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Cost-availability ratio modeling of two-dimensional extended warranty for multi-component systems with fault correlation

Xinjian Gao[a], Enzhi Dong[a], Tielu Gao[a], Kexin Jiang[b], Bo Zhu[a], Yu Wang[a] and Zhonghua Cheng[a*]

[a]*Army Engineering University of PLA, China*
[b]*Shijiazhuang Flying College of PLAAF, China*

C H R O N I C L E

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A B S T R A C T

The existing research on multi-component systems mostly assumes that the faults between components are independent and ignores their practical correlation, which will inevitably affect the calculation of system warranty cost and warranty availability. In order to quantitatively analyze the impact of fault independence and fault correlation between components on the minimum two-dimensional extended warranty (EW) cost-availability ratio of the system, this paper establishes a two-dimensional EW cost model and availability model for multi-component systems considering fault correlation based on incomplete periodic preventive maintenance (PM), and forms a warranty cost-availability ratio model accordingly. Subsequently, the artificial bee colony (ABC) intelligent optimization algorithm was introduced to solve the model, and a case study was conducted on the transmission system of a certain new energy vehicle. Through numerical comparison, it was found that considering fault correlation compared to the assumption of fault independence would increase the warranty cost-availability ratio of the system by 20%, providing more practical warranty references for users and manufacturers, and verifying the superiority of the model. Finally, a sensitivity analysis was conducted on the model to guide its more effective implementation and application.

1. Introduction

1.1 Research background

With the continuous advancement of technology, modern equipment such as new energy vehicles, intelligent robots, and photovoltaic modules typically consist of multi-component systems. In such systems, components interact synergistically to achieve specific functions. Due to their complex structures, high integration, and strong systemic nature, multi-component systems exhibit tight interdependencies among components, significantly complicating warranty management (Fan et al., 2025). Fault correlation refers to the phenomenon where the failure of one component influences the state of others, potentially increasing their failure rates (Zhou et al., 2015). Here, “state” encompasses various measurable metrics, including service life, failure rate, and reliability (Kim et al., 2025). For instance, in a transmission system, gudgeon block failures directly impact the condition of the gear wheel, altering their failure progression. Given the prevalence of fault correlations in multi-component systems, their study holds substantial theoretical and practical significance (Yousefian et al., 2025).

The EW is an extension of the initial warranty period, which refers to a period of follow-up repair service work carried out by the manufacturer and the user after signing an EW service contract at the end of the initial warranty period (Dai et al., 2025). The fundamental distinction between initial and extended warranties lies in their contractual nature: the initial warranty is provided as a "complimentary" service with predetermined terms fixed at the point of sale, whereas the EW is "purchased" voluntarily by customers with terms and conditions negotiated between both parties (Dong et al., 2024). The two-dimensional warranty represents an advanced warranty mechanism that determines coverage duration through two distinct parameters: temporal dimension (usage time) and intensity dimension (usage degree). A typical automotive application would specify

* Corresponding author
E-mail sci_sci@yeah.net (Z. Cheng)
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coverage as "4 years or 50,000 kilometers," with warranty services terminating upon reaching either threshold. This dual-parameter approach demonstrates superior alignment with actual equipment utilization patterns compared to conventional one-dimensional warranties (Wang et al., 2021). In recent years, with continuous improvements in manufacturers' after-sales service systems, two-dimensional EW has emerged as a significant research focus in the field of warranty management (He et al., 2020). However, most research on two-dimensional EW focuses on minimizing warranty cost as the decision-making objective, neglecting users' attention to device availability, which is not conducive to improving user satisfaction and manufacturers' market competitiveness. Users hope to obtain greater warranty availability with lower warranty cost during the two-dimensional EW period, that is, they tend to minimize the cost-availability ratio of the EW for the device. Therefore, for multi-component systems within the two-dimensional EW period, scientifically developing a warranty plan to minimize the cost-availability of equipment warranty is a direction worthy of research and has broad application value.

1.2 Literature review

1.2.1 Research status of two-dimensional warranty

Extensive research has been conducted on two-dimensional initial warranty policies for equipment. Wang et al. (2017) provided a comprehensive review and synthesis of theoretical approaches and practical applications related to two-dimensional initial warranty. Huang et al. (2015) focused on repairable products, employing a bivariate approach to construct product failure rate functions. Their study analyzed equipment degradation processes based on bivariate Weibull distribution, simultaneously considering both calendar time and usage intensity to establish a two-dimensional initial warranty cost model for repairable products. Wang et al. (2020) designed the optimal PM strategy for the product's two-dimensional initial warranty period to minimize the warranty cost of the seller. Wang et al. (2022) developed a dual stochastic Poisson process model for product failures, assuming continuous manufacturer monitoring of cumulative usage following a Gamma process. Their work introduced concepts of first passage time and Gamma bridge to derive cost functions under initial warranty schemes. In the context of two-dimensional initial warranty, Wang et al. (2017) investigated optimal warranty strategies based on periodic PM. They established a novel warranty model featuring cost-sharing between users and manufacturers for PM expenses.

In the field of two-dimensional EW for equipment, Huang et al. (2017) analyzed users' repair activity records during the initial warranty period and classified them into three types. They proposed a customized two-dimensional EW policy and demonstrated through case studies that scientifically categorizing users not only reduces EW cost but also helps manufacturers gain a competitive advantage in the market. Meanwhile, Su et al. (2016) considered the varying timing of consumers' purchases of EW services and implemented an imperfect PM strategy for equipment during the EW period, deriving the optimal PM interval. Gao et al. (2025) proposed a joint PM strategy for EW of complex product systems, which not only leverages the manufacturer's technical expertise in repairs but also enhances users' independent maintenance capabilities. Wang et al. (2020) incorporated different users' usage rates and failure histories into their two-dimensional EW decision-making model. They constructed a cost model based on minimal repairs to calculate the expected warranty cost and the manufacturer's expected profit. However, whether in the context of two-dimensional initial warrantie or EW for equipment, most existing research focuses on single-component systems, and there remains a scarcity of references on EW decision modeling for multi-component systems.

1.2.2 Research status of multi-component systems

In the field of structural correlation in multi-component systems, Olde Keizer et al. (2017) conducted a comprehensive review of condition-based maintenance modeling and applications for structurally correlated multi-component systems. Junbao et al. (2015) established relationships between maintenance time and cost among components while considering structural correlation, employing a minimal repair strategy and Monte Carlo simulation to derive the optimal opportunistic maintenance plan for the system during its operational life. To ensure system reliability and safety, Iung et al. (2016) studied structurally correlated multi-component systems and proposed a novel opportunistic maintenance strategy, followed by optimization analysis of the maintenance plan. Dong et al. (2019) considered the structural correlation of parallel redundant systems and developed an optimal PM strategy based on the system reliability function. Dinh et al. (2020) focused on engineering applications, successively proposing condition-based and selective maintenance strategies for structurally correlated multi-component systems. He constructed a failure rate function incorporating disassembly impacts and validated the effectiveness and rationality of the maintenance strategies through case studies.

In the study of economic correlation in multi-component systems, Einabadi et al. (2023) introduced a new mathematical programming model for systems with economic correlation, considering opportunity grouping of maintenance activities to minimize direct and indirect maintenance costs. Wang et al. (2018) conducted a detailed analysis of multi-component system composition and proposed an optimization model distinguishing between repairable and non-repairable components. Using a marginal analysis-based algorithm, the model identified components requiring maintenance and their corresponding strategies. Shahanaghi et al. (2013) took a certain type of commercial vehicle as the research object and regarded it as a multi-component system composed of five subsystems. By modeling the failure time and utilization rate of each subsystem and discussing its impact on the system maintenance cost, the EW cost of the system can be accurately solved. Sun et al. (2021) integrated both economic and structural correlation in numerical control machine tools, aiming to minimize maintenance cost, and derived the system's optimal maintenance scheme via a genetic algorithm. Despite extensive research on multi-component systems, most studies focus on structural and economic correlation, while investigations into fault correlation remain limited.

Based on the aforementioned literature review, current two-dimensional warranty research primarily focuses on single-component systems, while studies on multi-component systems remain relatively scarce. Moreover, existing research on multi-component systems lacks sufficient consideration of fault correlation. Therefore, investigating fault correlation within the two-dimensional EW period for multi-component systems is a highly valuable and meaningful direction. The rest of this paper is structured as follows: Section 2 analyzes fault correlation and makes assumptions about the model; Section 3 constructs a two-dimensional EW cost and availability model, forming a cost-availability ratio model for the EW; Section 4 designs an ABC intelligent optimization algorithm for the solution of the model; Section 5 conducts a case study on the transmission system of new energy vehicles and performs a sensitivity analysis on the warranty cost-availability ratio model; Section 6 summarizes the conclusions of this research and provides an outlook for future research directions.

2. Problem description

According to the results generated by fault correlation, it can be mainly divided into two categories: direct fault correlation and indirect fault correlation. Direct fault correlation refers to when a component in the system fails, it will directly cause other components to fail. Indirect fault correlation is related to the failure rate, that is, when a component fails, it will raise other components' failure rate to some extent, but does not directly cause other components to fail (Murthy et al., 1985). This article adopts incomplete PM and corrective maintenance (CM) strategies, with PM intervals and two-dimensional EW periods as decision variables, and the multi-component system's minimum warranty cost-availability ratio during EW period as the decision objective. In the case study, the significance and role of considering the correlation of component fault are verified by comparing the system's minimum warranty cost-availability ratio under the condition of component fault correlation and the condition of component fault non-correlation.

2.1 Fault correlation analysis

For multi-component systems with fault correlation, each component's actual failure rate consists of two parts: original failure rate and correlated failure rate. The original failure rate denotes the component's inherent failure rate, which is established by the design and manufacturing quality; the correlated failure rate represents the failure rate resulting from the fault of other components within the system (Dong et al., 2022). For a multi-component system containing q individual components, under the condition of fault correlation, the actual failure rate of each component can be expressed as:

$$\begin{cases} \chi_a(t \mid r) = \chi_{a0}(t \mid r) + \chi_{ab}(t \mid r) \\ \chi_{ab}(t \mid r) = \sigma_{ab}(t \mid r) \times \chi_b(t \mid r) \end{cases} \tag{1}$$

Among them, $\chi_a(t \mid r)$ is the actual failure rate of component a, $1 \le a \le q$ and $a \in N^+$; $\chi_{a0}(t \mid r)$ is the original failure rate of component a; $\chi_{ab}(t \mid r)$ is the correlated failure rate of component b to component a, $b \in \{b \mid b = 1,2,3,...,q \text{ and } b \ne a\}$. The actual failure rate of component b is denoted as $\chi_b(t \mid r)$, and the failure impact coefficient of component b on component a is denoted as $\sigma_{ab}(t \mid r)$, $0 \le \sigma_{ab}(t \mid r) \le 1$. When the value of $\sigma_{ab}(t \mid r)$ is 0, it means that component b has no influence on component a. When the value of $\sigma_{ab}(t \mid r)$ is 1, it indicates that the actual failure rate of component a is the sum of the original failure rate of component a and the actual failure rate of component b.

This paper primarily investigates the fault correlation of a two-component system with critical component and important component. The warranty strategy involves only performing CM on the system after a failure in initial warranty period, while implementing incomplete periodic PM and CM after failures during the EW period. The failure rates of the critical component and important component are denoted by $\chi_z(t \mid r)$ and $\chi_\varphi(t \mid r)$, respectively. The important component's failure will increase the critical component's failure rate to some extent (indirect fault correlation); however, the critical component's failure will directly lead to the important component's failure (direct fault correlation), resulting in a system-wide failure that requires corrective maintenance.

2.2 Model assumptions

This research is predicated on the following fundamental assumptions:

(1) The time and cost of a single PM are fixed, and the time and cost of a single CM are also fixed.
(2) Incomplete PM is an improvement maintenance that reduces the component's failure rate; CM is the minimum maintenance that does not change the component's failure rate.

(3) The usage rate of the same user is fixed during device usage, while the usage rates of different users follow a two parameter Weibull distribution.
(4) Purchase two-dimensional EW service at the same time as purchasing the device, without considering the situation of purchasing EW service at the end of the initial warranty.

3. Model construction

3.1 Failure rate model

Generally speaking, the system's reliability is associated with its usage rate, and reliability indicators are designed under a certain level of usage intensity. Therefore, changes in usage rate during the usage phase will affect the system reliability and failure rate. Lawless et al. (1995) employed Accelerated Failure Time (AFT) theory in conjunction with proportional hazards modeling to quantify the relationship between operational usage rate and system failure mechanisms. This analytical approach is well-established in reliability engineering and has been extensively validated through practical applications. Considering this situation, the article constructs a failure rate function for components on the basis of the AFT model. Assuming that τ_d and τ respectively represent the time of the first failure at the design usage rate r_d and the actual usage rate r, then the relationship between them can be expressed as:

$$\frac{\tau}{\tau_d} = (\frac{r_d}{r})^{\phi} \tag{2}$$

Assuming the cumulative fault distribution function is $F_d(t;\beta_d,\omega)$ at the design usage rate r_d, where the fault distribution's shape and scale parameters are denoted by β_d and ω, respectively. When the usage rate is r, the shape parameter ω remains unchanged, but the scale parameter β is:

$$\beta = \beta_d(\frac{r_d}{r})^{\phi} \tag{3}$$

Among them, the AFT model's parameter is $\phi \geq 1$. If the usage rate is r, the component's cumulative fault distribution function is:

$$F(t;\beta,\omega) = F_d(t;\beta_d(\frac{r_d}{r})^{\phi},\omega) \tag{4}$$

Given that the shape parameter ω remains constant regardless of the usage rate, in the expression below, it is neglected. When the usage rate is r, the component's failure rate function is:

$$\chi(t \mid r) = \chi(t;\beta) = \frac{f(t;\beta)}{\overline{F}(t;\beta)} \tag{5}$$

The effect of incompletet PM lies between “as bad as old” and “as good as new” (Tong et al., 2014). This paper expresses the effect of incompletet PM using the virtual age method, meaning each incompletet PM reduces the actual service age of the system by a certain period (Zhao et al., 2017). Let λ be the improvement factor of incompletet PM. Assuming the l-th incompletet PM is performed at time t, during the l-th PM interval, the system's failure rate is:

$$\chi(t \mid r) = \chi[(t-(l-1)\lambda T\ ) \mid r] \tag{6}$$

Among them, T is the interval period for incompletet PM. By applying the virtual age method, the Fig. 1 illustrates changes in the system's failure rate following each incomplete PM.

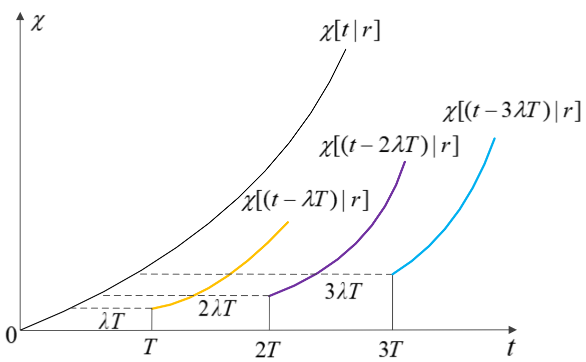


Fig. 1. The effect of incompletet PM

The system adopts a CM strategy for failures occurring during both initial warranty and EW period. The number of system failures in a certain period is:

$$E[k(t)] = \int_0^t \chi(v \mid r) dv \tag{7}$$

where $k(t)$ is the number of system failures within time $[0,t]$, and $\chi(v \mid r)$ is the failure rate of the system.

3.2 Two-dimensional EW cost model

3.2.1 The start time $S_{\Pi}(r)$ and end time $S_{\amalg}(r)$ of two-dimensional EW

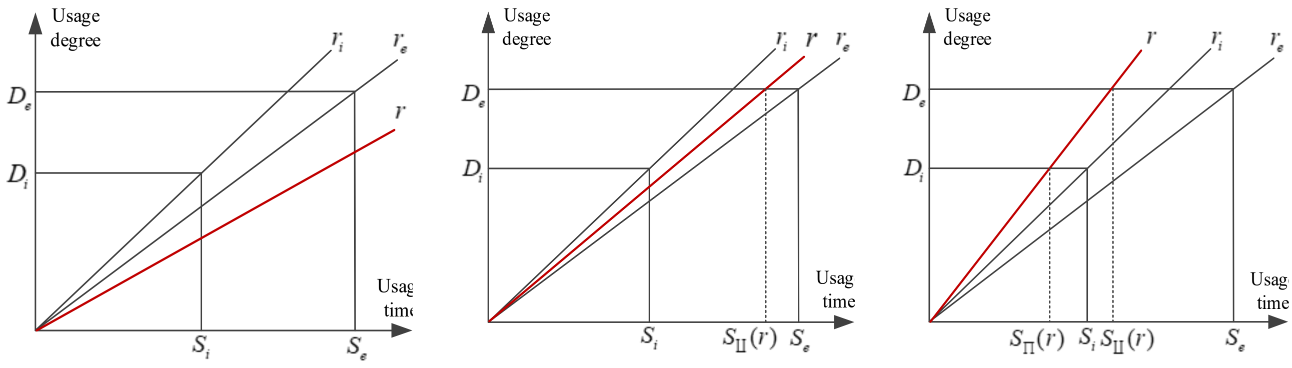
S_e and D_e represent the usage time and usage degree range of EW respectively, while S_i and D_i denote the usage time and usage degree range of initial warranty respectively. The shape parameters r_e and r_i for the EW region and initial warranty region are as follows:

$$r_e = \frac{D_e}{S_e}, \quad r_i = \frac{D_i}{S_i} \tag{8}$$

During EW period, the number of multi-component system's PM actions is:

$$m = \text{int}\left\{[S_{\amalg}(r) - S_{\Pi}(r)]/(T + T_p)\right\} \tag{9}$$

Among them, "int" is the floor function, T indicates the interval of PM, T_p indicates the time required for a single PM. When the usage rate is r, $S_{\Pi}(r)$ and $S_{\amalg}(r)$ denote the start time and end time of EW period respectively. Their expressions are determined based on different values of r, as specifically illustrated in Fig. 2 and Fig. 3.



(a) $r_i > r_e > r$ (b) $r_i > r > r_e$ (c) $r > r_i > r_e$

Fig. 2. Schematic diagram of EW period when $r_e \le r_i$

In Fig. 2, when $r_e \le r_i$, the piecewise function of $S_{\Pi}(r)$ and $S_{\amalg}(r)$ with the value of r is:

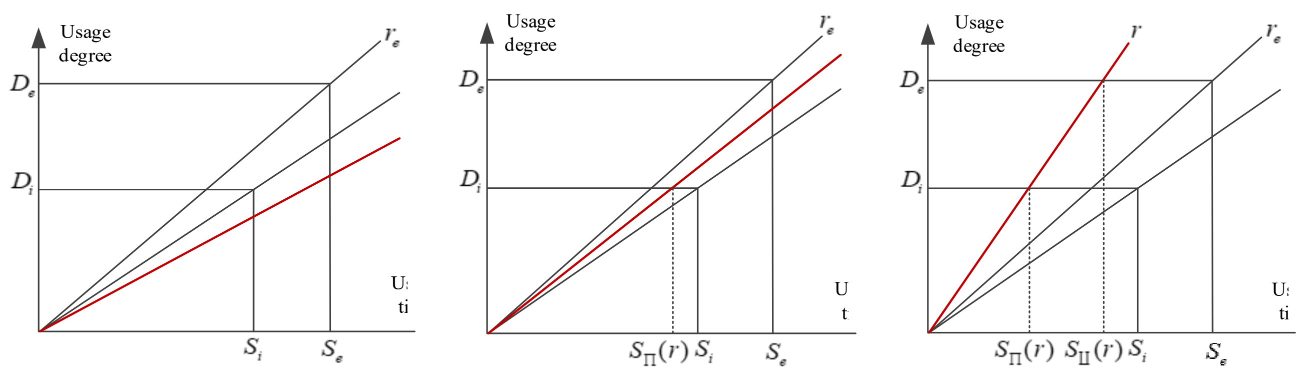
$$S_{\Pi}(r) = \begin{cases} S_i & r_l \le r \le r_i \\ \dfrac{D_i}{r} & r_i \le r \le r_u \end{cases} \tag{10}$$

$$S_{\amalg}(r) = \begin{cases} S_e & r_l \le r \le r_e \\ \dfrac{D_e}{r} & r_e \le r \le r_u \end{cases} \tag{11}$$

where r_u and r_l are the maximum and minimum usage rates, respectively. Similarly, at that time $r_e > r_i$, the multi-component system's EW period is represented in Fig. 3. In Figure 3, when $r_e > r_i$, the piecewise function of $S_{\Pi}(r)$ and $S_{\amalg}(r)$ with the value of r is:

$$S_{\Pi}(r)=\begin{cases} S_i & r_l \le r \le r_i \\ \dfrac{D_i}{r} & r_i \le r \le r_u \end{cases} \tag{12}$$

$$S_{\amalg}(r)=\begin{cases} S_e & r_l \le r \le r_e \\ \dfrac{D_e}{r} & r_e \le r \le r_u \end{cases} \tag{13}$$



(a) $r_e > r_i > r$ (b) $r_e > r > r_i$ (c) $r > r_e > r_i$

Fig. 3. Schematic diagram of EW period when $r_e > r_i$

3.2.2 The warranty cost during the l-th PM interval $[(l-1)(T+T_p), l(T+T_p)]$

During two-dimensional EW period, the multi-component system's warranty cost is mainly composed of CM cost and PM cost, which can be expressed as:

$$EC[T, S_{\Pi}(r), S_{\amalg}(r)] = mC_p + \sum_{l=1}^{m} C_{fl}[(l-1)(T+T_p), l(T+T_p)] + C_f[S_{\Pi}(r)+m(T+T_p), S_{\amalg}(r)] \tag{14}$$

Among them, the cost of a single PM is C_p, the cost of CM during the l-th PM interval is $C_{fl}[(l-1)(T+T_p), l(T+T_p)]$. $C_f[S_{\Pi}(r)+m(T+T_p), S_{\amalg}(r)]$ represents the cost of CM during the time period $[S_{\Pi}(r)+m(T+T_p), S_{\amalg}(r)]$, that is, the cost of CM from the last PM to the end of the EW period. During the l-th PM interval, the important component's failure rate function is:

$$\chi_{l\varphi} = \begin{cases} \chi_{\varphi}(t \mid r) & l=1 \\ \chi_{\varphi}\{[t-\lambda(S_i+(l-1)T] \mid r\} & l=2,3,4,\ldots,m \end{cases} \tag{15}$$

The failure rate of critical component will increase to a certain extent when important component fails. According to the fault correlation analysis in Section 2.1, the failure rate function of critical component in the l-th PM interval period is:

$$\chi_{lz} = \begin{cases} \chi_z(t \mid r) + \dfrac{\sigma}{2} n_{l\varphi}\chi_{l\varphi}(t \mid r) & l=1 \\ \chi_z\{[t-\lambda(S_i+(l-1)T)] \mid r\} + \\ \sigma\{\sum_{j=1}^{l}\{n_{j\varphi}\chi_{\varphi}[(t-\lambda(W+(j-1)T) \mid r\} - \dfrac{1}{2} n_{l\varphi}\chi_{l\varphi}(t \mid r)\} & l=2,3,4,\ldots,m \end{cases} \tag{16}$$

In this formula, $n_{l\varphi}$ denotes the number of failures of critical component during the l-th PM interval. Since the critical component and important component in multi-component system are connected in series, the total CM cost includes the CM cost of critical component and the CM cost of important component. In engineering practice, the shape parameter r_e of the EW period and the shape parameter r_i of the initial warranty period exhibit different quantitative relationships. Therefore, during the l-th PM interval, the multi-component system's CM cost has two scenarios.

At the scenario of $r_e \le r_i$, as depicted in Fig. 2, since the actual usage rate is not a fixed value, the CM cost of multi-component system under different actual usage rates need to be discussed.

(1) When $r_l \le r < r_e$, the system's CM cost during the l -th PM interval is:

$$EC_{f_1}(T)=C_{f\varphi}\int_{r_l}^{r_e}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_l}^{r_e}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{17}$$

In the formula, $C_{f\varphi}$ is the important component's CM cost, C_{fz} is the critical component's CM cost, and system usage rate is $h(r)$.

(2) When $r_e \le r < r_i$, the system's CM cost during the l -th PM interval is:

$$EC_{f_2}(T)=C_{f\varphi}\int_{r_e}^{r_i}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_e}^{r_i}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{18}$$

(3) When $r_i \le r \le r_u$, the system's CM cost during the l -th PM interval is:

$$EC_{f_3}(T)=C_{f\varphi}\int_{r_i}^{r_u}\int_{\frac{D_i}{r}+(l-1)(T+T_p)}^{\frac{D_i}{r}+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_i}^{r_u}\int_{\frac{D_i}{r}+(l-1)(T+T_p)}^{\frac{D_i}{r}+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{19}$$

Under the condition of $r_e \le r_i$, the cost of CM during the l -th PM interval for multi-component system is:

$$EC_{flx}(T)=EC_{f_1}(T)+EC_{f_2}(T)+EC_{f_3}(T) \tag{20}$$

At the scenario of $r_e > r_i$, as shown in Fig. 3, similar to $r_e \le r_i$, it is necessary to discuss the CM cost of multi-component system under different actual usage rates.

(4) When $r_l \le r < r_i$, the system's CM cost during the l -th PM interval is:

$$EC_{f_4}(T)=C_{f\varphi}\int_{r_l}^{r_i}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_l}^{r_i}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{21}$$

(5) When $r_i \le r < r_e$, the system's CM cost during the l -th PM interval is:

$$EC_{f_5}(T)=C_{f\varphi}\int_{r_i}^{r_e}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_i}^{r_e}\int_{S_i+(l-1)(T+T_p)}^{S_i+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{22}$$

(6) When $r_e \le r \le r_u$, the system's CM cost during the l -th PM interval is:

$$EC_{f_6}(T)=C_{f\varphi}\int_{r_e}^{r_u}\int_{\frac{D_i}{r}+(l-1)(T+T_p)}^{\frac{D_i}{r}+lT+(l-1)T_p}\chi_{l\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_e}^{r_u}\int_{\frac{D_i}{r}+(l-1)(T+T_p)}^{\frac{D_i}{r}+lT+(l-1)T_p}\chi_{lz}\cdot h(r)dtdr \tag{23}$$

Under the condition of $r_e > r_i$, the cost of CM during the l -th PM interval for multi-component system is:

$$EC_{fly}(T)=EC_{f_4}(T)+EC_{f_5}(T)+EC_{f_6}(T) \tag{24}$$

3.2.3 The warranty cost of CM between the last PM and the end of EW period

Since the m -th PM time may not necessarily be the end of EW, it is necessary to separately calculate the CM cost for multi-component systems from the m -th PM to the end of EW. This means calculating the system's CM cost within the period $[S_{\Pi}+m(T+T_p),S_{\Pi}]$. The analytical approach resembles the method used to determine the CM cost for multi-component system during the l -th PM interval, with the findings illustrated in Table 1.

Table 1

The multi-component system's CM cost in period $[S_{\Pi}+m(T+T_p),S_{\amalg}]$

			The equation for calculating CM cost	
$r_e \le r_i$	$r_l \le r < r_e$	$EC_{f_{(1)}}(T)$	$C_{f\varphi}\int_{r_l}^{r_e}\int_{S_i+m(T+T_p)}^{S_e}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_l}^{r_e}\int_{S_i+m(T+T_p)}^{S_e}\chi_{(m+1)z}\cdot h(r)dtdr$	(25)
	$r_e \le r < r_i$	$EC_{f_{(2)}}(T)$	$C_{f\varphi}\int_{r_e}^{r_i}\int_{S_i+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_e}^{r_i}\int_{S_i+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)z}\cdot h(r)dtdr$	(26)
	$r_i \le r \le r_u$	$EC_{f_{(3)}}(T)$	$C_{f\varphi}\int_{r_i}^{r_u}\int_{\frac{D_i}{r}+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_i}^{r_u}\int_{\frac{D_i}{r}+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)z}\cdot h(r)dtdr$	(27)
$r_e > r_i$	$r_l \le r < r_i$	$EC_{f_{(4)}}(T)$	$C_{f\varphi}\int_{r_l}^{r_i}\int_{S_i+m(T+T_p)}^{S_e}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_l}^{r_i}\int_{S_i+m(T+T_p)}^{S_e}\chi_{(m+1)z}\cdot h(r)dtdr$	(28)
	$r_i \le r < r_e$	$EC_{f_{(5)}}(T)$	$C_{f\varphi}\int_{r_i}^{r_e}\int_{S_i+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_i}^{r_e}\int_{S_i+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)z}\cdot h(r)dtdr$	(29)
	$r_e \le r \le r_u$	$EC_{f_{(6)}}(T)$	$C_{f\varphi}\int_{r_e}^{r_u}\int_{\frac{D_i}{r}+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)\varphi}\cdot h(r)dtdr+C_{fz}\int_{r_e}^{r_u}\int_{\frac{D_i}{r}+m(T+T_p)}^{\frac{D_e}{r}}\chi_{(m+1)z}\cdot h(r)dtdr$	(30)

When $r_e \le r_i$, the cost of CM for multi-component systems in period $[S_{\Pi}+m(T+T_p),S_{\amalg}]$ is:

$$EC_{f(x)}(T)=EC_{f_{(1)}}(T)+EC_{f_{(2)}}(T)+EC_{f_{(3)}}(T) \tag{31}$$

When $r_e > r_i$, the cost of CM for multi-component systems in period $[S_{\Pi}+m(T+T_p),S_{\amalg}]$ is:

$$EC_{f(y)}(T)=EC_{f_{(4)}}(T)+EC_{f_{(5)}}(T)+EC_{f_{(6)}}(T) \tag{32}$$

Therefore, the system's expected warranty cost under two-dimensional EW is:

$$EC(T)=\begin{cases} mC_p+\sum_{l=1}^{m}EC_{flx}(T)+EC_{f(x)}(T) & r_e \le r_i \\ mC_p+\sum_{l=1}^{m}EC_{fly}(T)+EC_{f(y)}(T) & r_e > r_i \end{cases} \tag{33}$$

3.3 Two-dimensional EW availability model

The system's usable time is the EW period minus the system's downtime, and the warrwnty availability is the ratio of the system's usable time to the EW period. Therefore, the multi-component system's expected warranty availability over the two-dimensional EW period is:

$$EA(T)=\frac{(S_e-S_i)-EO(T)}{S_e-S_i} \tag{34}$$

In the formula, $EO(T)$ represents the system's expected downtime in EW period, its derivation methodology parallels the computational model employed for determining expected warranty cost. It only requires replacing the important component's

CM cost $C_{f\varphi}$, critical component's CM cost C_{fz}, and the system's PM cost C_p in the two-dimensional EW cost model with important component's CM time $T_{f\varphi}$, critical component's CM time T_{fz}, and the system's PM time T_p, respectively.
Based on the system's EW cost and EW availability model, the EW cost-availability ratio model of the system can be established. The EW cost-availability ratio is the ratio of system's EW cost to system's availability:

$$R = \frac{EC(T, S_e, D_e)}{EA(T, S_e, D_e)} \quad (35)$$

The decision objective of this paper is to minimize the EW cost-availability ratio R of the system. The decision variables are the PM interval T, the EW usage time period S_e, and the EW usage degree period D_e.

4. Algorithm design

The cost-availability ratio model of EW established in this article has three decision variables, namely PM interval T, EW usage time period S_e, and EW usage degree period D_e. Considering the complexity of the model and the multi variable global optimization problem, this paper uses the ABC algorithm to solve the model. The ABC algorithm is an innovative global optimization method, aimed at solving multivariate function optimization problems. As a social insect, bees exhibit extremely complex behavior in groups composed of simple individuals, even though the behavior of individual bees is extremely simple. Real bee populations can efficiently collect nectar from food sources in any environment, while also adapting to changes in the environment. The ABC algorithm is proposed based on this collective intelligence and consisted of leader bees, scouter bees and follower bees. In the ABC algorithm, bees exhibit two fundamental behaviors: recruiting bees for a food source and abandoning a food source. The ABC algorithm mainly includes the following steps.

① Food source initialization: Randomly generate a certain number of initial solutions (i.e. food source locations) and calculate the fitness value of each solution.

$$v_{ij} = x_{\min j} + rand[0,1](x_{\max j} - x_{\min j}) \quad (36)$$

Among them, v_{ij} is the j-th dimensional value of the i-th food source, $x_{\min j}$ is the j-th dimension's minimum value, and $x_{\max j}$ is the j-th dimension's maximum value.

② Leader stage: Each leader performs a neighborhood search near its current solution, generates a new solution, and calculates its fitness value. If the fitness value of the new solution is superior to original solution, it updates the current solution.

$$v_{ij} = x_{ij} + \xi_{ij}(x_{ij} - x_{kj}) \quad (37)$$

x_k represents the neighborhood food source, takes the value of $\{1, 2, 3...N\}$, and $k \neq i$. ξ_{ij} is a random number with the value in [-1,1].

③ Follower stage: Bees follow the leader and select paths for search according to the fitness value given by the leader. Local searches are conducted near the selected solution to generate new solutions and calculate their fitness values. If the new solution's fitness value is superior, the solution will be updated. The probability of the n-th food source being selected is p_n.

$$p_n = \frac{f_n(x_n^-)}{\sum_{n=1}^{N} f_n(x_n^-)} \quad (38)$$

④ Scouter stage: If the search times of a solution exceed the set threshold and no better solution is found, the solution will be marked as a scouter, and a new solution will be randomly generated for search. The n-th food source's objective value is $fit_n(x_n^-)$, and the n-th food source's fitness value is $f_n(x_n^-)$.

$$fit_n(x_n^-) = \begin{cases} 1/[1 + f_n(x_n^-)] & f_n(x_n^-) \geq 0 \\ 1 - f_n(x_n^-) & f_n(x_n^-) < 0 \end{cases} \quad (39)$$

⑤ Iterative update: Repeat the above steps until converging at the prescribed maximum iteration limit.

Table 2 presents the pseudocode implementation of the ABC algorithm. Set the swarm size of the algorithm to 30, with 12 leader bees, 15 follower bees, and 3 scouter bees. The maximum number of iterations for the algorithm is 200, the number of food sources is 15, the memory limit is 50, the mutation parameter is 0.6, and the crossover parameter is 0.8. The fitness value of the algorithm is the EW cost-availability ratio $R(T, S_e, D_e)$, and the solution of the algorithm consists of three decision variables T, S_e, D_e. Based on this, by writing an MATLAB program, the PM interval and EW period that minimize the EW cost-availability ratio can be determined.

Table 2

Pseudo-code for running the algorithm

```
import numpy as np
def abc_algorithm(f, bounds, N=30, limit=50, max_iter=200):
D = len(bounds)
        X = np.random.rand(N, D)
    # Initializing
for i in range(N):
        X[i] = bounds[:, 0] + X[i] * (bounds[:, 1] - bounds[:, 0])
    fitness = np.array([f(x) for x in X])
    BestPosition = X[np.argmin(fitness)]
    BestFitness = np.min(fitness)
    trial = np.zeros(N)
      for t in range(max_iter):
      # Leader stage
    for i in range(N):
        while k == i:
          k = np.random.randint(N)
        phi = np.random.uniform(-1, 1, D)
        NewPosition = X[i] + phi * (X[i] - X[k])
        NewPosition = np.clip(NewPosition, bounds[:, 0], bounds[:, 1])
        NewFitness = f(NewPosition)
        if NewFitness < fitness[i]:
          X[i] = NewPosition
          fitness[i] = NewFitness
          trial[i] = 0
        else:
          trial[i] += 1
      # Follower stage
      prob = (0.9 * (1 / fitness) / np.sum(1 / fitness)) + 0.1
       for i in range(N):
        if np.random.rand() < prob[i]:
         while k == i:
           k = np.random.randint(N)
          phi = np.random.uniform(-1, 1, D)
          NewPosition = X[i] + phi * (X[i] - X[k])
          NewPosition = np.clip(NewPosition, bounds[:, 0], bounds[:, 1])
          NewFitness = f(NewPosition)
      # Scouter stage
      for i in range(N):
        if trial[i] > limit:
         X[i] = bounds[:, 0] + np.random.rand(D) * (bounds[:, 1] - bounds[:, 0])
          fitness[i] = f(X[i])
          trial[i] = 0
        if fitness[i] < BestFitness:
        BestFitness = fitness[i]
        BestPosition = X[i]
      return BestPosition, BestFitness
BestPosition, BestFitness = abc_algorithm(f, bounds)
print("{BestPosition},{BestFitness}")
```

Fig. 4. The operational flowchart of ABC algorithm.

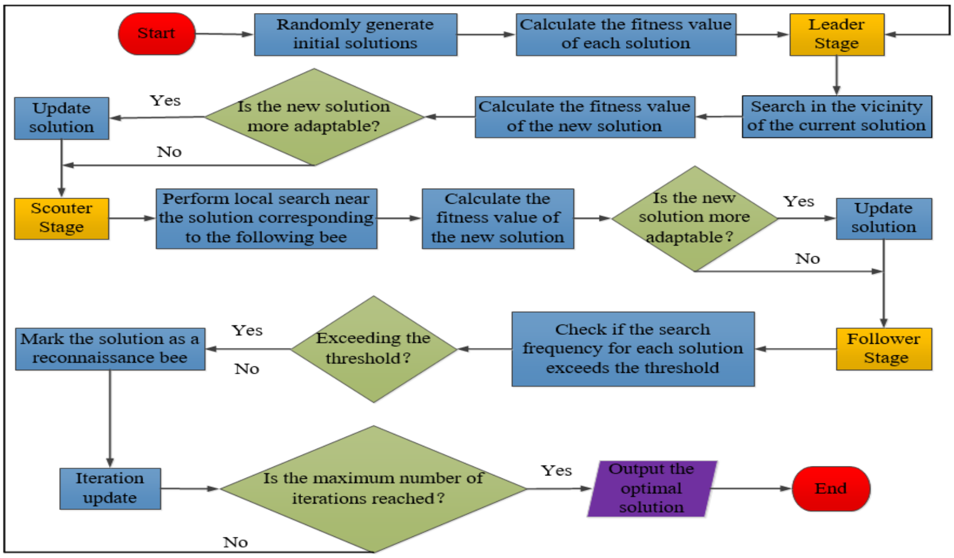


Fig. 4. The flow chart of ABC algorithm

5. Case study

This section takes the transmission system of a certain new energy vehicle as a case study. The transmission system mainly consists of multiple components such as gudgeon block, gear wheel, clutche, synchronizer, and sensor. Among these, the gudgeon block is a critical component, primarily used to support the smooth rotation of the gear wheel, reduce gear wheel wear, and also serve to dampen vibrations and provide cushioning; the gear wheel is an important part, mainly transmitting power through meshing, achieving changes in speed and torque, and driving the gudgeon block to rotate. As shown in Fig. 5, when a gudgeon block fails, the gear wheel's rotation lacks support, leading directly to gear wheel failure (direct failure-related); if the gear wheel fails, it will to some extent increase the gudgeon block's failure rate, but does not directly induce the gudgeon block failure (indirect failure-related).

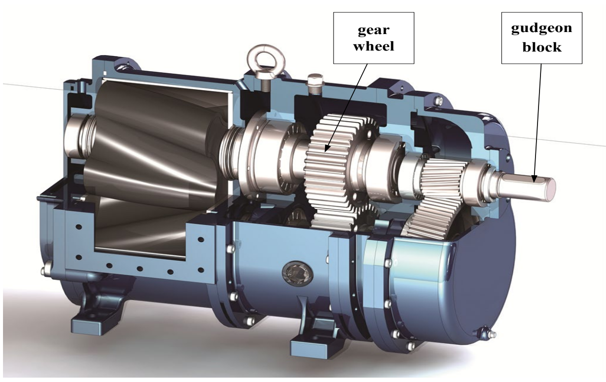


Fig. 5. The transmission system of new energy vehicles

According to the failure rate model established in Section 3.1, it is assumed that the cumulative distribution function of gudgeon block failure at design usage rate r_d is:

$$F_d(t;\beta_d,\omega) = 1 - \exp[-(t/\beta_d)^{\omega}] \tag{40}$$

Then the gudgeon block's failure cumulative distribution function at actual usage rate r is:

$$F(t;\beta,\omega) = F_d(t;\beta_d(\frac{r_d}{r})^{\phi},\omega) = 1 - \exp[-(\frac{r}{r_d})^{\phi\omega}\frac{t^{\omega}}{\beta_d^{\omega}}] \tag{41}$$

The gudgeon block's failure rate function is:

$$\chi_z(t;r) = \frac{f(t;\beta)}{\overline{F}(t;\beta)} = \omega(\frac{r}{r_d})^{\phi\omega}\frac{t^{\omega-1}}{\beta_d^{\omega}} \tag{42}$$

The cumulative distribution scale parameter of gudgeon block failure under design usage rate $r_d = 1.1 \times 10^4 km/year$ is $\beta_d = 0.7$, the shape parameter is $\omega = 1.2$, and the AFT model parameter is $\phi = 1.3$. The usage rate of the gear wheel obeys the double parameter Weibull distribution, that is:

$$h(r) = \frac{\alpha}{\gamma}\left(\frac{r}{\gamma}\right)^{\alpha-1} e^{-\left(\frac{r}{\gamma}\right)^{\alpha}} \quad (r_l < r < r_u) \tag{43}$$

In the formula, the shape parameter is $\alpha = 1.9$, and scale parameter is $\gamma = 5.1$. The transmission system's actual usage rate r is $1.6 \times 10^4 km/year$, the lowest usage rate is $r_l = 0.2 \times 10^4 km/year$ and the highest usage rate is $r_u = 9 \times 10^4 km/year$.

5.1 Comparative analysis

It is known that the two-dimensional initial warranty period of this transmission system is $S_i = 2years$, $D_i = 2 \times 10^4 km$; The CM time for gudgeon block is $T_{fz} = 15days$, with CM cost being $C_{fz} = 1900CNY$. The CM time for gear wheel is $T_{f\varphi} = 8days$, with CM cost being $C_{f\varphi} = 1300CNY$, and the failure rate of the gear wheel is $\chi_{\varphi} = 3.3 \times 10^{-4} / day$. The PM time for the transmission system is $T_p = 6days$, with PM cost being $C_p = 400CNY$. Based on maintenance experience and data analysis, it is found that gear wheel failures can lead to an increase in gudgeon block's failure rate, with the fault correlation coefficient being $\sigma = 0.5$, and the improvement factor for PM being $\lambda = 0.6$.

Set the EW S_e period for the transmission system to range between $[2years, 6.5years]$, with an incremental step of $0.5years$; set the usage degree D_e to range between $[2 \times 10^4 km, 6.5 \times 10^4 km]$, with an incremental step of $0.5 \times 10^4 km$; set the PM interval T to range between $[0.1years, 1.9years]$, with an incremental step of $0.2years$. By inputting these parameters into the EW cost-availability ratio model and applying the ABC algorithm, the minimum EW cost-availability ratio can be derived. After 69 iterations of the algorithm, it was found that when the PM interval is $0.5years$ and EW period is $[4years, 5 \times 10^4 km]$, the EW cost-availability ratio of the transmission system reaches its minimum value of 16325. At this point, the EW availability and EW cost are 0.730 and 11917 CNY, respectively. The iterative process of ABC algorithm is illustrated in Fig. 6.

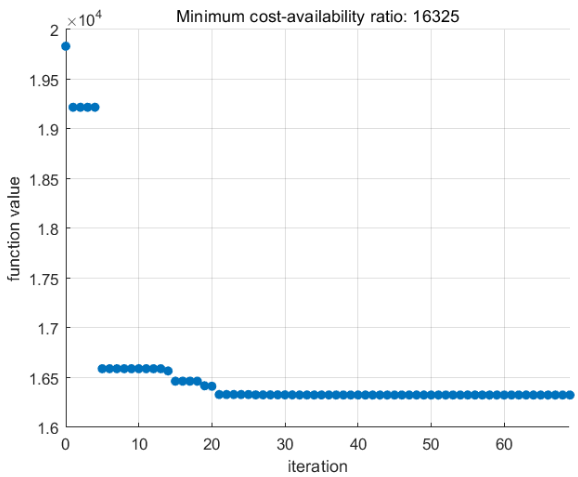


Fig. 6. The algorithm iteration operation diagram

The article considers the fault correlation between two components, and the fault correlation coefficient is $\sigma = 0.5$. If the fault correlation is ignored, assuming that the faults between components are independent, then the fault correlation coefficient would be $\sigma = 0$. The specific values of PM interval T and EW period (S_e, D_e) that minimize the EW cost-availability ratio R are represented in Table 3.

Table 3
The optimal warranty scheme for transmission system

	$T/years$	$S_e/years$	$D_e/\times 10^4 km$	EC/CNY	EA	R
Fault correlation	0.5	4.0	5.0	11917	0.730	16325

Fault independence	0.9	5.5	6.0	10561	0.809	13054

The transmission system's EW cost-availability ratio under the fault correlation between components, as well as the corresponding EW cost and EW availability are shown in Fig. 7.

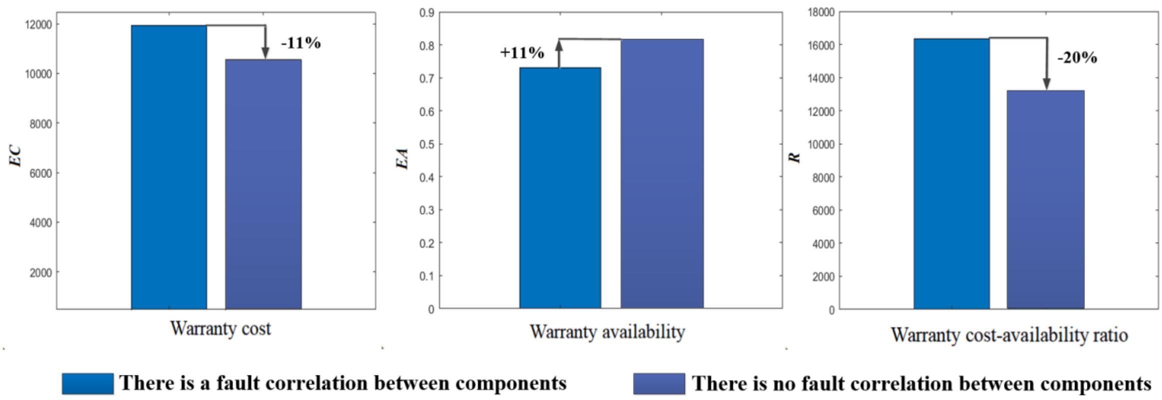


Fig. 7. Comparison between component fault correlation and fault independence

In Fig. 7, when the fault correlation between components is not considered, the system exhibits an 11% reduction in EW cost, an 11% improvement in EW availability, and a 20% decrease in the EW cost-availability ratio. Although the data may seem better than considering the correlation between faults between components, the assumption that faults between system components are independent of each other is unrealistic. This also proves that ignoring fault correlation will increase the EW availability expectation of user and reduce the EW cost expectation of manufacturer, leading to unacceptable analysis and decision-making errors. In practical applications, assuming faults independence between components in the EW scheme will raise manufacturer's warranty cost risk, causing more equipment failures and reducing user's satisfaction with EW services.

5.2 Result analysis

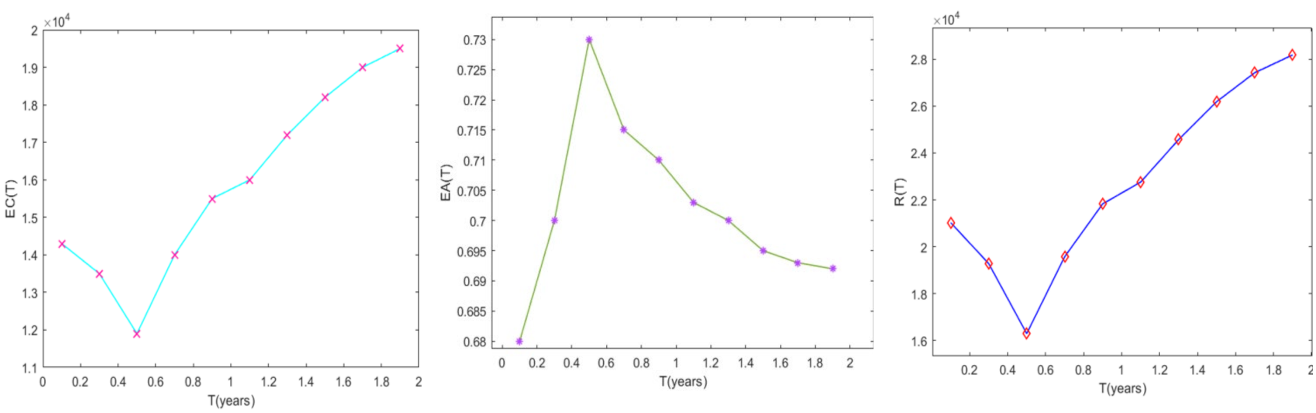
5.2.1 The relationship between EW cost-availability ratio $R(T)$ and PM interval T

The fixed EW period S_e is 4 *years*, and D_e is 5×10^4 *km*. The numerical relationship between transmission system's EW cost $EC(T)$, EW availability $EA(T)$, $R(T)$ and PM interval T is shown in Table 4.

Table 4
Numerical relationship between $EC(T)$, $EA(T)$, $R(T)$ and T

T/*years*	0.1	0.3	**0.5**	0.7	0.9	1.1	1.3	1.5	1.7	1.9
$EC(T)$	14315	13528	**11917**	14006	15539	16071	17268	18244	19028	19511
$EA(T)$	0.681	0.700	**0.730**	0.715	0.710	0.703	0.701	0.695	0.693	0.692
$R(T)$	21021	19326	**16325**	19589	21886	22861	24633	26250	27457	28195

The variation trends of $EC(T)$, $EA(T)$, and $R(T)$ with T are shown in Fig. 8.



(a) The relationship between $EC(T)$ with T

(b) The relationship between $EA(T)$ with T

(c) The relationship between $R(T)$ with T

Fig. 8. The relationship between $EC(T)$, $EA(T)$, $R(T)$ and T

Fig. 8(a) and Fig. 8(b) demonstrate that as the PM interval T increases, the EW cost $EC(T)$ of the transmission system first decreases and then increases, and the availability $EA(T)$ first increases and then decreases. This is because when the PM interval T is too long, it will increase the system's failure rate, causing more downtime due to CM, thereby reducing system's availability; When the interval T between PM is too short, it will make PM too frequent and cause more downtime due to PM, reducing system's availability. Similarly, when T is too large, more CM cost will be incurred due to insufficient PM; When T is too small, more PM cost will be incurred due to excessive PM, which will cause the system's EW cost to go up. Figure 8(c) demonstrates that with the increase of PM interval T, the EW cost-availability ratio $R(T)$ of the transmission system first decreases and then increases. Therefore, there exists an optimal PM interval $T = 0.5\,years$ that minimizes the system's EW cost-availability ratio to 16325.

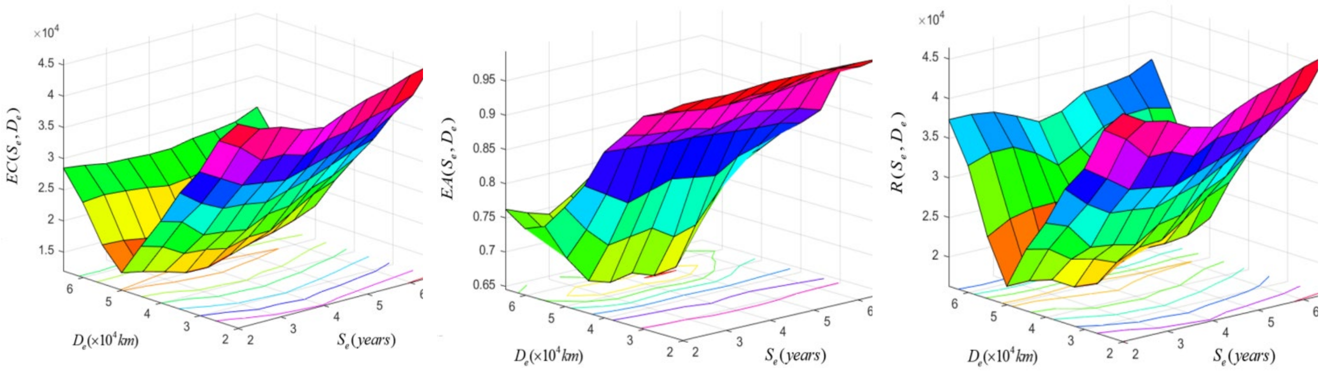
5.2.2 The relationship between EW cost-availability ratio $R(S_e, D_e)$ and S_e, D_e

Similar to Section 5.2.1, The fixed PM interval T is 0.5 *years*, the numerical relationship between transmission system's EW cost-availability ratio $R(S_e, D_e)$ and EW period S_e, D_e is shown in Table 5.

Table 5
Numerical relationship between $R(S_e, D_e)$ and S_e, D_e

D_e /×10^4*km*	S_e /*years*									
	2.0	2.5	3.0	3.5	**4.0**	4.5	5.0	5.5	6.0	6.5
2.0	45127	43434	42835	41372	40751	42296	43533	44355	45607	46465
2.5	42268	40415	37500	37500	36269	37863	38144	39139	40000	41237
3.0	36592	34664	32638	32207	31756	32804	33841	34737	35654	37657
3.5	33250	30668	28879	28634	28233	29139	30011	31602	33278	34831
4.0	28175	27119	23799	23436	22756	24658	25195	27440	28752	31054
4.5	24096	23457	21519	20875	19753	20994	21250	23171	25301	26548
5.0	18646	18667	18125	16552	**16325**	18169	19361	19542	19713	21235
5.5	24128	25714	26360	26462	25922	29612	30097	29429	27454	27436
6.0	32319	33288	33315	31892	29708	31200	33218	34069	34237	33553
6.5	37333	37333	36953	35789	34404	35065	36811	37086	37556	38961

The variation trends of transmission system's EW cost $EC(S_e, D_e)$, EW availability $EA(S_e, D_e)$ and $R(S_e, D_e)$ with S_e, D_e are shown in Fig. 9.



(a) The variation of $EC(S_e, D_e)$ with S_e, D_e

(b) The variation of $EA(S_e, D_e)$ with S_e, D_e

(c) The variation of $R(S_e, D_e)$ with S_e, D_e

Fig. 9. The relationship between $EC(S_e, D_e)$, $EA(S_e, D_e)$, $R(S_e, D_e)$ and S_e, D_e

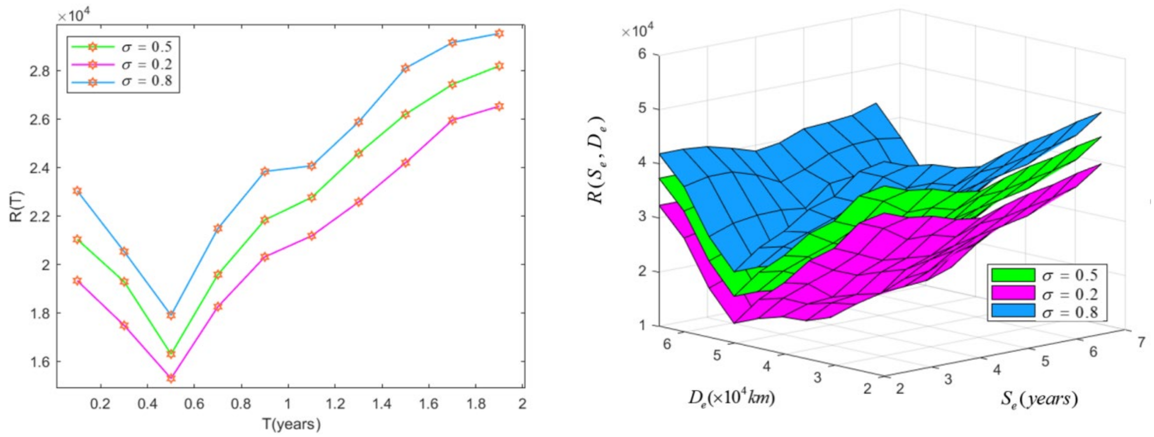
From Fig. 9, it can be seen that the effects of $EC(S_e, D_e)$, $EA(S_e, D_e)$ and $R(S_e, D_e)$ with respect to D_e are very significant, while the effects with respect to S_e are relatively weak. This indicates that compared to the usage time S_e of the system, users should pay more attention to the usage degree D_e when choosing the two-dimensional EW service. Meanwhile, as shown in Figure 9(c), there is an optimal EW period $(S_e, D_e) = (4\,years, 5\times 10^4\,km)$ for the transmission system, which minimizes the system's EW cost-availability ratio to 16325

In Fig. 9 (c), when S_e remains constant, as D_e increases, $R(S_e, D_e)$ initially declines and then subsequently rises. When D_e is larger, it means that the user has a greater usage degree of the transmission system, which increases the system's failure rate, reduces the system's availability, and increases warranty cost, thereby increasing the system's EW cost-availability ratio. When D_e is smaller, it means that the user's usage degree of the transmission system is relatively low. Although this will diminish the failure rate of system, it will waste the PM cost and PM time, thereby increasing the system's EW cost-availability ratio. If D_e is too small, PM not only fails to achieve the desired improvement effect, it also causes the warranty cost and system downtime to go up. Therefore, the value of D_e should be within a reasonable range to minimize the system's EW cost-availability ratio.

5.3 Sensitivity analysis

5.3.1 The influence of fault correlation coefficient σ on EW cost-availability ratio R

The fixed EW period S_e is 4 *years*, and D_e is 5×10^4 *km*, under different PM interval T, the variation relationship between $R(T)$ and σ is shown in Figure 10(a). The fixed PM interval T is 0.5 *years*, under different EW period S_e, D_e, the variation relationship between $R(S_e, D_e)$ and σ is shown in Figure 10(b).



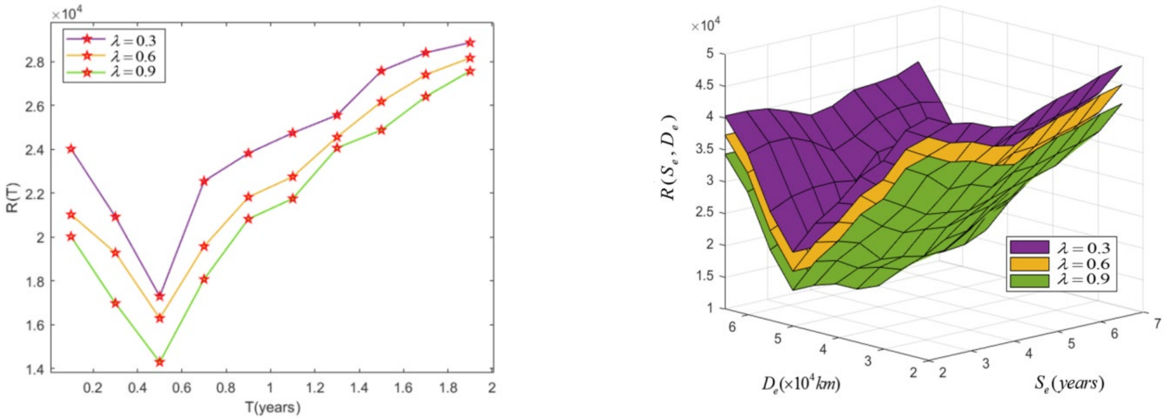
(a) The relationship between $R(T)$ and σ (b) The relationship between $R(S_e, D_e)$ and σ

Fig. 10. The relationship between EW cost-availability ratio and fault correlation coefficient

As depicted in Figure 10, when fault correlation coefficient σ is reduced from 0.8 to 0.2, the minimum EW cost-availability ratio R of the transmission system is reduced from 18029 to 15210; that is, when σ is reduced by 1%, R is reduced by 0.21%. This suggests that manufacturer should strive to optimize the structure and design of equipment to achieve smaller fault correlation coefficient, reduce the EW cost-availability ratio of equipment, and improve user satisfaction.

5.3.2 The influence of PM improvement factor λ on EW cost-availability ratio R

The fixed EW period S_e is 4 *years*, and D_e is 5×10^4 *km*, under different PM interval T, the variation relationship between $R(T)$ and λ is shown in Figure 11(a). The fixed PM interval T is 0.5 *years*, under different EW period S_e, D_e, the variation relationship between $R(S_e, D_e)$ and λ is shown in Fig. 11(b).



(a) The relationship between $R(T)$ and λ (b) The relationship between $R(S_e, D_e)$ and λ

Fig. 11. The relationship between EW cost-availability ratio and PM improvement factor

As depicted in Fig. 11, when PM improvement factor λ is increased from 0.3 to 0.9, the minimum EW cost-availability ratio R of the transmission system is reduced from 17536 to 14439; that is, when λ is increased by 1%, R is reduced by 0.09%. This suggests that manufacturer should strive to enhance their maintenance techniques and capabilities to achieve a larger PM improvement factor, reduce the EW cost-availability ratio of equipment, and improve user satisfaction.

6. Conclusion

This article builds the EW cost and availability model for multi-component systems with inter component fault correlation, and subsequently forms the EW cost-availability ratio model. The model takes minimizing the two-dimensional EW cost-availability ratio as the decision objective, with PM interval T, EW usage time period S_e, and EW usage degree period D_e as decision variables. Considering the model's complexity, this research introduces an ABC intelligent algorithm for the model solution.

In the case study, it was found that: (1) compared to assuming that the component fault independence, considering fault correlation would increase the system's EW cost by 11%, reduce EW availability by 11%, and increase EW cost-availability ratio by 20%. This fully demonstrates that ignoring the fault correlation between components can seriously deviate from the expectations of the user and the manufacturer for warranty, leading to erroneous warranty decisions. (2) The system's EW cost-availability ratio is significantly affected by the PM interval T and EW period (S_e, D_e), and the values of T and (S_e, D_e) should be moderate. Being too large or too small is not conducive to reducing the EW cost-availability ratio of the system. (3) Manufacturer should strive to improve their PM technology level in order to achieve higher PM improvement factor λ; at the same time, manufacturer should try to optimize the structure and design of the system as much as possible to reduce the fault correlation coefficient σ.

The future extension directions of this research include: (1) Exploring more complex fault correlation relationships among components in multi-component systems. (2) Investigating more effective algorithmic solutions could be a promising avenue for future research.

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