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Pricing and emission reduction decisions for remanufacturing supply chain based on consumer preferences and blockchain technology

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A B S T R A C T

The present study constructs a remanufacturing supply chain framework that encompasses both manufacturer and retailer. We conduct an in-depth investigation into the influence exerted by blockchain platforms in the supply chain, in response to carbon reduction challenges initiated by manufacturers. By introducing parameters such as the proportion of joint emission reduction investment costs and blockchain unit verification fees, we perform a comprehensive analysis of the effects produced by consumer sensitivity and blockchain platforms on sales prices, recycling prices, carbon emission reduction, market demand, amount of recycled, and profits under different emission reduction modes. Research has revealed that irrespective of whether the manufacturer adopts blockchain platforms, wholesale prices, retail prices, carbon emission reduction, market demand, manufacturer's profit in the joint emission reduction model are always higher than those in individual emission reduction model, while recycling prices and amount of recycled remain unchanged. With retailer's share of the carbon reduction investment cost kept within an appropriate range, both profits of retailer and supply chain increase under joint emission reduction model. When a manufacturer introduces blockchain technology platforms, unit verification fees only affect wholesale prices and have no impact on retail prices, recycling prices, carbon emission reduction, and profits.

1. Introduction

In the wake of rapid economic development and a significant rise in living standards, the total energy consumption for national industrial production and daily consumption continues to increase, leading to numerous atmospheric environmental problems such as haze and global climate anomalies. The international community has attached great importance to this issue and reached a global consensus on low-carbon development.Governments worldwide are vigorously promoting low-carbon economic development and advocating for the adoption of new development concepts to achieve carbon reduction. The most widely recognized effective way to curb carbon emissions is through remanufacturing, which focuses on reproducing recycled used products through standardized industrial processes. Remanufactured products possess similar quality and functionality as new products (Chen & Chen, 2021), while their production process can save approximately 40% of costs and reduce emissions by 70% (Zhao & Meng, 2021).

The development of the remanufacturing supply chain relies on effective cooperation between downstream and upstream entities. However, manufacturers often focus on researching and directly manufacturing low-carbon products aimed at reducing emissions across upstream and downstream operations. Meanwhile, large retailers typically prefer to cultivate consumers' low-carbon awareness, increase consumer demand, and encourage upstream manufacturers to adopt low-carbon and emission-reducing production methods. Despite these efforts, the market still faces challenges such as information opacity and lack of traceability. These issues may lead to high recycling costs for used products, weak consumer willingness to purchase remanufactured products, and high investment costs for emission reduction technologies.

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The emergence and application of blockchain technology offer new technical support to address these challenges. Blockchain provides traceability, openness, and transparency (Saberi et al., 2019), enabling comprehensive information transmission for products throughout the entire supply chain network (Li et al., 2021). Blockchain can resolve trust issues, enhance operational efficiency in the remanufacturing supply chain, and facilitate collaboration between upstream and downstream entities in fulfilling corporate environmental responsibility.

Based on the above considerations, this study takes the remanufacturing supply chain as the research object, considering consumer sensitivity and blockchain technology platforms. The study primarily focuses on exploring three key issues:

1. How do blockchain technology platforms introduced by manufacturers affect carbon reduction decisions and pricing when manufacturers independently reduce emissions?
2. If manufacturers introduce blockchain platforms, how does this impact carbon reduction decisions and pricing when manufacturers and retailers collaborate to reduce emissions?
3. How do important parameters related to emission reduction influence the optimal decisions of the participants?

The remainder of this paper is organized as follows. Section 2 presents a comprehensive review of the literature. Based on the problem description and assumptions outlined in Section 3, Section 4 details the development of four decision-making models. In Section 5, numerical examples are employed to conduct an in-depth analysis and discussion of parameter sensitivity. The final section provides the conclusion of this study.

2. Literature review

2.1 Strategic decisions on pricing and carbon reduction for remanufacturing supply chain

Existing scholars have conducted research from multiple perspectives, including government regulations (Liu et al., 2024), recycling channels for used products (Liang et al., 2025), supply chain coordination (Chen & Xing, 2024), corporate social responsibility (Cheng et al., 2025), and optimization of remanufacturing models (Liu et al., 2025). They have also explored the impact of factors such as the level of recycling and remanufacturing efforts (Lu et al., 2025), logistics service level in the remanufacturing supply chain (Feng, 2024), consumer preferences for remanufactured products (Hu et al., 2025), and trust and information transparency among supply chain members (Xie et al., 2025) on pricing and emission reduction decisions for remanufactured products.

The problem of a lack of trust and insufficient information transparency among members in the remanufacturing supply chain is becoming increasingly prominent. This not only increases the difficulty of remanufacturing used products and reducing carbon emissions but also hinders the improvement of market acceptance for remanufactured products. Consequently, it becomes challenging for enterprises to accurately formulate scientific and reasonable product pricing and emission reduction strategies (Wen & Siqin, 2020; Zhang & Yu, 2025; Dong et al., 2021).

In response to pricing mechanism failures and carbon reduction efficiency losses caused by trust heterogeneity and information asymmetry in the remanufacturing supply chain, current academic research has largely focused on soft governance approaches. First, researchers have designed two-way information feedback mechanisms based on consumer behavior theory to enhance supply chain transparency through demand-side data empowerment (Wen & Siqin, 2020). Second, scholars have relied on innovations in government regulatory tools rooted in new public management theory, emphasizing flexible policy instruments such as propaganda and education to reshape the environmental awareness of market entities (Zhang & Yu, 2025). Third, researchers have constructed optimal compensation mechanisms based on contract theory, aiming to achieve vertical synergy in the supply chain through benefit redistribution (Dong et al., 2021).

However, these governance models, which depend on the bounded rationality of market entities and the soft constraints of administrative regulations, exhibit significant governance blind spots in resolving structural contradictions such as deep trust fractures among supply chain members. As a result, they struggle to form a sustained optimization path that achieves Pareto improvement. Therefore, it is urgent to establish a rigid governance system with mandatory constraints. Through institutionalized arrangements and standardized execution, such a system can provide a legally effective decision-making support framework for the remanufacturing supply chain, thereby achieving a balanced integration of environmental, economic, and social benefits.

2.2 Blockchain-Powered Supply Chain Management

As a decentralized and traceable distributed database, blockchain has established reliable applications in areas such as organic food traceability systems (Casino et al., 2021; Niu et al., 2021) and resource recycling and conservation (Saberi et al., 2018). In supply chain management, research has shown that blockchain enables more efficient and secure supply chain operations through information sharing, information tracing, and trust building (Li & Chen, 2021). Biswas et al. (2023) developed a blockchain-based mechanism to facilitate information sharing for collaborative emission reduction across supply chains. Blockchain technology can significantly enhance supply chain transparency (Choi et al., 2019). Xu et al. (2022) proposed a resource allocation method for cloud manufacturing that leverages blockchain technology. The findings of Yu et al. (2022) shed light on the spillover effects of blockchain technology on the labor force in secondary supply chains. Centobelli et al. (2021) studied the effects of blockchain on circular logistics platforms. Nonetheless, research remains scarce on the dual role of blockchain in both transforming the remanufacturing supply chain and enabling low-carbon emission reduction.

2.3 Research on Carbon Emission Reduction Decisions

In the academic evolution of circular economy research on supply chain management, early studies primarily focused on a single model incorporating extended producer responsibility and corporate social responsibility (Cheng et al., 2025). Currently, scholars are concentrating on the research context of carbon tax policies and carbon quota trading mechanisms (Li & Li, 2024; Li et al., 2024). Some cutting-edge research integrates corporate voluntary social responsibility commitments with mandatory carbon policy constraints, investigating the optimal pathway for achieving lifecycle carbon reduction across the supply chain (Li et al., 2025).

To study carbon reduction in remanufacturing supply chains, Zhu et al. (2024) developed a duopoly game model between manufacturers and retailers. Their work aimed to assess how carbon taxes, low-carbon preferences, and capacity constraints influence carbon reduction outcomes. Zhang and Han (2024) established game models between producers and remanufacturers based on consumer environmental concern and governmental carbon pricing mechanisms under three scenarios: non-emission reduction strategy, emission reduction strategy, and supply chain coordination. Their research showed that an increase in consumer environmental concern can encourage manufacturers to reduce prices for new products and patent fees. For remanufacturers, a high market acceptance of remanufactured products is a prerequisite for their prices to respond positively to rising consumer environmental awareness. An intriguing finding is that the optimal emission reduction rate for new products does not consistently rise with growing consumer environmental awareness; this relationship is mediated by the gap in unit carbon emissions between new and remanufactured products.

To address carbon emissions in a multi-retailer manufacturing-remanufacturing setting, Handa et al. (2024) developed a sustainable inventory model under carbon tax policies, seeking to mitigate emissions from production, storage, waste handling, and product deterioration. Zhang and Chen (2022) considered different cooperation models between low-carbon manufacturers and retailers, constructing a Stackelberg game model incorporating government subsidies to derive the coordinated equilibrium solution for a supply chain dominated by low-carbon manufacturers. The research of Shi et al. (2024) indicates that, under specific parameter conditions where the reciprocity coefficient falls within a reasonable range, a combined advertising and cost-sharing contract is effective in coordinating low-carbon supply chains, regardless of the manufacturer's individual inclination toward reciprocity. Wang et al. (2017) conducted a comparative analysis of wholesale price and cost-sharing contracts for low-carbon supply chain coordination. Zhang et al. (2025) proposed that if the unit blockchain investment cost falls below a specified threshold, blockchain is likely to accelerate the shift in consumer demand from new products toward remanufactured ones, while also achieving Pareto improvement in the profits of both manufacturers. The adoption of blockchain can significantly increase supply chain transparency (Azzi et al., 2019).

In summary, current literature on carbon emissions pays limited attention to the longitudinal comparison of different emission reduction modes. Few studies consider the impact of blockchain technology on decision-making in relevant literature on joint emission reduction. In the literature on the impact of blockchain technology on low-carbon emission reduction, quantitative research on blockchain technology is rarely considered. This paper introduces blockchain into the remanufacturing supply chain, designs reasonable variables and parameters to study the impact of blockchain technology on carbon reduction strategies, and provides theoretical guidance for the application of blockchain in pricing and emission reduction.

3. Model description and assumptions

3.1 Model description and framework

A remanufacturing supply chain consisting of a single manufacturer and a single retailer is established. The manufacturer is responsible for producing new products, recycling used products for remanufacturing, and selling both new and remanufactured products to the retailer at the same wholesale price. The retailer, in turn, sells these products to consumers at the same retail price. The manufacturer acts as the Stackelberg leader, while the retailer serves as the follower.

Based on whether the manufacturer introduces blockchain technology platforms and whether the manufacturer and retailer engage in joint emission reduction, this paper examines four distinct models: the MND model, MNL model, MBD model, and MBL model.

- In the **MND model**, the manufacturer does not introduce blockchain technology platforms and undertakes emission reduction independently, bearing the associated costs of emission reduction technology.

- In the **MNL model**, the manufacturer also does not introduce blockchain technology platforms; however, the manufacturer and retailer adopt a joint emission reduction model, in which both parties share the cost of emission reduction.

- In the **MBD model**, the manufacturer introduces blockchain technology platforms to verify the authenticity of their low-carbon products. In this scenario, the manufacturer independently reduces emissions and incurs the corresponding emission reduction technology costs.

- In the **MBL model**, the manufacturer introduces blockchain technology platforms to verify the authenticity of their low-carbon products, while the retailer participates in joint emission reduction through a cost-sharing arrangement, with both manufacturer and retailer sharing the emission reduction costs.

Accordingly, this paper explores the application value of blockchain technology in the remanufacturing supply chain and investigates the decision-making behavior of enterprises regarding its adoption. The framework illustrating the impact of pricing and carbon reduction decisions in the remanufacturing supply chain, considering consumer sensitivity and blockchain technology, is presented in Fig. 1.

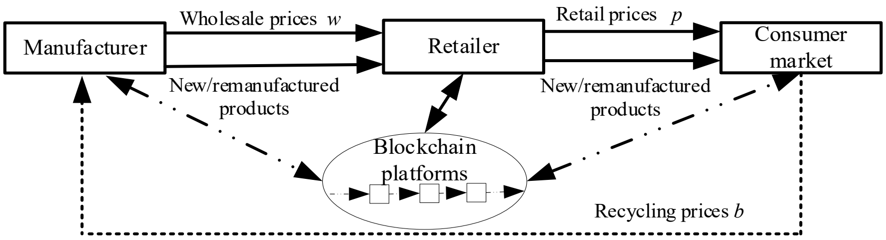

Fig. 1. Model framework

3.2 Model parameters and basic assumptions

Table 1 lists the model parameters mentioned in this paper.

Table 1
Model parameters

Notation	Definition
i	$i \in$ {MND, MNL,MBD,MBL}, representing four decision models: MND, MNL, MBD, and MBL;
c_n	Unit cost required by manufacturer to produce new products;
c_z	Unit cost required for manufacturer to produce remanufactured products without adopting blockchain technology;
c_o	If blockchain technology fails to be adopted, manufacturer can save costs by remanufacturing used products, namely: $c_o = c_n - c_z > 0$;
ω	Sensitivity coefficient of consumers to recycling prices;
θ	Consumer low-carbon preference coefficient;
σ	Without using blockchain technology, $1 - \sigma$ represents the probability of authentication being false for the authenticity of low-carbon products;
φ	Consumer market capacity, $\varphi > 0$;
ψ	Voluntary return of used products in the consumer market, $\psi > 0$;
μ	Cost coefficient for carbon reduction;
δ	Parameter for reducing the unit cost of remanufactured products when applying blockchain technology;
k	Sensitivity coefficient of consumers to retail prices;
q	Remanufacturing rate of recycled used products reflects the quality of recycled used products, and $0 \leq q \leq 1$;
t	Duration of product testing and evaluation without using blockchain technology;
γ	When manufacturer adopts blockchain technology, retailer pays unit verification fees to manufacturer;
α	Sensitivity coefficient of consumers to duration of product testing and evaluation;
β	Sensitivity coefficient of consumers to the false probability of product testing and evaluation results reflects their trust in low-carbon products;
T	Duration of product testing and evaluation with using blockchain technology;
ρ	Manufacturer introducing blockchain technology requires a fixed one-time fee to be paid;
τ	Proportion of retailer sharing the investment cost of carbon reduction;
g	When manufacturer introduces blockchain technology, the parameter for increasing the amount of used product recycling increases;
w^i	In model i, unit wholesale prices of new and remanufactured products;
p^i	In model i, unit retail prices of new and remanufactured products;
b^i	In model i, unit recycling prices of used products;
e^i	In model i, carbon emission reduction per unit product during the production process of new and remanufactured products;
d^i	In model i, demand for new and remanufactured products in consumer market;
D^i	In model i, amount of recycled used products;
f_m^i	In model i, manufacturer's profit;
f_r^i	In model i, retailer's profit;
f_s^i	In model i, total profit of the supply chain.

According to the modeling requirements, some assumptions are formulated in advance.

Assumption 1 Supply chain only produces a single low-carbon product. Moreover, the manufacturer solely emits carbon during the production phase;

Assumption 2 Consumers who exhibit low - carbon products preferences are willing to pay premium prices for them;

Assumption 3 According to the reference (D'Aspremont & Jacquemin, 1988), input cost of carbon emission reduction technology is represented as $\frac{1}{2}\mu(e^i)^2$;

Assumption 4 Blockchain technology can record and track energy consumption data at different stages and production processes; Supply chain members can jointly verify and monitor carbon emission data, reducing the risk of data errors and inconsistencies, therefore $t > T$ (Michaud et al., 2013);

Assumption 5 We posit blockchain technology guarantees perpetual truth in low - carbon product detection and evaluation, that is, $\sigma = 1$(Michaud et al., 2013);

Assumption 6 When applying blockchain technology, the cost of remanufacturing products per unit is reduced, that is, the cost of remanufacturing products when applying blockchain technology is $(1-\delta)c_z$. At this point, cost savings in remanufacturing used products is $c_o + \delta c_z$;

Assumption 7 When blockchain technology is not used, i.e. $i \in$ {MND, MNL}, the market demand is $d^i = \varphi - kp^i + \theta e^i - \alpha t - \beta(1-\sigma)$; When blockchain technology is used, i.e. $i \in$ {MBD, MBL}, $d^i = \varphi - kp^i + \theta e^i - \alpha T$;

Assumption 8 When the blockchain technology is not used, i.e. $i \in$ {MND, MNL}, the amount of recycled used products is $D^i = \psi + \omega b^i$; When blockchain technology is used, i.e. $i \in$ {MBD, MBL}, the amount of recycled used products is $D^i = (1+g)(\psi + \omega b^i)$.

4. Model construction and solution

4.1 MND model

In MND model, manufacturer failures to introduce blockchain technology platforms and is responsible for reducing emission independently. In order to maximize profits, manufacturer first determines w^{MND}, b^{MND}, and e^{MND}. Then retailer decides p^{MND}. The profit functions are listed below as Eqs. (1-3).

$$f_m^{MND} = (w^{MND} - c_n)d^{MND} + (c_o - b^{MND})qD^{MND} - \frac{1}{2}\mu(e^{MND})^2 \tag{1}$$

$$f_r^{MND} = (p^{MND} - w^{MND})d^{MND} \tag{2}$$

$$f_s^{MND} = f_m^{MND} + f_r^{MND} \tag{3}$$

Proposition 1 In the MND model, when $\theta^2 < 4k\mu$ is present, the equilibrium results of decisions are shown in Eqs. (4-7).

$$w^{MND*} = \frac{2\mu[\varphi - \alpha t + \beta(\sigma - 1) + kc_n] - \theta^2 c_n}{4k\mu - \theta^2} \tag{4}$$

$$b^{MND*} = \frac{c_o\omega - \psi}{2\omega} \tag{5}$$

$$e^{MND*} = \frac{\theta[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]}{4k\mu - \theta^2} \tag{6}$$

$$p^{MND*} = \frac{c_n(k\mu - \theta^2) + 3\mu[\varphi - \alpha t + \beta(\sigma - 1)]}{4k\mu - \theta^2} \tag{7}$$

The market demand, the amount of recycled used products, and profits are shown in Eqs. (8-12), respectively.

$$d^{MND*} = \frac{k\mu[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]}{4k\mu - \theta^2} \tag{8}$$

$$D^{MND*} = \frac{\psi + c_o\omega}{2} \tag{9}$$

$$f_m^{MND*} = \frac{\mu[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2(4k\mu - \theta^2)} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{10}$$

$$f_r^{MND*} = \frac{k\mu^2[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{(4k\mu - \theta^2)^2} \tag{11}$$

$$f_s^{MND*} = \frac{(6k\mu^2 - \mu\theta^2)[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2(4k\mu - \theta^2)^2} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{12}$$

4.2 MNL model

The manufacturer failures to introduce blockchain technology platforms and jointly reduce emission with the retailer in MNL model. In pursuit of maximizing the bottom-line, manufacturer first determines w^{MNL}, b^{MNL}, and e^{MNL}. Then retailer decides p^{MNL}. The profit functions can be found in Eqs. (13-15).

$$f_m^{MNL} = (w^{MNL} - c_n)d^{MNL} + (c_o - b^{MNL})qD^{MNL} - \frac{1}{2}\mu(1 - \tau)(e^{MNL})^2 \tag{13}$$

$$f_r^{MNL} = (p^{MNL} - w^{MNL})d^{MNL} - \frac{1}{2}\mu\tau(e^{MNL})^2 \tag{14}$$

$$f_s^{MNL} = f_m^{MNL} + f_r^{MNL} \tag{15}$$

Proposition 2 In the MNL model, when $\theta^2 < 4k\mu(1 - \tau)$ is present, the equilibrium results of decisions are shown in Eqs. (16-19).

$$w^{MNL*} = \frac{2\mu(1 - \tau)[\varphi - \alpha t + \beta(\sigma - 1) + kc_n] - \theta^2 c_n}{4k\mu(1 - \tau) - \theta^2} \tag{16}$$

$$b^{MNL*} = \frac{c_o\omega - \psi}{2\omega} \tag{17}$$

$$e^{MNL*} = \frac{\theta[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]}{4k\mu(1 - \tau) - \theta^2} \tag{18}$$

$$p^{MNL*} = \frac{c_n[k\mu(1 - \tau) - \theta^2] + 3\mu(1 - \tau)[\varphi - \alpha t + \beta(\sigma - 1)]}{4k\mu(1 - \tau) - \theta^2} \tag{19}$$

The market demand, the amount of recycled used products, and profits are shown in Eq. (20-24), respectively.

$$d^{MNL*} = \frac{k\mu(1 - \tau)[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]}{4k\mu(1 - \tau) - \theta^2} \tag{20}$$

$$D^{MNL*} = \frac{\psi + c_o\omega}{2} \tag{21}$$

$$f_m^{MNL*} = \frac{\mu(1 - \tau)[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2[4k\mu(1 - \tau) - \theta^2]} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{22}$$

$$f_r^{MNL*} = \frac{[2k\mu^2(1 - \tau)^2 - \mu\tau\theta^2][\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2[4k\mu(1 - \tau) - \theta^2]^2} \tag{23}$$

$$f_s^{MNL*} = \frac{[6k\mu^2(1 - \tau)^2 - \mu\theta^2][\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2[4k\mu(1 - \tau) - \theta^2]^2} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{24}$$

By analyzing Proposition 1 and Proposition 2, we derive Corollary 1 and 2.

Corollary 1 When manufacturer failures to introduce blockchain technology platforms, i.e. $i \in$ {MND, MNL}, sensitivity coefficient β has different effects on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, f_r^{i*}, and f_s^{i*}. The specific results are as follows:

$$\frac{\partial w^{i*}}{\partial \beta}<0,\quad \frac{\partial p^{i*}}{\partial \beta}<0,\quad \frac{\partial b^{i*}}{\partial \beta}=0,\quad \frac{\partial e^{i*}}{\partial \beta}<0,\quad \frac{\partial d^{i*}}{\partial \beta}<0,\quad \frac{\partial D^{i*}}{\partial \beta}=0,$$

$$\frac{\partial f_m^{i*}}{\partial \beta}<0,\quad \frac{\partial f_r^{i*}}{\partial \beta}<0,\quad \frac{\partial f_s^{i*}}{\partial \beta}<0$$

Corollary 1 indicates that when the manufacturer refrains from introducing blockchain technology platforms—regardless of whether the manufacturer and retailer jointly reduce emissions—a reduction in the sensitivity coefficient of consumers to false product testing and evaluation results leads to an increase in consumer trust in low-carbon products. As consumer trust grows, the market demand for these products has the potential to expand accordingly, and profits can increase. However, the recycling prices and the quantity of recycled used products remain unaffected.

To enhance consumer trust in low-carbon products, the manufacturer needs to invest greater effort in producing low-carbon products, which raises the level of carbon emission reduction. In response, the manufacturer increases wholesale prices to maintain profitability, while the retailer similarly raises retail prices to preserve their own profit margins.

Corollary 2 When manufacturer failures to introduce blockchain technology platforms, i.e. $i \in \{\text{MND, MNL}\}$, there are different relationships between w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, and f_r^{i*}. The particular results are enumerated in the following manner:

(1)$w^{MNL*} > w^{MND*}$, $p^{MNL*} > p^{MND*}$, $b^{MNL*} = b^{MND*}$, $e^{MNL*} > e^{MND*}$, $d^{MNL*} > d^{MND*}$, $D^{MNL*} = D^{MND*}$;

(2)$f_m^{MNL*} > f_m^{MND*}$;

(3)When $0 < \tau < \dfrac{\theta^2(4k\mu-\theta^2)}{2k\mu(8k\mu-\theta^2)}$, $\quad f_r^{MNL*} > f_r^{MND*}$.

Corollary 2 suggests that if the manufacturer fails to introduce blockchain platforms, compared to the scenario where the manufacturer reduces emissions independently, the participation of the retailer in joint emission reduction leads to increases in wholesale and retail prices, carbon emission reduction, market demand, and the manufacturer's profit. However, recycling prices and the quantity of recycled products remain unchanged. Notably, the retailer's profit will increase only if the cost-sharing ratio remains below a given threshold; otherwise, it will decrease. From this, it can be seen that in terms of incentivizing enterprises to reduce emissions, it may be necessary to encourage collaborative efforts among firms and to tailor the proportion of carbon reduction investment costs borne by the retailer based on actual demand. Ultimately, such coordination can facilitate the production of low-carbon products across the entire market and promote sustainable development.

4.3 MBD model

In MBD model, manufacturer introduces blockchain technology platforms and applies them to verify the authenticity of the low-carbon products. This process is carried out independently by manufacturer to reduce emission. In pursuit of maximizing the bottom-line, manufacturer decides w^{MBD}, b^{MBD}, e^{MBD}. Retailer determines p^{MBD}. The profit functions can be found in Eqs. (25-27).

$$f_m^{MBD} = (w^{MBD} - c_n + \gamma)d^{MBD} + (c_o + \delta c_z - b^{MBD})qD^{MBD} - \frac{1}{2}\mu(e^{MBD})^2 - \rho \tag{25}$$

$$f_r^{MBD} = (p^{MBD} - w^{MBD} - \gamma)d^{MBD} \tag{26}$$

$$f_s^{MBD} = f_m^{MBD} + f_r^{MBD} \tag{27}$$

Proposition 3 In the MBD model, when $\theta^2 < 4k\mu$ is present, the equilibrium results of decisions are shown in Eqs. (28-31).

$$w^{MBD*} = \frac{2\mu[\varphi - \alpha T - 2k\gamma + kc_n] - \theta^2(c_n - \gamma)}{4k\mu - \theta^2} \tag{28}$$

$$b^{MBD*} = \frac{\omega(c_o + \delta c_z) - \psi}{2\omega} \tag{29}$$

$$e^{MBD*} = \frac{\theta[\varphi - \alpha T - kc_n]}{4k\mu - \theta^2} \tag{30}$$

$$p^{MBD*} = \frac{c_n(k\mu - \theta^2) + 3\mu[\varphi - \alpha T]}{4k\mu - \theta^2} \tag{31}$$

The market demand, the amount of recycled used products, and profits are shown in Eqs. (32-36), respectively.

$$d^{MBD*} = \frac{k\mu[\varphi - \alpha T - kc_n]}{4k\mu - \theta^2} \tag{32}$$

$$D^{MBD*} = \frac{(1+g)[\psi + \omega(c_o + \delta c_z)]}{2} \tag{33}$$

$$f_m^{MBD*} = \frac{\mu[\varphi - \alpha T - kc_n]^2}{2(4k\mu - \theta^2)} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{34}$$

$$f_r^{MBD*} = \frac{k\mu^2(\varphi - \alpha T - kc_n)^2}{(4k\mu - \theta^2)^2} \tag{35}$$

$$f_s^{MBD*} = \frac{(6k\mu^2 - \mu\theta^2)(\varphi - \alpha T - kc_n)^2}{2(4k\mu - \theta^2)^2} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{36}$$

4.4 MBL model

In MBL model, manufacturer introduces blockchain technology platforms and applies them to verify the authenticity of the low-carbon products. This process involves retailer participating in joint emissions reduction through a cost sharing model. In order to maximize profits, manufacturer first determines w^{MBL}, b^{MBL}, and e^{MBL}. Then retailer decides p^{MBL}. The profit functions can be found in Eqs. (37-39).

$$f_m^{MBL} = (w^{MBL} - c_n + \gamma)d^{MBL} + (c_o + \delta c_z - b^{MBL})qD^{MBL} - \frac{1}{2}\mu(1-\tau)(e^{MBD})^2 - \rho \tag{37}$$

$$f_r^{MBL} = (p^{MBL} - w^{MBL} - \gamma)d^{MBL} - \frac{1}{2}\mu\tau(e^{MBD})^2 \tag{38}$$

$$f_s^{MBD} = f_m^{MBD} + f_r^{MBD} \tag{39}$$

Proposition 4 In the MBL model, when $\theta^2 < 4k\mu(1-\tau)$ is present, the equilibrium results of decisions are shown in Eqs. (40-43).

$$w^{MBL*} = \frac{2\mu(1-\tau)(\varphi - \alpha T - 2k\gamma + kc_n) - \theta^2(c_n - \gamma)}{4k\mu(1-\tau) - \theta^2} \tag{40}$$

$$b^{MBL*} = \frac{\omega(c_o + \delta c_z) - \psi}{2\omega} \tag{41}$$

$$e^{MBL*} = \frac{\theta(\varphi - \alpha T - kc_n)}{4k\mu(1-\tau) - \theta^2} \tag{42}$$

$$p^{MBL*} = \frac{c_n[k\mu(1-\tau) - \theta^2] + 3\mu(1-\tau)(\varphi - \alpha T)}{4k\mu(1-\tau) - \theta^2} \tag{43}$$

The market demand, the amount of recycled used products, and profits are shown in Eqs. (44-48), respectively.

$$d^{MBL*} = \frac{k\mu(1-\tau)(\varphi - \alpha T - kc_n)}{4k\mu(1-\tau) - \theta^2} \tag{44}$$

$$D^{MBL*} = \frac{(1+g)[\psi + \omega(c_o + \delta c_z)]}{2} \tag{45}$$

$$f_m^{MBL*} = \frac{\mu(1-\tau)[\varphi - \alpha T - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{46}$$

$$f_r^{MBL*} = \frac{[2k\mu^2(1-\tau)^2 - \mu\tau\theta^2][\varphi - \alpha T - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]^2} \tag{47}$$

$$f_s^{MBL*} = \frac{[6k\mu^2(1-\tau)^2 - \mu\theta^2][\varphi - \alpha T - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]^2} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{48}$$

Analyzing Propositions 3 and 4, we derive Corollary 3 ～ Corollary 5.

Corollary 3 When manufacturer introduces blockchain technology platforms, i.e.$i \in \{\mathrm{MBD}, \mathrm{MBL}\}$, unit verification fee γ has impacts on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, f_r^{i*}, and f_s^{i*}. The particular results are enumerated in the following manner:

$$\frac{\partial w^{i*}}{\partial \gamma} < 0, \frac{\partial p^{i*}}{\partial \gamma} = \frac{\partial b^{i*}}{\partial \gamma} = \frac{\partial e^{i*}}{\partial \gamma} = \frac{\partial d^{i*}}{\partial \gamma} = \frac{\partial D^{i*}}{\partial \gamma} = \frac{\partial f_m^{i*}}{\partial \gamma} = \frac{\partial f_r^{i*}}{\partial \gamma} = \frac{\partial f_s^{i*}}{\partial \gamma} = 0$$

Corollary 3 indicates that when the manufacturer introduces blockchain technology platforms, regardless of whether the manufacturer and retailer jointly reduce emissions, increasing the unit verification fee γ can reduce the wholesale prices of products. However, retail prices, recycling prices, carbon emission reduction, market demand, the quantity of recycled products, and profits remain unaffected.

In fact, after the manufacturer introduces blockchain technology platforms, the adjustment of verification fees has three main impacts on the supply chain:

(1) From a cost-structure perspective, the automated verification enabled by smart contracts reduces labor costs, and adjustments in wholesale prices help compensate the retailer for any increase in marginal costs.

(2) In terms of information asymmetry, the tamper-proof feature of blockchain enhances transparency and reduces the risk of information errors.

(3) With regard to institutional constraints, the integration of blockchain with the extended producer responsibility system strengthens the supervision of recycling processes, thereby creating a policy incentive loop.

Corollary 4 When manufacturer introduces a blockchain technology platforms, i.e. $i \in \{\mathrm{MBD}, \mathrm{MBL}\}$, the fixed fee ρ that manufacturer needs to pay at once for introducing the blockchain technology platforms has different impacts on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_r^{i*}, f_m^{i} , and f_s^{i*}. The specific results are as follows:

$$\frac{\partial w^{i*}}{\partial \rho} = \frac{\partial p^{i*}}{\partial \rho} = \frac{\partial b^{i*}}{\partial \rho} = \frac{\partial e^{i*}}{\partial \rho} = \frac{\partial d^{i*}}{\partial \rho} = \frac{\partial D^{i*}}{\partial \rho} = \frac{\partial f_r^{i*}}{\partial \rho} = 0, \frac{\partial f_m^{i}}{\partial \rho} < 0, \frac{\partial f_s^{i*}}{\partial \rho} < 0$$

Corollary 4 indicates that when the manufacturer adopts blockchain platforms, reducing the fixed one-time fees required for introducing blockchain technology platforms does not affect wholesale and retail prices, recycling prices, carbon emission reduction, market demand, the quantity of recycled products, or the retailer's profit. However, it leads to a significant increase in the profits of both the manufacturer and the supply chain as a whole. The manufacturer is able to reduce the fixed one-time cost of introducing blockchain technology platforms without affecting supply chain prices and operations due to several factors: the modular architecture enables component reuse, significantly lowering costs; the automation of smart contract processes reduces the need for manual intervention; and the distributed platform breaks down data silos, improving overall efficiency. These benefits are further supported by policy subsidies that help offset initial investment costs. This cost-reduction strategy enhances the manufacturer's net profit while simultaneously boosting the overall profitability of the supply chain, all without disrupting retail, recycling, or other operational links.

Corollary 5 When the manufacturer introduces blockchain technology platforms, i.e. $i \in \{\mathrm{MBD}, \mathrm{MBL}\}$, there are different relationships between w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_r^{i*} , and f_m^{i}. The specific results are as follows:

(1) $w^{MBL*} > w^{MBD*}$, $p^{MBL*} > p^{MBD*}$,$b^{MBL*} = b^{MBD*}$,$e^{MBL*} > e^{MBD*}$,$d^{MBL*} > d^{MBD*}$, $D^{MBL*} = D^{MBD*}$;

(2) $f_m^{MBL*} > f_m^{MBD*}$;

(3) When $0 < \tau < \dfrac{\theta^2(4k\mu - \theta^2)}{2k\mu(8k\mu - \theta^2)}$,$f_r^{MBL*} > f_r^{MBD*}$.

Corollary 5 states that if the manufacturer introduces blockchain technology platforms, compared to the scenario where the manufacturer reduces emissions independently, the participation of the retailer in joint emission reduction leads to increases in wholesale and retail prices, while recycling prices remain unchanged. Carbon emission reduction is enhanced, market demand increases, and the quantity of recycled products does not change. The manufacturer's profit increases under joint emission reduction. However, the retailer's profit will increase only if the cost-sharing ratio remains below a given threshold; otherwise, it will decrease. When the manufacturer introduces blockchain technology and collaborates with the retailer to reduce emissions, wholesale and retail prices rise due to increased emission reduction costs and a heightened consumer willingness to pay for environmentally friendly products. Recycling prices, however, remain unchanged due to policy stability. Carbon emission reductions are significantly enhanced through blockchain-enabled transparency and tracking, combined with technological investment from both parties, leading to an increase in market demand driven by greater consumer trust.

The manufacturer's profit growth is attributed to higher sales volume and an effective cost-sharing mechanism, while the retailer's profit depends on the sharing ratio. If the ratio falls below the threshold, the retailer's revenue covers the cost; otherwise, profits decline. To ensure mutual benefit, the manufacturer should establish reasonable sharing ratios through contracts and utilize blockchain to supervise emission reduction implementation. The retailer, in turn, needs to assess their own cost-bearing capacity and confirm price elasticity through market research. Both parties can jointly apply for government subsidies to reduce initial investment, while employing blockchain to optimize supply chain management, ultimately aiming to enhance overall efficiency.

Analyzing Propositions 1～4, we derive Corollary 6～Corollary 8.

Corollary 6 Consumer low-carbon preference coefficient θ has different impacts on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, f_r^{i*}, and f_s^{i*}. The specific results are as follows:

(1)When $i \in$ {MND, MNL,MBD,MBL}, $\frac{\partial w^{i*}}{\partial \theta} > 0, \frac{\partial p^{i*}}{\partial \theta} > 0, \frac{\partial b^{i*}}{\partial \theta} = 0, \frac{\partial e^{i*}}{\partial \theta} > 0, \quad \frac{\partial d^{i*}}{\partial \theta} > 0, \frac{\partial D^{i*}}{\partial \theta} = 0, \frac{\partial f_m^{i*}}{\partial \theta} > 0;$

(2)When $i \in$ {MND, MBD}, $\frac{\partial f_r^{i*}}{\partial \theta} > 0, \frac{\partial f_s^{i*}}{\partial \theta} > 0;$

(3)When $i \in$ { MNL,MBL}, if $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}, \frac{\partial f_r^{i*}}{\partial \theta} > 0, \frac{\partial f_s^{i*}}{\partial \theta} > 0.$

Corollary 6 suggests that if consumers' environmental consciousness in the entire market is strong, or as consumers become more inclined to purchase low-carbon products, demand for these products will rise accordingly, driving an increase in the profits of both the manufacturer and the retailer. To further encourage consumers to purchase low-carbon products, the manufacturer needs to invest effort in promoting the importance of environmental protection or strive to develop new low-carbon products with high quality and creativity. To compensate for these investments, the manufacturer may increase wholesale prices to maintain profit margins, prompting the retailer to correspondingly raise retail prices to preserve its own profitability. Whether or not blockchain technology is adopted, consumers' preference for low-carbon products, or their environmental awareness, serves as a positive factor for the supply chain. In contrast, the increased unit cost associated with introducing blockchain technology acts as a negative factor for all parties involved. Additionally, if consumers have a low preference for low-carbon products, the retailer will increase retail prices to maintain profits as the cost of introducing blockchain rises. However, if consumers have a high preference for low-carbon products, an increase in the cost of adopting blockchain technology leads the retailer to actually lower retail prices, thereby passing on some value to consumers.

Corollary 7 Cost coefficient for carbon reduction μ has different impacts on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, f_r^{i*}, and f_s^{i*}. The specific results are as follows:

(1)When $i \in$ {MND, MNL,MBD,MBL}, $\frac{\partial w^{i*}}{\partial \mu} < 0, \frac{\partial b^{i*}}{\partial \mu} = 0, \frac{\partial e^{i*}}{\partial \mu} < 0, \frac{\partial p^{i*}}{\partial \mu} < 0, \frac{\partial d^{i*}}{\partial \mu} < 0, \frac{\partial D^{i*}}{\partial \mu} = 0, \frac{\partial f_m^{i*}}{\partial \mu} < 0;$

(2)When $i \in$ {MND, MBD}, $\frac{\partial f_r^{i*}}{\partial \mu} < 0, \frac{\partial f_s^{i*}}{\partial \mu} < 0;$

(3)When $i \in$ { MNL,MBL}, if $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}, \frac{\partial f_r^{i*}}{\partial \mu} < 0, \frac{\partial f_s^{i*}}{\partial \mu} < 0.$

Corollary 7 indicates that in all four models, as the cost coefficient for carbon reduction μ decreases, wholesale and retail prices, carbon emission reduction, market demand, and the manufacturer's profit all increase, while recycling prices and the quantity of recycled products remain unchanged. When the manufacturer reduces emissions independently, a decrease in μ leads to an increase in the profits of both the retailer and the overall supply chain. However, when the manufacturer and retailer jointly reduce emissions, a decrease in μ may result in either an increase or a decrease in the profits of the retailer and the supply chain. As μ decreases, the manufacturer's technical efficiency improves and scale effects emerge, which reduces emission reduction costs. This encourages wholesale and retail prices to rise due to the enhanced premium potential of environmentally friendly products. Meanwhile, carbon emission reductions continue to increase as a result of technological optimization, and consumers' environmental preferences stimulate growth in market demand. Due to the rigid constraints of the extended producer responsibility system, recycling prices and the quantity of recycled products remain stable. In the case of independent emission reduction, the manufacturer's cost savings directly boost profits, and the retailer benefits from efficiency gains transmitted through the supply chain. In the joint emission reduction scenario, if the retailer's cost-sharing ratio is low, the retailer can cover costs through increased sales, and profits rise as the cost coefficient decreases. Conversely, if the sharing ratio is too high, profits are diluted. Therefore, the manufacturer should dynamically adjust the pace of investment in emission reduction technologies and utilize blockchain to track carbon footprints, thereby strengthening consumer trust. The retailer needs to determine the appropriate level of participation based on the cost-sharing threshold and verify price elasticity through market research. At the same time, both parties should jointly apply for policy subsidies to mitigate the risks associated with joint emission reduction.

Corollary 8 When $i \in \{\text{MND, MNL, MBD, MBL}\}$, the sensitivity coefficient α has different effects on w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_m^{i*}, f_r^{i*}, and f_s^{i*}. That is:

$$\frac{\partial w^{i*}}{\partial \alpha} < 0, \quad \frac{\partial b^{i*}}{\partial \alpha} = 0, \quad \frac{\partial e^{i*}}{\partial \alpha} < 0, \quad \frac{\partial p^{i*}}{\partial \alpha} < 0, \quad \frac{\partial d^{i*}}{\partial \alpha} < 0, \quad \frac{\partial D^{i*}}{\partial \alpha} = 0,$$

$$\frac{\partial f_m^{i*}}{\partial \alpha} < 0, \quad \frac{\partial f_r^{i*}}{\partial \alpha} < 0, \quad \frac{\partial f_s^{i*}}{\partial \alpha} < 0$$

Corollary 8 shows that if consumers in the entire market do not care about the time cost associated with product testing and evaluation, demand for low-carbon products will increase initially. This rise in demand, in turn, drives an increase in the profits of both the manufacturer and the retailer. To further encourage consumers to purchase low-carbon products, the manufacturer needs to invest greater effort in promoting the importance of environmental protection or strive to develop new low-carbon products with high quality and creativity. As a result, the manufacturer will raise wholesale prices to maintain profit margins, prompting the retailer to correspondingly increase retail prices to preserve its own profitability. Regardless of whether blockchain technology is adopted, changes in the parameter α do not affect recycling prices or the quantity of recycled used products.

By analyzing Proposition 1 and Proposition 3 in depth, Corollary 9 is obtained.

Corollary 9 When manufacturer reduces emission separately, i.e. $i \in \{\text{MBD, MND}\}$, w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_r^{i*}, and f_m^{i*} have different relationships. The specific results are as follows:

(1)When $\alpha > \dfrac{2\mu\beta(\sigma-1)+\gamma(4k\mu-\theta^2)}{2\mu(t-T)}$, $w^{MBD*} > w^{MND*}$;

(2) $p^{MBD*} > p^{MND*}$, $b^{MBD*} > b^{MND*}$, $e^{MBD*} > e^{MND*}$, $d^{MBD*} > d^{MND*}$, $D^{MBD*} > D^{MND*}$, $f_r^{MBD*} > f_r^{MND*}$;

(3)When $\rho < \frac{\mu[\alpha(t-T)+\beta(1-\sigma)][2\varphi-\alpha(t+T)-2kc_n-\beta(1-\sigma)]}{2(4k\mu-\theta^2)} + \frac{q(1+g)[\psi+\omega(c_o+\delta c_z)]^2}{4\omega}$, $f_m^{MBD*} > f_m^{MND*}$.

Corollary 9 suggests that when the manufacturer chooses the separate emission reduction model, compared to the scenario where the manufacturer fails to introduce blockchain technology platforms, the adoption of blockchain leads to increases in retail prices, recycling prices, carbon emission reduction, market demand, the quantity of recycled products, and the retailer's profit. However, wholesale prices will increase only when the sensitivity coefficient α exceeds a certain threshold; otherwise, they will decrease. This may be explained by the fact that when testing the authenticity of low-carbon products produced by the manufacturer, a large sensitivity coefficient α indicates that consumers face a lengthy waiting time for the supply chain to produce qualified genuine products, and they are highly concerned about this additional time cost. In such a situation, low-carbon consumers are willing to pay higher prices to offset the time costs they would otherwise incur. Blockchain technology drives variations in the profits of all supply chain members. Furthermore, when the fixed cost of introducing blockchain falls below a certain threshold, blockchain technology also leads to an increase in the manufacturer's profit.

By analyzing Proposition 2 and Proposition 4 in depth, Corollary 10 is obtained.

Corollary 10 When manufacturer and retailer jointly reduce emission, i.e. $i \in \{\text{MBL, MNL}\}$, w^{i*}, p^{i*}, b^{i*}, e^{i*}, d^{i*}, D^{i*}, f_r^{i*}, and f_m^{i*} have different relationships. The specific results are as follows:

(1) When $\alpha > \dfrac{2\mu(1-\tau)[\beta(\sigma-1)+2k\gamma]-\theta^2\gamma}{2\mu(1-\tau)(t-T)}$, $w^{MBL*} > w^{MNL*}$;

(2) $p^{MBL*} > p^{MNL*}$, $b^{MBL*} > b^{MNL*}$, $e^{MBL*} > e^{MNL*}$, $d^{MBL*} > d^{MNL*}$, $D^{MBL*} > D^{MNL*}$, $f_r^{MBL*} > f_r^{MNL*}$;

(3) When $\rho < \frac{\mu[\alpha(t-T)+\beta(1-\sigma)][2\varphi-\alpha(t+T)-2kc_n-\beta(1-\sigma)]}{2(4k\mu-\theta^2)} + \frac{q(1+g)[\psi+\omega(c_o+\delta c_z)]^2}{4\omega}$, $f_m^{MBL*} > f_m^{MNL*}$.

Corollary 10 indicates that with joint emission reduction, after manufacturer introduces blockchain technology platforms, retail prices, recycling prices, carbon reduction emission, market demand, amount of recycled, retailer's profit will increase. But only when sensitivity coefficient of consumers to duration of product testing and evaluation α surpasses a given threshold, wholesale prices of the products will rise, and vice versa, it will decrease. This reveals that when the coefficient α is small, it means that the entire platform's inspection time for low-carbon products will be shortened, while also ensuring the authenticity of the products, which will attract more low-carbon preference consumers. When the coefficient α surpasses a given threshold, blockchain technology will drive profits of all members of supply chain to increase. This indirectly reflects that only when the fixed costs generated by investing in blockchain technology are controlled within a controllable range, will the enthusiasm for introducing blockchain into supply chain be mobilized.

5. Numerical analysis and discussion

By using the decision model, the manufacturer and retailer can obtain optimal dynamic decisions. Since some results do not allow for an intuitive understanding of the relationships between variables, numerical analysis is employed in this section to provide further substantiation. Table 2 presents the main parameter values.

Table 2
Main values

Parameter	c_n	c_z	q	δ	σ	φ	ψ	μ	ρ	g
Values	30	20	0.8	0.6	0.7	150	3	0.6	180	1
Parameter	θ	k	ω	t	T	α	β	γ	τ	c_o
Values	0.5	0.6	0.8	2	1	0.85	0.3	0.9	0.1	10

The optimal results are displayed in Table 3. When the parameter values meet certain conditions, if the manufacturer introduces the blockchain technology platform, and manufacturer and retailer engage in a joint effort to mitigate emission, $w^{MBL*}$is the smallest, while $e^{MBL*}$and $b^{MBL*}$are the largest. This approach is beneficial for encouraging consumers to engage in recycling activities for used products and proactively opt for low - carbon products, thus greatly increasing recycling volume and market demand of products. Under the stimulation of consumption, the profits (f_m^{MBL*},f_r^{MBL*}, f_s^{MBL*}) are optimal.

Table 3
Optimal solutions of four models

Model i	w^{i*}	b^{i*}	e^{i*}	p^{i*}	d^{i*}	D^{i*}	f_m^{i*}	f_r^{i*}	f_s^{i*}
MND	161.304	3.125	54.710	226.956	39.391	5.5	4304.530	2586.119	6890.649
MNL	164.442	3.125	62.242	231.664	40.333	5.5	4406.688	2594.994	7001.682
MBD	161.352	9.125	55.105	228.378	39.676	20.6	4368.396	2623.593	6991.989
MBL	155.413	9.125	62.691	233.120	40.630	20.6	4472.034	2632.597	7104.631

5.1 Impact of relevant parameters on carbon emission reduction e^{i*}

(1) Impact of θ on e^{i*}

Consumer low-carbon preference, as an important factor in stimulating carbon emission reduction, exerts an influence on the market demand by regulating consumer willingness to consume. Consequently, this affects the manufacturer's inclination and exertions regarding investment in emission reduction.

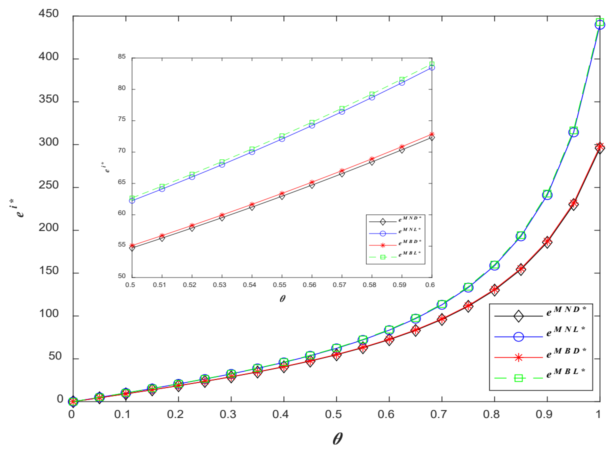

Fig. 2. Impact of θ on e^{i*}

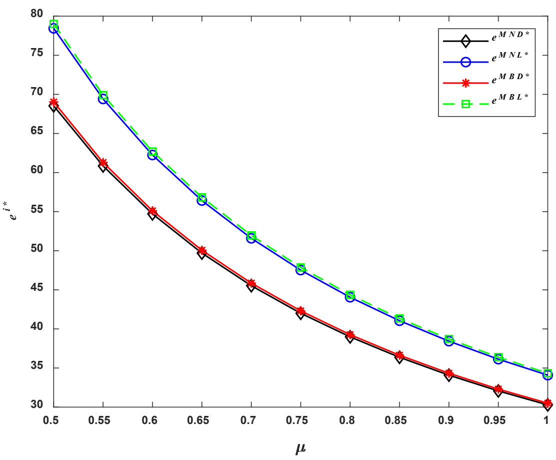

Fig. 3. Impact of μ on e^{i*}

Fig. 2 shows impact of consumer low-carbon preference coefficient θ on carbon emission reduction e^{i*} in four models. Fig. 2 shows that in all four models, as θ increases, e^{i*} also increases. If θ increases, consumers' low-carbon preferences and demand increase, prompting manufacturer to invest more in emission reduction technologies, thereby increasing e^{i*}. From a horizontal perspective, when $\theta \in (0, 0.5)$ is present, the growth rate of e^{i} remains relatively consistent. When $\theta \in (0.5, 1)$ is present, the growth rates of e^{MNL*} and e^{MBL*} are much higher than those of e^{MND*} and e^{MBD*}, with e^{MBL*} having the fastest growth rate. When the number of low-carbon consumers exceeds that of non low-carbon consumers, manufacturer and retailer reduce emission, thereby increasing carbon reduction per unit product. From a vertical perspective, when $\theta \in (0, 1)$is present, we can obtain $e^{MBL*} > e^{MNL*} > e^{MBD*} > e^{MND*}$. The introduction of blockchain platforms by manufacturer and establishment of low-carbon product traceability and certification functions can promote supply chain emission reduction. Simultaneously, if manufacturer and retailer jointly reduce emission, supply chain node should pay attention to promotion among the defining features of blockchain, such as trustlessness, so that consumers' perception of blockchain technology can be improved and impact of blockchain technology on consumers' low-carbon preferences can be enhanced.

(2) Impact of μ on e^{i*}

As a direct economic driver for emission reduction, the carbon reduction cost directly impacts the production expenses and profitability of nodes within the remanufacturing supply chain. This, in turn, shapes manufacturers' willingness to invest in carbon-cutting initiatives. Emission reduction cost coefficient μ influences optimal carbon emission level e^{i*} differently across the four models, as illustrated in Fig. 3. In all four models, as μ decreases, e^{i*} increases. From a horizontal perspective, as μ increases, the growth rate of e^{i*} becomes faster and faster. From a vertical perspective, for any given μ, there is $e^{MBL*} > e^{MNL*} > e^{MBD*} > e^{MND*}$. The improvement in the manufacturer's emission reduction technology efficiency has led to a decrease in marginal emission reduction costs, prompting enterprises to invest more resources in optimizing emission reduction efforts. This accelerates breakthroughs in emission reduction technology and results in faster growth rates. Horizontally speaking, the transparency incentive provided by blockchain and the division of labor advantage inherent in joint emission reduction form a synergistic effect, enabling more rapid improvements in emission reduction efficiency. In a longitudinal comparison, the joint emission reduction model overcomes the limitations of a single entity's technology through resource integration, while the introduction of blockchain reduces redundant investment via information sharing. The combination of the two yields the best emission reduction outcome, whereas independent emission reduction results in diminished efficiency due to information silos and technical constraints. The manufacturer should prioritize the deployment of blockchain to enhance cross-link collaboration and dynamically adjust technology investment based on cost coefficients. The retailer needs to clarify the cost-sharing ratio in joint emission reduction through contract design, utilize blockchain's traceability function to ensure investment transparency, and jointly build a carbon emission reduction database with the manufacturer to optimize decision-making efficiency.

(3) Impact of α on e^{i*}

Sensitivity coefficient of consumers to duration of product testing and evaluation α is a key factor in stimulating emission reduction e^{i*}. By adjusting cost of emission reduction, it affects production costs and profits of remanufacturing supply chain nodes, thereby influencing manufacturer's willingness to invest in emissions reduction. Fig. 4 shows the impact α on e^{i*} in four models.

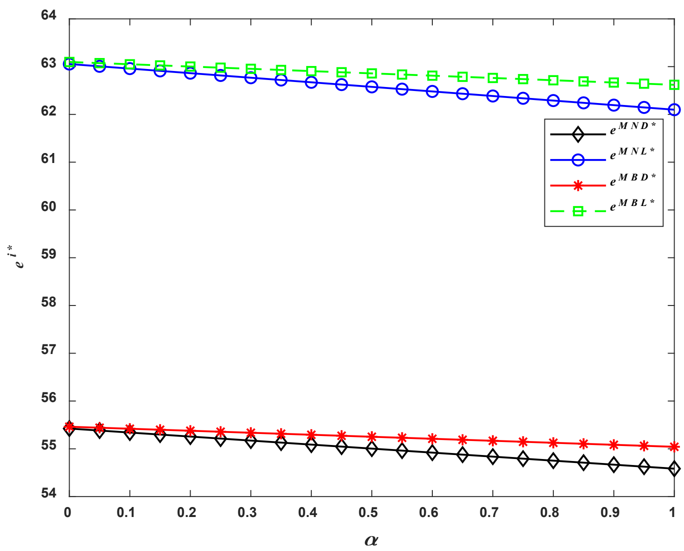

Fig. 4. Impact of α on e^{i*}

Fig. 4 shows that in all four models, as α decreases, e^{i*} increases. From a horizontal perspective, when $\alpha \in (0,1)$ changes, as α changes, e^{MND*} and e^{MNL*} change faster, while e^{MBL*} and e^{MBD*} change very smoothly. From a vertical perspective, when $\alpha \in (0,1)$, if α has the same value, we can obtain $e^{MBL*} > e^{MNL*} > e^{MBD*} > e^{MND*}$. As consumers' sensitivity to testing and evaluation time decreases, their decision tolerance increases, and companies have more ample time to optimize emission reduction technologies, promoting a raise in unit carbon emission. From a horizontal comparison perspective, if the manufacturer does not introduce blockchain technology platforms, information lag forces enterprises to quickly adjust their emission reduction strategies in response to demand fluctuations, resulting in more drastic changes in carbon emissions. In contrast, the real-time data sharing capability of blockchain smooths out information asymmetry, making emission reduction investments more stable. Vertically speaking, the joint emission reduction model overcomes technological bottlenecks through resource integration, and the introduction of blockchain technology platforms further strengthens cross-link collaboration efficiency. Therefore, the introduction of blockchain in joint emission reduction consistently maintains the optimal emission reduction effect. The manufacturer should accelerate the deployment of blockchain to stabilize the pace of emission reduction investment, while establishing a dynamic response mechanism to adapt to changes in sensitivity coefficients—such as increasing investment in rapid detection technology during periods of high sensitivity. The retailer needs to strengthen real-time data collaboration with the manufacturer, optimize demand forecasting using blockchain traceability functions, and clarify the linkage clauses between time sensitivity and emission reduction targets within joint emission reduction contracts.

Based on Figs. (2–4), the deployment of blockchain platforms by the manufacturer is beneficial in incentivizing companies to invest in emission reduction. Therefore, governments can leverage fiscal subsidies, tax reductions, and other incentives to lower the application cost of blockchain technology. This approach directly encourages its broader adoption. Prior to adopting blockchain technology, enterprises should undertake a thorough cost-benefit assessment, evaluating both its financial and technical viability. If the implementation costs of blockchain are prohibitively high and cannot be offset by the derived benefits, its economic feasibility becomes questionable. In such cases, companies can consider collaborating with other enterprises to share the application costs, thereby enabling blockchain technology to be applied on a wider scale.

5.2 Impact of relevant parameters on profits

(1) Impact of θ and τ on profits

Figure 5 shows impact of consumer low-carbon preference coefficient θ and retailer sharing of carbon emission reduction investment cost ratio τ on f_m^i, f_r^i, and f_s^i . From Fig. 5 (a), it can be observed that as θ increases, f_m^{MND*}, f_m^{MNL*}, f_m^{MBD*}, and f_m^{MBL*}all increase. The growth rate experiences acceleration. As τ increases, f_m^{MND*} and f_m^{MBD*} remain unchanged, while f_m^{MNL*} and f_m^{MBL*} increase. If we take the appropriate θ, there exists $\tau = \tau_0$. When $\tau \in (0, \tau_0)$, $f_m^{MBL*} > f_m^{MBD*} > f_m^{MNL*} > f_m^{MND*}$. When $\tau \in (\tau_0, 0.3)$,$f_m^{MBL*} > f_m^{MNL*} > f_m^{MBD*} > f_m^{MND*}$. From Fig. 5 (b), it is apparent that due to $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}$, as θ increases, f_r^{MND*}, f_r^{MNL*}, f_r^{MBD*}, and f_r^{MBL*} all increase. The growth rate experiences acceleration. As τ increases, f_r^{MND*} and f_r^{MBD*} remain unchanged, while f_r^{MNL*} and f_r^{MBL*} first increase and then decrease. If we take the appropriate θ, there exists $\tau = \tau_1$. When $\tau \in (0, \tau_1)$, $f_r^{MBL*} > f_r^{MBD*} > f_r^{MNL*} > f_r^{MND*}$. There exists$\tau = \tau_2$. When $\tau \in (\tau_1, \tau_2)$, $f_r^{MBD*} > f_r^{MBL*} > f_r^{MND*} > f_r^{MNL*}$. When $\tau \in (\tau_2, 0.3)$, $f_r^{MBD*} > f_r^{MND*} > f_r^{MBL*} > f_r^{MNL*}$. From Figure 5 (c), it can be observed that due to $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}$, as θ increases, f_s^{MND*}, f_s^{MNL*}, f_s^{MBD*}, and f_s^{MBL*} all increase. The growth rate experiences acceleration. As τ increases, f_s^{MND*} and f_s^{MBD*} remain unchanged, while f_s^{MNL*} and f_s^{MBL*} both increase. If we take the appropriate θ, there exists $\tau = \tau_3$. When $\tau \in (0, \tau_3)$, $f_s^{MBL*} > f_s^{MBD*} > f_s^{MNL*} > f_s^{MND*}$. When $\tau \in (\tau_3, 0.3)$, $f_s^{MBL*} > f_s^{MNL*} > f_s^{MBD*} > f_s^{MND*}$.

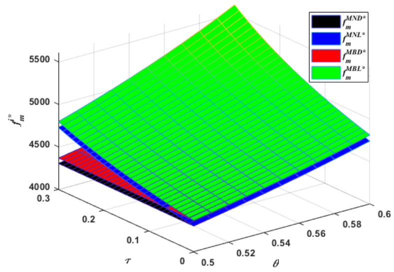

Fig. 5 (a) Impact of θ and τ on f_m^{i*}

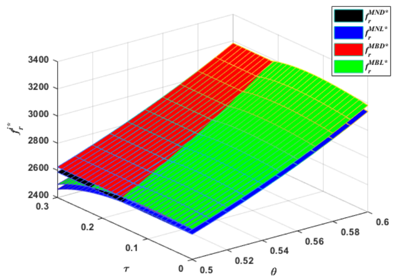

Fig. 5 (b) Impact of θ and τ on f_r^{i*}

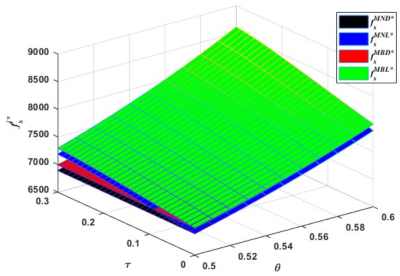

Fig. 5 (c) Impact of θ and τ on f_s^{i*}

(2) Impact of μ and τ on profits

Figure 6 shows impact of carbon emission reduction cost coefficient μ and the proportion of carbon emission reduction investment cost shared by the retailer τ on f_m^i, f_r^i, and f_s^i . From Fig. 6 (a), it can be observed that as μ decreases, f_m^{MND*}, f_m^{MNL*}, f_m^{MBD*}, and f_m^{MBL*} all increase. An accelerating trend in the rate of increase is observed. As τ increases, f_m^{MND*} and f_m^{MBD*} remain constant, while f_m^{MNL*} and f_m^{MBL*} both increase. An accelerating trend in the rate of increase is observed. If we take the appropriate μ, there exists $\tau = \tau_4$. When $\tau \in (0, \tau_4)$, $f_m^{MBL*} > f_m^{MBD*} > f_m^{MNL*} > f_m^{MND*}$. When $\tau \in (\tau_4, 0.3)$, $f_m^{MBL*} > f_m^{MNL*} > f_m^{MBD*} > f_m^{MND*}$. From Fig. 6 (b), it can be observed that due to $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}$, as μ decreases, f_r^{MND*}, f_r^{MNL*}, f_r^{MBD*}, and f_r^{MBL*} all increase. An accelerating trend in the rate of increase is observed. As τ increases, f_r^{MND*} and f_r^{MBD*} remain constant, while f_r^{MNL*} and f_r^{MBL*} first increase and then decrease. If we take the appropriate μ, there exists $\tau = \tau_5$. When $\tau \in (0, \tau_5)$, $f_r^{MBL*} > f_r^{MBD*} > f_r^{MNL*} > f_r^{MND*}$. There exists $\tau = \tau_6$. When $\tau \in (\tau_5, \tau_6)$, $f_r^{MBD*} > f_r^{MBL*} > f_r^{MND*} > f_r^{MNL*}$. When $\tau \in (\tau_6, 0.3)$, $f_r^{MBD*} > f_r^{MND*} > f_r^{MBL*} > f_r^{MNL*}$. From Fig. 6 (c), it can be observed that due to $\theta^2 < \frac{4k\mu(1-\tau)(1-2\tau)}{\tau}$, as μ decreases, f_s^{MND*}, f_s^{MNL*}, f_s^{MBD*}, and f_s^{MBL*} all increase. An accelerating trend in the rate of increase is observed. As τ increases, f_s^{MND*} and f_s^{MBD*} remain constant, while f_s^{MNL*} and f_s^{MBL*} both increase. If we take the appropriate μ, there exists $\tau = \tau_7$. When $\tau \in (0, \tau_7)$, $f_s^{MBL*} > f_s^{MBD*} > f_s^{MNL*} > f_s^{MND*}$. When $\tau \in (\tau_7, 0.3)$, $f_s^{MBL*} > f_s^{MNL*} > f_s^{MBD*} > f_s^{MND*}$. Fig. 6 illustrates that if manufacturer strives to improve carbon reduction technologies and reduce carbon reduction costs, the overall profits of manufacturer, retailer, and supply chains will all increase. For manufacturer, regardless of τ, introduction of blockchain platforms and joint emissions reduction will bring the highest profits to manufacturer. But for retailer, only when the value of τ is appropriate, introducing blockchain technology platforms and jointly reducing emissions can maximum profits be achieved.

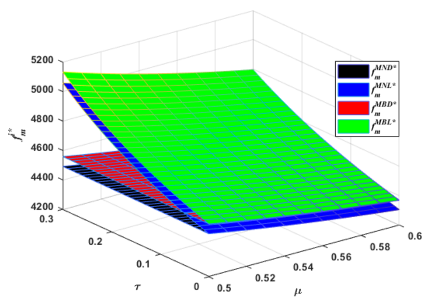

Fig. 6 (a) Impact of μ and τ on f_m^{i*}

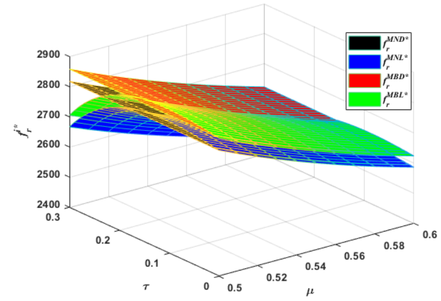

Fig. 6 (b) Impact of μ and τ on f_r^{i*}

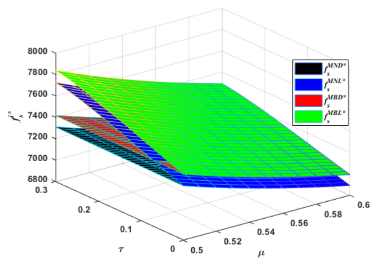

Fig. 6 (c) Impact of μ and τ on f_s^{i*}

(3) Impact of α and ρ on profits

Fig. 7 shows impact of sensitivity coefficient of consumers to duration of product testing and evaluation α, and fixed fee ρ that manufacturer needs to pay once to introduce blockchain technology on f_m^i, f_r^i, and f_s^i in four models. From Fig. 7 (a), it can be observed that as α decreases, f_m^{MND*}, f_m^{MNL*}, f_m^{MBD*}, and f_m^{MBL*} all increase. As ρ decreases, f_m^{MND*}, f_m^{MNL*}, f_m^{MBD*}, and f_m^{MBL*} all increase. If we take the appropriate α, there exists $\rho = \rho_0$. When $\rho \in (100, \rho_0)$, $f_m^{MBL*} > f_m^{MBD*} > f_m^{MNL*} > f_m^{MND*}$. There exists $\rho = \rho_1$. When $\rho \in (\rho_0, \rho_1)$, $f_m^{MBL*} > f_m^{MNL*} > f_m^{MBD*} > f_m^{MND*}$. When $\rho \in (\rho_1, 300)$, $f_m^{MNL*} > f_m^{MBL*} > f_m^{MND*} > f_m^{MBD*}$. From Fig. 7 (b), it can be observed that as α decreases, f_r^{MND*}, f_r^{MNL*}, f_r^{MBD*}, and f_r^{MBL*} all increase. As ρ changes, f_r^{MND*}, f_r^{MNL*}, f_r^{MBD*}, and f_r^{MBL*} remain unchanged. If the parameter value satisfies $0 < \tau < \frac{\theta^2(4k\mu-\theta^2)}{2k\mu(8k\mu-\theta^2)}$, then $f_r^{MBL*} > f_r^{MBD*} > f_r^{MNL*} > f_r^{MND*}$. From Fig. 7 (c), it can be observed that as α decreases, f_s^{MND*}, f_s^{MNL*}, f_s^{MBD*}, and f_s^{MBL*} all increase. As ρ decreases, f_s^{MND*}, f_s^{MNL*}, f_s^{MBD*}, and f_s^{MBL*} all increase. If the appropriate parameter value is taken, there exists $\rho = \rho_2$. When $\rho \in (100, \rho_2)$, $f_s^{MBL*} > f_s^{MBD*} > f_s^{MNL*} > f_s^{MND*}$. There exists $\rho = \rho_3$. When $\rho \in (\rho_2, \rho_3)$, $f_s^{MBL*} > f_s^{MNL*} > f_s^{MBD*} > f_s^{MND*}$. When $\rho \in (\rho_3, 300)$, $f_s^{MNL*} > f_s^{MBL*} > f_s^{MND*} > f_s^{MBD*}$. Fig. 7 illustrates that in promoting application of blockchain, manufacturer should strive to reduce fixed one-time cost of introducing it, and consumers should reduce their sensitivity to the time required for product testing and evaluation. Enterprises can choose appropriate strategies based on market demand and decisions of other members, such as jointly reducing emissions or jointly applying blockchain technology with partners to achieve competitive advantage and overall profit improvement.

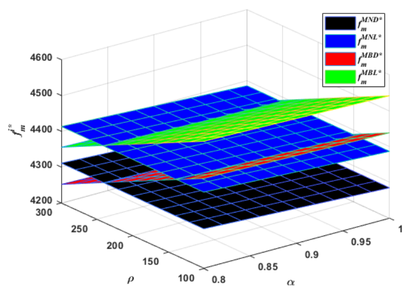

Fig.7(a) Impact of α and ρ on f_m^{i*}

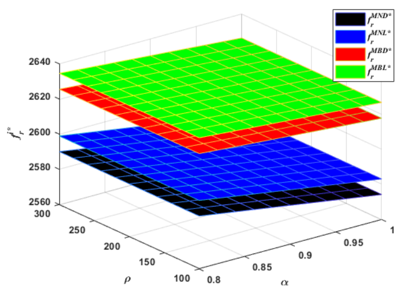

Fig. 7(b) Impact of α and ρ on f_r^{i*}

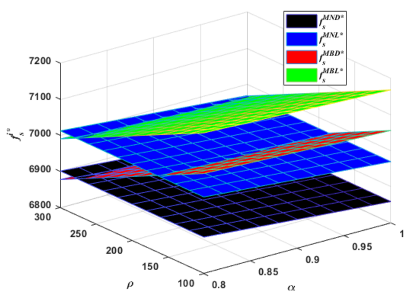

Fig. 7(c) Impact of α and ρ on f_s^{i*}

6. Conclusions

This paper focuses on the secondary single channel recycling and remanufacturing supply chain system. Moreover, we introduce multiple relevant parameters of blockchain taking into account the low - carbon emission reduction mode. We explored ramifications of blockchain platforms in the choice of emission reduction modalities in a manufacturer - led game. The principal findings derived are presented as below.

(1) In scenarios without blockchain platforms, key metrics typically reach higher levels under joint emission reduction model. Specifically, wholesale prices, carbon emission reduction, retail prices, market demand, and manufacturer's profit all exceed those observed in the manufacturer-led independent reduction model. However, recycling prices and quantity of used products do not change. In the scenario where the cost-sharing coefficient remains below a specific threshold value, profits of retailer and supply chain in joint emission reduction model both increase.

(2) When manufacturer introduces blockchain platforms, unit verification fees will only cause fluctuations in wholesale prices, but retail prices, recycling prices, carbon reduction emission, market demand, amount of recycled, and profits are not affected. Compared with the independent emission reduction model of manufacturer, carbon emission reduction, wholesale prices, retail prices, and market demand in joint emission reduction model have all increased, but recycling prices and amount have not changed. The manufacturer's profit in joint emission reduction model is always higher than that in the manufacturer's independent emission reduction model. If cost sharing coefficient is lower than a specific threshold, profits of retailer and supply chain in joint emission reduction model are both higher than those in manufacturer's independent emission reduction model.

(3) In manufacturer's individual emission reduction model, retail prices, recycling prices, carbon reduction emission, market demand, amount of recycled, profit of retailer all increase under introducing of blockchain. If sensitivity coefficient of consumers to duration of product testing and evaluation surpasses a specified cutoff value, manufacturer using blockchain technology have higher prices. If manufacturer introduces blockchain technology platforms and pays lower fixed fees, it's profit and overall supply chain's profit are higher when using blockchain technology.

(4) In joint emission reduction model, introducing of blockchain increases retail prices, recycling prices, carbon reduction emission, market demand, amount of recycled, and profit of retailer. When sensitivity coefficient of consumers to duration of product testing and evaluation surpasses a specified cutoff value, manufacturer using blockchain technology have higher prices. If manufacturer introduces blockchain technology platforms and pays lower fixed fees, it's profit and overall supply chain's profit are higher when using blockchain technology.

The findings above yield important implications for the deployment of blockchain technology in low-carbon reduction frameworks.

For manufacturer, the decision to adopt blockchain technology platforms is independent of emission reduction models. However minimal the retailer's cost-sharing proportion may be, manufacturer tends to choose joint emission reduction models. When manufacturer decides to introduce blockchain technology platforms, choosing a joint emission reduction model strengthens the enterprise's competitive advantage in the long run. If the emission reduction mode is fixed, whether manufacturer makes the decision to introduce blockchain technology platforms mainly depends on sensitivity coefficient of consumers to duration of product testing and evaluation, and the size of the fixed cost invested by manufacturer in blockchain technology.

For retailer, regardless of whether manufacturer introduces blockchain technology verification platforms, choosing a joint emission reduction model is the optimal decision under a low cost-sharing ratio. On the contrary, opting out of the joint emission reduction model emerges as optimal strategy. If emission reduction mode is fixed, while consumers have a low sensitivity coefficient of consumers to duration of product testing and evaluation, it is more advantageous to introduce models under the blockchain technology inspection platforms.

Considering overall supply chain, whether manufacturer introduces blockchain platforms or not, the joint emission reduction model outperforms the individual model. The introduction of blockchain technology platforms leads to higher profits of manufacturer, irrespective of whether a standalone or a joint emission reduction model is employed under a fixed emission reduction mode.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

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Appendix

A. Proof procedure for Proposition 1

Proof: The reverse solving method is adopted. By substituting $d^{MND} = \varphi - kp^{MND} + \theta e^{MND} - \alpha t - \beta(1-\sigma)$ into equation $f_r^{MND} = (p^{MND} - w^{MND})d^{MND}$, we obtain $f_r^{MND} = (p^{MND} - w^{MND})[\varphi - kp^{MND} + \theta e^{MND} - \alpha t - \beta(1-\sigma)]$. Initial-order partial derivative is:

$$\frac{\partial f_r^{MND}}{\partial p^{MND}} = \varphi - kp^{MND} + \theta e^{MND} - \alpha t - \beta(1-\sigma) - k(p^{MND} - w^{MND})$$

Second-order partial derivative is:

$$\frac{\partial^2 f_r^{MND}}{\partial (p^{MND})^2} = -2k < 0$$

f_r^{MND} is strictly concave in p^{MND}. We calculate initial-order partial derivative of f_r^{MND} to p^{MND} and make it 0. Thus:

$$p^{MND} = \frac{\varphi + \theta e^{MND} - \alpha t - \beta(1-\sigma) + kw^{MND}}{2k}$$

By substituting it into equation $f_m^{MND} = (w^{MND} - c_n)d^{MND} + (c_o - b^{MND})qD^{MND} - \frac{1}{2}\mu(e^{MND})^2$, we obtain the Hessian matrix of function f_m^{MND} as:

$$H(w^{MND}, b^{MND}, e^{MND}) = \begin{bmatrix} -k & 0 & \frac{\theta}{2} \\ 0 & -2q\omega & 0 \\ \frac{\theta}{2} & 0 & -\mu \end{bmatrix}$$

We can see that there is a first-order principal minor $H_1(w^{MND}, b^{MND}, e^{MND}) = -k < 0$ and a second-order principal minor $H_2(w^{MND}, b^{MND}, e^{MND}) = 2kq\omega > 0$. When $\theta^2 < 4k\mu$ is present, there is a third-order principal minor

$H_3(w^{MND}, b^{MND}, e^{MND}) = q\omega(\frac{\theta^2}{2} - 2k\mu) < 0$. We determine that f_m^{MND} is strictly concave in w^{MND}, b^{MND}, and e^{MND}. Therefore, we obtain w^{MND*}, b^{MND*}, and e^{MND*} from the first-order partial derivative of function f_m^{MND} being 0.

$$w^{MND*}=\frac{2\mu[\varphi-\alpha t+\beta(\sigma-1)+kc_n]-\theta^2 c_n}{4k\mu-\theta^2} \tag{1}$$

$$b^{MND*}=\frac{c_o\omega-\psi}{2\omega} \tag{2}$$

$$e^{MND*}=\frac{\theta[\varphi-\alpha t+\beta(\sigma-1)-kc_n]}{4k\mu-\theta^2} \tag{3}$$

Therefore,

$$p^{MND*}=\frac{c_n(k\mu-\theta^2)+3\mu[\varphi-\alpha t+\beta(\sigma-1)]}{4k\mu-\theta^2} \tag{4}$$

$$d^{MND*} = \frac{k\mu[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]}{4k\mu - \theta^2} \tag{5}$$

$$D^{MND*} = \frac{\psi + c_o\omega}{2} \tag{6}$$

$$f_m^{MND*} = \frac{\mu[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{2(4k\mu - \theta^2)} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{7}$$

$$f_r^{MND*} = \frac{k\mu^2[\varphi - \alpha t + \beta(\sigma - 1) - kc_n]^2}{(4k\mu - \theta^2)^2} \tag{8}$$

$$f_s^{MND*} = f_m^{MND*} + f_r^{MND*} \tag{9}$$

The validity of Proposition 1 has been established.

B.Proof procedure for Proposition 2

Proof: By substituting $d^{MNL} = \varphi - kp^{MNL} + \theta e^{MNL} - \alpha t - \beta(1-\sigma)$ into equation $f_r^{MNL} = (p^{MNL} - w^{MNL})d^{MNL} - \frac{1}{2}\mu\tau(e^{MNL})^2$, we obtain f_r^{MNL}=$(p^{MNL} - w^{MNL})[\varphi - kp^{MNL} + \theta e^{MNL} - \alpha t - \beta(1-\sigma)] - \frac{1}{2}\mu\tau(e^{MNL})^2$. Hence,

$$\frac{\partial f_r^{MNL}}{\partial p^{MNL}} = \varphi - kp^{MNL} + \theta e^{MNL} - \alpha t - \beta(1-\sigma) - k(p^{MNL} - w^{MNL})$$

Second-order partial derivative is:

$$\frac{\partial^2 f_r^{MNL}}{\partial (p^{MNL})^2} = -2k < 0$$

f_r^{MNL} is strictly concave in p^{MNL}. We calculate initial-order partial derivative of f_r^{MNL} to p^{MNL} and make it 0. Thus:

$$p^{MNL} = \frac{\varphi + \theta e^{MNL} - \alpha t - \beta(1-\sigma) + kw^{MNL}}{2k}$$

By substituting it into equation $f_m^{MNL} = (w^{MNL} - c_n)d^{MNL} + (c_o - b^{MNL})qD^{MNL} - \frac{1}{2}\mu(1-\tau)(e^{MNL})^2$, we obtain the Hessian matrix of function f_m^{MNL} as:

$$H(w^{MNL},\ b^{MNL},\ e^{MNL}) = \begin{bmatrix} -k & 0 & \frac{\theta}{2} \\ 0 & -2q\omega & 0 \\ \frac{\theta}{2} & 0 & \mu(\tau-1) \end{bmatrix}$$

We can see that there is a first-order principal minor $H_1(w^{MNL}, b^{MNL}, e^{MNL}) = -k < 0$ and a second-order principal minor $H_2(w^{MNL}, b^{MNL}, e^{MNL}) = 2kq\omega > 0$. When $\theta^2 < 4k\mu(1-\tau)$ is present, there is a third-order principal minor $H_3(w^{MNL}, b^{MNL}, e^{MNL}) = q\omega[\frac{\theta^2}{2} - 2k\mu(1-\tau)] < 0$. We determine that f_m^{MNL} is strictly concave in w^{MNL}, b^{MNL}, and e^{MNL}. Therefore, we obtain w^{MNL*}, b^{MNL*}, and e^{MNL*} from the first-order partial derivative of function f_m^{MNL} being 0.

$$w^{MNL*} = \frac{2\mu(1-\tau)[\varphi - \alpha t + \beta(\sigma-1) + kc_n] - \theta^2 c_n}{4k\mu(1-\tau) - \theta^2} \tag{10}$$

$$b^{MNL*} = \frac{c_o\omega - \psi}{2\omega} \tag{11}$$

$$e^{MNL*} = \frac{\theta[\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{4k\mu(1-\tau) - \theta^2} \tag{12}$$

Therefore,

$$p^{MNL*} = \frac{c_n[k\mu(1-\tau) - \theta^2] + 3\mu(1-\tau)[\varphi - \alpha t + \beta(\sigma-1)]}{4k\mu(1-\tau) - \theta^2} \tag{13}$$

$$d^{MNL*} = \frac{k\mu(1-\tau)[\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{4k\mu(1-\tau) - \theta^2} \tag{14}$$

$$D^{MNL*} = \frac{\psi + c_o\omega}{2} \tag{15}$$

$$f_m^{MNL*} = \frac{\mu(1-\tau)[\varphi - \alpha t + \beta(\sigma-1) - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]} + \frac{q(\psi + c_o\omega)^2}{4\omega} \tag{16}$$

$$f_r^{MNL*} = \frac{[2k\mu^2(1-\tau)^2 - \mu\tau\theta^2][\varphi - \alpha t + \beta(\sigma-1) - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]^2} \tag{17}$$

$$f_s^{MNL*} = f_m^{MNL*} + f_r^{MNL*} \tag{18}$$

Proposition 2 is proven.

C. Demonstration process of Corollary 1

Proof: (1) In the MND model, we know from Table 1, $\mu > 0, \theta > 0, k > 0, \sigma - 1 < 0$. We know from Proposition 1, $4k\mu - \theta^2 > 0, \varphi - \alpha t + \beta(\sigma-1) - kc_n > 0$. Therefore,

$$\frac{\partial w^{MND}}{\partial \beta} = \frac{2\mu(\sigma-1)}{4k\mu - \theta^2} < 0, \frac{\partial b^{MND}}{\partial \beta} = 0, \frac{\partial e^{MND}}{\partial \beta} = \frac{\theta(\sigma-1)}{4k\mu - \theta^2} < 0, \quad \frac{\partial p^{MND}}{\partial \beta} = \frac{3\mu(\sigma-1)}{4k\mu - \theta^2} < 0, \frac{\partial d^{MND}}{\partial \beta} = \frac{k\mu(\sigma-1)}{4k\mu - \theta^2} < 0, \frac{\partial D^{MND}}{\partial \beta} = 0,$$

$$\frac{\partial f_m^{MND}}{\partial \beta} = \frac{2\mu(\sigma-1)[\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{2(4k\mu - \theta^2)} < 0, \quad \frac{\partial f_r^{MND}}{\partial \beta} = \frac{2k\mu^2(\sigma-1)[\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{(4k\mu - \theta^2)^2} < 0,$$

$$\frac{\partial f_s^{MND}}{\partial \beta} = \frac{\partial f_m^{MND}}{\partial \beta} + \frac{\partial f_r^{MND}}{\partial \beta} < 0;$$

In the MNL model, we know from Table 1, $\mu > 0, \theta > 0, k > 0, \sigma - 1 < 0, 1 - \tau > 0$. We know from Proposition 2, $4k\mu(1-\tau) - \theta^2 > 0, \varphi - \alpha t + \beta(\sigma-1) - kc_n > 0, 2k\mu^2(1-\tau)^2 - \mu\tau\theta^2 > 0$. Therefore,

$$\frac{\partial w^{MNL}}{\partial \beta} = \frac{2\mu(\sigma-1)(1-\tau)}{4k\mu(1-\tau) - \theta^2} < 0, \frac{\partial b^{MNL}}{\partial \beta} = 0, \frac{\partial e^{MNL}}{\partial \beta} = \frac{\theta(\sigma-1)}{4k\mu(1-\tau) - \theta^2} < 0,$$

$$\frac{\partial p^{MNL}}{\partial \beta} = \frac{3\mu(\sigma-1)(1-\tau)}{4k\mu(1-\tau) - \theta^2} < 0, \frac{\partial d^{MNL}}{\partial \beta} = \frac{k\mu(\sigma-1)(1-\tau)}{4k\mu(1-\tau) - \theta^2} < 0, \frac{\partial D^{MNL}}{\partial \beta} = 0,$$

$$\frac{\partial f_m^{MNL}}{\partial \beta} = \frac{2\mu(1-\tau)(\sigma-1)[\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{2[4k\mu(1-\tau) - \theta^2]} < 0,$$

$$\frac{\partial f_r^{MNL}}{\partial \beta} = \frac{2(\sigma-1)[2k\mu^2(1-\tau)^2 - \mu\tau\theta^2][\varphi - \alpha t + \beta(\sigma-1) - kc_n]}{2[4k\mu(1-\tau) - \theta^2]^2} < 0,$$

$$\frac{\partial f_s^{MNL}}{\partial \beta}=\frac{\partial f_m^{MNL}}{\partial \beta}+\frac{\partial f_r^{MNL}}{\partial \beta}<0.$$

Combining (1) and (2), corollary 1 is proven.

D. Demonstration process of Corollary 2

Proof: We know from Table 1, $\mu>0,\theta>0,k>0,\tau>0$. We know from Proposition 1 and Proposition 2, $(4k\mu-\theta^2)>0, 4k\mu(1-\tau)-\theta^2>0, \varphi-\alpha t+\beta(\sigma-1)-kc_n>0, 2k\mu^2(1-\tau)^2-\mu\tau\theta^2>0$. Therefore,

$$w^{MNL}-w^{MND}=\frac{2\mu\tau\theta^2[\varphi-\alpha t+\beta(\sigma-1)-kc_n]}{(4k\mu-\theta^2)[4k\mu(1-\tau)-\theta^2]}>0,\quad p^{MNL}-p^{MND}=\frac{3\mu\tau\theta^2[\varphi-\alpha t+\beta(\sigma-1)-kc_n]}{(4k\mu-\theta^2)[4k\mu(1-\tau)-\theta^2]}>0,$$

$$b^{MNL}=b^{MND}=\frac{c_o\omega-\psi}{2\omega},\quad e^{MNL}-e^{MND}=\frac{2\mu\tau\theta^2[\varphi-\alpha t+\beta(\sigma-1)-kc_n]}{(4k\mu-\theta^2)[4k\mu(1-\tau)-\theta^2]}>0,$$

$$d^{MNL}-d^{MND}=\frac{k\mu\tau\theta^2[\varphi-\alpha t+\beta(\sigma-1)-kc_n]}{(4k\mu-\theta^2)[4k\mu(1-\tau)-\theta^2]}>0,\quad D^{MNL}=D^{MND}=\frac{\psi+c_o\omega}{2},$$

$$f_m^{MNL}-f_m^{MND}=\frac{\mu\tau\theta^2[\varphi-\alpha t+\beta(\sigma-1)-kc_n]^2}{2(4k\mu-\theta^2)[4k\mu(1-\tau)-\theta^2]}>0,$$

When $0<\tau<\dfrac{\theta^2(4k\mu-\theta^2)}{2k\mu(8k\mu-\theta^2)}$,

$$f_r^{MNL}-f_r^{MND}=\frac{\mu\tau\theta^2[\theta^2(4k\mu-\theta^2)-2k\mu\tau(8k\mu-\theta^2)][\varphi-\alpha t+\beta(\sigma-1)-kc_n]^2}{2(4k\mu-\theta^2)^2[4k\mu(1-\tau)-\theta^2]^2}$$

Overall, Corollary 2 is proven.

E. Demonstration process of Proposition 3

Proof :The reverse solving method is adopted.

By substituting $d^{MBD}=\varphi-kp^{MBD}+\theta e^{MBD}-\alpha T$ into equation $f_r^{MBD}=(p^{MBD}-w^{MBD}-\gamma)d^{MBD}$, we obtain $f_r^{MBD}=(p^{MBD}-w^{MBD}-\gamma)[\varphi-kp^{MBD}+\theta e^{MBD}-\alpha T]$. Hence,

$$\frac{\partial f_r^{MBD}}{\partial p^{MBD}}=\varphi-kp^{MBD}+\theta e^{MBD}-\alpha T-k(p^{MBD}-w^{MBD}-\gamma)$$

Second-order partial derivative is $\dfrac{\partial^2 f_r^{MBD}}{\partial (p^{MBD})^2}=-2k<0.$

f_r^{MBD} is strictly concave in p^{MBD}. We calculate initial-order partial derivative of f_r^{MBD} to p^{MBD} and make it 0. Thus:

$$p^{MBD}=\frac{\varphi+\theta e^{MBD}-\alpha T+k(w^{MBD}+\gamma)}{2k}$$

By substituting it into equation $f_m^{MBD}=(w^{MBD}-c_n+\gamma)d^{MBD}+(c_o+\delta c_z-b^{MBD})qD^{MBD}-\frac{1}{2}\mu(e^{MBD})^2-\rho$, we obtain the Hessian matrix of function f_m^{MBD} as:

$$H(w^{MBD},b^{MBD},e^{MBD})=\begin{bmatrix}-k & 0 & \frac{\theta}{2}\\ 0 & -2q\omega(1+g) & 0\\ \frac{\theta}{2} & 0 & -\mu\end{bmatrix}$$

We can see that there is a first-order principal minor $H_1(w^{MBD},b^{MBD},e^{MBD})=-k<0$ and a second-order principal minor $H_2(w^{MBD},b^{MBD},e^{MBD})=2kq\omega(1+g)>0$. When $\theta^2<4k\mu$ is present, there is a third-order principal minor $H_3(w^{MBD},b^{MBD},e^{MBD})=q\omega(1+g)(\frac{\theta^2}{2}-2k\mu)<0$. We determine that f_m^{MBD} is strictly concave in w^{MBD}, b^{MBD}, and e^{MBD}. Therefore, we obtain w^{MBD*}, b^{MBD*}, and e^{MBD*} from the first-order partial derivative of function f_m^{MBD} being 0.

$$w^{MBD*} = \frac{2\mu[\varphi - \alpha T - 2k\gamma + kc_n] - \theta^2(c_n - \gamma)}{4k\mu - \theta^2} \tag{19}$$

$$b^{MBD*} = \frac{\omega(c_o + \delta c_z) - \psi}{2\omega} \tag{20}$$

$$e^{MBD*} = \frac{\theta[\varphi - \alpha T - kc_n]}{4k\mu - \theta^2} \tag{21}$$

Therefore,

$$p^{MBD*} = \frac{c_n(k\mu - \theta^2) + 3\mu[\varphi - \alpha T]}{4k\mu - \theta^2} \tag{22}$$

$$d^{MBD*} = \frac{k\mu[\varphi - \alpha T - kc_n]}{4k\mu - \theta^2} \tag{23}$$

$$D^{MBD*} = \frac{(1+g)[\psi + \omega(c_o + \delta c_z)]}{2} \tag{24}$$

$$f_m^{MBD*} = \frac{\mu[\varphi - \alpha T - kc_n]^2}{2(4k\mu - \theta^2)} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{25}$$

$$f_r^{MBD*} = \frac{k\mu^2(\varphi - \alpha T - kc_n)^2}{(4k\mu - \theta^2)^2} \tag{26}$$

Proposition 3 is proven.

F. Proof procedure for Proposition 4

Proof: The reverse solving method is adopted. By substituting $d^{MBL} = \varphi - kp^{MBL} + \theta e^{MBL} - \alpha T$ into equation $f_r^{MBL} = (p^{MBL} - w^{MBL} - \gamma)d^{MBL} - \frac{1}{2}\mu\tau(e^{MBD})^2$, we obtain $f_r^{MBL} = (p^{MBL} - w^{MBL})[\varphi - kp^{MBL} + \theta e^{MBL} - \alpha T] - \frac{1}{2}\mu\tau(e^{MBL})^2$. Hence,

$$\frac{\partial f_r^{MBL}}{\partial p^{MBL}} = \varphi - kp^{MBL} + \theta e^{MBL} - \alpha T - k(p^{MBL} - w^{MBL} - \gamma)$$

Second-order partial derivative is:

$$\frac{\partial^2 f_r^{MBL}}{\partial (p^{MBL})^2} = -2k < 0$$

The function f_r^{MBL} is strictly concave in p^{MBL}. We calculate initial-order partial derivative of f_r^{MBL} to p^{MBL} and make it 0. Thus:

$$p^{MBL} = \frac{\varphi + \theta e^{MBL} - \alpha T + k(w^{MBL} + \gamma)}{2k}$$

By substituting it into equation $f_m^{MBL} = (w^{MBL} - c_n + \gamma)d^{MBL} + (c_o + \delta c_z - b^{MBL})qD^{MBL} - \frac{1}{2}\mu(1-\tau)(e^{MBD})^2 - \rho$, we obtain the Hessian matrix of function f_m^{MBL} as:

$$H(w^{MBL},\ b^{MBL},\ e^{MBL}) = \begin{bmatrix} -k & 0 & \frac{\theta}{2} \\ 0 & -2q\omega(1+g) & 0 \\ \frac{\theta}{2} & 0 & \mu(\tau - 1) \end{bmatrix}$$

We can see that there is a first-order principal minor $H_1(w^{MBL},\ b^{MBL},\ e^{MBL}) = -k < 0$ and a second-order principal minor $H_2(w^{MBL}, b^{MBL}, e^{MBL}) = 2kq\omega(1+g) > 0$. When $\theta^2 < 4k\mu(1-\tau)$ is present, there is a third-order principal minor $H_3(w^{MBL}, b^{MBL}, e^{MBL}) = q\omega(1+g)[\frac{\theta^2}{2} - 2k\mu(1-\tau)] < 0$. We determine that f_m^{MBL} is strictly concave in w^{MBL}, b^{MBL}, and e^{MBL}. Therefore, we obtain w^{MBL*}, b^{MBL*}, and e^{MBL*} from the first-order partial derivative of function f_m^{MBL} being 0.

$$w^{MBL*} = \frac{2\mu(1-\tau)(\varphi - \alpha T - 2k\gamma + kc_n) - \theta^2(c_n - \gamma)}{4k\mu(1-\tau) - \theta^2} \tag{27}$$

$$b^{MBL*} = \frac{\omega(c_o + \delta c_z) - \psi}{2\omega} \tag{28}$$

$$e^{MBL*} = \frac{\theta(\varphi - \alpha T - kc_n)}{4k\mu(1-\tau) - \theta^2} \tag{29}$$

Therefore,

$$p^{MBL*} = \frac{c_n[k\mu(1-\tau) - \theta^2] + 3\mu(1-\tau)(\varphi - \alpha T)}{4k\mu(1-\tau) - \theta^2} \tag{30}$$

$$d^{MBL*} = \frac{k\mu(1-\tau)(\varphi - \alpha T - kc_n)}{4k\mu(1-\tau) - \theta^2} \tag{31}$$

$$D^{MBL*} = \frac{(1+g)[\psi + \omega(c_o + \delta c_z)]}{2} \tag{32}$$

$$f_m^{MBL*} = \frac{\mu(1-\tau)[\varphi - \alpha T - kc_n]^2}{2[4k\mu(1-\tau) - \theta^2]} + \frac{q(1+g)[\psi + \omega(c_o + \delta c_z)]^2}{4\omega} - \rho \tag{33}$$

$$f_r^{MBL*} = \frac{[2k\mu^2(1-\tau)^2 - \mu\tau\theta^2][\varphi - \alpha T]^2}{2[4k\mu(1-\tau) - \theta^2]^2} \tag{34}$$

Proposition 4 is proven.

The proof methods for Corollary 3, 4, 6~8 are the same as Corollary 1; The proof method for Corollary 5, Corollary 9, and Corollary 10 is the same as Corollary 2. This paper will not elaborate further.