4-10b^2s^2+bs^3+s^4)c_n+4b^4(4b^2-5s^2)c_r}{4b^2(8b^4-10b^2s^2+s^4)}$;

$p_d^1 = \frac{(8b^3+8b^2s-2bs^2-s^3)a_n+2bs^2a_r+(8b^4-10b^2s^2+bs^3+s^4)c_n-2b^2s^2c_r}{2(8b^4-10b^2s^2+s^4)}$;

$p_r^1 = \frac{s(20b^4+12b^3s-12b^2s^2-2bs^3+s^4)a_n+4b(6b^4-7b^2s^2+s^4)a_r+s(-b+s)(-2b^2-2bs+s^2)(2b^2+4bs+s^2)c_n+4b^4(2b^2-3s^2)c_r}{4b^2(8b^4-10b^2s^2+s^4)}$;

$p_n^1 = \frac{(24b^4+16b^3s-12b^2s^2-2bs^3+s^4)a_n+4b^3sa_r+(8b^5+8b^4s-8b^3s^2-10b^2s^3+bs^4+s^5)c_n-4b^4sc_r}{4b^2(8b^4-10b^2s^2+s^4)}$;

$\pi_R^1 = \frac{1}{16b(8b^4-10b^2s^2+s^4)^2}\Big((64b^8-144b^6s^2+32b^5s^3+128b^4s^4-8b^3s^5-24b^2s^6+s^8)a_n^2+16(4b^2-s^2)(b^3-bs^2)^2a_r^2-16(4b^2-s^2)(b^3-bs^2)^2a_r(-sc_n+2bc_r)-2(b-s)a_n(-8b^2s(b+s)^2(4b^2-s^2)a_r+(64b^8-176b^6s^2-32b^5s^3+104b^4s^4+8b^3s^5-16b^2s^6+s^8)c_n+8b^3s(b+s)^2(4b^2-s^2)c_r)+(b-s)^2\big((64b^8-144b^6s^2+32b^5s^3+128b^4s^4-8b^3s^5-24b^2s^6+s^8)c_n^2-16b^3s(b+s)^2(4b^2-s^2)c_nc_r+16b^4(b+s)^2(4b^2-s^2)c_r^2\big)\Big)$;

$\pi_O^1 = \frac{1}{8b(8b^4-10b^2s^2+s^4)}\Big((24b^4+32b^3s+4b^2s^2-4bs^3-s^4)a_n^2-2a_n(-4b^2s(b+s)a_r+(24b^5+8b^4s-32b^3s^2-12b^2s^3+3bs^4+s^5)c_n+4b^3s(b+s)c_r)+(b-s)\big(8b^2(b+s)a_r^2+(24b^5+8b^4s-28b^3s^2-8b^2s^3+3bs^4+s^5)c_n^2-8b^3s(b+s)c_nc_r+8b^4(b+s)c_r^2-8b^2(b+s)a_r(-sc_n+2bc_r)\big)\Big)$.

Proof of Proposition 3

By comparing the new product prices in the retail channels under the baseline scenario and the scenario where manufacturers sell directly, it can be concluded that:

$p_n^1 - p_n = \frac{\frac{1}{4}\left(-\frac{3(b+s)}{b^2-s^2}-c_n\right)+(24b^4+20b^3s-12b^2s^2-2bs^3+s^4+(8b^5+8b^4s-8b^3s^2-10b^2s^3+bs^4+s^5)c_n-4b^4sc_r)}{4b(8b^4-10b^2s^2+s^4)}$

Make it greater than 0, and the range it satisfies is $0 < s < s_1, b_1 < b < b_2$； $s_1 < s < s_2, b_1 < b < b_2$； $s_2 \le s < s_3$， $b_1 < b < 1$； Among them, $s_1 \approx 0.455$， $s_2 \approx 0.767$， $s_3 \approx 0.936$， $b_1 = Root[s^4 - 10s^2b^2 + 8b^4 = 0,4]$， $b_2 = Root[s^4 - 10s^2b^2 + 2sb^3 + 4b^4 = 0,4]$.

Proof of Proposition 4

By comparing the profits of OEM under the baseline scenario and the scenario where manufacturers sell directly, it can be concluded that:

$\Pi_O^1 - \Pi_O = -\frac{1}{8(b-s)(b+s)}\big(2b+2(s+(-b+s)(b+s)c_n)+2(-b^2+s^2)c_r+(b-s)(b+s)(bc_n^2-2sc_nc_r+bc_r^2)\big)+\frac{1}{8b(8b^4-10b^2s^2+s^4)}\Big(24b^4+32b^3s+4b^2s^2-4bs^3-s^4-2(-4b^2s(b+s)+(24b^5+8b^4s-32b^3s^2-12b^2s^3+3bs^4+s^5)c_n+4b^3s(b+s)c_r)+(b-s)\big(8b^2(b+s)+(24b^5+8b^4s-28b^3s^2-8b^2s^3+3bs^4+s^5)c_n^2-8b^3s(b+s)c_nc_r+8b^4(b+s)c_r^2-8b^2(b+s)(-sc_n+2bc_r)\big)\Big)$.

Make it greater than 0, and the range it satisfies is $0 < s < s_1, b_1 < b < 1$，其中$s_1 \approx 0.455$， $b_1 = Root[s^4 - 10s^2b^2 + 8b^4 = 0,4]$.

Proof of Theorem 3

Following the proof strategy of Theorem 2, we can obtain the equilibrium solution in this scenario as:

$$q_d^2=-\frac{(2b+s)(-a_n+(b-s)(c_n-\lambda_o\tau_o)-\tau_o\varphi_o)}{4b};\ q_r^2=-\frac{b^2(b+s)\left(-sa_n+(b-s)\left(-2a_r-sc_n+2bc_r+s\lambda_o\tau_o-2\tau_c(b\lambda_c+\varphi_c)\right)-s\tau_o\varphi_o\right.}{8b^4-10b^2s^2+s^4};\ q_n^2=\frac{1}{4(8b^4-10b^2s^2+s^4)}\left((8b^4-12b^2s^2-2bs^3+s^4)a_n-(b-s)(4bs(b+s)a_r+(8b^4-8b^2s^2+2bs^3+s^4)c_n-4b^3sc_r-4b^2s^2c_r+4b^3s\lambda_c\tau_c+4b^2s^2\lambda_c\tau_c-8b^4\lambda_o\tau_o+8b^2s^2\lambda_o\tau_o-2bs^3\lambda_o\tau_o-s^4\lambda_o\tau_o+4bs(b+s)\tau_c\varphi_c)+(8b^4-12b^2s^2-2bs^3+s^4)\tau_o\varphi_o\right);\ w_n^2=\frac{1}{8b^4-10b^2s^2+s^4}\left((2b-s)(b+s)(2b+s)a_n+s^3a_r+4b^4c_n-5b^2s^2c_n+s^4c_n-bs^3c_r+bs^3\lambda_c\tau_c-4b^4\lambda_o\tau_o+5b^2s^2\lambda_o\tau_o-s^4\lambda_o\tau_o+s^3\tau_c\varphi_c+(2b-s)(b+s)(2b+s)\tau_o\varphi_o\right);\ w_r^2=\frac{1}{4b^2(8b^4-10b^2s^2+s^4)}(s(16b^4+8b^3s-12b^2s^2-2bs^3+s^4)a_n+4b(4b^4-5b^2s^2+s^4)a_r+8b^4s^2c_n-10b^2s^4c_n+bs^5c_n+s^6c_n+16b^6c_r-20b^4s^2c_r-16b^6\lambda_c\tau_c+20b^4s^2\lambda_c\tau_c-8b^4s^2\lambda_o\tau_o+10b^2s^4\lambda_o\tau_o-bs^5\lambda_o\tau_o-s^6\lambda_o\tau_o+16b^5\tau_c\varphi_c-20b^3s^2\tau_c\varphi_c+4bs^4\tau_c\varphi_c+s(16b^4+8b^3s-12b^2s^2-2bs^3+s^4)\tau_o\varphi_o);\ p_n^2=\frac{1}{4b(8b^4-10b^2s^2+s^4)}\left((24b^4+16b^3s-12b^2s^2-2bs^3+s^4)a_n+4b^3sa_r+8b^5c_n+8b^4sc_n-8b^3s^2c_n-10b^2s^3c_n+bs^4c_n+s^5c_n-4b^4sc_r+4b^4s\lambda_c\tau_c-8b^5\lambda_o\tau_o-8b^4s\lambda_o\tau_o+8b^3s^2\lambda_o\tau_o+10b^2s^3\lambda_o\tau_o-bs^4\lambda_o\tau_o-s^5\lambda_o\tau_o+4b^3s\tau_c\varphi_c+(24b^4+16b^3s-12b^2s^2-2bs^3+s^4)\tau_o\varphi_o\right);\ p_d^2=\frac{1}{2(8b^4-10b^2s^2+s^4)}\left((8b^3+8b^2s-2bs^2-s^3)a_n+2bs^2a_r+8b^4c_n-10b^2s^2c_n+bs^3c_n+s^4c_n-2b^2s^2c_r+2b^2s^2\lambda_c\tau_c-8b^4\lambda_o\tau_o+10b^2s^2\lambda_o\tau_o-bs^3\lambda_o\tau_o-s^4\lambda_o\tau_o+2bs^2\tau_c\varphi_c+(8b^3+8b^2s-2bs^2-s^3)\tau_o\varphi_o\right);\ p_r^2=\frac{1}{4b^2(8b^4-10b^2s^2+s^4)}(s(20b^4+12b^3s-12b^2s^2-2bs^3+s^4)a_n+4b(6b^4-7b^2s^2+s^4)a_r+4b^5sc_n+8b^4s^2c_n-4b^3s^3c_n-10b^2s^4c_n+bs^5c_n+s^6c_n+8b^6c_r-12b^4s^2c_r-8b^6\lambda_c\tau_c+12b^4s^2\lambda_c\tau_c-4b^5s\lambda_o\tau_o-8b^4s^2\lambda_o\tau_o+4b^3s^3\lambda_o\tau_o+10b^2s^4\lambda_o\tau_o-bs^5\lambda_o\tau_o-s^6\lambda_o\tau_o+24b^5\tau_c\varphi_c-28b^3s^2\tau_c\varphi_c+4bs^4\tau_c\varphi_c+s(20b^4+12b^3s-12b^2s^2-2bs^3+s^4)\tau_o\varphi_o);\ \pi_O^2=\frac{1}{8b(8b^4-10b^2s^2+s^4)}\Big((24b^4+32b^3s+4b^2s^2-4bs^3-s^4)a_n^2+8b^2(b-s)(b+s)a_r^2+24b^6c_n^2-16b^5sc_n^2-36b^4s^2c_n^2+20b^3s^3c_n^2+11b^2s^4c_n^2-2bs^5c_n^2-s^6c_n^2-8b^5sc_nc_r+8b^3s^3c_nc_r+8b^6c_r^2-8b^4s^2c_r^2-32b^5\kappa^2k_A+40b^3s^2\kappa^2k_A-4bs^4\kappa^2k_A+8b^5sc_n\lambda_c\tau_c-8b^3s^3c_n\lambda_c\tau_c-16b^6c_r\lambda_c\tau_c+16b^4s^2c_r\lambda_c\tau_c+8b^6\lambda_c^2\tau_c^2-8b^4s^2\lambda_c^2\tau_c^2-48b^6c_n\lambda_o\tau_o+32b^5sc_n\lambda_o\tau_o+72b^4s^2c_n\lambda_o\tau_o-40b^3s^3c_n\lambda_o\tau_o-22b^2s^4c_n\lambda_o\tau_o+4bs^5c_n\lambda_o\tau_o+2s^6c_n\lambda_o\tau_o+8b^5sc_r\lambda_o\tau_o-8b^3s^3c_r\lambda_o\tau_o-8b^5s\lambda_c\lambda_o\tau_c\tau_o+8b^3s^3\lambda_c\lambda_o\tau_c\tau_o+24b^6\lambda_o^2\tau_o^2-16b^5s\lambda_o^2\tau_o^2-36b^4s^2\lambda_o^2\tau_o^2+20b^3s^3\lambda_o^2\tau_o^2+11b^2s^4\lambda_o^2\tau_o^2-2bs^5\lambda_o^2\tau_o^2-s^6\lambda_o^2\tau_o^2+8b^4sc_n\tau_c\varphi_c-8b^2s^3c_n\tau_c\varphi_c-16b^5c_r\tau_c\varphi_c+16b^3s^2c_r\tau_c\varphi_c+16b^5\lambda_c\tau_c^2\varphi_c-16b^3s^2\lambda_c\tau_c^2\varphi_c-8b^4s\lambda_o\tau_c\tau_o\varphi_c+8b^2s^3\lambda_o\tau_c\tau_o\varphi_c+8b^4\tau_c^2\varphi_c^2-8b^2s^2\tau_c^2\varphi_c^2+2\tau_o\big(-\big((24b^5+8b^4s-32b^3s^2-12b^2s^3+3bs^4+s^5)c_n\big)-4b^3s(b+s)c_r+4b^4s\lambda_c\tau_c+4b^3s^2\lambda_c\tau_c+24b^5\lambda_o\tau_o+8b^4s\lambda_o\tau_o-32b^3s^2\lambda_o\tau_o-12b^2s^3\lambda_o\tau_o+3bs^4\lambda_o\tau_o+s^5\lambda_o\tau_o+4b^2s(b+s)\tau_c\varphi_c\big)\varphi_o+(24b^4+32b^3s+4b^2s^2-4bs^3-s^4)\tau_o^2\varphi_o^2+8b^2(b+s)a_r\left(-\Big((b-s)\big(-sc_n+2bc_r+s\lambda_o\tau_o-2\tau_c(b\lambda_c+\varphi_c)\big)\Big)+s\tau_o\varphi_o\right)+2a_n(4b^2s(b+s)a_r-(24b^5+8b^4s-32b^3s^2-12b^2s^3+3bs^4+s^5)c_n-4b^4sc_r-4b^3s^2c_r+4b^4s\lambda_c\tau_c+4b^3s^2\lambda_c\tau_c+24b^5\lambda_o\tau_o+8b^4s\lambda_o\tau_o-32b^3s^2\lambda_o\tau_o-12b^2s^3\lambda_o\tau_o+3bs^4\lambda_o\tau_o+s^5\lambda_o\tau_o+4b^3s\tau_c\varphi_c+4b^2s^2\tau_c\varphi_c+(24b^4+32b^3s+4b^2s^2-4bs^3-s^4)\tau_o\varphi_o)\Big);\ \pi_R^2=\frac{1}{16b(8b^4-10b^2s^2+s^4)^2}\Big((64b^8-144b^6s^2+32b^5s^3+128b^4s^4-8b^3s^5-24b^2s^6+s^8)a_n^2+16b^2(b^2-s^2)^2(4b^2-s^2)a_r^2+64b^{10}c_n^2-128b^9sc_n^2-80b^8s^2c_n^2+320b^7s^3c_n^2-80b^6s^4c_n^2-232b^5s^5c_n^2+120b^4s^6c_n^2+40b^3s^7c_n^2-23b^2s^8c_n^2-2bs^9c_n^2+s^{10}c_n^2-64b^9sc_nc_r+144b^7s^3c_nc_r-96b^5s^5c_nc_r+16b^3s^7c_nc_r+64b^{10}c_r^2-144b^8s^2c_r^2+96b^6s^4c_r^2-16b^4s^6c_r^2+64b^9sc_n\lambda_c\tau_c-144b^7s^3c_n\lambda_c\tau_c+96b^5s^5c_n\lambda_c\tau_c-16b^3s^7c_n\lambda_c\tau_c-128b^{10}c_r\lambda_c\tau_c+288b^8s^2c_r\lambda_c\tau_c-192b^6s^4c_r\lambda_c\tau_c+32b^4s^6c_r\lambda_c\tau_c+64b^{10}\lambda_c^2\tau_c^2-144b^8s^2\lambda_c^2\tau_c^2+96b^6s^4\lambda_c^2\tau_c^2-16b^4s^6\lambda_c^2\tau_c^2-128b^{10}c_n\lambda_o\tau_o+256b^9sc_n\lambda_o\tau_o+160b^8s^2c_n\lambda_o\tau_o-640b^7s^3c_n\lambda_o\tau_o+160b^6s^4c_n\lambda_o\tau_o+464b^5s^5c_n\lambda_o\tau_o-240b^4s^6c_n\lambda_o\tau_o-80b^3s^7c_n\lambda_o\tau_o+46b^2s^8c_n\lambda_o\tau_o+4bs^9c_n\lambda_o\tau_o-2s^{10}c_n\lambda_o\tau_o+64b^9sc_r\lambda_o\tau_o-144b^7s^3c_r\lambda_o\tau_o+96b^5s^5c_r\lambda_o\tau_o-16b^3s^7c_r\lambda_o\tau_o-64b^9s\lambda_c\lambda_o\tau_c\tau_o+144b^7s^3\lambda_c\lambda_o\tau_c\tau_o-96b^5s^5\lambda_c\lambda_o\tau_c\tau_o+16b^3s^7\lambda_c\lambda_o\tau_c\tau_o+64b^{10}\lambda_o^2\tau_o^2-128b^9s\lambda_o^2\tau_o^2-80b^8s^2\lambda_o^2\tau_o^2+320b^7s^3\lambda_o^2\tau_o^2-80b^6s^4\lambda_o^2\tau_o^2-232b^5s^5\lambda_o^2\tau_o^2+120b^4s^6\lambda_o^2\tau_o^2+40b^3s^7\lambda_o^2\tau_o^2-23b^2s^8\lambda_o^2\tau_o^2-2bs^9\lambda_o^2\tau_o^2+s^{10}\lambda_o^2\tau_o^2+64b^8sc_n\tau_c\varphi_c-144b^6s^3c_n\tau_c\varphi_c+96b^4s^5c_n\tau_c\varphi_c-16b^2s^7c_n\tau_c\varphi_c-128b^9c_r\tau_c\varphi_c+288b^7s^2c_r\tau_c\varphi_c-192b^5s^4c_r\tau_c\varphi_c+32b^3s^6c_r\tau_c\varphi_c+128b^9\lambda_c\tau_c^2\varphi_c-288b^7s^2\lambda_c\tau_c^2\varphi_c+192b^5s^4\lambda_c\tau_c^2\varphi_c-32b^3s^6\lambda_c\tau_c^2\varphi_c-64b^8s\lambda_o\tau_c\tau_o\varphi_c+144b^6s^3\lambda_o\tau_c\tau_o\varphi_c-96b^4s^5\lambda_o\tau_c\tau_o\varphi_c+16b^2s^7\lambda_o\tau_c\tau_o\varphi_c+64b^8\tau_c^2\varphi_c^2-144b^6s^2\tau_c^2\varphi_c^2+96b^4s^4\tau_c^2\varphi_c^2-16b^2s^6\tau_c^2\varphi_c^2+2(b-s)\tau_o\big(-\big((64b^8-176b^6s^2-32b^5s^3+104b^4s^4+8b^3s^5-16b^2s^6+s^8)c_n\big)-8b^3s(b+s)^2(4b^2-s^2)c_r+32b^7s\lambda_c\tau_c+64b^6s^2\lambda_c\tau_c+24b^5s^3\lambda_c\tau_c-16b^4s^4\lambda_c\tau_c-8b^3s^5\lambda_c\tau_c+64b^8\lambda_o\tau_o-176b^6s^2\lambda_o\tau_o-32b^5s^3\lambda_o\tau_o+104b^4s^4\lambda_o\tau_o+8b^3s^5\lambda_o\tau_o-16b^2s^6\lambda_o\tau_o+s^8\lambda_o\tau_o+8b^2s(b+s)^2(4b^2-s^2)\tau_c\varphi_c\big)\varphi_o+(64b^8-144b^6s^2+32b^5s^3+128b^4s^4-8b^3s^5-24b^2s^6+s^8)\tau_o^2\varphi_o^2-16b^2(b-s)(2b-s)(b+s)^2(2b+s)a_r\left((b-s)\big(-sc_n+2bc_r+s\lambda_o\tau_o-2\tau_c(b\lambda_c+\varphi_c)\big)-s\tau_o\varphi_o\right)-2a_n(-8b^2(b-s)(2b-s)s(b+s)^2(2b+s)a_r+(b-s)(64b^8-176b^6s^2-32b^5s^3+104b^4s^4+8b^3s^5-16b^2s^6+s^8)c_n+32b^8sc_r+32b^7s^2c_r-40b^6s^3c_r-40b^5s^4c_r+8b^4s^5c_r+8b^3s^6c_r-32b^8s\lambda_c\tau_c-32b^7s^2\lambda_c\tau_c+40b^6s^3\lambda_c\tau_c+40b^5s^4\lambda_c\tau_c-8b^4s^5\lambda_c\tau_c-8b^3s^6\lambda_c\tau_c-64b^9\lambda_o\tau_o+64b^8s\lambda_o\tau_o+176b^7s^2\lambda_o\tau_o-144b^6s^3\lambda_o\tau_o-\Big)$$

$136b^5s^4\lambda_o\tau_o + 96b^4s^5\lambda_o\tau_o + 24b^3s^6\lambda_o\tau_o - 16b^2s^7\lambda_o\tau_o - bs^8\lambda_o\tau_o + s^9\lambda_o\tau_o - 32b^7s\tau_c\varphi_c - 32b^6s^2\tau_c\varphi_c + 40b^5s^3\tau_c\varphi_c + 40b^4s^4\tau_c\varphi_c - 8b^3s^5\tau_c\varphi_c - 8b^2s^6\tau_c\varphi_c - (64b^8 - 144b^6s^2 + 32b^5s^3 + 128b^4s^4 - 8b^3s^5 - 24b^2s^6 + s^8)\tau_o\varphi_o)\Big)$.

Proof of Proposition 5

The impact of OEM's AI innovation level τ_o on supply chain demands:

(a) $\frac{\partial q_n^2}{\partial \tau_o} = \frac{\left(-\left((b-s)(-8b^4\lambda_o+8b^2s^2\lambda_o-2bs^3\lambda_o-s^4\lambda_o)\right)+(8b^4-12b^2s^2-2bs^3+s^4)\varphi_o\right.}{4(8b^4-10b^2s^2+s^4)} < 0$ holds true under condition $s_1 \le s < 1, 0 < \varphi_o < \varphi_{o1}, \lambda_{o1} < \lambda_o < 1$;

(b) $\frac{\partial q_r^2}{\partial \tau_o} = -\frac{b^2(b+s)\left((b-s)s\lambda_o-s\varphi_o\right)}{8b^4-10b^2s^2+s^4} < 0$ holds true under condition $s_1 \le s < 1, 0 < \varphi_o < \varphi_{o2}, 0 < \lambda_o < \lambda_{o2}$ or $s_1 \le s < 1, \varphi_{o2} \le \varphi_o < 1$;

(c) $\frac{\partial q_d^2}{\partial \tau_o} = -\frac{(2b+s)\left(-\left((b-s)\lambda_o\right)-\varphi_o\right)}{4b} < 0$ is always true.

Among them, $s_1 \approx 0.936$, $\varphi_{o1} = \frac{-8b^5+8b^4s+8b^3s^2-10b^2s^3+bs^4+s^5}{8b^4-12b^2s^2-2bs^3+s^4}$, $\lambda_{o1} = \frac{-8b^4\varphi_o+12b^2s^2\varphi_o+2bs^3\varphi_o-s^4\varphi_o}{8b^5-8b^4s-8b^3s^2+10b^2s^3-bs^4-s^5}$, $\varphi_{o2} = b - s$, $\lambda_{o2} = \frac{\varphi_o}{b-s}$.

Proof of Proposition 6

The impact of TPR's AI innovation level τ_c on supply chain demands:

(a) $\frac{\partial q_n^2}{\partial \tau_c} = -\frac{(b-s)\left(4b^3s\lambda_c+4b^2s^2\lambda_c+4bs(b+s)\varphi_c\right)}{4(8b^4-10b^2s^2+s^4)} < 0$ holds true under condition $s_1 \le s < 1, \varphi_c < \varphi_{c1}$;

(b) $\frac{\partial q_r^2}{\partial \tau_c} = \frac{2b^2(b-s)(b+s)(b\lambda_c+\varphi_c)}{8b^4-10b^2s^2+s^4} < 0$ holds true under condition $s_1 \le s < 1, \varphi_c > \varphi_{c1}$;

(c) $\frac{\partial q_d^2}{\partial \tau_c} = 0$.

Among them, $s_1 \approx 0.936$, $\varphi_{c1} = -b\lambda_c$.

Proof of Proposition 7 and Proposition 8

By comparing the profits of OEM and retailers R in models M1 and M2, we can obtain:

$$\Delta\Pi_O = \Pi_O^2 - \Pi_O^1 = \frac{A}{8b(8b^4 - 10b^2s^2 + s^4)}$$

$$\Delta\Pi_R = \Pi_R^2 - \Pi_R^1 = \frac{B}{16b(8b^4 - 10b^2s^2 + s^4)^2}$$

Among them, $A = -4b(8b^4 - 10b^2s^2 + s^4)\kappa^2k_A + 8b^4(b-s)(b+s)\lambda_c^2\tau_c^2 + 48b^5\lambda_o\tau_o + 8b^4s\lambda_o\tau_o - 64b^3s^2\lambda_o\tau_o - 16b^2s^3\lambda_o\tau_o + 6bs^4\lambda_o\tau_o + 2s^5\lambda_o\tau_o - 48b^6c_n\lambda_o\tau_o + 32b^5sc_n\lambda_o\tau_o + 72b^4s^2c_n\lambda_o\tau_o - 40b^3s^3c_n\lambda_o\tau_o - 22b^2s^4c_n\lambda_o\tau_o + 4bs^5c_n\lambda_o\tau_o + 2s^6c_n\lambda_o\tau_o + 8b^5sc_r\lambda_o\tau_o - 8b^3s^3c_r\lambda_o\tau_o + 24b^6\lambda_o^2\tau_o^2 - 16b^5s\lambda_o^2\tau_o^2 - 36b^4s^2\lambda_o^2\tau_o^2 + 20b^3s^3\lambda_o^2\tau_o^2 + 11b^2s^4\lambda_o^2\tau_o^2 - 2bs^5\lambda_o^2\tau_o^2 - s^6\lambda_o^2\tau_o^2 + 16b^4\tau_c\varphi_c + 8b^3s\tau_c\varphi_c - 8b^2s^2\tau_c\varphi_c + 8b^4sc_n\tau_c\varphi_c - 8b^2s^3c_n\tau_c\varphi_c - 16b^5c_r\tau_c\varphi_c + 16b^3s^2c_r\tau_c\varphi_c - 8b^4s\lambda_o\tau_c\tau_o\varphi_c + 8b^2s^3\lambda_o\tau_c\tau_o\varphi_c + 8b^4\tau_c^2\varphi_c^2 - 8b^2s^2\tau_c^2\varphi_c^2 + 2\tau_o\big((2b+s)(12b^3 + 12b^2s - 2bs^2 - s^3) - (24b^5 + 8b^4s - 32b^3s^2 - 12b^2s^3 + 3bs^4 + s^5)c_n - 4b^3s(b+s)c_r + (24b^5 + 8b^4s - 32b^3s^2 - 12b^2s^3 + 3bs^4 + s^5)\lambda_o\tau_o + 4b^2s(b+s)\tau_c\varphi_c\big)\varphi_o + (24b^4 + 32b^3s + 4b^2s^2 - 4bs^3 - s^4)\tau_o^2\varphi_o^2 - 8b^3(b+s)\lambda_c\tau_c(-2b + s + (b-s)(-sc_n + 2bc_r + s\lambda_o\tau_o - 2\tau_c\varphi_c) - s\tau_o\varphi_o)$, $B = 16b^4(b^2-s^2)^2(4b^2-s^2)\lambda_c^2\tau_c^2 + (b-s)^2(64b^8 - 144b^6s^2 + 32b^5s^3 + 128b^4s^4 - 8b^3s^5 - 24b^2s^6 + s^8)\lambda_o^2\tau_o^2 + 128b^8\tau_c\varphi_c + 64b^7s\tau_c\varphi_c - 224b^6s^2\tau_c\varphi_c - 80b^5s^3\tau_c\varphi_c + 112b^4s^4\tau_c\varphi_c + 16b^3s^5\tau_c\varphi_c - 16b^2s^6\tau_c\varphi_c + 64b^8sc_n\tau_c\varphi_c - 144b^6s^3c_n\tau_c\varphi_c + 96b^4s^5c_n\tau_c\varphi_c - 16b^2s^7c_n\tau_c\varphi_c - 128b^9c_r\tau_c\varphi_c + 288b^7s^2c_r\tau_c\varphi_c - 192b^5s^4c_r\tau_c\varphi_c + 32b^3s^6c_r\tau_c\varphi_c + 64b^8\tau_c^2\varphi_c^2 - 144b^6s^2\tau_c^2\varphi_c^2 + 96b^4s^4\tau_c^2\varphi_c^2 - 16b^2s^6\tau_c^2\varphi_c^2 - 2\tau_o\Big(-64b^8 - 32b^7s + 112b^6s^2 + 8b^5s^3 - 88b^4s^4 + 16b^2s^6 - s^8 + (b-s)\big((64b^8 - 176b^6s^2 - 32b^5s^3 + 104b^4s^4 + 8b^3s^5 - 16b^2s^6 + s^8)c_n + 8b^2s(b+s)^2(4b^2-s^2)(bc_r - \tau_c\varphi_c)\big)\Big)\varphi_o + (64b^8 - 144b^6s^2 + 32b^5s^3 + 128b^4s^4 - 8b^3s^5 - 24b^2s^6 + s^8)\tau_o^2\varphi_o^2 - 16b^3(b-s)(2b-s)(b+$

$s)^2(2b+s)\lambda_c\tau_c(-2b+s+(b-s)(-sc_n+2bc_r+s\lambda_o\tau_o-2\tau_c\varphi_c)-s\tau_o\varphi_o)-2(b-s)\lambda_o\tau_o(-64b^8+32b^7s+208b^6s^2-8b^5s^3-144b^4s^4+24b^2s^6-s^8+(b-s)(64b^8-144b^6s^2+32b^5s^3+128b^4s^4-8b^3s^5-24b^2s^6+s^8)c_n-8b^2(b-s)(2b-s)s(b+s)^2(2b+s)(bc_r-\tau_c\varphi_c)-(64b^8-176b^6s^2-32b^5s^3+104b^4s^4+8b^3s^5-16b^2s^6+s^8)\tau_o\varphi_o)$.

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