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Technological innovation in trade-in supply chain: Enterprise operations and consumer reactions

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A B S T R A C T

Trade-in services, coupled with technological innovation for product update, are widely adopted by businesses. However, the practical implications of this strategy for enterprise operations and consumer purchasing behavior remain unclear, necessitating further exploration of how firms should respond. This study investigates a manufacturer offering trade-in services by comparing two scenarios: one where the manufacturer implements technological innovation and another where it refrains from doing so. Through the development of decision-making models and a comparative analysis of game-theoretic results, we examine the effects on enterprise operations and consumer responses to technological innovation. Additionally, we conduct a factor analysis to assess the determinants of technological innovation's impact. Our findings reveal that, under trade-in services, technological innovation enhances the manufacturer's profitability but may also lead to supplier hitchhiking. Both new and existing consumers exhibit homogeneous responses to innovation; however, under certain conditions, technological innovation may trigger consumer resistance. Furthermore, trade-in services can generate a synergistic effect with technological innovation, amplifying both its positive and negative consequences. Based on these insights, we propose operational adjustments to mitigate the identified adverse effects. This research provides managerial guidance for optimizing decision-making and addressing consumer reactions when implementing technological innovation in trade-in supply chains.

1. Introduction

Since the onset of the industrial era in human civilization, the manufacturing domain has undergone swift advancement, providing abundant products for mankind, yet concurrently aggravating resource depletion, environmental degradation, and climatic shifts. Hence, the sustainable progression of human society necessitates enhancing the harmonious coexistence of the manufacturing sector and the environment (Jayaram & Avittathur, 2015; Khan et al., 2021). The green economic model characterized by low energy consumption, low pollution, and low emissions stands as an inevitable choice for fostering both environmental preservation and human development in the present era (Pretty, 2013; Tang & Solangi, 2023). As the pivotal facet of the green economic model, the circular economy has evolved into a crucial pathway for unraveling resource constraints, addressing climate change, and cultivating novel focal points for economic growth (Schröder et al., 2020; Kurniawan et al., 2024). Trade-in service for household consumer goods is a typical circular economy model. It not only promotes the recycling of used products, driving the industry towards green and low-carbon transformation but also stimulates consumers' willingness to buy new products, thereby promoting consumption growth (Dou and Choi, 2021; Assurant, 2022).

According to statistics, about 65% of the global greenhouse gas emissions come from household consumption, so it is very potential and urgent to reduce the carbon emissions of household consumption (Ivanova et al., 2020; Lian et al., 2024). Since the 21st century, many enterprises have been implementing trade-in services to foster a circular economy and diminish carbon emissions associated with household consumption. For example, in August 2013, Apple launched an iPhone trade-in program in the U.S. market. In August 2015, Huawei initiated "The Green Action 2.0 Program", a mobile phone trade-in and recycling plan. In June 2018, Gree launched a trade-in service for household air conditioners. In May 2023, JD held "The Mid-Year 618 Shopping Festival". In the first ten minutes alone, the transaction volume of home appliances and furnishings through the

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trade-in service surged by over 120% compared to the previous year. This not only contributed to carbon emission reduction but also stimulated consumption growth (Wang & Yang, 2023). Moreover, according to a report released by Assurant, a record high $ 4.3 billion was returned to U.S. consumers in 2023 through mobile device trade-in services, and more than half of the top traded models are 5G devices turned in during the year (Assurant, 2024). Overall, in recent years, the trade-in service market has shown sustained growth, with consumer demand for new products increasing significantly.

Indeed, the flourishing development of trade-in services is not only propelled by the ethos of green and low-carbon consumption, but also associated with consumers' emotional preferences and usage needs for new products (Bellezza et al., 2017; Li et al., 2023). In the wake of recent strides in science and technology, the speed of product innovation has accelerated, amplifying consumers' perception of the devaluation of their own products (Okada, 2001; Jung et al., 2022). Consequently, the willingness of consumers to exchange used products for new ones at an additional cost has increased. However, through practical observation, it is noted that the current innovative products generally come with a higher price tag compared to the previous products. The higher price of some new products might potentially dampen consumers' enthusiasm for trade-in (McCarthy, 2019). What's more, technological innovation may also entail risks and potential losses for enterprises (Chen and Ibhagui, 2019). That might be associated with the fact that technological innovation within a business is highly likely to trigger alterations in its operations and elicit societal effects such as consumer responses (Simpson et al., 2006; Heidenreich & Kraemer, 2015; Heidenreich et al., 2016). Therefore, in the practical realm of business operations, studying and answering the following questions holds practical guiding significance for enterprises that are practicing trade-in services to implement products upgrading and their market launch:

Q1. What are the effects of technological innovation strategies adopted by trade-in service providers on their operational decision-making, organizational performance, and consumer purchasing behavior?
Q2. Does the trade-in service intervene in the effects of technological innovation?
Q3. Based on the above results, what strategic operational adjustments should enterprises adopt to mitigate the identified negative effects?

Currently, research in this field is primarily bifurcated into two independent aspects: the research on trade-in service and the research on technological innovation. In terms of the research on trade-in service, the primary focus lies within the realm of market strategy analysis. For example, some studies explore optimal marketing strategies, pricing mechanisms, and repurchase strategies for companies from different perspectives such as consumer heterogeneity (Li et al., 2022; Wang et al., 2023; Hu & Tang, 2024), corporate competition (Zhao et al., 2021; Tang et al., 2023; Shi & Chen, 2023), remanufacture (Ma et al., 2020; Wu et al., 2021) and warranty services (Cao et al., 2022; Vafaeinejad and Sajadieh, 2022). Undoubtedly, these studies greatly contribute to the constructive advancement of product marketing strategies for trade-in enterprises. Nevertheless, a notable insufficiency exists in terms of pertinent guidance pertaining to technological innovation decision and its effects on trade-in enterprises. In addition, there is some research that delves into the problems between innovation technology and trade-in service. Notably, Yang et al. (2023) analyze the influence of the manufacturer's product innovation strategy on the retailer's trade-in decision and the decision to enter the secondary market. Hu et al. (2019) discuss the optimal price and trade-in rebate of the innovation products considering strategic consumers and trade-in products within a time limit. While the above research discusses the innovation of products, it fails to delve into the technological innovation decision undertaken by a trade-in enterprise, and its effects on supply chain enterprises and consumers. Furthermore, taking into account the practical implications of technology spillover effect on the operations of partner firm within a value chain (Isaksson et al., 2016; Zhou et al., 2020), it will be more practical to examine the technological innovation decision of a trade-in enterprise and its effects from supply chain perspective.

There exists a substantial body of research on technological innovation, with particular relevance to this article being the examination of the technological innovation decision of enterprises and its effects on enterprises and consumers. In the research on technological innovation decision making, Vish et al. (2019) analyze the investment anchor decision for various objectives from investments in innovation to market coverage to profit. Dimakopoulou et al. (2023) examine the enterprise's eco-innovation decision and the joint impact of it and R&D collaboration on enterprise innovation efficiency. Liu et al. (2023) discuss supply chain product innovation considering jointly held options under uncertain demand. While numerous studies have delved into the technological innovation decision making of enterprises, regrettably, the effect of trade-in service is overlooked in these scholarly endeavors. In the research on the effect of technological innovation on enterprises, some scholars find that technological innovation has a significant positive effect on enterprise operation and economic performance (Ren et al., 2022; Li et al., 2023). Conversely, other studies shed light on the potential risks associated with *R&D* and innovation, which can introduce uncertainties that subsequently affect enterprise performance (Chen & Ibhagui, 2019; Leung et al., 2020). Regarding research on effects of technological innovation on consumers, Jiang et al. (2022) discover that technological innovation has a positive effect on consumers' purchase intention by transmitting the social value, hedonic value and novelty value of products. On the contrary, findings by Claudy et al. (2015), Heidenreich et al. (2016) and Joachim et al. (2018) indicate that technological innovation can potentially elicit resistance among consumers. The research conducted by Fox and Griffy (2023) reveals that social groups' responses to innovation are influenced by various factors, including their cultural beliefs, institutional values and the types of innovation. Regarding the effects of technological innovation on enterprises and consumers, the disparity in existing research conclusions may stem from different inherent characteristics of the underlying circumstances, and taking different influence factors into consideration. Consequently, to obtain an instructive conclusion, it becomes imperative to analyze the effects of technological innovation by comprehensively considering the fundamental situation and characteristics of the research problem. The marketing under the trade in service has its uniqueness compared with ordinary marketing. Primarily, its marketing is characterized by price discrimination (Agrawal et al., 2016). Within this domain, the market consumers will naturally be segmented into two categories: new consumers and existing consumers. The existing consumers have trade-in rebates through returning their used products when buying new products, whereas new consumers do not, thereby naturally establishing a form of price discrimination (Ray et al., 2005; Li et al., 2022; Zhou et al., 2022). Secondly, in its marketing process, existing consumers have unique buying behaviors. Existing consumers, owing to their possession of used products, exhibit distinct preferences and behaviors when it comes to purchasing new products. Their willingness to acquire new offerings differs from that of new consumers, as they take into account the depreciation status of their used products before making a purchasing decision (Levinthal and Purohit, 1989; Zhao et al., 2021). This consideration of the depreciation status is a crucial factor that influences their decision-making process. Hence, elucidating the technological innovation decisions and its effects on enterprises and consumers in the context of trade-in proves arduous with the existing research findings. In view of the characteristics of trade-in service, it is necessary to especially delve into the technological innovation decision of trade-in enterprises and its precise effects on enterprises and consumers.

Therefore, due to the dearth of existing research and practical needs, this paper explores a two-tier supply chain with a supplier and a manufacturer, focusing on two scenarios: the manufacturer implements technological innovation strategy and he does not. It establishes respective decision making models for the trade-in supply chain and analyzes the optimal decision of stakeholders in both cases. By comparing the decision changes, it investigates how technological innovation affects trade-in enterprises' operations and consumer purchasing attitude. Furthermore, it delves into the relationship between factor changes and technological innovation's effects, while proposing strategic mitigation measures for enterprises to address adverse effects. This paper helps to understand the effects of technological innovation on trade-in marketing, enriches theory on the relation between innovation and consumer choice, and adds to trade-in and tech innovation research.

2. Problem description

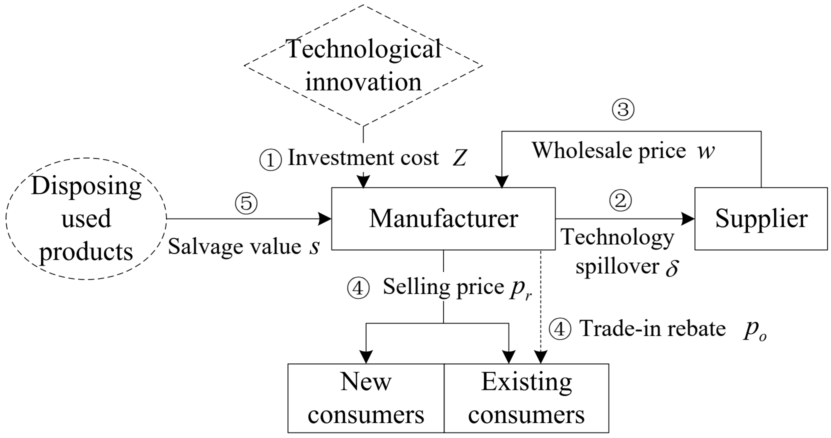
This study delves into a two-tier supply chain comprising a supplier and a manufacturer that implements a trade-in program for consumers. Under this program, market consumers are bifurcated into two types: new consumers and existing consumers, with the latter being eligible for additional discount when participating in the trade-in program. Furthermore, to gain greater competitive advantage, the manufacturer intends to integrate technological innovation into its trade-in program. In order to reveal the effects of technological innovation on supply chain decision making, consumer behavior, and societal welfare, this study investigates two distinct scenarios:

I. No technological innovation strategy

In this scenario, the manufacturer does not engage in technological innovation strategy, so that there is no technological innovation investment cost, and the supply chain system only involves the production and sale stage depicted as ③-⑤ in Fig. 1. Firstly, the supplier provides semi-finished products to the manufacturer at a wholesale price w. Then, the manufacturer produces products with the original technology and provides them to market consumers at a direct selling price p_r. However, unlike the new consumers, the existing consumers can get a trade-in rebate p_o by returning their used products in the trade-in program. And the manufacturer can obtain a unit salvage value s of the used product by disposing of it in some ways, such as selling and remanufacturing.

II. Technological innovation strategy

In this scenario, to strengthen its competitive position, the manufacturer adopts technological innovation strategy. This strategy entails the supply chain system comprising two stages: (a) The technological innovation stage. It is shown as ① and ② in Fig. 1. The manufacturer invests in technological innovation at a cost Z, $Z > 0$, to develop advanced production capabilities for new product offerings. Additionally, due to technological innovation spillover effect, its coefficient denoted as δ, the supplier's production is influenced within the supply chain by the technological innovation emanating from the manufacturer. (b) The production and sale stage. At this stage, the supply chain operation processes are similar to those in the prior scenario, depicted as ③-⑤ in Fig. 1. The supplier furnishes the manufacturer with semi-finished products, which are then transformed into new products and marketed to consumers by the manufacturer. In addition, the existing consumers get the trade-in rebate, and the manufacturer obtains the unit salvage value of used product. With a notable difference, the manufacturer will adopt the innovative technology to produce new products.



Note. ①~⑤ denote the sequences of decision-making behaviors and influence generated by members in the supply chain.

Fig. 1. Supply chain operation processes of research scenarios

Drawing inspiration from scholars such as Feng et al. (2019) and Xiao et al. (2020), the total market demand for new products is set at 1, segmented into new consumers and existing consumers with used products. The market share of the existing consumers is denoted as α, $0 \le \alpha \le 1$, while that of the new consumers is represented as $1-\alpha$. Furthermore, the new consumers' payment willingness is set as v, which follows the uniform distribution in the range [0, 1]. Therefore, the utility function for the new consumers purchasing new products is expressed as

$$U_n = v - p_r \tag{1}$$

As the new consumers will only purchase the products when their utility is greater than 0, we get the condition $v > p_r$ from Eq. (1). For the existing consumers, their willingness to pay for new products may vary significantly due to their distinct purchasing conditions, compared to the new consumers. The existing consumers, holding used products, will consider the residual value of the existing products before purchasing new ones. The lower the residual value of used products, signifying the greater wear and tear of used products, the stronger willingness of existing consumers to buy new products. Suppose that the wear and tear of the used product into depreciation relative to the new generation product and define that the depreciation degree of used product with the symbol γ, $0 < \gamma < 1$ (Huang et al., 2023). Based on the consumer value perception principle, it can be inferred that the existing consumers' willingness to exchange the used products for new ones is the residual value of used product, i.e., γv. Therefore, the utility function for the existing consumers purchasing new products is expressed as

$$U_o = \gamma v - (p_r - p_o) \tag{2}$$

Similar to the new consumers, we get another condition $v > (p_r - p_o)/\gamma$ from Eq. (2). According to the above conditions, the purchasing probabilities of the new consumers and the existing consumers are $1-p_r$ and $1-(p_r - p_o)/\gamma$, respectively. To guarantee the study's effectiveness, this paper narrowly confines its scope to the situation where both new and existing consumers purchase new products. Thus, the purchasing probabilities of the two types of consumers both need to be greater than zero, so there are constraints $p_r < 1$ and $p_r < \gamma + p_o$. Besides, in line with the fundamental premise that manufacturers gain from disposing of used products, the sum of the depreciated value of used products and the trade-in rebate should be less than 1, i.e., $\gamma + p_o < 1$. Additionally, the trade-in rebate p_o must fulfill $p_o < p_r$. Therefore, the total constraint condition is $p_o < p_r < \gamma + p_o < 1$. Based on the market share, the purchasing probabilities, the total constraint condition, and the demand functions are

$$\begin{cases} D_n = (1-\alpha)(1-p_r) \\ D_o = \alpha\left(1 - \dfrac{p_r - p_o}{\gamma}\right) \\ s.t.\ \ p_o < p_r < \gamma + p_o < 1 \end{cases} \tag{3}$$

The notations involved in this paper are shown in Table 1.

Table 1
Model notations

Notations	Meaning
i	$i = n, o$, where n denotes the new consumers and o denotes the existing consumers
j	$j = s, m$, where s denotes the supplier and m denotes the manufacturer
w	The wholesale price of semi-manufactured product
p_r	The direct selling price of manufactured product
p_o	The trade-in rebate
Z	The technological innovation investment cost
c_j	The production cost of j
δ	The spillover coefficient of technological innovation
η	The efficiency coefficient of technological innovation
α	The market share of the existing consumers
γ	The depreciation degree of used product
s	The salvage value of used product
v	The new consumers' payment willingness
D_i	The demand of i
π_j	The profit of j
CV_i	The compensating variation of i
V_i	The consumer surplus of i
E	The total consumer surplus

3. Model analysis

Based on the aforementioned two scenarios, this section develops corresponding game models. It delves into the optimal decision making of enterprises participating in the trade-in supply chain. This analysis lays the groundwork for subsequent discussions, encompassing alterations in business operations and consumer behavioral responses.

3.1. Model 1: no technological innovation strategy

Suppose the production cost of the supplier's semi-finished product as c_s and the production cost of the manufacturer's product as c_m . We obtain the profit functions of the supplier and the manufacturer respectively, as shown below.

$$\max \pi_s = (w - c_s)(D_n + D_o) \tag{4}$$
$$\max \pi_m = (p_r - w - c_m)D_n + (p_r - p_o - w - c_m + s)D_o \tag{5}$$

According to the sequential rule of Stackelberg game, the supplier initially determines the optimal pricing to maximize its own profit. Subsequently, the manufacturer responds to this pricing decision by formulating its own optimal decision accordingly. Using Eqs. (3)–(5) and applying the backward induction method, the following Theorem 1 can be obtained.

Theorem 1. Under no technological innovation strategy, the optimal decision making for the supplier and the manufacturer to achieve maximum profits is w^* and $(p_r^*,\ p_o^*)$ as

$$\begin{cases} w^* = \left[\alpha s + \gamma + \tau(c_s - c_m)\right] / 2\tau \\ p_r^* = \left[2\tau + \alpha s + \gamma + \tau(c_s + c_m)\right] / 4\tau \\ p_o^* = (1 + s - \gamma)/2 \end{cases} \tag{6}$$

In the Eq. (6), there exists symbolic expression $\tau = \alpha + \gamma - \alpha\gamma$. And elsewhere in the paper, τ stands for the same content as here.

Proof. See Appendix A.

Based on the optimal decision making above, the optimal demand of the new consumers and the existing consumers can be got as

$$D_n^*=(1-\alpha)\left[2\tau-\gamma-\alpha s-\tau(c_s+c_m)\right]/4\tau \tag{7}$$

$$D_o^*=\alpha\left[s(2\tau-\alpha)+\gamma(2\tau-1)-\tau(c_s+c_m)\right]/4\gamma\tau \tag{8}$$

The optimal profits of the supplier and the manufacturer can be got as

$$\pi_s^*=\left[\alpha s+\gamma-\tau(c_s+c_m)\right]^2/8\gamma\tau \tag{9}$$

$$\pi_m^*=\frac{1}{16\gamma\tau}\left\{\begin{array}{l}2\alpha\gamma(1-\alpha)\left[2(1-\gamma)^2-3s\right]+\alpha s(s+2\gamma)(4\tau-3\alpha)\\+\gamma^2+\tau(c_s+c_m)\left[\tau+2\gamma+2\alpha s+2\alpha\gamma(1-c_s-c_m)\right]\end{array}\right\} \tag{10}$$

According to Theorem 1, **Corollary 1** can be obtained.

Corollary 1. Under no technological innovation strategy, the optimal pricing p_r^* and w^* of the manufacturer and supplier generally exhibit a consistent trend of change in response to variations of the factors. They increase with the increase of s and γ, decrease with the increase of α, and are directly proportional to their own costs. However, their behavior diverges when considering the costs of the other party. Specifically, p_r^* is directly proportional to c_s, whereas w^* is inversely proportional to c_m. The trade-in rebate p_o^* is only related to s and γ. As s increases, it increases; however, as γ increases, it decreases.

Proof. The proof of Corollary 1 is straightforward by analyzing the first-order derivatives of each decision making with respect to each influencing factor, and is omitted.

From **Corollary 1**, some interesting conjectures can be derived:

(a) To ensure a competitive price in the market, compared to the supplier, the manufacturer may display a more pronounced inclination to collaborate with the supplier in strategies aimed at reducing production costs for both parties. Conversely, the supplier may try to encroach on the additional benefits from the cooperative cost reduction by raising wholesale price.

(b) The increased depreciation of the used products will push up pricing at various stages of the supply chain, and diminish the trade-in rebates for the existing consumers. Consequently, compared to the new consumers, the existing customers will suffer from double influence. To mitigate this, under the trade-in policy, they may make every effort to maintain and preserve their used products to sustain a relatively newer condition. In addition, since an increase in the market share of the existing consumers can lead to lower prices, the existing consumers would prefer the traded-in products to be mainstream rather than novel and rare.

Conclusion 1. Under no technological innovation strategy, to lower the retail price, the manufacturer should concentrate on reducing its own production cost. If a collaborative effort is pursued to cut costs with the supplier, it's crucial to guard against the risk of the supplier subsequently increasing the wholesale price. This hazard can be avoided through contracts, agreements, and other mechanisms to regulate the supplier's actions. To augment consumer satisfaction, the manufacturer could contemplate incorporating product maintenance services or training items into the spectrum of consumer service provisions, such as initiating a dedicated maintenance avenue, organizing complimentary product maintenance seminars. To stimulate consumer enthusiasm and participation in trade-in programs, the manufacturer should conduct market research on the market share of various products. Based on this, he can implement a trade-in policy, targeting popular products with a substantial market share among the existing customers.

3.2 Model 2: technological innovation strategy

Assuming the adoption of innovative technology, the production cost of the manufacturer is expected to decrease by Δc, satisfying $\Delta c>0$. Affected by technological innovation spillover, the reduction in the production cost of the supplier will be $\delta\Delta c$, where δ is the spillover coefficient of technological innovation, satisfying $0\le\delta\le 1$. Consequently, under technological innovation strategy, the actual production costs will be $\hat{c}_m=c_m-\Delta c$ and $\hat{c}_s=c_s-\delta\Delta c$, respectively.

Referring to the research by Biancardi and Villani (2018), we assume that there exists a function relation $Z = \frac{1}{2}\eta\Delta c^2$ between the technological innovation investment cost and the production cost, where η is the efficiency coefficient of technological innovation, satisfying $\eta > 0$. The smaller value for the parameter η indicates the higher innovation efficiency. From the function relation, we can get $\Delta c = \sqrt{2Z/\eta}$. Therefore, the production costs can vary as

$$\begin{cases} \hat{c}_m = c_m - \sqrt{2Z/\eta} \\ \hat{c}_s = c_s - \delta\sqrt{2Z/\eta} \end{cases} \tag{11}$$

Thus, we obtain the profit functions of the supplier and the manufacturer as

$$\max \hat{\pi}_s = \left[w - \left(c_s - \delta\sqrt{2Z/\eta}\right)\right](D_n + D_o) \tag{12}$$

$$\max \hat{\pi}_m = \left[p_r - w - \left(c_m - \sqrt{2Z/\eta}\right)\right]D_n + \left[p_r - w - p_0 + s - \left(c_m - \sqrt{2Z/\eta}\right)\right]D_o - Z \tag{13}$$

Applying the backward induction method, solve the optimal pricing of the manufacturer and supplier in the production and sale stage first. Then determine the optimal technological innovation investment cost decision of the manufacturer in the technological innovation stage. According to Equations (3), (12) and (13), Theorem 2 is obtained by applying the backward induction method.

Theorem 2. Under technological innovation strategy, aiming to achieve maximum profits, the two participants in the supply chain formulate their decision making as

$$Z^* = \frac{\eta(1+\delta)^2\left[\alpha s + \gamma - \tau(c_m + c_s)\right]^2}{2\left[8\eta\gamma - \tau(1+\delta)^2\right]^2} \tag{14}$$

$$\hat{w}^* = \frac{\begin{Bmatrix} \gamma^2 c_s\left[\delta\alpha^2 - (1-\alpha)^2\right] + (1+\delta)\left[\delta\tau(c_m\tau - \alpha s - \gamma) - \alpha^2 c_s\right] \\ -2\alpha\gamma c_s(1+\delta)(1-\tau) - 4\eta\gamma\left[\tau(c_m - c_s) - \alpha s - \gamma\right] \end{Bmatrix}}{\tau\left[8\eta\gamma - \tau(1+\delta)^2\right]} \tag{15}$$

$$\begin{cases} \hat{p}_r^* = \dfrac{4\eta\gamma\left[\alpha s + \gamma + \tau(2 + c_s + c_m)\right] - \tau(1+\delta)^2(\alpha s + \tau + \gamma)}{2\tau\left[8\eta\gamma - \tau(1+\delta)^2\right]} \\ \hat{p}_o^* = (1 + s - \gamma)/2 \end{cases} \tag{16}$$

Proof. See Appendix A.

According to the optimal decision making and Eq. (3), the optimal demand of the new consumer and the existing consumer can be deduced as

$$\hat{D}_n^* = \frac{(1-\alpha)\left[4\eta\gamma(c_s + c_m) + \alpha(1+\delta)^2(1-\gamma-s)\right]}{2\left[8\eta\gamma - \tau(1+\delta)^2\right]} \tag{17}$$

$$\hat{D}_o^* = \frac{4\eta\alpha\gamma\left[2(1+\gamma+s) - (c_s + c_m)\right] + \alpha\left[\gamma(1-\tau) - s(\alpha+\tau)\right](1+\delta)^2}{2\gamma\left[8\eta\gamma - \tau(1+\delta)^2\right]} \tag{18}$$

Based on the demand function in Eq. (17), and considering the physical meaning that customer demand must be positive in actual business operations, we introduce the constraint condition $8\eta\gamma - \tau(1+\delta)^2 > 0$ to ensure the rationality of the solution. All subsequent discussions will be conducted under this constraint. Similarly, the optimal profits of the two participants in the supply chain can be got as

$$\hat{\pi}_s^* = \frac{8\eta^2\gamma\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]^2}{\tau\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]^2} \tag{19}$$

$$\hat{\pi}_m^* = \frac{\left\{\begin{array}{l} 8\eta\gamma\left(1-\alpha\right)\left[\alpha\left(1-\gamma\right)^2+s^2\right]+2\eta\left(c_s+c_m\right)^2\left[\alpha^2\left(1+\gamma\right)^2+\gamma\left(2\tau-\gamma\right)\right] \\ -2\eta\left[2\alpha\gamma s\left(3-4\tau\right)+2\tau\left(\gamma+\alpha s\right)\left(c_s+c_m\right)-\alpha^2 s^2-\gamma^2\right]-\alpha\left(1+\delta\right)^2 \\ \left[\tau\left(1-\alpha+\alpha\tau\right)+s\left(2-s\right)\left(\alpha^2-\tau\right)-2s\gamma^2\left(1-\alpha^2\right)+4\gamma s\left(\tau-\alpha^2\right)\right] \end{array}\right\}}{4\tau\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]} \tag{20}$$

Then we can derive Corollary 2 and Corollary 3.

Corollary 2. Under technological innovation strategy, the decision of trade-in rebate $\hat{p}_o^*$ offered to the existing consumer by the manufacturer is similar to the situation described in Corollary 1, depending only on s and γ. However, the specific variations in Z^* and $\hat{p}_r^*$ with changes in technological innovation factors (i.e. δ and η), cost factors (i.e. c_s and c_m), and trade-in factors (i.e. α and γ) are as follows:

(1) Z^* increases with δ and decreases with η. $\hat{p}_r^*$ decreases with δ and increases with η in the situation of $\tau \in (0,\overline{\tau_2})$, and the relationships of them are inverse in the situation of $\tau \in (\overline{\tau_2},1]$.

(2) Z^* decreases with c_s, c_m, and increases with s in the situation of $\tau \in (0,\overline{\tau_2})$, but the relationships of them are inverse in the situation of $\tau \in (\overline{\tau_2},1]$. $\hat{p}_r^*$ increases with c_s and c_m, and the rate of changes of them are the same. Furthermore, $\hat{p}_r^*$ increases with s in the situation of $\tau \in (0,4\overline{\tau_1})$, and decreases with s in the situation of $\tau \in (4\overline{\tau_1},1]$.

(3) Z^* increases with α when $f_1<1$, and it increase with γ when $f_2>1$. $\hat{p}_r^*$ increases with α when $f_3>0$, and increases with γ when $f_4>0$. Conversely, they are the opposite.

In the above content, there exists symbolic expressions $\overline{\tau_1}=\frac{\eta\gamma}{\left(1+\delta\right)^2}$, $\overline{\tau_2}=\frac{\alpha s+\gamma}{c_s+c_m}$,

$$f_1=\frac{\left(1-\gamma\right)\left(1+\delta\right)^2\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]^2}{\left[\left(1-\gamma\right)\left(c_s+c_m\right)-s\right]\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]},\quad f_2=\frac{\left[\left(1-\alpha\right)\left(1+\delta\right)^2-8\eta\right]\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]^2}{\left[\left(1-\alpha\right)\left(c_s+c_m\right)-1\right]\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]},$$

$$f_3=\frac{4\eta s\gamma+\left(1-\gamma\right)\left(2+c_s+c_m\right)-\left[s\gamma+\left(1-\gamma\right)\left(2\alpha s+2\tau+\gamma\right)\right]\left(1+\delta\right)^2}{4\eta\gamma\left[\alpha s+\gamma+\tau\left(2+c_s+c_m\right)\right]-\tau\left(\alpha s+\tau+\gamma\right)\left(1+\delta\right)^2}-\frac{2\left(1-\gamma\right)\left[4\eta\gamma-\tau\left(1+\delta\right)^2\right]}{\tau\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]},$$

$$f_4=\frac{4\eta\left[\alpha s+2\gamma+\left(2\tau-\alpha\right)\left(2+c_s+c_m\right)\right]-\left[\alpha+\left(1-\alpha\right)\left(\alpha s+2\tau+2\gamma\right)\right]\left(1+\delta\right)^2}{4\eta\gamma\left[\alpha s+\gamma+\tau\left(2+c_s+c_m\right)\right]-\tau\left(\alpha s+\tau+\gamma\right)\left(1+\delta\right)^2}-\frac{8\eta\left(2\tau-\alpha\right)-\left(1-\alpha\right)\left(1+\delta\right)^2}{\tau\left[8\eta\gamma-\tau\left(1+\delta\right)^2\right]}.$$

Proof. The proof of Corollary 2 can be readily derived by taking the first-order partial derivative of Z^* with respect to each influencing factor. Therefore, the detailed steps are omitted for brevity.

Corollary 3. As $\hat{w}^*$ set by the supplier is an increasing function of Z, the relationships of $\hat{w}^*$ with all the factors such as δ, η are consistent with the situation of Z^*.

Proof. It is straightforward to obtain $\frac{\partial w}{\partial Z}=\frac{1}{2}\left(1-\delta\right)>0$ and confirm that w is proportional to Z, when $w=\frac{\alpha s+\gamma+\tau\left[c_s-c_m+\sqrt{2Z/\eta}\left(1-\delta\right)\right]}{2\tau}$ is given by the proof of Theorem 2.

Based on Corollary 2 and 3, we observe the following particular phenomena:

(a) The trade-in rebate offered by the manufacturer is unaffected by the technological innovation decision and its related factors. This indicates that the rebate amount remains consistent for the existing consumers, irrespective of whether the manufacturer implements a technological innovation strategy or not.

(b) The increase of technological innovation spillover will lead the manufacturer to invest more in technological innovation, and also results in the manufacturer facing a higher wholesale price imposed by the supplier.

(c) The direct selling price is not solely influenced by the manufacturer's own cost. Instead, it is jointly influenced by the costs of all members in the supply chain with the same degree effect.

Conclusion 2. After implementing the technological innovation strategy, the manufacturer does not need to adjust the trade-in rebate. This is helpful for stabilizing the market. However, since the direct selling price is influenced by the overall supply chain cost, the manufacturer should actively collaborate with his supply chain partner to collectively reduce the costs, thereby stabilizing or reducing the direct selling price. It is noteworthy that if the manufacturer intends to assist the supplier in cost reduction through increased innovation spillover, he should be prepared for a higher technological innovation investment fund. In addition, alternative approaches and channels such as joint procurement and collaborative production can be explored to cut down the costs.

4. Decision comparison and effects analysis

In this section, we will delve into the decision making changes of supply chain participants and consumer reactions towards technological innovation. The objective is to investigate the effects of technological innovation on trade-in marketing and scrutinize the relationships between those effects and influencing factors. This endeavor aims to furnish valuable insights for businesses and government practices.

4.1 Changes in enterprises' decision making and profits

Based on Theorem 1 and Theorem 3, we can infer the changes in the enterprises' decision making after the implementation of technological innovation, and get some corollaries and conclusions are as follows.

Corollary 4. After the implementation of technological innovation strategy, significant changes have occurred in the manufacturer's optimal decision making, manifesting as follows:

(1) The change in the direct selling price in the market is

$$\Delta p_r = \hat{p}_r^* - p_r^* \begin{cases} <0, & \text{if } \gamma \in \left(0, \dfrac{\alpha(c_s + c_m - s)}{1-(1-\alpha)(c_s + c_m)}\right) \\ =0, & \text{if } \gamma = \dfrac{\alpha(c_s + c_m - s)}{1-(1-\alpha)(c_s + c_m)} \\ >0, & \text{if } \gamma \in \left(\dfrac{\alpha(c_s + c_m - s)}{1-(1-\alpha)(c_s + c_m)}, 1\right) \end{cases}.$$

(2) The trade-in rebate is unaffected by the technological innovation, aligning with the original decision, i.e., $\Delta p_o = \hat{p}_o^* - p_o^* = 0$.

(3) Since $Z=0$ under no technological innovation strategy, the technological innovation investment cost consistently takes the form of an increase, i.e., $\Delta Z = Z^* > 0$.

Proof. According to Eq. (6) and Eq. (16), we get $\Delta p_r = -\dfrac{(1+\delta)^2\left[\alpha s + \gamma - (\alpha + \gamma - \alpha\gamma)(c_s + c_m)\right]}{4\left[8\eta\gamma - \tau(1+\delta)^2\right]}$. Since $8\eta\gamma - \tau(1+\delta)^2 > 0$ is known, it is straightforward to verify the point (1) in Corollary 4 with analysis of the positive and negative value situation for $\alpha s + \gamma - (\alpha + \gamma - \alpha\gamma)(c_s + c_m)$. The proof of points (2) and (3) is straightforward and is omitted.

Corollary 5. As $\Delta Z = Z^*$, the relationships between ΔZ and the influencing factors are the same to those described in Corollary 2 with regard to Z^*. And that for Δp_r manifests as follows:

(1) Δp_r decreases with δ and increases with η in the situation of $\tau \in (0, \overline{\tau_2})$, but they are the opposite relationships in the situation of $\tau \in (\overline{\tau_2}, 1]$.

(2) If $\delta < 2\sqrt{\frac{2\eta\left[(1-\gamma)(c_s+c_m)-s\right]}{1-\gamma-s}}-1$, Δp_r increases with α. And if $\delta > 2\sqrt{\frac{2\eta(c_s+c_m-s)}{1+\alpha s-s}}-1$, Δp_r increases with γ. Conversely, they will be the opposite.

(3) Δp_r always increase with c_m and c_s, and always decreases with s.

Proof. The proof of Corollary 5 can be readily derived by taking the first-order partial derivative of Δp_r with respect to each influencing factor. Therefore, the detailed steps are omitted for brevity.

Combining Corollaries 4 and 5, two observations can be obtained:

(a) The technological innovation strategy does not necessarily lead to an increase in product price. But the direct selling price generally changes. The manufacturer may implement price increase or decrease depending on specific circumstances. The state of depreciation in existing consumers' used products has a crucial effect on this. When the depreciation is high, the technological innovation strategy may lead to price increase, while it may result in price decrease when the depreciation is low.

(b) The magnitude of change in direct selling price is further influenced by influencing factors. Under certain conditions, a rise in technological innovation spillover and improved innovation efficiency can intensify this change. Interestingly, when the spillover of technological innovation is large, an increase in the market share of the existing consumers and accelerated depreciation of used products tend to mitigate this change. Notably, the trade-in rebate is unaffected by the technological innovation strategy, which further supports to the actions to be taken by the manufacturers mentioned in conclusion 2.

Corollary 6. After implementing the technological innovation strategy, the adjustment of the supplier's wholesale price tends to differ significantly from that of the manufacturer's direct selling price. The only similarity is that if $\gamma=\frac{\alpha(c_s+c_m-s)}{1-(1-\alpha)(c_s+c_m)}$, there exists $\hat{w}^*=w^*$. In addition, the relationships between Δw and the influencing factors are as follows:

(1) If $\tau \in \left(8\delta\overline{\tau_1},\ \overline{\tau_2}\right)$, Δw increases with δ, and it is opposite in other cases. If $\tau \in (\overline{\tau_2},1]$, Δw increases with η, and it is opposite in other cases.

(2) If $\delta > 2\sqrt{\frac{2\eta\left[(1-\gamma)(c_s+c_m)-s\right]}{1-\gamma-s}}-1$, Δw increase with α. It is opposite in other cases. If $\delta < 2\sqrt{\frac{2\eta(c_s+c_m-s)}{1+\alpha s-s}}-1$, Δw increases with γ. It is opposite in other cases.

(3) Δw always decreases with c_m and c_s, and always increases with s.

Proof. According to Equations (6) and (15), we get $\Delta w=\frac{(1-\delta^2)\left[\alpha s+\gamma-(\alpha+\gamma-\alpha\gamma)(c_s+c_m)\right]}{2\left[8\eta\gamma-\tau(1+\delta)^2\right]}$. Compared with Δp_r in the proof of Corollary 4, Δw is the opposite of it. Therefore, it is straightforward to verify the difference and similarity between Δw and Δp_r. In addition, the proof of the relationships between Δw and the influencing factors can be readily derived by taking the first-order partial derivatives of Δw with respect to the influencing factors. Therefore, the detailed steps are omitted for brevity.

Upon comparing Corollaries 4 to 6, two noteworthy findings emerge:

(a) After implementing the technological innovation strategy, the supplier will also implement price increase or decrease depending on specific circumstances. However, his actions are significantly different from those of the manufacturer. Specifically, the supplier tends to decrease the wholesale price when the depreciation of used products is high, whereas he will increase it when the depreciation is low. This leads to an increase in unit benefit for the manufacturer in the former case, and the opposite in the latter case.

(b) Although the relationships between the magnitude of change in the wholesale price and the influencing factors are mathematically opposite to the situation of the direct selling price, the meanings embodied by them are consistent in practical terms. They all reflect a state of intensification or mitigation, as observed by analyzing their absolute values.

Conclusion 3. After implementing the technological innovation strategy, the supplier and manufacturer may choose to adjust their prices upwards or downwards depending on the specific circumstances, yet their actions exhibit significant differences, even diverge. This leads to extreme profit outcomes for both sides. While this reflects the optimal decision in the competitive game between them, pursuing cooperation for Pareto improvement would be a better outcome for both sides. Additionally, they should vigilantly monitor the magnitude of price changes and, if necessary, adopt reasonable measures such as modifying packaging and introducing gradual price adjustments to alleviate consumers' resistance caused by significant price fluctuations.

Continuing to observe changes in enterprises' profits, the effects of implementing the technological innovation strategy on enterprises' profits can be assessed. Based on Theorem 1 and Theorem 3, the following corollaries and conclusions can be derived.

Corollary 7. After implementing technological innovation strategy, the optimal profits of the supplier, the manufacturer, and the whole supply chain will undergo corresponding changes, as shown in Table 2, where $\Delta\pi_s = \hat{\pi}_s^* - \pi_s^*$, $\Delta\pi_m = \hat{\pi}_m^* - \pi_m^*$, $\Delta\pi = \left(\hat{\pi}_s^* + \hat{\pi}_m^*\right) - \left(\pi_s^* + \pi_m^*\right)$.

Table 2
Profit changes

Category	$\Delta\pi_s$	$\Delta\pi_m$	$\Delta\pi$
Difference	+	+	+

Note. "+" means that the difference is greater than 0.

Proof. See Appendix A.

From Corollary 7, it can be inferred that implementing technological innovation strategy can lead to greater profits for all members in the supply chain. Even though the supplier does not directly participate in technological innovation, he still benefits from it. It can be surmised that, in practical application, the supplier, motivated by the allure of profit, would vigorously encourage the manufacturer to engage in technological innovation and development to ride along, aiming to seek greater benefit for him.

Corollary 8. According to Corollary 7, the profits of enterprises will increase. However, the magnitude of the increase varies due to factors, as detailed in Table 3.

Table 3
Relationships between profit increase and influencing factors

Conditions	profit increase and influencing factors											
	$\Delta\pi_s$				$\Delta\pi_m$				$\Delta\pi$			
	δ	η	c_j	s	δ	η	c_j	s	δ	η	c_j	s
$\tau < \overline{\tau_2}$	↗	↘	↘	↗	↗	↘	↘	↗	↗	↘	↘	↗
$\tau > \overline{\tau_2}$	↗	↘	↗	↘	↗	↘	↗	↘	↗	↘	↗	↘

Notes. $j = s, m$. $\overline{\tau_2} = (\alpha s + \gamma)/(c_s + c_m)$. "↗"means increase and "↘"means decrease.

Proof. The proof of Corollary 8 is straightforward by further analysis of the first-order derivatives of each profit increase with respect to each influencing factor based on the proof of Corollary 7. Therefore, the detailed steps are omitted for brevity.

Based on Table 3, it can be observed that in terms of the relationships with various factors, $\Delta\pi_s$, $\Delta\pi_m$ and $\Delta\pi$ consistently demonstrate alignment. But the effects differ by different factor. $\Delta\pi_s$, $\Delta\pi_m$ and $\Delta\pi$ are all positively correlated with δ, and negatively correlated with η. The relationships between them with c_j are constrained by the relationship of size between τ and $\overline{\tau_2}$, resulting in diametrically opposed outcomes. If $\tau < \overline{\tau_2}$, they exhibit a negative correlation, while if $\tau > \overline{\tau_2}$, they show a positive correlation. Additionally, the relationships they have with s is in stark contrast to the one they have with c_j, exhibiting an opposing pattern.

From Corollary 8, it is found that a consistent rise in the spillover coefficient of technological innovation invariably promotes a sustained augmentation of the manufacturer's profit. This indicates that the innovation spillover enables the manufacturer to attain higher profit, serving as a beneficial factor for the manufacturer. On the contrary, an increase in the efficiency coefficient of technological innovation results in a decline in the manufacturer's profit. Hence, regarding technological innovation spillover and supplier free-riding, the manufacturer does not need to be overly concerned. Instead, he should focus on how to improve the innovation efficiency, i.e., reducing the efficiency coefficient of technological innovation.

To more intuitively analyze the driving effect of the trade-in factors α and γ on the changes in enterprises' profits, without loss of generality, we set $c_s = 0.2$, $c_m = 0.4$, $s = 0.05$, $\delta = 0.03$, and $\eta = 0.5$. Under feasible solution conditions, we get the relationships between $\Delta\pi_s$, $\Delta\pi_m$ and $\Delta\pi$ with α and γ respectively, as shown in Fig. 2 and Fig. 3.

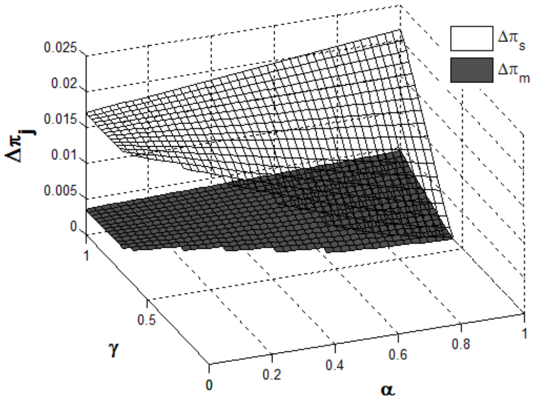


Fig. 2. The relationships between $\Delta\pi_j$ with α, γ

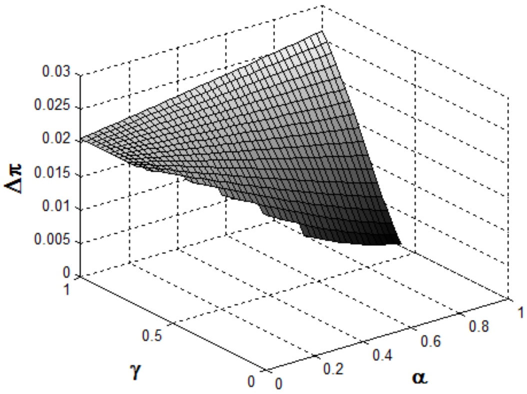


Fig. 3. The relationships between $\Delta\pi$ with α, γ

From Fig. 2 and Fig. 3, there are some observations:

(a) After implementing the technological innovation strategy, the supplier experiences a substantially larger profit increase compared to the manufacturer, potentially motivating him to encourage the manufacturer to carrying out technological innovation for free-riding.

(b) The profit increase for the supplier and the manufacturer show similar relationships with α and γ. They increase with γ. When γ is relatively small, they decrease with α. But when γ is relatively large, they increase with α. This occurs because as the increase in the market share of the existing consumers reduces the market share of the new consumers, thereby mitigating the growth in enterprises' profits. However, when the depreciation of the used products is severe, the existing consumers are more inclined to upgrade their products. In such situation, the increased market share of the existing consumers offsets and even surpasses the profit losses resulting from the reduced market share of the new consumers.

(c) The profit increase of the whole supply chain follows the performance of the supplier and the manufacturer.

Conclusion 4. In the context of trade-in, further implementing the technological innovation strategy is a win-win plan for both the supplier and the manufacturer. The anticipated surge in profit for the supplier will foster a profound interest in the strategy. Consequently, the manufacturer should mobilize the enthusiasm of the supplier to engage in the strategy, fostering collaborative innovation between both parties, thereby avoiding the risks resulting from independent technological innovation. In addition, even though the supplier and manufacturer adopt different operation decision after implementing the strategy, their profit changes and relationships with the influencing factors tend to be consistent. Therefore, they can collaborate to investigate change trends of the influencing factors and mitigate some adverse effects from them.

4.2 Consumers' reactions

Consumers' reactions to technological innovation is primarily reflected in their compensating variation, as well as changes in demand and surpluses. The following will focus on exploring the effects of the strategy on consumers around the three aspects, aiming to analyze the consumers' responses.

Compensating variation, also known as income compensation change, is a method of measuring utility change from a monetary perspective. It refers to the necessary income change (increase or decrease) that compensates consumers to return to their original consumption state (i.e., the same utility as before). This income change exactly compensates for the effect of price change on consumers. According to Eq. (1) and Eq. (2), we get the compensating variation of the two type consumers regarding to an increase in the direct selling price after implementing the strategy (see Fig. 4). In the figure, CV_n can be calculated by $CV_n = U_n^* - \hat{U}_n^*$, and CV_o is calculated by $CV_o = U_o^* - \hat{U}_o^*$. Moreover, as both the slopes are the same, there is $CV_n = CV_o$.

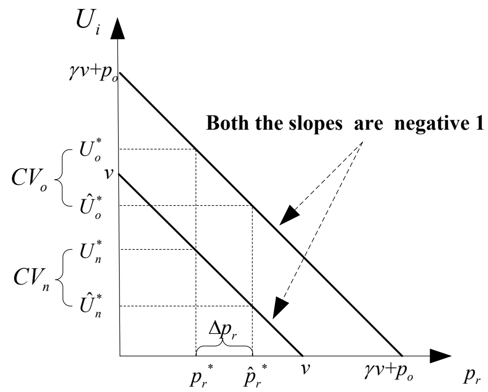


Fig. 4. The compensating variation (*CV*) of consumers

From Fig. 4, we extract two key contents. Firstly, upon the introduction of technological innovation, both the new consumers and existing consumers experience identical change in compensation. This signifies that their gains or losses from transactions are equivalent, bringing about a uniform reaction to technological innovation. Secondly, after implementing technological innovation, if the manufacturer increases the direct selling price, both the two types of consumers must increase their income to counterbalance the negative effect of it. This is prone to elicit consumer resistance, which may in turn hinder the sale of the new products. Conversely, if the price is reduced, new sales are propitious to transaction gains for the consumers, triggering a positive market response.

Conclusion 5. The reaction of the new consumers towards technological innovation is the same as that of the existing consumers, unaffected by no rebate for purchasing. Upon a price hike, both the two types of consumers incur transaction losses. Therefore, when the manufacturer decides to increase price, it will be better for him to additionally introduce a transitional and unified shopping welfare policy for both the new and existing consumers in the early stage of the new product launch. This policy could include allowances, rebates, or other benefits equal to the compensation change, enabling consumers to sustain their previous consumption levels temporarily. It helps compensate for the consumer losses caused by the price increase, alleviate consumer resistance to innovation, and smoothly pull through the difficulty of raising prices.

Corollary 9. After implementing the technological innovation strategy, the changes in demand of the two types of consumers are shown in Table 4, where $\Delta D_o = \hat{D}_o^* - D_o^*$ and $\Delta D_n = \hat{D}_n^* - D_n^*$. Moreover, we find $\frac{\Delta D_o}{\Delta D_n} = \frac{\alpha}{\gamma(1-\alpha)}$.

Table 4
Changes in demand of both types of consumers

Conditions	ΔD_o	ΔD_n
$\tau < \overline{\tau_2}$	+	+
$\tau = \overline{\tau_2}$	0	0
$\tau > \overline{\tau_2}$	–	–

Notes. "+" means the difference is greater than 0. "–" means the difference is less than 0.

Proof. See Appendix A.

From Corollary 9, there are some findings:

(a) After implementing technological innovation, the increase or decrease in demand of product for both types of consumers is influenced by the same conditions. Only when $\tau < \overline{\tau_2}$, will the demand of both types of consumers increase and consumers respond positively to technological innovation.

(b) By rearranging $\frac{\alpha}{\gamma(1-\alpha)}$, it can convert into $\frac{\alpha/\gamma}{1-\alpha}$. Thus from a comparative perspective, it can be deduced that the changes in demand of both types of consumers are inherently linked to their respective original states, referring to the situations they were in prior to the implementation of the technological innovation strategy. That is, the demand of the existing consumers is influenced by α/γ, and that of the new consumers is influenced by $1-\alpha$. Moreover, they both proportionally fluctuate with the factors of these original states. Therefore, it indicates that, upon the introduction of technological innovation, only the original states have an effect on the demand changes of consumers, reflecting the consumers' reactions. Both types of consumers themselves do not exhibit additional response to the technological innovation strategy. Therefore, both types of consumers exhibit the same acceptance and reaction to the strategy in terms of their internal feelings. That further validates

Conclusion 5.

(c) The comparison of the size between ΔD_o and ΔD_n hinges on α and γ. Therefore, for the manufacturer embarking on technological innovation, the status importance of the two types of consumers is related to the original market share and the depreciation level of used products.

Corollary 10. The relationships between ΔD_o, ΔD_n and influencing factors are exactly opposite to the situation of Δp_r in Corollary 5. Moreover, the proportion of the partial derivatives of ΔD_o, ΔD_n with respect to δ is the same as that to η, that is, $\frac{\alpha}{\gamma(1-\alpha)}$.

Proof. The proof of Corollary 10 is straightforward by further analysis of the first-order derivatives of each demand change with respect to each influencing factor based on the proof of Corollary 9. The detailed steps are omitted for brevity.

In Corollary 10, the changes in demand for both types of consumers are influenced by various factors in a manner opposite to the situation of selling price. This aligns with the economic phenomenon where price and demand typically exhibit an inverse relationship. Furthermore, based on $\frac{\alpha}{\gamma(1-\alpha)}$, it is evident that, from a comparative perspective, the growth rate of demand resulting from technological innovation factors (i.e., δ and η) is related to the original states. This indicates that, compared to the scenario of trade-in program with no technological innovation, the changes in demand will be influenced by the trade-in factors (i.e., α and γ) again, besides technological innovation factors. Therefore, it can be clearly that trade-in will interact with technological innovation, giving rise to a compounded effect on consumers.

Conclusion 6. Implementing technological innovation does not necessarily have a positive market response. It may also lead to consumer innovation resistance, a situation shaped by external conditions such as consumer market proportion and depreciation state of used products. To alleviate this resistance, gain consumers' support, and foster successful technological innovation implementation, enterprises should undertake a thorough analysis of pertinent conditions beforehand. Furthermore, while the nature of technological innovation is fundamentally similar for both types of consumers, the factors of trade-in and technological innovation will interact to produce a compounded effect, magnifying the effects of technological innovation on the consumers. This interaction will also affect the significance of both types of consumers in the implementation of technological innovation. Therefore, before implementing technological innovation, the manufacturer should clearly identify the primary consumer type they should serve, based on the consumer market share and depreciation of used products. Then they can innovate and improve products according to the needs and preferences of these consumers, and align later sales, advertising, and strategies accordingly.

The following will further analyze consumers' reactions towards technological innovation from the perspective of consumer surplus welfare.

Consumer surplus represents the difference between the maximum price a consumer is prepared to pay for a specific good or service and the actual price they pay in the market.

Consumer surplus refers to the difference between the maximum price a consumer is willing to pay for a certain good and the actual market price. According to the concept, combining Theorem 1 with Eq. (7) and Eq. (8), and applying the Newton-Leibniz method, the consumer surpluses of the two types of consumers can be got under no technological innovation strategy as

$$V_n^* = \int_0^{D_n^*} p_r^*\left(D_n^*\right) dD_n^* - p_r^* D_n^* = \frac{D_n^{*2}}{2(1-\alpha)} = \frac{(1-\alpha)\left[2\tau - \alpha s - \gamma - \tau\left(c_s + c_m\right)\right]^2}{32\tau^2} \tag{21}$$

$$V_o^* = \int_0^{D_o^*} \left[\gamma\left(1 - \frac{D_o^*}{\alpha}\right) + p_o^*\right] dD_o^* - p_r^* D_o^* = \frac{\gamma D_o^*}{2\alpha} = \frac{\tau\left(2s + 2\gamma - c_s - c_m\right) - \alpha s - \gamma}{8\tau} \tag{22}$$

Similarly, under technological innovation strategy, the consumer surpluses can be got as

$$\hat{V}_n^* = \frac{\hat{D}_n^{*2}}{2(1-\alpha)} = \frac{(1-\alpha)\left[4\eta\gamma\left(c_s + c_m\right) + \alpha(1-\gamma-s)(1+\delta)^2\right]^2}{8\left[8\eta\gamma - \tau(1+\delta)^2\right]^2} \tag{23}$$

$$\hat{V}_o^* = \frac{\gamma \hat{D}_o^*}{2\alpha} = \frac{4\eta\gamma\left[2(1+s+\gamma)-c_s-c_m\right]+\left[\gamma(1-\tau)-s(\alpha+\tau)\right](1+\delta)^2}{4\left(8\eta\gamma-\tau(1+\delta)^2\right)} \tag{24}$$

Based on Eqs. (21)–(24), we obtain Corollary 11 as follow.

Corollary 11. Only when $\tau < \overline{\tau_2}$, there are $\hat{V}_n^* > V_n^*$, $\hat{V}_o^* > V_o^*$ and $\hat{E} > E$, where $E = V_n^* + V_o^*$ and $\hat{E} = \hat{V}_n^* + \hat{V}_o^*$. Conversely, the opposite holds.

Proof. See Appendix A.

According to Corollary 11, both types of consumers will actively respond to the technological innovation strategy undertaken by the enterprise only with certain conditions. This once again corroborates the consumer attitudes towards technological innovation as discussed in Conclusion 6. Therefore, whether from the perspective of changes in demand or consumers' welfare, consumers consistently exhibit a conditional positive response to technological innovation.

Conclusion 7. While it is financially advantageous for enterprises to further implement technological innovation under a trade-in program, such a move can trigger a decline in the consumer market if consumers resist such innovation, ultimately having detrimental effects on long-term growth. Therefore, from a buyer-seller interest-balancing perspective, enterprises should patiently await for the maturity of corresponding conditions and timing before implementing technological innovation, as this approach is more acceptable to consumers.

To understand the relationships between total consumer surplus and technological innovation factors under technological innovation strategy, relevant relationships are established under the feasible solution conditions and basic parameter settings, as illustrated in Fig. 5 and Fig. 6.

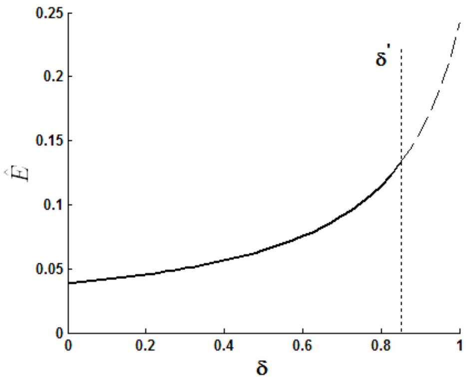


Fig. 5. The relationship between $\hat{E}$ and δ when $\eta = 0.7$

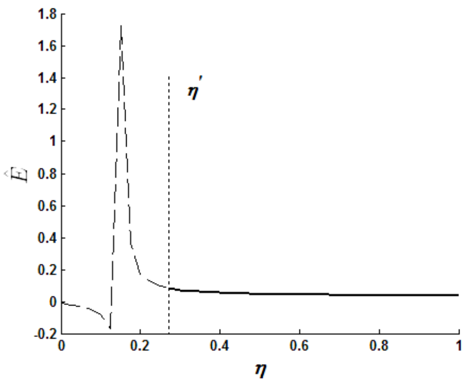


Fig. 6. The relationship between $\hat{E}$ and η when $\delta = 0.03$

In Fig. 5 and Fig. 6, δ' and η' represent the critical points for feasible region. The dashed portions of the figures indicate the absence of feasible solutions, which are not within the scope of the study. From the figures, two key insights emerge. Firstly, the spillover coefficient of technological innovation has a positive effect on the total consumer surplus, suggesting that the spillover of technological innovation consistently benefits consumers. This spillover is a positive factor for both enterprises and consumers. Secondly, the efficiency coefficient of technological innovation negatively affects consumer surplus. That is because a rise in this coefficient actually signifies a decrease in the enterprise's innovation efficiency and an increase in technological innovation investment cost. Thus it indirectly leads to a higher selling price, causing consumers to resist innovation.

Conclusion 8. Innovation spillover, when augmented, bolsters consumer backing for technological innovation. Conversely, a dip in innovation efficiency triggers consumer resistance to such innovation. Combining this with Corollary 8, the manufacturer undertaking technological innovation ought to consistently boost innovation spillover, elevate the comprehensive innovation capability of the supply chain, and pay attention to improving technological innovation efficiency to gain greater support from diverse stakeholders.

5. Conclusion

In the context of manufacturers adopting a trade-in program, this paper undertakes a comparative analysis to examine alterations in the operation decision and the profits for supply chain participants, and consumer reactions, after the implementation of technological innovation. The aim is to explore the effects of enterprise's technological innovation behavior in the marketing of trade-in programs and investigate the relationships between the effect variations and influencing factors.

The following are the main findings and managerial insights derived:

(1) Under the trade-in program, further implementing technological innovation is a win-win plan for the supplier and manufacturer. Nevertheless, it may trigger consumer innovation resistance, leading to a contraction of the consumer market and a decline in consumer surplus. Therefore, in practical business operations, enterprises should balance consumer experience, corporate social responsibility, and their own profits to prudently determine the timing for implementing technological innovation.

(2) Both new and existing consumers exhibit the same level of acceptance and reaction towards technological innovation. However, under technological innovation strategy, the original trade-in factors are amplified in their influence on consumer purchases, creating a compounding effect through the synergistic interaction of the trade-in and technological innovation. This ultimately shapes the positions of the two types of consumers in the implementation of the technological innovation strategy. Therefore, a high degree of the depreciation of used products or a high market share of the existing consumers can highlight the advantages of technological innovation. Enterprises should assess the specific circumstances of consumer market proportion and the depreciation status of used products, clarify the importance of the consumer types, and accordingly conduct targeted innovation research and development as well as marketing efforts.

(3) Under technological innovation strategy, innovation spillover and efficiency improvement in the supply chain are beneficial to enterprises and consumers. In operation, manufacturers should actively cooperate with suppliers to comprehensively promote the overall innovation of the supply chain.

This paper discusses the issue of technological innovation effects for enterprises in the context of trade-in programs, offering valuable insights and guidance for relevant enterprises and governments. Future research may potentially delve deeper into collaborative technological innovation among enterprises in the trade-in supply chain, or broaden the scope to encompass multiple sales cycles, like taking two cycles as the research focus.

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Declarations

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Appendix A. Proofs of theorems and corollaries

Proof of Theorem 1. Solve the optimal decision making problem using the backward induction method. Firstly, according to Equation (5), we get Hessian matrix of π_m with respect to p_r and p_o to be $H = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial {p_r}^2} & \frac{\partial^2 \pi_m}{\partial p_r p_o} \\ \frac{\partial^2 \pi_m}{\partial p_o p_r} & \frac{\partial^2 \pi_m}{\partial {p_o}^2} \end{bmatrix} = \begin{bmatrix} -2(1-\alpha)-\frac{2\alpha}{\gamma} & \frac{2\alpha}{\gamma} \\ \frac{2\alpha}{\gamma} & -\frac{2\alpha}{\gamma} \end{bmatrix}$. Then by obtaining the first-order principal matrix $|H_1| = -2(1-\alpha) - \frac{2\alpha}{\gamma} < 0$ and the second-order principal matrix $|H| = \frac{4\alpha(1-\alpha)}{\gamma} > 0$, it is confirmed that π_m is a strictly joint convex function of p_r and p_o. Secondly, the first-order partial derivatives of π_m with respect to p_r and p_o are $\frac{\partial \pi_m}{\partial p_r} = 1+(1-\alpha)(w+c_m-2p_r)-\frac{2\alpha(p_r-p_o)-\alpha(w+c_m-s)}{\gamma}$ and $\frac{\partial \pi_m}{\partial p_o} = \alpha\left(\frac{2(p_r-p_o)-(w+c_m-s)}{\gamma}-1\right)$, respectively. Let $\frac{\partial \pi_m}{\partial p_r}=0$ and $\frac{\partial \pi_m}{\partial p_o}=0$, we obtain the results $p_r = \frac{1+w+c_m}{2}$ and $p_o = \frac{1+s-\gamma}{2}$ by solving them jointly. Substituting the above two results into Equation (4), we get $\pi_s = \frac{(w-c_s)\left[\alpha s+\gamma-\tau(c_m+w)\right]}{2\gamma}$ and the second-order partial derivative $\frac{\partial^2 \pi_s}{\partial w^2} = -(1-\alpha) - \frac{\alpha}{\gamma} < 0$. Let $\frac{\partial \pi_s}{\partial w} = 0$ and solve it, and we can get w^*, as shown in Theorem 1. Substituting w^* into $p_r = \frac{1+w+c_m}{2}$ and $p_o = \frac{1+s-\gamma}{2}$, we get p_r^* and p_o^*, as shown in Theorem 1. □

Proof of Theorem 2. Solve the optimal decision making problem using the backward induction method. Firstly, analyze the decision making process of the production and sale stage. Similar to the proof of Theorem 1, we get Hessian matrix of π_m with respect to p_r and p_o and confirm that π_m is a strictly joint convex function of p_r and p_o by $|H_1| = -2(1-\alpha) - \frac{2\alpha}{\gamma} < 0$ and $|H| = \frac{4\alpha(1-\alpha)}{\gamma} > 0$. Then we solve the first-order partial derivatives $\frac{\partial \hat{\pi}_m}{\partial p_r}=0$ and $\frac{\partial \hat{\pi}_m}{\partial p_o}=0$ jointly, thus get $p_r = \frac{1+w+c_m-\sqrt{2Z/\eta}}{2}$ and $p_o = \frac{1+s-\gamma}{2}$. Substituting them into Equation (12), we get the new functional formula for $\hat{\pi}_s$ easily and obtain $\frac{\partial \hat{\pi}_s^2}{\partial w^2} < 0$. Making $\frac{\partial \hat{\pi}_s}{\partial w} = 0$, we get $w = \frac{\alpha s+\gamma+\tau\left[c_s-c_m+\sqrt{2Z/\eta}(1-\delta)\right]}{2\tau}$. Substituting it into p_r and p_o, we get $p_r = \frac{\alpha s+\gamma+\tau\left[2+c_s+3c_m-(3+\delta)\sqrt{2Z/\eta}\right]}{4\tau}$ and $p_o = \frac{1+s-\gamma}{2}$. Secondly, analyze the decision making process of the technological innovation stage. Substituting the newly obtained formulas of w, p_r and p_o into Equation (13), we get $\hat{\pi}_m = \frac{\alpha}{2}\left(\gamma+s-c_m+\sqrt{2Z/\eta}-\frac{H}{2}\right)\left[1-\frac{1}{2\gamma}\left(c_m+\gamma-s-\sqrt{2Z/\eta}+\frac{H}{2}\right)\right] + \frac{\alpha}{4}\left(1-c_m+\sqrt{2Z/\eta}-\frac{H}{2}\right)^2 - Z$, where $H = \frac{\alpha s+\gamma}{\tau} + (1-\delta)\sqrt{2Z/\eta} - c_m + c_s$. As the complexity of the radical sign of $\sqrt{2Z/\eta}$ and the known formula $Z = \frac{1}{2}\eta\Delta c^2$, we convert the function of $\hat{\pi}_m$ into an ascending power function to be $\hat{\pi}_m=\frac{\alpha}{2}\left(\gamma+s-c_m+\Delta c-\frac{H}{2}\right)\left[1-\frac{1}{2\gamma}\left(c_m+\gamma-s-\Delta c+\frac{H}{2}\right)\right]+\frac{\alpha}{4}\left(1-c_m+\Delta c-\frac{H}{2}\right)^2-\frac{1}{2}\eta\Delta c^2$. Then we the first-order partial derivatives of $\hat{\pi}_m$ with respect to Δc and let $\frac{\partial\hat{\pi}_m}{\partial\Delta c}=0$, we get $\Delta c^*=\frac{(1+\delta)\left[\alpha s+\gamma-\tau\left(c_m+c_s\right)\right]}{8\eta\gamma-\tau(1+\delta)^2}$. As $Z=\frac{1}{2}\eta\Delta c^2$, we obtain $Z^*=\frac{\eta(1+\delta)^2\left[\alpha s+\gamma-\tau\left(c_m+c_s\right)\right]^2}{2\left[8\eta\gamma-\tau(1+\delta)^2\right]^2}$. Then, substituting Z^* into the above w, p_r and p_o, we get $\hat{w}^*$, $\hat{p}_r^*$ and $\hat{p}_o^*$ easily, as shown in Theorem 2.

Proof of Corollary 7. According to Equations (9), (10), (19) and (20), we get $\Delta\pi_s=\frac{(1+\delta)^2\left[16\eta\gamma-\tau(1+\delta)^2\right]\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]^2}{8\gamma\left[8\eta\gamma-\tau(1+\delta)^2\right]^2}$, $\Delta\pi_m=\frac{(1+\delta)^2\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]^2}{16\gamma\left[8\eta\gamma-\tau(1+\delta)^2\right]}$ and $\Delta\pi=\frac{(1+\delta)^2\left[40\eta\gamma-3\tau(1+\delta)^2\right]\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]^2}{16\gamma\left[8\eta\gamma-\tau(1+\delta)^2\right]^2}$. Given that $8\eta\gamma-\tau\left(1+\delta\right)^2>0$, it is readily to obtain $\Delta\pi_s>0$, $\Delta\pi_m>0$ and $\Delta\pi>0$. □

Proof of Corollary 9. According to Equations (7), (8), (17) and (18), we get $\Delta D_n=\hat{D}_n^*-D_n^*=\frac{(1-\alpha)(1+\delta)^2\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]}{4\left[8\eta\gamma-\tau(1+\delta)^2\right]}$, $\Delta D_o=\hat{D}_o^*-D_o^*=\frac{\alpha(1+\delta)^2\left[\alpha s+\gamma-\tau\left(c_s+c_m\right)\right]}{4\gamma\left[8\eta\gamma-\tau(1+\delta)^2\right]}$. Given that $8\eta\gamma-\tau\left(1+\delta\right)^2>0$, by analyzing the positive and negative value situation for $\alpha s+\gamma-\tau\left(c_s+c_m\right)$, we obtain some conclusions: (i) If $\tau<\overline{\tau_2}$ (where $\overline{\tau_2}=\frac{\alpha s+\gamma}{c_s+c_m}$), there exists a positive value for ΔD_n and ΔD_o, i.e., $\Delta D_n>0$ and $\Delta D_o>0$. (ii) If $\tau=\overline{\tau_2}$, then ΔD_n and ΔD_o are equal to zero, i.e., $\Delta D_n=0$ and $\Delta D_o=0$. (iii) If $\tau>\overline{\tau_2}$, then ΔD_n and ΔD_o take on negative values, i.e., $\Delta D_n<0$ and $\Delta D_o<0$. □

Proof of Corollary 11. According to Eqs. (21)–(24), we get $\hat{V}_n^*-V_n^*=\frac{\left(\hat{D}_n^*+D_n^*\right)\left(\hat{D}_n^*-D_n^*\right)}{2(1-\alpha)}$, $\hat{V}_o^*-V_o^*=\frac{\gamma\left(\hat{D}_o^*-D_o^*\right)}{2\alpha}$ and $\hat{E}-E=\frac{\left(\hat{D}_n^*+D_n^*\right)\left(\hat{D}_n^*-D_n^*\right)}{2(1-\alpha)}+\frac{\gamma\left(\hat{D}_o^*-D_o^*\right)}{2\alpha}$. Based on the proof of Corollary 9, we can find that only if $\tau<\overline{\tau_2}$, there exists $\hat{V}_n^*-V_n^*>0$, $\hat{V}_o^*>V_o^*>0$ and $\hat{E}-E>0$. This completes the proof. □