

Pricing and inventory coordination in cross-border e-commerce supply chains based on revenue sharing contracts**Xiaohui Hu^a, Yu Lu^a, Jingxian He^a, Fuchang Li^{a,b*} and Tianhao Xu^{a*}**^a*School of Economics, Yunnan Normal University, Kunming 650500, China*^b*Key Laboratory of Digital Finance Development and Management of Yunnan Provincial Department of Education, Kunming 650500, China***CHRONICLE***Article history:*

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Keywords:

*Cross-border e-commerce**Pricing strategy**Inventory mechanism**Export tax rebate**Import duty***ABSTRACT**

Against the backdrop of the rapid development of cross-border e-commerce, pricing and inventory coordination are the core links in its supply chain management, which are of great significance for improving the efficiency of enterprise resource allocation, balancing the interests of the chain's internal entities, and enhancing the resilience of the supply chain. This study examines the impact of government policies on supply chain operations by analyzing cooperative and non-cooperative situations between overseas suppliers and domestic retailers. In the centralized decision-making model, overseas suppliers and in-country retailers fully cooperate in determining commodity prices, inventory levels and revenue distribution. In the decentralized decision-making model, both parties make decisions independently for their own benefit. By comparing the supply chain operation under the two models, it is found that the centralized decision-making model can maximize the overall profit of the supply chain. To further optimize supply chain coordination, this study introduces the revenue sharing contract model. In this model, the retailer shares part of the revenue to the supplier in order to incentivize the supplier to reduce the wholesale price, thus realizing the overall profit of the supply chain. At the same time, it is also agreed in the contract that the retailer's excess revenue is shared to the supplier at a certain percentage to balance the interests of both parties. Through comparative analysis, under the revenue sharing contract, the price of goods is more competitive, consumer demand is stimulated, and the profit of the whole supply chain is improved. It is further found that under the centralized decision-making model, there exists an optimal export tax rebate rate and import tariff rate that maximizes the supply chain profit. In addition, commodity pricing is negatively correlated with the export tax rebate rate and positively correlated with the import tariff; inventory is positively correlated with the export tax rebate rate and negatively correlated with the import tariff. This provides a theoretical basis for the government to formulate relevant policies. Finally, the theoretical conclusions of this study are verified through numerical examples. The results show that the revenue-sharing contract can effectively coordinate cross-border supply chains and improve overall profits. The government should fully consider the impact of export tax rebates and import tariffs when formulating relevant policies to promote the healthy development of cross-border supply chains.

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1. Introduction

Cross-border e-commerce is based on the background of the rise of the Internet, the goods of different customs subjects can be traded through e-commerce platforms, and then through international logistics to complete the distribution of a commercial activity. Compared with the traditional foreign trade mode, cross-border e-commerce has the advantages of low cost, strong control and fast response, which significantly reduces the threshold for the internationalization of small and medium-sized enterprises and strongly promotes the transformation and upgrading of traditional foreign trade enterprises (Pan et al., 2023). E-commerce activities create significant opportunities for small and large enterprises to operate and provide products and services on a global scale (Cha et al., 2023). Eduardsen et al. (2023) believes that e-commerce is now an important channel for many companies looking to expand into international markets. About 12% of global trade takes place within cross-border

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e-commerce platforms, and 86% of tech startups report some type of cross-border activity. For enterprises, the open, multi-dimensional and three-dimensional multilateral economic and trade cooperation mode built by cross-border e-commerce greatly broadens the path to enter the international market, greatly promotes the optimal allocation of multilateral resources and the mutual benefit and win-win situation between enterprises. For consumers, cross-border e-commerce makes it very easy for them to get information from other countries and buy good and cheap goods. For foreign trade enterprises, e-commerce can reduce distance-related costs more in service trade than in goods trade, which indicates that cross-border e-commerce plays an important role in improving the competitiveness of service trade. Meanwhile, the government also plays a significant role in it. Urata(2018) demonstrates that the cross-border e-commerce support policy proposed by the government can improve the enthusiasm of foreign trade enterprises, formulate the strategy of free shipping, so that enterprises can obtain more orders and gain more profits. Raising the threshold of free shipping within a certain range will help improve the average order price and gross profit rate of enterprises, and reduce the cost of enterprises. Yin et al. (2023) believes that with the proposal of the "Belt and Road" Initiative, China's cross-border e-commerce has a huge impact on its export of goods and services to the "Belt and Road" countries. With the improvement of the level of cross-border e-commerce, the impact of distance on trade in services (goods) shows a trend of decreasing (increasing) year by year, while the impact on trade in services (goods) shows a trend of increasing (decreasing) year by year. Promoting the sustainable development of cross-border e-commerce can increase the export volume. China attaches great importance to the development of cross-border e-commerce and has formulated various policies to facilitate transactions between different countries, which has made cross-border e-commerce an important force in the Belt and Road Initiative. The development of cross-border e-commerce has a great impact on the economy, so it is necessary to promote the rapid development of e-commerce. In order to promote inclusive e-commerce participation, policies must be put in place to improve the country's formal institutions, facilitate the entry of less developed countries into the e-commerce sector, and promote the adoption of e-commerce by small and medium-sized enterprises. In addition, the role of cross-border e-commerce was particularly evident during the global pandemic. Liu(2021) analyzes that with the continuous upgrading of global trade interconnection and foreign trade, cross-border e-commerce businesses quietly spread all over the world. In the case of the global epidemic, cross-border e-commerce business is safer and more convenient, so that global consumers can buy their favorite goods without leaving their homes. Han et al. (2024) states that the trade volume brought by cross-border e-commerce industry is rising in the foreign trade volume and has become the main channel of foreign trade.

As a strategic driver of economic integration and trade globalization, cross-border e-commerce possesses immense potential and profitability, fundamentally transforming international trade (Zhou et al., 2023). In order to create greater profit space for our foreign trade enterprises and create more wealth for the society(Qiao & Qi, 2018). In 2023, the cross-border e-commerce import and export volume of Ningbo reached 230.2 billion yuan, increasing by 14.8% year-on-year and contributing 1.6 percentage points to the growth of the city's total foreign trade exports. According to the data released by the General Administration of Customs in 2015-2025, the total volume of cross-border e-commerce imports and exports has increased year by year, rising from 36.02 billion yuan in 2015 to 23.700 billion yuan in 2024, with a year-on-year growth of 5.7% in 2025. Feindt et al. (2002) believes that in a complex social environment, the development of e-commerce has increased more positions and promoted employment. According to the data of the National Engineering Laboratory of E-commerce Transaction Technology, in 2024, the number of e-commerce practitioners in our country increased by approximately 2.94% year-on-year, reaching 70 million. The development of cross-border e-commerce will inevitably become the development direction of cross-border trade in the future and play a huge role in promoting economic growth. Cha et al. (2023) deems that e-commerce activity is on the rise, creating significant opportunities for small and large businesses to operate and provide products and services globally, with the global cross-border e-commerce market estimated to be worth around \$780 billion in 2019 and expected to reach around \$4,842 billion by 2026. Similarly, other estimates suggest that there will be about 18 billion cross-border e-commerce orders in 2025, compared to 3.2 billion in 2015. Cross-border e-commerce is gradually evolving into a significant force driving the liberalization of global trade and economic integration, exerting a profound influence on national competitiveness, enterprise internationalization, and consumer welfare.

The pricing of cross-border e-commerce is a very complex issue, because it involves multiple subjects and links, including production, import and export, transportation, sales, etc. In this process, due to the uncertainty of foreign demand for cross-border goods, the price competition in the consumer market is fierce, and Chinese goods win in the export market with better quality and lower price. However, pricing is an important decision for the profit of multiple parties, which is often affected by multiple factors, such as cost, market demand and competition. Azadeh et al. (2017) demonstrates that pricing is the core issue of cross-border e-commerce research. Therefore, the pricing of cross-border e-commerce needs to consider multiple factors in order to achieve the optimal pricing strategy. First, the pricing of cross-border e-commerce needs to take into account the cost factor. Since cross-border e-commerce involves multiple links, including production, import and export, transportation, etc., the cost will increase accordingly. In pricing, these cost factors need to be taken into account comprehensively to determine the final selling price. In addition, it is also necessary to consider the impact of exchange rate fluctuations, taxes and other factors on the cost. Secondly, the pricing of cross-border e-commerce needs to take into account market demand factors. The market demand of cross-border e-commerce is often uncertain, so it is necessary to take into account changes in market demand and pricing strategies of competitors when pricing. Li et al. (2014) believes that when the market demand is high, the price can be appropriately raised to obtain higher profits. Finally, the pricing of cross-border e-commerce needs to take into account the impact of multiple agents. Cross-border e-commerce involves many subjects,

including manufacturers, importers and exporters, transporters, and sellers. In pricing, the interests of these subjects need to be taken into account comprehensively to achieve the optimal pricing strategy. At the same time, the rights and obligations of each entity need to be taken into account to ensure the stable operation of the entire supply chain.

At present, the pricing of cross-border e-commerce is mainly carried out from three aspects: cross-border logistics costs, exchange rate fluctuations and market competition. The logistics costs of cross-border e-commerce are relatively high, including sea transport, air transport, tariffs, insurance and other costs, so suppliers need to consider these cost factors to develop pricing strategies. Due to exchange rate fluctuations, the monetary value of cross-border e-commerce can be affected, so suppliers need to consider exchange rate risk to develop pricing strategies. The cross-border e-commerce market is highly competitive, and suppliers need to consider the price and quality of competitors in order to develop competitive pricing strategies. However, there are still some problems in the pricing of cross-border e-commerce that need to be optimized. For example, pricing strategy is not flexible enough, pricing data is not accurate enough, personalized pricing is lacking, foreign demand and consumer preferences are not fully considered, and factors such as inventory are not considered in cross-border commodity pricing. Shen(2021) believes that due to the complexity of cross-border e-commerce supply chain, suppliers need to flexibly adjust pricing strategies to adapt to market demand and changes. However, some suppliers' pricing strategies are too rigid to be adjusted in a timely manner. Pricing decisions need to rely on accurate data support, however, because cross-border e-commerce involves multiple countries and regions, the collection and analysis of data becomes more difficult, so suppliers need to find more accurate data sources. The needs and preferences of cross-border e-commerce consumers vary, so suppliers need to adopt personalized pricing strategies to meet consumer needs. However, some vendors lack the ability to personalize pricing. Therefore, cross-border e-commerce supply chains need to optimize pricing strategies to improve market competitiveness and consumer satisfaction.

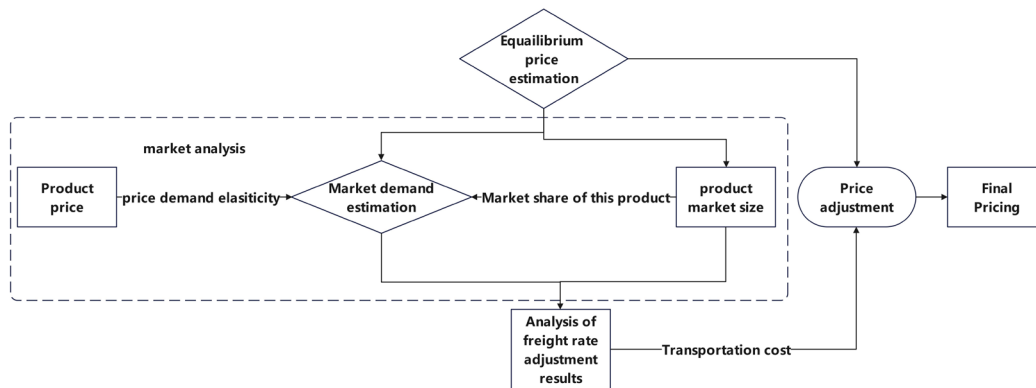


Fig. 1. Traditional Pricing flow chart

The inventory management of cross-border e-commerce is a very important issue, which directly affects the operation efficiency and profit of enterprises. He et al. (2020) deems that when making inventory decisions, enterprises need to take into account multiple factors, including warehouse rental or self-construction, commodity delivery time, order quantity, etc. In cross-border e-commerce, due to the uncertain delivery time of goods, enterprises need to be more cautious and flexible in inventory management. First of all, the rental or self-construction of warehouses is a key factor in the inventory management of enterprises. When choosing a warehouse, companies need to consider factors such as the location, area, and facilities of the warehouse, as well as the cost of renting or self-building. When choosing a warehouse, enterprises need to take these factors into account to ensure the efficiency and cost of inventory management. Secondly, goods delivery time and order volume are also important factors in enterprise inventory management. Das et al. (2023) believes that in cross-border e-commerce, the delivery time of goods is uncertain, which requires enterprises to be more cautious and flexible in inventory management. In the case of uncertain order volume, enterprises need to timely adjust the inventory of goods according to market demand and supply capacity to ensure the balance of supply and demand. Finally, the inventory shortage or backlog of goods in the warehouse is also a problem for enterprise inventory management. If there is a shortage or backlog of goods in the warehouse, it means that the inventory is not optimal at this time, and the enterprise needs to adjust (Zhu et al., 2022). When adjusting inventory, enterprises need to comprehensively consider market demand, supply capacity, warehouse capacity and other factors to ensure the efficiency and cost of inventory management.

The inventory decision of a cross-border e-commerce supply chain mainly considers the following aspects, maintaining the appropriate inventory level, inventory management and inventory distribution. Suppliers need to reduce inventory levels as much as possible while ensuring timely delivery. Therefore, suppliers need to make inventory planning according to market demand and sales forecast. Suppliers need to manage the inventory, including regular inventory, goods inspection, fresh-keeping, etc. Inventory management helps to avoid loss and waste of goods. Cross-border e-commerce involves many countries and regions, and suppliers need to determine the distribution of inventory according to market demand and logistics costs and other factors. At present, there are still a series of problems in the inventory management of cross-border e-commerce supply chains, such as too much or too little inventory, inaccurate inventory data, and non-standard inventory management. Some suppliers may over-maintain inventory, leading to inventory overhang and capital occupation; Other suppliers may

choose too little inventory to meet consumer demand in a timely manner. Inventory decisions need to rely on accurate data support, however, because cross-border e-commerce involves multiple countries and regions, the collection and analysis of data becomes more difficult, so suppliers need to find more accurate data sources. Some suppliers lack effective inventory management methods and processes, resulting in loss and waste of goods. Luo et al. (2022) demonstrates that the inventory decision of a cross-border e-commerce supply chain is very important, because it is directly related to the supplier's capital occupation, logistics costs and consumer satisfaction. Too much inventory will occupy too much capital, resulting in capital turnover, and too little inventory will lead to failure to meet consumer demand. Therefore, inventory management needs to find a balance point, both to meet consumer demand, but also to reduce capital occupation as much as possible, cross-border e-commerce logistics costs are relatively high, including sea transport, air transport, tariffs, insurance and other costs. Miao et al. (2019) believes that inventory management needs to consider logistics cost factors and reduce logistics cost through reasonable inventory planning and inventory distribution. Inventory management is also directly related to consumer satisfaction. Too much inventory can lead to obsolescence, damage, or waste, while too little inventory can lead to failure to meet consumer demand in a timely manner. Therefore, inventory management requires inventory planning according to market demand and sales forecast to meet consumer demand.

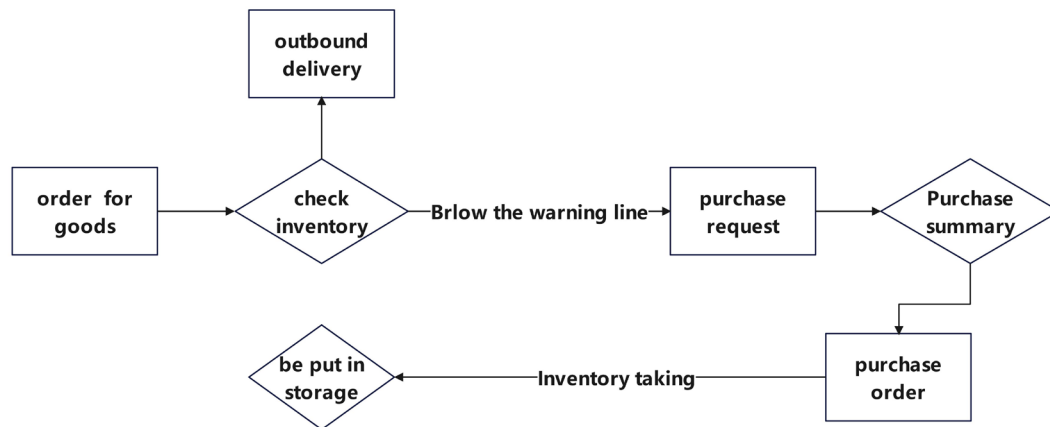


Fig. 2. Traditional Inventory decision flow chart

Ahi et al. (2022) demonstrates that traditional pricing and inventory are not optimal pricing and inventory, and the coordination of pricing and inventory in cross-border e-commerce overcomes the limitations of traditional research on pricing and inventory. Pricing and inventory coordination can help suppliers reduce capital footprint. Through reasonable pricing strategy and inventory planning, suppliers can reduce inventory levels as much as possible, thus reducing capital occupation, and pricing and inventory coordination can also reduce logistics costs. Through reasonable pricing strategy and inventory planning, suppliers can reduce inventory overstocking and logistics costs, improve logistics efficiency, and pricing and inventory coordination can also improve consumer satisfaction. Through reasonable pricing strategy and inventory planning, suppliers can meet consumer demand in a timely manner and improve consumer satisfaction. Pricing and inventory coordination are very important for cross-border e-commerce supply chains. Reasonable pricing strategy and inventory planning can help suppliers improve capital utilization efficiency, reduce logistics costs, improve inventory turnover and meet consumer demand, so as to improve suppliers' market competitiveness and consumer satisfaction. In addition, the integration and optimization of inventory and pricing will affect the competitiveness of enterprises.

Revenue sharing contracts can effectively coordinate cross-border supply chain pricing and inventory, as it can motivate all participants in the supply chain to work together to achieve optimal pricing and inventory decisions. The following are specific reasons: Encourages collaboration: Revenue sharing contracts can motivate all participants in the supply chain to work together to achieve optimal pricing and inventory decisions. This is because revenue sharing contracts allow all participants in the supply chain to share profits, thereby motivating them to work together to achieve optimal pricing and inventory decisions. Optimizes pricing and inventory decisions: Revenue sharing contracts can optimize pricing and inventory decisions, thereby improving the efficiency and effectiveness of the supply chain. This is because revenue sharing contracts allow all participants in the supply chain to jointly consider factors such as market demand, sales forecasts, and logistics costs, thereby achieving optimal pricing and inventory decisions. Reduces costs and risks: Revenue sharing contracts can reduce costs and risks in the supply chain. This is because revenue sharing contracts allow all participants in the supply chain to jointly bear costs and risks, thereby reducing costs and risks in the supply chain. In summary, revenue sharing contracts can effectively coordinate cross-border supply chain pricing and inventory. However, the mechanism of how revenue sharing contracts can be integrated and coordinated is not yet clear. Therefore, this article aims to conduct research on this topic.

The contributions of our work to the literature on commodity pricing and inventory in cross-border e-commerce are as follows: First, we obtain optimal pricing and inventory decisions by solving optimization methods and through joint decision-making, which have not been effectively addressed in the existing literature. Secondly, there are few studies on the pricing and inventory of cross-border e-commerce in the existing literature, and this paper also considers consumer psychology. Through

the research of this paper, it is found that cross-border e-commerce can obtain the optimal pricing and inventory under the joint decision of pricing and inventory, so that the multiple parties of the supply chain can obtain the maximum profit. Finally, through the analysis of this paper, it is found that tariffs will affect the decisions of suppliers and retailers, and the sharing ratio will also affect the interests of both parties and even the profits of the supply chain.

Based on this, assuming that the cross-border e-commerce supply chain is a two-tier supply chain composed of suppliers and retailers, and suppliers only provide one product to retailers. Market demand depends on price and inventory levels. Using game theory, we established four models, namely centralized, decentralized, retailer-led revenue sharing, and surplus revenue sharing, and analyzed them from both analytical and numerical perspectives to determine the optimal decision-making. The main purpose of this study is to find answers to the following questions:

What inventory level will retailers maintain? How should cross-border goods be priced?
 Why should retailers share revenue with manufacturers? What is the economic impact on retailers?
 Will sharing of surplus income, including consumers, have a positive impact?
 How do import tariffs and export rebates affect supply chain profits?
 How does revenue sharing ratio affect supply chain profits?

The remaining sections of this article are arranged as follows: the literature review will be presented in the next section. Section 3 provides a description of the problem, symbols, and assumptions. Section 4 develops the proposed models and analyzes the optimal decision-making and profitability of channel members. In Section 5, numerical illustrations are provided. Section 6 summarizes the entire article and provides guidance for further research.

2. Literature review

2.1 *cross-border e-commerce supply chain pricing*

Yan et al. (2022) believes that logistics service and product price are two factors that affect consumers' purchasing behavior. Guo(2022) demonstrates that commodity pricing will affect consumers' purchase decisions. When making purchase decisions, customers are not only concerned about price, but make a comprehensive comparison of the benefits and costs of alternative products, and then choose products. When pricing, enterprises must also analyze the benefits and costs of products according to customers' ideas. In agricultural production, reasonable pricing is beneficial for farmers to maximize profits in production activities. If farmers decide the price of their inputs strategically, their returns will be higher. Under revenue-sharing contracts, the value of strategic pricing to farmers decreases with the increase of market competition. Under wholesale contracts, the value of strategic pricing to farmers may increase or decrease in market competition. In addition, Jain et al. (2023) believes that different geographical locations of farmers and different pricing rules will have different impacts on farmers' interests. The same is true of cross-border e-commerce products, and reasonable pricing decisions can help enterprises get higher returns. Shi-Zhen et al. (2016) demonstrates that the practice of e-commerce shows that the trust and convenience perceived by online consumers are affected by the order of payment and delivery. Online sellers can improve the shopping experience of online consumers by providing different payment schemes. Therefore, sellers' pricing of goods will have an impact on consumers' purchasing behavior. Dasgupta et al. (2003) demonstrates that charging different prices to buyers with different purchase standards improves the profit of sellers. When all sellers in the market use a tiered pricing strategy, price wars among sellers that cause regular price fluctuations in the market are also prevented. The study found that when consumers realize that others (consumers in a dominant position) are buying the same product online for a better price. Liu et al. (2022) believes that at present, the pricing of service portfolios by network service providers in the market is not scientific, and arbitrary and temporary pricing is still relatively common. Dynamic pricing is also becoming a common pricing model in supply chain product pricing, and the Internet is transforming the automotive industry as traditional manufacturer and dealer structures face a growing threat from third-party e-tailers. Manufacturers can address these challenges with dynamic pricing and direct-to-customer business models. Biller et al. (2005) argues that, in fact, by coordinating production and inventory decisions with dynamic pricing, firms can increase profits and improve supply chain performance.

2.2 *cross-border e-commerce supply chain inventory*

According to Wang et al. (2022), inventory management (e.g. loss of sales) is a central issue in supply chain management. Najafnejhad et al. (2021) In supply chain management, supplier management of inventory plays a crucial role in the production system. It is not only important for suppliers to manage inventory, but also necessary for retailers to manage inventory. In addition, Cai et al. (2024) believes that the sales effort of enterprises will affect product inventory, and the risk aversion of inventory managers has a significant impact on the inventory management mode preference of supply chain members. The research of Li et al. (2015) shows that in dual-channel, consumer demand is affected by inventory level, and the customer cannot ignore the dependence of demand on inventory level when ordering goods. Chen et al. (2024) Persistent supply chain problems endanger the prosperity of businesses and their suppliers due to supply chain instability, which increases the likelihood of overstocking (having too many goods that consumers don't want) or overstocking (not actually having enough goods that buyers like). Jauhari et al. (2021) believes in finding the optimal shipment quantity, production allocation, shipment

number, safety factor, defect rate and productivity to minimize supply chain costs. Similarly, in the cross-border e-commerce supply chain, enterprises can effectively reduce their costs and maximize their profits by finding the optimal inventory level. Zhou et al. (2024) believes that the inventory level of products will also be affected by many external factors, for example, the long journey of cross-border goods will be affected by the transportation time. The delay of transportation time and the change of inventory adjustment time will have an impact on the inventory level of each node enterprise in the supply chain (Kano, 2018).

As seen, some of the literature mentioned above considers the impact of consumer behavior on product pricing or inventory, while another part of the literature considers the impact of producer production scale on product pricing. However, there is relatively little literature that simultaneously considers product pricing and inventory, and even fewer studies on the factors affecting profits in cross-border e-commerce supply chains. Most of the existing research on pricing or inventory based on revenue-sharing contracts is focused on suppliers as leaders. Based on this, this paper introduces tariffs in the study of pricing and inventory in cross-border e-commerce, explores the relationship between supply chain profits and pricing and inventory, and establishes a revenue-sharing contract model led by retailers to investigate the changes in supply chain profits with the sharing ratio.

3. Related work

3.1 Problem description

In this section, a cross-border e-commerce supply chain consisting of one supplier and one retailer was analyzed under the principles of profit maximization for both parties, considering cooperation and non-cooperation scenarios and the role of revenue-sharing contracts. Specifically, in the case of cooperation, both parties consider maximizing supply chain profits rather than their own interests. In the case of non-cooperation, both parties are selfish and only consider maximizing their own profits. As the leader of the supply chain, the decision-making party makes decisions, and the follower adjusts their decisions accordingly to maximize profits in this situation. The overseas supplier sells products to the retailer at a wholesale price W , and the retailer incurs a sales cost of c per item sold to consumers. In addition, a storage fee of CK is generated for each item in inventory after it is received. Consumers have a basic demand of a , but market demand is affected by price and inventory. With the rapid development of e-commerce in the context of the Internet, cross-border purchases have entered people's lives, and cross-border e-commerce has emerged. The self-operated transaction model of cross-border e-commerce, in which product suppliers send goods to retailers for platform sales, such as JD.com's self-operated model, and domestic retailers purchase goods from foreign suppliers and sell them directly to consumers, such as the four major brands VANKYO, VANTOP, SNAPTAIN, and HEIMVISION on the Wantro website, has expanded consumer purchasing channels. To promote the rapid development of cross-border e-commerce, the government has introduced a series of supportive policies, such as logistics subsidies and tariff reductions. In this paper, import tariffs and export tax rebates are considered, and the import tariff rate is higher than the export tax rebate rate. Consumers have a sensitivity to price of $0 < b < 1$ and a sensitivity to inventory of $0 < \theta < 1$. Based on this, it is assumed that the cross-border e-commerce supply chain is a second-tier supply chain consisting of one supplier and one retailer, and that the supplier only provides one product to the retailer. The goal of this paper is to seek an optimal pricing and inventory strategy that maximizes profits for both the cross-border e-commerce and overseas supplier during the continuous operation period, namely, maximizing the value of sales revenue minus costs. It is assumed that the supplier's production capacity is infinite and can satisfy the retailer's demand for goods at any time, so the supplier's inventory is not considered and only the retailer's inventory is considered. It is assumed that the retailer's purchase quantity is fixed at K each time and all previous inventory can be sold before the next purchase. The portion of consumer demand that has cleared customs is delivered directly to consumers, while the remaining portion incurs inventory costs in the warehouse. The manufacturer faces a price-dependent demand from the retailer and supplies finished products to the retailer. The retailer directly transacts with consumers and delivers the goods to them as quickly as possible based on their demand, as shown in Fig. 3.

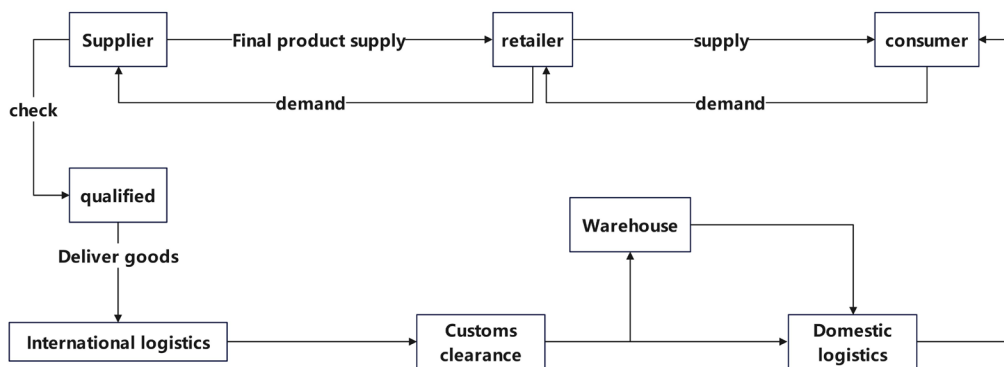


Fig. 3. Flow chart

Assuming that the maximum objectives of both the supplier and retailer under decentralized decision-making are to maximize their own interests, as independent decision-makers, the supplier acts as the leader and the retailer as the follower in decentralized decision-making. The supplier and retailer are in a dynamic game of complete information, and time is continuous, which is suitable for the Stackelberg game model. Under revenue-sharing contracts, the retailer acts as the leader and the supplier as the follower, and they are also in a dynamic game of complete information, which is suitable for the Stackelberg game model. λ represents the inventory investment cost coefficient, which is the ratio of actual investment to total investment, where total investment includes storage subsidies provided by the government, and λK^2 represents the inventory investment cost paid by the retailer for building a warehouse. Assuming that the purchase quantity is K , the number of goods entering the warehouse is $K-D$. When the supplier exports goods, there is an export tax rebate policy, and when the retailer imports goods, it needs to pay import tariffs. The export tax rebate rate is t_1 , and the unit import goods are taxed at t_2 . The export tax rebate amount obtained by the supplier is $t_1 w K$, and the tariff cost paid by the retailer is $t_2 (K-D)$. Assuming that the market demand D is related to the basic demand a , the sensitivity of consumers to price b , and the sensitivity of consumers to inventory level θ , consumers will worry about not being able to buy the product and choose to believe in the quality of the product and place an order immediately when the inventory level is low. When the inventory level is high, consumers have a psychological perception that the product is due to an inventory backlog caused by poor sales, and the demand of consumers is negatively correlated with the inventory level. Therefore, $D=a-bP-\theta K$. Assuming that the inventory cost of the product is calculated per piece.

3.2 Notations and assumptions

The following notations are used for developing the proposed model:

symbol	meaning
w	Unit wholesale price
p	Unit selling price
D	Market demand
c	Unit cost of products sold
C_k	Cost per unit of product inventory
t_1	Export tax rebate rate for unit products of suppliers
t_2	Specific duty on each unit of goods imported by retailers
π_m	Supplier profit
π_R	Retailer profit
π_g	The overall profit of the supply chain
b	Consumer sensitivity to price
a	Basic needs
θ	Consumer sensitivity to inventory
K	Product inventory level
β	The proportion of revenue sharing
λ	Inventory investment cost factor

The following assumptions are made for establishing the proposed model:

Assumption 1 The manufacturer and the retailer are responsible for maximizing their own objectives. They work as independent decision makers with the manufacturer as the Stackelberg leader and the retailer as the Stackelberg follower in case of decentralized policy.

Assumption 2 In order to make sure the non-negativity of the optimal decisions, it is assumed that $a > bP - \theta k$, $\theta^2 < w + 4bw\lambda$, $\theta^2 < 2 + 4b\lambda$, $\theta < 2t_1$.

Assumption 3 The inventory cost of goods is calculated on a piece-by-piece basis.

Assumption 4 The retailer's purchase quantity is fixed at K each time, it can sell all the goods in the previous purchase before the next purchase.

Assumption 5 Overseas suppliers have unlimited capacity to produce and supply products, and can supply the corresponding quantity of products according to the needs of buyers at any time.

Based on the above assumptions, the profit functions of the supplier and retailer are derived as follows:

$$\Pi_m = (w - c)D + t_1 w K \quad (1)$$

$$\Pi_R = PD - wD - C_K (K - D) - t_2 K - \frac{1}{2} \lambda K^2 \quad (2)$$

4. Model development and analysis

This paper will study the optimal pricing and inventory coordination strategies of cross-border e-commerce suppliers and retailers by establishing four models: the centralized decision-making model, the decentralized decision-making model, the retailer-led revenue-sharing contract model, and the sharing of excess profits revenue-sharing contract model. By studying the behavior of suppliers and retailers under centralized and decentralized decision-making, a win-win optimal pricing and inventory strategy can be obtained through comparison.

4.1 Pricing and inventory coordination of cross-border e-commerce supply chain under decentralized decision-making

Under decentralized decision-making, both the supplier and retailer act as rational economic agents and first consider maximizing their own interests. In the Stackelberg game model, the supplier acts as the leader and decides the wholesale price to be offered to the retailer, while the retailer acts as the follower and first decides the selling price and inventory level of the product.

Proposition 1: Under decentralized decision-making, there exists an optimal wholesale price w for the supplier, and an optimal inventory level and optimal product pricing for the retailer

$$w_f = \frac{2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c) + b\theta M(\theta - 2t_1) - bCF + 2abC}{2bC} \quad (3)$$

$$P_f = \frac{2bC[CF + \theta^2 M + 2b\theta(C_K + t_2)] + (bC - b\theta^2) \left[\frac{2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c)}{+b\theta M(\theta - 2t_1) - bCF + 2abC} \right]}{4b^2C^2} \quad (4)$$

$$K_f = \frac{b\theta[2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c) + b\theta M(\theta - 2t_1) - bCF + 2abC] - 2bC[2b(C_K + t_2) - \theta M]}{2bC^2} \quad (5)$$

Among them, $F = a - bC_K$.

Proof Proofs of Proposition 1 and subsequent propositions are given in Appendix.

Corollary 1: Under centralized and decentralized decision-making, the retailer's product pricing will gradually decrease with the increase of the export tax rebate rate and gradually increase with the increase of the import tariff rate. The retailer's inventory will also increase with the increase of the export tax rebate rate and decrease with the increase of the import tariff rate.

$$\text{When making centralized decisions, } \frac{\partial P_j}{\partial t_1} = \frac{-2bw\theta}{2bC} < 0, \frac{\partial P_j}{\partial t_2} = \frac{1}{2bC} > 0, \frac{\partial K_j}{\partial t_1} = \frac{2bw}{C} > 0, \frac{\partial K_j}{\partial t_2} = \frac{-2b}{C} < 0,$$

When you decentralize decision making,

$$\frac{\partial P_f}{\partial t_1} = \frac{b(C - \theta^2) - 4b^2(C_K + t_2) - 2b\theta M}{4b^2C^2} < 0, \frac{\partial P_f}{\partial t_2} = \frac{4b^2\theta C + 2b^3(c - \theta^2)(\theta - 2t_1)}{4b^2C^2} > 0, \frac{\partial K}{\partial t_1} = \frac{b^2\theta[4b(C_K + t_2) + 2\theta M]}{2bC^2} > 0$$

$$\frac{\partial K}{\partial t_2} = \frac{b\theta[2b^2(\theta - 2t_1)] - 4b^2C}{2bC^2} < 0.$$

As can be seen from the above, the export tax rebate rate t_1 is a decreasing function of the price P , gradually decreasing with the increase of the tax rebate rate, while the import tariff rate t_2 is an increasing function of the price P , and the product pricing will gradually increase with the increase of the tariff rate. The export tax rebate rate t_1 is an increasing function of the inventory K , and the inventory will gradually increase with the increase of the export tax rebate rate, while the import tariff rate t_2 is a decreasing function of the inventory K , and the inventory will gradually decrease with the increase of the import tariff rate. The reason for this situation may be that the export tax rebate reduces the cost of the retailer, and the product pricing of the retailer decreases accordingly, making the product sell well, so the inventory increases; conversely, the same is true.

4.2 Pricing and inventory coordination of cross-border e-commerce supply chain under centralized decision-making

Under centralized decision-making, the common goal of the supplier and retailer is to maximize the profit of the supply chain. At this time, the wholesale price that the supplier offers to the retailer does not matter. Therefore, the profit function of the supply chain is:

$$\Pi g = PD - cD - C_K(K - D) + t_1 wK - t_2 K - \frac{1}{2} \lambda K^2 \quad (6)$$

Proposition 2: Under centralized decision-making, there exists a unique optimal solution between cross-border e-commerce suppliers and retailers, which maximizes the profits of both parties.

$$P_j = \frac{AC - B\theta^2 + 2b\theta(C_K - t_1w + t_2)}{2bC} \tag{7}$$

$$K_j = \frac{B\theta - 2b(C_K - t_1w + t_2)}{C} \tag{8}$$

Among them, $A = a + bc - bC_K$, $B = bc - bC_K - a$, $C = 2b\lambda - \theta^2$

By substituting Eq. (4) and Eq. (5) into Eq. (3), the profit of supply chain under centralized decision-making is

$$\begin{aligned} & [AC - B\theta^2 + 2b\theta(C_K - t_1w + t_2) - E][2aC - AC - B\theta^2 + 2b\theta(C_K - t_1w + t_2)] \\ \Pi_g^j = & \frac{-4bBC\theta(C_K - t_1w + t_2) - 8b^2(C_K - t_1w + t_2)^2 - 4b\lambda[B\theta - 2b(C_K - t_1w + t_2)]^2}{4bC^2} \end{aligned} \tag{9}$$

Among them, $E = 2bC(c - C_K)$.

Corollary 2: In cross-border e-commerce, it is not necessarily better for the export tax rebate rate to be higher or for the import tariff rate to be higher. There exists a unique optimal tax rebate rate and import tax rate that maximizes the profit of the supply chain.

$$\begin{aligned} & (-2bw\theta)[2aC - AC - B\theta^2 + 2b\theta(C_K + t_2)] + (-2bw\theta)[AC - B\theta^2 + 2b\theta(C_K + t_2) - E] \\ t_1 = & \frac{+4b\theta BCw + 16b^2w(C_K + t_2) - 16b^2\lambda w[B\theta - 2b(C_K + t_2)]}{8b^2w^2(4b\lambda + 2 - \theta^2)} \end{aligned} \tag{10}$$

$$\begin{aligned} & 2b\theta[2aC - AC - B\theta^2 + 2b\theta(C_K - t_1w)] + 2b\theta[AC - B\theta^2 + 2b\theta(C_K - t_1w) - E] \\ t_2 = & \frac{-4bBC\theta - 16b^2(C_K - t_1w) + 16b^2\lambda[B\theta - 2b(C_K - t_1w)]}{8b^2(4b\lambda + 2 - \theta^2)} \end{aligned} \tag{11}$$

So to sum up, because $\frac{\partial^2 \Pi_g}{\partial t_1^2} < 0$, $\frac{\partial^2 \Pi_g}{\partial t_2^2} < 0$, With the increase of the export tax rebate rate, more companies will join the cross-

border e-commerce industry, which increases competition among companies and forces some companies to start price wars, resulting in reduced profits. Therefore, it is not necessarily better for the export subsidy amount to be higher. If the import tariff rate is too low, foreign goods will flood in, causing domestic products to be unsalable, and domestic products will not be able to go out. With the increase of the import tariff rate, some foreign companies will give up cross-border sales due to high tax rates. If the tax rate keeps increasing and some products are preferred by domestic consumers, cross-border e-commerce will try to pass on the import tariffs to consumers, which will make price-sensitive consumers give up cross-border purchases and thus be unfavorable to the development of cross-border e-commerce. Revenue sharing contracts can coordinate pricing and inventory management through a reasonable pricing strategy, coordinated inventory management, sharing information and data, and a reasonable income distribution plan. Through revenue-sharing contracts, all links in the supply chain can work together to achieve win-win goals.

4.3 Pricing and inventory coordination of cross-border e-commerce supply chain under revenue-sharing contract

4.3.1 The revenue-sharing ratio is determined by the retailer

A good contract can help both parties maximize profits. Under a revenue-sharing contract, the retailer signs a contract with the supplier to obtain more homogeneous products at a lower price. They agree to share a portion of the revenue with the supplier, where the retailer retains a portion β ($0 < \beta < 1$) of the sales revenue and shares $1 - \beta$ of the revenue with the supplier. Therefore, the profit functions of the retailer and supplier are as follows:

$$\Pi_r^i = (\beta P - w)D - C_K(K - D) - t_2K - \frac{1}{2}\lambda K^2 \tag{12}$$

$$\Pi_m^I = (w - c)D + t_1 w K + (1 - \beta)PD \quad (13)$$

If the revenue-sharing model is led by the retailer, then under this model, the retailer has a better understanding of consumer demand. Therefore, the supplier has to accept the revenue-sharing ratio agreed upon by the retailer in the revenue-sharing model. Therefore, under this model, the optimal pricing and inventory of the retailer should be calculated first:

$$P_i = \frac{2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)}{2b\beta(2b\lambda - \beta\theta^2)} \quad (14)$$

$$K_i = \frac{bw\theta - a\beta\theta - G - 2bt_2}{2b\lambda - \beta\theta^2} \quad (15)$$

Among them, $G = bC_K(2 + \theta)$. The profit functions of the supplier is:

$$\Pi_m^I = \frac{2b\beta(w - c)(\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 4b\beta^2 t_1 (bw\theta - a\beta\theta - G - 2bt_2) + (1 - \beta)[2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)](\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2)}{4b\beta^2(2b\lambda - \beta\theta^2)} \quad (16)$$

Among them, $H = 2a(1 - b\lambda)$, $I = 2b^2\lambda$, $J = G\theta - b\theta^2 C_K$, $L = 2b\theta$.

The profit function of the supplier is a convex function, so there exists an optimal solution for the wholesale price of the supplier.

$$w_i = \frac{2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + 4b^2\beta^2\theta t_1 + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I[2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2)]}{4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2\beta^2\theta t_1} \quad (17)$$

The price and inventory of the retailer are:

$$P_i = \frac{2ab\beta\lambda - 2b^2\lambda C_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) + 2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + (2b^2\lambda - 2b\beta\theta^2) + 4b^2\beta^2\theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I[2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2)]}{2b\beta(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2\beta^2\theta t_1} \quad (18)$$

$$K_i = \frac{2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + 4b^2\beta^2\theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I[2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2)] - a\beta\theta - G - 2bt_2}{2b\lambda - \beta\theta^2} \quad (19)$$

Proposition 3: Under a cooperative contract, as the revenue-sharing ratio increases, the profit of the retailer improves, but to some extent, it deteriorates the profit of the supplier.

The profit functions of the retailer is:

$$\Pi_r^I = \frac{[(\beta H + C_K I - wI + \beta J + \beta t_2 L) - 2bw(2b\lambda - \beta\theta^2)](\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta(2b\lambda - \beta\theta^2)} - C_K \frac{2\beta(bw\theta - a\beta\theta - G - 2bt_2) - (2b\lambda - \beta\theta^2)(\beta H + C_K I - wI + \beta J + \beta t_2 L)}{2\beta(2b\lambda - \beta\theta^2)} \quad (20)$$

$$- \frac{2t_2(bw\theta - a\beta\theta - G - 2bt_2)(2b\lambda - \beta\theta^2) - \lambda(bw\theta - a\beta\theta - G - 2bt_2)^2}{2(2b\lambda - \beta\theta^2)^2}$$

$$\frac{\partial \Pi_m^I}{\partial \beta} = - \frac{[2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)](\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta^2(2b\lambda - \beta\theta^2)} < 0 \quad (21)$$

$$\frac{\partial \Pi_r^I}{\partial \beta} = \frac{\left[\begin{matrix} 2ab\beta\lambda + 2b^2\lambda(w - C_K) \\ + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2) \end{matrix} \right] (\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta^2(2b\lambda - \beta\theta^2)} > 0 \tag{22}$$

Therefore, it can be concluded that when the price and demand for the product are positive, the revenue-sharing ratio is a monotonically increasing function of the profit function of the retailer. The larger the share retained by the retailer, the smaller the share shared with the supplier, and the greater the profit obtained by the retailer, while the profit obtained by the supplier will decrease.

Corollary 3: Under a revenue-sharing contract, the wholesale price of overseas suppliers will be affected by tariffs, decreasing with the increase of import tariffs, and the export tax rebate rate has a minimum limit.

$$\frac{\partial w}{\partial t_1} = \frac{\left[\begin{matrix} -4b\beta^2(a\beta\theta + G + 2bt_2) + 4b^2\beta^2\theta w \\ + 8b^2\beta^2\theta \left[\begin{matrix} 2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1(a\theta\beta + G + 2bt_2) \\ + 4b^2\beta^2\theta t_1 w + (1-\beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) \end{matrix} \right] \\ - (1-\beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right] \end{matrix} \right]}{\left[4b\beta I(2b\lambda - \beta\theta^2) + 2I(1-\beta)(I - 2b\beta\theta^2) - 8b^2\beta^2\theta t_1 \right]^2} \tag{23}$$

$$\frac{\partial^2 w}{\partial t_1^2} = \frac{16b^2\beta^2\theta \left[\begin{matrix} -4b\beta^2(a\beta\theta + G + 2bt_2) + 4b^2\beta^2\theta w \\ + 8b^2\beta^2\theta \left[\begin{matrix} 2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1(a\theta\beta + G + 2bt_2) \\ + 4b^2\beta^2\theta t_1 w + (1-\beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) \end{matrix} \right] \\ - (1-\beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K \\ + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right] \end{matrix} \right]}{4b\beta I(2b\lambda - \beta\theta^2)} \tag{24}$$

$$\frac{\partial^2 w}{\partial t_1^2} = \frac{+2I(1-\beta)(I - 2b\beta\theta^2) - 8b^2\beta^2\theta t_1}{\left[4b\beta I(2b\lambda - \beta\theta^2) + 2I(1-\beta)(I - 2b\beta\theta^2) - 8b^2\beta^2\theta t_1 \right]^4} > 0$$

$$\frac{\partial w}{\partial t_2} = \frac{2b\beta^2 L X_1 - 8b^2\beta^2 t_1 + X_1 X_2 X_3 \beta L - 2b\beta\theta I X_2}{4b\beta I X_1 + 2I X_2 X_3 - 8b^2\beta^2\theta t_1} < 0 \tag{25}$$

Among them, $X_1 = 2b\lambda - \beta\theta^2$, $X_2 = 1 - \beta$, $X_3 = I - 2b\beta\theta^2$

From the above reasoning, it can be concluded that the wholesale price is a monotonically increasing function of the import tariff. Therefore, as the import tariff rate increases, the wholesale price of the supplier will also increase, ultimately leading to an increase in the selling price of the product when sold by the retailer to the consumer. The wholesale price is a quadratic function of the export tax rebate rate, and the second derivative of the export tax rebate rate is greater than zero, so the function is a concave function with a unique minimum value, which is the lower limit value of the tax rebate rate.

4.3.2 The revenue-sharing ratio is determined by the excess income

Under this revenue-sharing contract, the supplier wholesales the product to the retailer at a price lower than w , denoted as w_1 . The retailer carries out promotional activities and sells the product to consumers at a price lower than the selling price p . If the retailer sells more products at the lower wholesale price provided by the supplier, the retailer needs to distribute the excess income to the supplier in proportion to β ($0 < \beta < 1$). At this point, the demand function of consumers is $D_1 = a - bP_1 - \theta K_1$, and the excess demand is $D_1 - D$. Therefore, the profit functions of the retailer and supplier are as follows:

$$\Pi_m^c = (w_1 - c)D_1 + t_1 w_1 K_1 + (D_1 - D)P_1(1-\beta) \tag{26}$$

$$\Pi_r^c = P_1 D_1 - w_1 D_1 - C_K(K_1 - D_1) - t_2 K_1 - \frac{1}{2} \lambda K_1^2 - (D_1 - D)P_1(1-\beta) \tag{27}$$

In this case, assuming that the market demand is known, the retailer will make efforts to sell the goods in order to obtain a lower wholesale price from the supplier and share $1-\beta$ of the excess income with the supplier. At this point, the retailer still determines the price and inventory of the product. the optimal pricing and inventory of the retailer should be calculated first:

$$P_c = \frac{A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2}{2b\beta(2b\beta - \beta^2\theta^2)} \tag{28}$$

$$K_c = \frac{A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1 - \beta)}{2b\beta - \beta^2\theta^2} \quad (29)$$

Among them, $Y = C_K(1 + \theta)$, $A_3 = b\theta C_K - 2bY$

$$X = 2a - C_K\theta^2 + 2\theta Y, A_1 = bX + \theta + b\theta^2 C_K - D\theta^2 - 2bD, A_2 = 2bD - \theta - 2b^2 C_K$$

There exists an optimal solution for the wholesale price of the supplier:

$$\begin{aligned} & 2\beta(2b\beta - \beta^2\theta^2)(\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4b^2\beta^2 c(2b\beta - \beta^2\theta^2) \\ & + 4\beta^2 t_1(2b\beta - \beta^2\theta^2)[A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1 - \beta)] \\ & + 4b\beta^3\theta t_1(2b\beta - \beta^2\theta^2) - 2b^2\beta(1 - \beta)[A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2] \\ w_1 = & \frac{+(1 - \beta)[\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)](-2\beta^2 b\theta^2 + 2b^2\beta)}{4b^2\beta(1 - \beta)(-2\beta^2 b\theta^2 + 2b^2\beta) - (2b\beta - \beta^2\theta^2)(8b\beta^3\theta t_1 - 8b^2\beta^2)} \end{aligned} \quad (30)$$

The price and inventory of the retailer are:

$$\begin{aligned} & A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2 \\ & 2\beta(2b\beta - \beta^2\theta^2)(\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4b^2\beta^2 c(2b\beta - \beta^2\theta^2) \\ & + 4\beta^2 t_1(2b\beta - \beta^2\theta^2)[A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1 - \beta)] \\ & + 4b\beta^3\theta t_1(2b\beta - \beta^2\theta^2) - 2b^2\beta(1 - \beta)[A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2] \\ P_c = & \frac{+(1 - \beta)[\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)](-2\beta^2 b\theta^2 + 2b^2\beta)}{2b\beta(2b\beta - \beta^2\theta^2)} \end{aligned} \quad (31)$$

$$\begin{aligned} & A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1 - \beta) \\ & 2\beta(2b\beta - \beta^2\theta^2)(\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4b^2\beta^2 c(2b\beta - \beta^2\theta^2) \\ & + 4\beta^2 t_1(2b\beta - \beta^2\theta^2)[A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1 - \beta)] \\ & + 4b\beta^3\theta t_1(2b\beta - \beta^2\theta^2) - 2b^2\beta(1 - \beta)[A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2] \\ K_c = & \frac{+(1 - \beta)[\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)](-2\beta^2 b\theta^2 + 2b^2\beta) + b\beta\theta}{2b\beta - \beta^2\theta^2} \end{aligned} \quad (32)$$

Proposition 4: Under a revenue-sharing contract, a mandatory revenue-sharing ratio leads to higher product pricing; when excess income sharing is agreed upon, the retailer's purchase quantity increases, and the profit of the supply chain also increases.

$$K_c - K_l = \frac{A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1 - \beta)}{2b\beta - \beta^2\theta^2} - \frac{bw\theta - a\beta\theta - G - 2bt_2}{2b\lambda - \beta\theta^2} > 0 \quad (33)$$

$$\begin{aligned} P_c - P_l = & \frac{A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2}{2b\beta(2b\beta - \beta^2\theta^2)} \\ & - \frac{2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)}{2b\beta(2b\lambda - \beta\theta^2)} < 0 \end{aligned} \quad (34)$$

When sharing excess income, the profit of supply chain

$$\Pi_g^c = (w_1 - c)D_1 + t_1 w_1 K_1 + P_l D_1 - w_1 D_1 - C_K(K_1 - D_1) - t_2 K_1 - \frac{1}{2}\lambda K_1^2 \quad (35)$$

The profit of the supply chain is

$$\Pi_g^l = (w - c)D + t_1 w K + P D - w D - C_K(K - D) - t_2 K - \frac{1}{2}\lambda K^2 \quad (36)$$

because $w_1 < w, P_1 < P, K_1 > K, D_1 > D, \Pi_g^c - \Pi_g^l > 0$,,

5. Numerical illustration

In order to verify the validity of the model, this paper focuses on the relationship between pricing, inventory and profit, and the relationship between tariff and commodity pricing, inventory and profit. Therefore, some examples are further used to study the cooperation under the revenue sharing contract. Through investigation, it is found that according to the average value of A company's cross-border products in off-peak seasons, as $sumew=20, C_k=10, c=5, a=500, b=0.8, \theta=0.2, t_1=0.11, t_2=0.13, \lambda=0.2$, and draw the relationship between the profit of supply chain and cross-border commodity pricing, inventory, import and export tax rate in the case of centralized decision making, as shown in Fig. 2 and Fig. 3. Fig. 2 shows that in the case of centralized decision, The relationship between profit and inventory in the cross-border e-commerce supply chain. As can be clearly seen from the figure, the relationship shows a fluctuation pattern, which means that the profit of the supply chain will fluctuate with the change of the inventory. The wave crest in the figure decreases to the right, but there is a unique highest point. This phenomenon shows that there is a single optimal pricing and inventory strategy for centralized decision-making that maximizes the profits of the entire supply chain. In the actual operation process, supply chain managers need to formulate appropriate pricing and inventory strategies according to market demand, cost and other relevant factors. By observing the volatility trends in Fig. 2, managers can find the optimal strategy to maximize profits. This has important practical significance for enterprises, because in the competitive market environment, enterprises need to constantly optimize supply chain management, reduce costs, and improve profitability. Fig. 3 shows the relationship between supply chain profit function and import and export tax rate. As you can see from the diagram, the relationship takes on a parabolic shape. This means that, within limits, there is a single tariff on imports and exports that maximises the profits of the supply chain. This finding has important reference value for policy makers and enterprise managers. For policymakers, appropriate import and export tariff policies will help promote the development of cross-border e-commerce and improve the efficiency and profitability of the entire supply chain. Policymakers can follow the parabolic relationship in Figure 3 to find the best import and export tax rates to maximize supply chain profits. For enterprise managers, understanding the relationship between import and export tax rates and supply chain profits is helpful to better cope with the changes of tax policies and optimize the operation strategy of enterprises. When the tax rate changes, enterprises can adjust their pricing and inventory strategies according to the paraboloid relationship in Fig. 3 to adapt to the new market environment and maximize profits. In conclusion, through the analysis in Fig. 4 and Fig. 5, it can be concluded that there is an optimal pricing and inventory strategy, as well as an optimal import and export tax rate in centralized decision-making, which maximizes the profit of the cross-border e-commerce supply chain. This conclusion has important theoretical guidance and practical significance for supply chain managers and policy makers. In decentralised decision-making, cross-border suppliers need to consider the impact of wholesale prices on their profits.

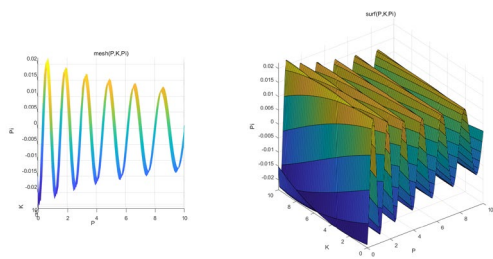


Fig. 4. Relationship between profit and pricing and inventory in centralized decision-making

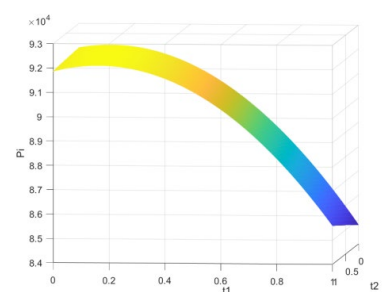


Fig. 5. Relation between profit function and import tariff and export rebate rate under centralized decision

As shown in Fig. 6, assuming W is unknown, the relationship between supplier profit and wholesale price is presented as a parabola with an upward opening. This indicates that within a certain range, there is an optimal value of wholesale price, which maximizes the profit of suppliers, and suppliers need to weigh a variety of factors when setting wholesale price. Although excessively high wholesale prices can bring higher profits to suppliers, they may also cause some retailers to give up purchasing products from the suppliers, thus affecting the stability of the entire supply chain. This is because retailers, as rational economic agents, seek to maximise their profits. When wholesale prices are too high, retailers may choose to buy products from other suppliers or adjust their sales strategies to reduce costs and increase profits. Therefore, suppliers need to reasonably set wholesale prices on the premise of ensuring their own profits, so as to maintain a good cooperative relationship with retailers. When wholesale prices are known, retailers need to develop appropriate commodity pricing and inventory strategies based on market demand, costs and other relevant factors. At this point, the relationship between the retailer's profit function and commodity pricing and inventory can be plotted. By analyzing this relationship, retailers can find the optimal pricing and inventory strategies to maximize profits. In the actual operation process, retailers may face a variety of uncertainties, such as fluctuations in market demand and changes in competitors' strategies. Therefore, retailers need to

constantly adjust their pricing and inventory strategies according to the actual situation to respond to market changes. Meanwhile, retailers also need to maintain good communication and collaboration with suppliers to ensure the stable operation of the supply chain. In the decentralized decision-making situation, the interest relationship between suppliers and retailers is more complex. In order to maintain the stability of the entire supply chain, suppliers should consider the interests of retailers while pursuing the maximization of their own profits. Retailers need to formulate appropriate pricing and inventory strategies to maximize their profits in the process of responding to market changes. In conclusion, in the case of decentralized decision-making, suppliers and retailers in the cross-border e-commerce supply chain need to reasonably formulate wholesale prices, commodity pricing and inventory strategies according to market demand, cost and other relevant factors, so as to maximize their respective profits. At the same time, both sides also need to maintain a good cooperative relationship to ensure the stable operation of the entire supply chain. Through the analysis of the relationship between supplier profit and wholesale price, as well as the study of the relationship between retailer profit function and commodity pricing and inventory, it is helpful to better understand the operation mechanism of cross-border e-commerce supply chain under decentralized decision-making, and provide effective guidance for actual operation, as shown in Fig. 7. Under the revenue sharing contract where the retailer forces the revenue sharing ratio, the profit of retailer and supplier is affected by the revenue sharing ratio β . Assume $K=100$ and $P=100$, as shown in Figure 6. Under the given data assumption, when the revenue-sharing ratio gradually increases, the retailer's profit will also increase, while the supplier's profit will decrease. The reason for this phenomenon may be that when the retailer shares part of the revenue to the supplier, the supplier is willing to wholesale the goods to the retailer at a lower price, thus reducing the cost of the retailer and increasing the profit of the retailer. However, suppliers suffered a certain loss of profits by selling goods wholesale at lower prices. In real life, suppliers may take other measures, such as improving production efficiency and reducing raw material costs, to increase their own profits. It is worth noting that under the revenue-sharing contract, the entire supply chain is in a profitable state and never suffers a loss. It is of great significance to maintain the stable operation of the supply chain.

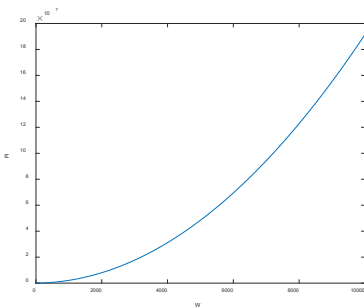


Fig. 6. Relation between supplier's profit and commodity wholesale price in decentralized decision making

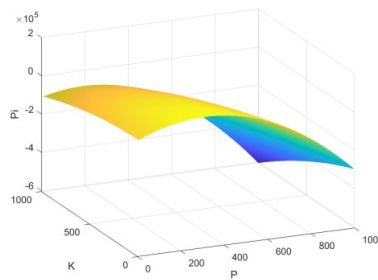


Fig. 7. Dispersion decision is retailer profit the relationship between commodity pricing

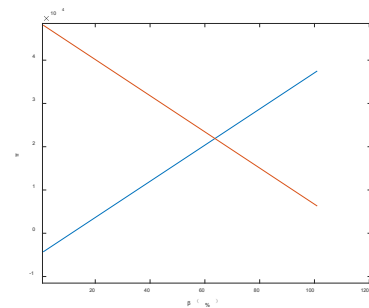


Fig. 8. Relationship between profit and income sharing ratio and inventory

At the same time, Fig. 8 also shows that the profit gap between retailers and suppliers shrinks as the proportion of revenue sharing increases. When $\beta=0.64$, the profit difference decreases to 0, that is, the profit of retailer and supplier reaches balance. However, when the retailer's profit is higher than the supplier's profit, the profit gap begins to widen again. This means that in some cases, a high proportion of revenue sharing may lead to uneven profit distribution, thus affecting the relationship between supply chain partners. To sum up, under the revenue-sharing contract in which the retailer forces the proportion of revenue shared, the profit of the retailer and the supplier is affected by the proportion of revenue shared. Reasonable revenue sharing ratio helps to maximize the profit of the supply chain and maintain a good relationship between supply chain partners. To achieve this goal, retailers and suppliers need to work closely together to adjust the revenue-sharing ratio based on market demand, costs and other relevant factors in order to maximize supply chain profits and partnership stability. In the actual operation process, both parties also need to pay attention to market changes and constantly optimize revenue sharing strategies to cope with the uncertain market environment. When $\beta=0.5$, $t_I=0.11$, the relationship between wholesale price and import tariff is drawn, as shown in Fig. 9. It can be seen from the figure that there is a negative correlation between import tariff and wholesale price, which indicates that when import tariff increases, retailers and suppliers in cross-border e-commerce supply chain will face a series of challenges. First, Retailers need to consider the cost of tariffs when buying products. Price-sensitive retailers may reduce their preference for this product because of the higher tariff costs and seek other, more competitive products. This will lead to a decline in the market demand for cross-border goods, thus affecting the profitability of retailers. Secondly, even if retailers are willing to bear the cost of tariffs to buy products, they are likely to pass on the cost of tariffs to consumers in the retail link. This will lead to higher retail prices for cross-border goods, and price-sensitive consumers may turn away from buying the goods in favor of alternatives with price advantages. This phenomenon further aggravates the decrease in cross-border commodity market demand. In order to cope with the decrease in market demand caused by tariff increase, suppliers may choose to lower wholesale prices to attract retailers to buy. Although such a strategy may lead to a decline in the profit of suppliers, through export tax rebates or export subsidies and other policies, suppliers can make up part of the loss, so as to maintain their profit level to a certain extent. However, lowering wholesale prices is not a long-term solution. Suppliers need to constantly improve their competitiveness, improve production efficiency, reduce costs, optimize product structure and other ways to cope with the increase of tariff costs. In addition, suppliers can also seek communication

and cooperation with government departments to obtain more policy support to reduce the impact of tariffs on their business. As shown in Fig. 8 and Fig. 9, commodity pricing and inventory are also affected by tariffs.

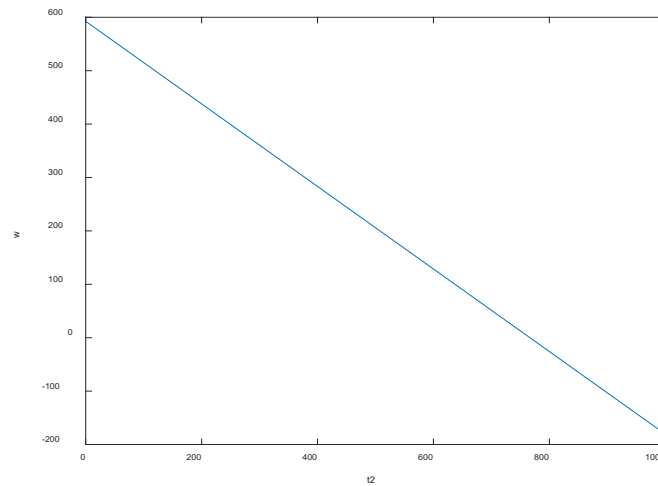


Fig. 9. Wholesale prices under revenue sharing contracts the relationship with import tariffs

As can be seen from Fig. 9, when the import tariff is determined, the price of the commodity is relatively lower when the export tax rebate rate is high. This is because the export tax rebate reduces the operating cost of the supplier to some extent, and the operating cost decreases. If the supplier adopts the sales strategy of small profit and high turnover, he will reduce the wholesale price and sell to the retailer. Retailers will adopt the same strategy to lower the retail price of their products and sell them to consumers. When the export tax rebate rate is fixed, with the increase of the import tariff rate, the tariff cost will increase, and merchants will try to transfer the tariff or not let their benefits change too much, which will lead to the increase of commodity prices. As can be seen from Fig. 10, when the export rebate rate is certain, the increase of import tariff will indeed have an impact on the cross-border e-commerce supply chain. A rise in commodity prices may reduce demand and build up inventories. This shows that it is crucial for companies to pay attention to inventory management in the face of tariff changes. Inventory management is an important part of supply chain management, which involves the control of inventory level, the optimization of inventory cost and the response to market demand. With import tariffs rising, companies need to take a number of measures to deal with inventory overhang. First of all, enterprises need to strengthen the forecast and analysis of market demand in order to grasp the market changes more accurately. Through in-depth understanding of market demand, enterprises can adjust production plans and inventory strategies in advance to reduce the risk of overstocking. Second, enterprises can reduce inventory costs by optimizing purchasing and production processes. For example, purchasing on demand, reducing production batches and improving production efficiency are adopted to reduce inventory holding costs and unsalable risks. In addition, enterprises can adjust product mix and pricing strategies to cope with the increase of tariff costs. For example, introducing more competitive products or adopting more flexible pricing strategies to entice consumers to buy and thus reduce inventory pressure. In the process of inventory management, enterprises also need to maintain good communication and collaboration with suppliers, retailers and other partners. By sharing information, coordinating planning and execution, all parties can jointly address the challenges posed by tariff changes and reduce the risk of overstocking. In conclusion, when the export tax rebate rate is fixed, the increase of import duties may lead to the rise of commodity prices and inventory overhang. To cope with this challenge, enterprises need to strengthen inventory management and reduce the risk of overstocking by forecasting market demand, optimizing purchasing and production processes, adjusting product mix and pricing strategies, etc. At the same time, enterprises also need to maintain good communication and collaboration with their supply chain partners to jointly cope with the impact of tariff changes.

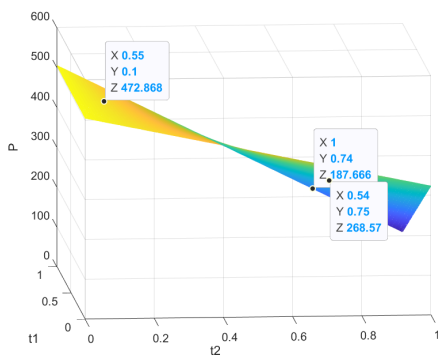


Fig. 10. Relationship between commodity pricing and tariff

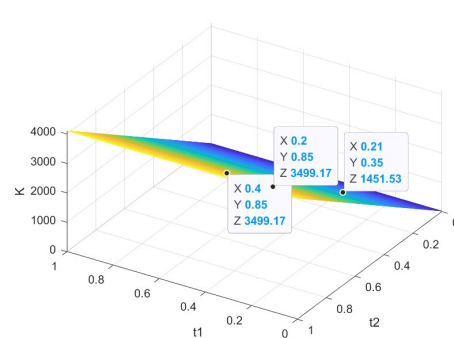


Fig. 11. Relationship between inventory and tariff

6. Conclusions

The characteristics of a cross-border e-commerce supply chain include global consumer demand and order production. Due to the unique nature of cross-border e-commerce, consumers can purchase goods from anywhere, while suppliers can produce goods from anywhere. This means that the supply chain must be able to manage and coordinate various production and logistics activities globally to meet consumer demands. In addition, cross-border e-commerce supply chains require high flexibility and adaptability to deal with the complexity and uncertainty of international trade. The supply chain must be able to cope with various currency, language, cultural, and legal differences, as well as handle international logistics and customs procedures. Finally, cross-border e-commerce supply chains also require high visibility and transparency to ensure that consumers and suppliers can track the status of orders and logistics processes. This helps to enhance consumer trust and satisfaction, while also reducing errors and delays in the supply chain. In summary, the cross-border e-commerce supply chain is a highly complex and global system that requires flexibility, adaptability, visibility, and transparency to meet global consumer demand and order production. One of the key factors behind the success of any modern supply chain is close collaboration among channel members. This study focuses on a two-tier sustainable supply chain, where the manufacturer and retailer have a division of labor and cooperation. Under centralized decision-making, the manufacturer and retailer cooperate only to pursue the maximization of supply chain profits. Under decentralized decision-making, the supplier and retailer act as rational businessmen, each considering the maximization of their own profits. Therefore, to explore the relationship between supply chain profits and the inventory and product pricing of retailers and suppliers, this study establishes profit functions under decentralized decision-making, centralized decision-making, and revenue-sharing contracts, and draws the following conclusions: 1. Whether under centralized or decentralized decision-making, the profits of cross-border e-commerce supply chains are affected by export tax rebates and import tariffs. It is not the case that higher export tax rebates or higher import tariffs are better; only the optimal tax rate can promote the development of e-commerce. 2. Under centralized and decentralized decision-making, there exists a unique optimal pricing and inventory that maximizes the profit of the cross-border e-commerce supply chain, and import tariffs and export tax rates affect the pricing and inventory of cross-border products. 3. Under a revenue-sharing contract signed between overseas suppliers and domestic retailers, the method of sharing excess income will have a certain impact on the profit of the supply chain. Sharing excess income with the supplier can increase the profit of the supply chain. 4. Under a revenue-sharing contract, the retailer improves their own business situation by sharing part of the income with the supplier, but it may harm the supplier to some extent. 5. Under different sharing methods in a revenue-sharing contract, the profit of the supply chain when sharing excess income is higher than when sharing income is mandatory. 6. When faced with an increase in tariffs, suppliers may sell products to retailers at a reduced price to prevent the products from becoming unsold. 7. In cross-border e-commerce, retailers can effectively reduce warehousing costs and ultimately reduce operating costs by coordinating with suppliers. The conclusions of this study are based on certain assumptions, and due to the complexity of the market environment, future research can consider the following aspects: on the one hand, consumer demand has a certain impact on channel selection, so based on consumer preferences, the substituted ability of domestic homogeneous products for cross-border products can be considered. On the other hand, due to the uncertainty of market demand, decision-making problems between cross-border retailers and suppliers can be studied under uncertain environments, and the impact of factors such as deferred payment on the profits of cross-border e-commerce supply chains can be researched.

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References

- Ahi, A. A., Sinkovics, N., Shildibekov, Y., Sinkovics, R. R., & Mehandjiev, N. (2022). Advanced technologies and international business: A multidisciplinary analysis of the literature. *International Business Review*, 31(4), 101967.
- Azadeh, A., Songhori, H., & Salehi, N. (2017). A unique optimization model for deterministic bundle pricing of two products with limited stock. *International Journal of System Assurance Engineering and Management*, 8(S2), 1154–1160.
- Biller, S., Chan, L. M. A., Simchi-Levi, D., & Swann, J. (2005). Dynamic pricing and the direct-to-customer model in the automotive industry. *Electronic Commerce Research*, 5(2), 309–334.
- Cai, J., Jia, L., Zhou, Q., & Yao, D. (2024). E-commerce supply chain inventory decisions and contract design considering sales effort and risk aversion. *Electronic Commerce Research*, 24(3), 1847–1888.
- Cha, H., Kotabe, M., & Wu, J. (2023). Reshaping internationalization strategy and control for global E-commerce and digital transactions: A hayekian perspective. *Management International Review*, 63(1), 161–192.
- Chen, Q., Tian, G., Zhuang, Q., & Zhou, F. (2024). RETRACTED ARTICLE: Manufacturers' optimization choice models for

- E-retailing markets: the role of the sale price and profit. *Electronic Commerce Research*, 24(S1), 11–11.
- Das, R., Barman, A., Roy, B., & De, P. K. (2023). Pricing and greening strategies in a dual-channel supply chain with cost and profit sharing contracts. *Environment, Development and Sustainability*, 25(6), 5053–5086.
- Dasgupta, P., & Melliar-Smith, P. (2003). Dynamic consumer profiling and tiered pricing using software agents. *Electronic Commerce Research*, 3, 115–127.
- Eduardsen, J., Marinova, S., Leonidou, L. C., & Christodoulides, P. (2023). Organizational influences and performance impact of cross-border E-commerce barriers: The moderating role of home country digital infrastructure and foreign market internet penetration. *Management International Review*, 63(3), 433–467.
- Feindt, S., Jeffcoate, J., & Chappell, C. (2002). Identifying success factors for rapid growth in SME E-commerce. *Small Business Economics*, 19(1), 51–62.
- Guo, L. (2022). Cross-border e-commerce platform for commodity automatic pricing model based on deep learning. *Electronic Commerce Research*, 22(1), 1–20.
- Han, S., Chen, S., Yang, K., Li, H., Yang, F., & Luo, Z. (2024). Free shipping policy for imported cross-border e-commerce platforms. *Annals of Operations Research*, 335(3), 1537–1566.
- He, Y., Huang, H., & Li, D. (2020). Inventory and pricing decisions for a dual-channel supply chain with deteriorating products. *Operational Research*, 20(3), 1461–1503.
- Jain, T., Hazra, J., & Cheng, T. C. E. (2023). Analysis of upstream pricing regulation and contract structure in an agriculture supply chain. *Annals of Operations Research*, 320(1), 85–122.
- Jauhari, W. A., Pujawan, I. N., & Suef, M. (2021). Sustainable energy storage management for a power plant-transmission station system with hybrid power generation and stochastic electricity demand. *IOP Conference Series: Materials Science and Engineering*, 1096(1), 12010.
- Kano, K. (2018). Consumer inventory and demand for storable goods: New evidence from a consumer survey. *The Japanese Economic Review*, 69(3), 284–305.
- Li, J., Liu, X., Wu, J., & Yang, F. (2014). Coordination of supply chain with a dominant retailer under demand disruptions. *Mathematical Problems in Engineering*, 2014(1).
- Li, T., Zhao, X., & Xie, J. (2015). Inventory management for dual sales channels with inventory-level-dependent demand. *Journal of the Operational Research Society*, 66(3), 488–499.
- Liu, L. (2021). The rise of data politics: Digital china and the world. *Studies in Comparative International Development*, 56(1), 45–67.
- Liu, Z., Guo, H., Zhao, Y., Hu, B., Ji, X., Lang, L., & Shi, L. (2022). Optimal pricing decision of composite service offered by network providers in E-commerce environment. *Electronic Commerce Research*, 22(1), 177–193.
- Luo, L., Ma, X., & Wang, Z. (2022). The moderate-reputation trap: Evidence from a chinese cross-border business-to-business e-commerce portal. *Asia Pacific Journal of Management*, 39(2), 395–432.
- Miao, Y., Du, R., Li, J., & Westland, J. C. (2019). A two-sided matching model in the context of B2B export cross-border e-commerce. *Electronic Commerce Research*, 19(4), 841–861.
- Najafnejhad, E., Tavassoli Roodsari, M., Sepahrom, S., & Jenabzadeh, M. (2021). A mathematical inventory model for a single-vendor multi-retailer supply chain based on the vendor management inventory policy. *International Journal of System Assurance Engineering and Management*, 12(3), 579–586.
- Pan, L., Fu, X., & Li, Y. (2023). SME participation in cross-border e-commerce as an entry mode to foreign markets: A driver of innovation or not? *Electronic Commerce Research*, 23(4), 2327–2356.
- Qiao, P., & Qi, Z. (2018). The application of VAR model in economic interaction: The case of China E-commerce and trade. *Wireless Personal Communications*, 103(1), 847–856.
- Shen, J. (2021). Uncertain two-echelon green supply chain models based on revenue sharing contract. *International Journal of Machine Learning and Cybernetics*, 12(7), 2059–2068.
- Shi-Zhen, B., Na, X. U., & Management, S. O. (2016). The study on inventory and pricing strategy for online sellers under different payment schemes. *Chinese Journal of Management Science*.
- Urata, S. (2018). Free trade agreements and patterns of trade in east Asia from the 1990s to 2010s. *East Asian Community Review*, 1(1–2), 61–73.
- Wang, Q., Peng, Y., & Yang, Y. (2022). Solving inventory management problems through deep reinforcement learning. *Journal of Systems Science and Systems Engineering*, 31(6), 677–689.
- Yan, B., Chen, X., Li, H., & Sun, Y. (2022). Managing homogeneous products in two competing supply chains under price and logistics service. *Electronic Commerce Research*, 22(3), 691–713.
- Yin, Z. H., & Choi, C. H. (2023). The effects of China's cross-border e-commerce on its exports: A comparative analysis of goods and services trade. *Electronic Commerce Research*, 23(1), 443–474.
- Zhou, F., Zhang, C., Chen, T., & Lim, M. K. (2023). An evolutionary game analysis on blockchain technology adoption in cross-border e-commerce. *Operations Management Research*, 16(4), 1766–1780.
- Zhou, Y., Li, H., Hu, S., & Yu, X. (2024). RETRACTION ARTICLE: Two-stage supply chain inventory management based on system dynamics model for reducing bullwhip effect of sulfur product. *Annals of Operations Research*, 337(S1), 5–5.
- Zhu, X., Xie, L., Lin, G., & Ma, X. (2022). Decision analysis of individual supplier in a vendor-managed inventory program with revenue-sharing contract. *International Journal of Industrial Engineering Computations*, 13(3), 405–420.

Appendix

The profit functions of suppliers and retailers are respectively

$$\Pi_m = (w - c)D + t_1 w K \quad (\text{A.1})$$

$$\Pi_R = PD - wD - C_K (K - D) - t_2 K - \frac{1}{2} \lambda K^2 \quad (\text{A.2})$$

The profit function of the supply chain is

$$\Pi_g = PD - cD - C_K (K - D) + t_1 w K - t_2 K - \frac{1}{2} \lambda K^2 \quad (\text{A.3})$$

Proof of Proposition 1 Now, Derive the formula (3)

$$\frac{\partial \Pi_g}{\partial P} = a - 2bP - \theta k + bc - bC_K \quad (\text{A.4})$$

$$\frac{\partial \Pi_g}{\partial K} = \theta(c - P) - C_K - \theta C_K + t_1 w - t_2 - \lambda K \quad (\text{A.5})$$

$$\frac{\partial^2 \Pi_g}{\partial P^2} = -2b < 0, \quad \frac{\partial^2 \Pi_g}{\partial K^2} = -\lambda < 0, \quad \frac{\partial^2 \Pi_g}{\partial P \partial K} = -\theta < 0.$$

Because the second derivative is less than zero, the expression of this function is convex and has a unique maximum point.

Let $\frac{\partial \Pi}{\partial p} = 0, \frac{\partial \Pi}{\partial K} = 0$. The optimal pricing is obtained as follows

$$P_j = \frac{AC - B\theta^2 + 2b\theta(C_K - t_1 w + t_2)}{2bC} \quad (\text{A.6})$$

The optimal inventory is

$$K_j = \frac{B\theta - 2b(C_K - t_1 w + t_2)}{C} \quad (\text{A.7})$$

Among them, $A = a + bc - bC_K$, $B = bc - bC_K - a$, $C = 2b\lambda - \theta^2$.

Proof of corollary 1

Substituting Eqs. (A.6-A.7) into Eq. (A.3), the profit of supply chain under centralized decision-making is

$$\begin{aligned} & [AC - B\theta^2 + 2b\theta(C_K - t_1 w + t_2) - E] [2aC - AC - B\theta^2 + 2b\theta(C_K - t_1 w + t_2)] \\ \Pi_g^j = & \frac{-4bBC\theta(C_K - t_1 w + t_2) - 8b^2(C_K - t_1 w + t_2)^2 - 4b\lambda[B\theta - 2b(C_K - t_1 w + t_2)]^2}{4bC^2} \end{aligned} \quad (\text{A.8})$$

Among them, $E = 2bC(c - C_K)$.

$$(-2bw\theta)[2aC - AC - B\theta^2 + 2b\theta(C_K - t_1 w + t_2)] + (-2bw\theta)[AC - B\theta^2 + 2b\theta(C_K - t_1 w + t_2) - E] \quad (\text{A.9})$$

$$\frac{\partial \Pi_g^j}{\partial t_1} = \frac{+4b\theta BCw + 16b^2 w(C_K - t_1 w + t_2) - 16b^2 \lambda w[B\theta - 2b(C_K - t_1 w + t_2)]}{4bC^2}$$

$$\frac{\partial^2 \Pi_g^j}{\partial t_1^2} = \frac{8b^2 w(\theta^2 - w - 4bw\lambda)}{4bC^2} \quad (\text{A.10})$$

because $\theta^2 < w + 4bw\lambda$, $\frac{\partial^2 \Pi_g^j}{\partial t_1^2} < 0$.

$$2b\theta[2aC - AC - B\theta^2 + 2b\theta(C_K - t_1w + t_2)] + 2b\theta[AC - B\theta^2 + 2b\theta(C_K - t_1w + t_2) - E] \tag{A.11}$$

$$\frac{\partial \Pi_g^j}{\partial t_2} = \frac{-4bBC\theta - 16b^2(C_K - t_1w + t_2) + 16b^2\lambda[B\theta - 2b(C_K - t_1w + t_2)]}{4bC^2}$$

$$\frac{\partial^2 \Pi_g^j}{\partial t_2^2} = \frac{8b^2(\theta^2 - 2 - 4b\lambda)}{4bC^2} \tag{A.12}$$

because $\theta^2 < 2 + 4b\lambda$, $\frac{\partial^2 \Pi_g^j}{\partial t_2^2} < 0$.

To sum up, In centralized decision-making, the profit function graph of supply chain is a convex function about T1 and T2, and there are only optimal export tax rebate rate and import tariff rate in the profit function of supply chain to maximize the

profit of supply chain. Let $\frac{\partial \Pi_g^j}{\partial t_1} = 0, \frac{\partial \Pi_g^j}{\partial t_2} = 0$, when

$$(-2bw\theta)[2aC - AC - B\theta^2 + 2b\theta(C_K + t_2)] + (-2bw\theta)[AC - B\theta^2 + 2b\theta(C_K + t_2) - E] \tag{A.13}$$

$$t_1 = \frac{+4b\theta BCw + 16b^2w(C_K + t_2) - 16b^2\lambda w[B\theta - 2b(C_K + t_2)]}{8b^2w^2(4b\lambda + 2 - \theta^2)}$$

$$2b\theta[2aC - AC - B\theta^2 + 2b\theta(C_K - t_1w)] + 2b\theta[AC - B\theta^2 + 2b\theta(C_K - t_1w) - E] \tag{A.14}$$

$$t_2 = \frac{-4bBC\theta - 16b^2(C_K - t_1w) + 16b^2\lambda[B\theta - 2b(C_K - t_1w)]}{8b^2(4b\lambda + 2 - \theta^2)}$$

The profit of the supply chain reaches the maximum.

Proof of Proposition 2

Take the first derivative of the retailer's profit function (2)

$$\frac{\partial \Pi_r}{\partial P} = a - 2bP - \theta K + bw - bC_K \tag{A.15}$$

$$\frac{\partial \Pi_r}{\partial K} = w\theta - C_K - P\theta - \theta C_K - \lambda K - t_2 \tag{A.16}$$

$$\frac{\partial^2 \Pi_r}{\partial P^2} = -2b < 0, \frac{\partial^2 \Pi_r}{\partial K^2} = -\lambda < 0, \frac{\partial^2 \Pi_r}{\partial P \partial K} = -\theta < 0.$$

The second derivative is less than 0, so the retailer's profit function is a convex function with a unique maximum point.

Let $\frac{\partial \Pi_r}{\partial P} = 0, \frac{\partial \Pi_r}{\partial K} = 0$, get the retailer's decision

$$P_f = \frac{C(F + bw) + \theta^2(M - bw) + 2b\theta(C_K + t_2)}{2bC} \tag{A.17}$$

$$K_f = \frac{-2b(C_K + t_2) - \theta(M - bw)}{C} \tag{A.18}$$

Among them, $M = a + bC_K$ □ According to the retailer's strategy, the supplier makes the optimal decision, and brings Eq. (A.11) and Eq. (A.12) into Eq. (A.1). The supplier's profit function is

$$\Pi_m^f = \frac{(w - c)[b\theta^2(M - bw) + 2b^2\theta(C_K + t_2) - bC(F + bw) + 2abC] - 4b^2t_1w(C_K + t_2) - 2b\theta t_1w(M - bw)}{2bC} \tag{A.19}$$

Take the first derivative of Eq. (A.13) and get

$$\frac{\partial \Pi_m^f}{\partial w} = \frac{[b\theta^2(M - bw) + 2b^2\theta(C_K + t_2) - bC(F + bw) + 2abC] - (w - c)(b^2\theta^2 + b^2C) - 4b^2t_1(C_K + t_2) - 2b\theta t_1(M - bw) + 2b^2\theta t_1w}{2bC} \tag{A.20}$$

$$\frac{\partial^2 \Pi_m^f}{\partial w^2} = \frac{2b^2(2\theta t_1 - \theta^2 - C)}{2bC} \tag{A.21}$$

Because $2\theta t_1 < \theta^2 + C$, Then there is a second-order function less than 0, so the profit function of the supplier is a convex function.

Let $\frac{\partial \Pi_m^f}{\partial w} = 0$, the optimal wholesale price of suppliers is

$$w_f = \frac{2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c) + b\theta M(\theta - 2t_1) - bCF + 2abC}{2bC} \quad (\text{A.22})$$

Among them, $F = a - bC_K$.

The retailer's optimal decision to bring Eq. (A.14) into Eq. (A.11) and Eq. (A.12) is as follows

$$P_i = \frac{2bC[CF + \theta^2 M + 2b\theta(C_K + t_2)] + (bc - b\theta^2)[2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c) + b\theta M(\theta - 2t_1) - bCF + 2abC]}{4b^2C^2} \quad (\text{A.23})$$

$$K_i = \frac{b\theta[2b^2(C_K + t_2)(\theta - 2t_1) + b^2c(\theta^2 + c) + b\theta M(\theta - 2t_1) - bCF + 2abC] - 2bC[2b(C_K + t_2) - \theta M]}{2bC^2} \quad (\text{A.24})$$

Proof of corollary 2

Centralized decision-making $\frac{\partial P_j}{\partial t_1} = \frac{-2bw\theta}{2bC} < 0$, $\frac{\partial P_j}{\partial t_2} = \frac{1}{2bC} > 0$, $\frac{\partial K_j}{\partial t_1} = \frac{2bw}{C} > 0$, $\frac{\partial K_j}{\partial t_2} = \frac{-2b}{C} < 0$.

Decentralized decision-making, because $\theta < 2t_1$,

$$\begin{aligned} \frac{\partial P_f}{\partial t_1} &= \frac{b(C - \theta^2) - 4b^2(C_K + t_2) - 2b\theta M}{4b^2C^2} < 0, & \frac{\partial P_f}{\partial t_2} &= \frac{4b^2\theta C + 2b^3(c - \theta^2)(\theta - 2t_1)}{4b^2C^2} > 0, \\ \frac{\partial K}{\partial t_1} &= \frac{b^2\theta[4b(C_K + t_2) + 2\theta M]}{2bC^2} > 0, & \frac{\partial K}{\partial t_2} &= \frac{b\theta[2b^2(\theta - 2t_1)] - 4b^2C}{2bC^2} < 0. \end{aligned}$$

Proof of Proposition 3

Under the revenue sharing contract, the proportion of revenue sharing is determined by retailers themselves. The profit function of retailers and suppliers is as follows

$$\Pi_r^i = (\beta P - w)D - C_K(K - D) - t_2 K - \frac{1}{2}\lambda K^2 \quad (\text{A.25})$$

$$\Pi_m^i = (w - c)D + t_1 w K + (1 - \beta)PD \quad (\text{A.26})$$

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$$P_i = \frac{2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)}{2b\beta(2b\lambda - \beta\theta^2)} \quad (\text{A.27})$$

$$K_i = \frac{bw\theta - a\beta\theta - G - 2bt_2}{2b\lambda - \beta\theta^2} \quad (\text{A.28})$$

Bringing Eq. (A.12) and Eq. (A.13) into Eq. (A.11) gives the supplier's profit function as follows

$$\Pi_m^i = \frac{2b\beta(w - c)(\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 4b\beta^2 t_1 w(bw\theta - a\beta\theta - G - 2bt_2) + (1 - \beta)[2ab\beta\lambda + 2b^2\lambda(w - C_K) + b\beta\theta^2(C_K - 2w) + \beta\theta(G + 2bt_2)](\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2)}{4b\beta^2(2b\lambda - \beta\theta^2)} \quad (\text{A.29})$$

where $H = 2a(1 - b\lambda)$, $I = 2b^2\lambda$, $J = G\theta - b\theta^2 C_K$, $L = 2b\theta$

Derive Eq. (A.29)

$$2b\beta(\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - 2b\beta I(2b\lambda - \beta\theta^2)(w - c) + 4b\beta^2 t_1 (bw\theta - a\theta\beta - G - 2bt_2) + 4b^2 \beta^2 \theta t_1 w + (1 - \beta)(2b^2 \lambda - 2b\beta\theta^2)(\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I \left[\begin{matrix} 2ab\beta\lambda + 2b^2 \lambda (w - C_K) \\ + b\beta\theta^2 (C_K - 2w) + \beta\theta(G + 2bt_2) \end{matrix} \right] \tag{A.30}$$

$$\frac{\partial \Pi_m^I}{\partial w} = \frac{4b\beta^2 (2b\lambda - \beta\theta^2)}{4b\beta^2 (2b\lambda - \beta\theta^2)}$$

$$\frac{\partial^2 \Pi_m^I}{\partial w^2} = \frac{-4b\beta I(2b\lambda - \beta\theta^2) + 8b^2 \beta^2 \theta t_1 - I(1 - \beta)(2b^2 \lambda - 2b\beta\theta^2) - (1 - \beta)I^2}{4b\beta^2 (2b\lambda - \beta\theta^2)} < 0 \tag{A.31}$$

The profit function of the supplier is a convex function, so $\frac{\partial \Pi_m^I}{\partial w} = 0$ is an optimal solution to the wholesale price of the supplier when.

$$w_1 = \frac{2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + 4b^2 \beta^2 \theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right]}{4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2 \beta^2 \theta t_1} \tag{A.32}$$

Bring w_1 into P_1 and K_1

$$P_1 = \frac{2ab\beta\lambda - 2b^2 \lambda C_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) + 2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + (2b^2 \lambda - 2b\beta\theta^2) \left[\begin{matrix} + 4b^2 \beta^2 \theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right] \end{matrix} \right]}{4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2 \beta^2 \theta t_1} \tag{A.33}$$

$$K_1 = \frac{2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) + b\theta \left[\begin{matrix} + 4b^2 \beta^2 \theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right] \end{matrix} \right] - a\beta\theta - G - 2bt_2}{4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2 \beta^2 \theta t_1} \tag{A.34}$$

The supplier's profit function is

$$\Pi_m^I = \frac{2b\beta(w - c)(\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 4b\beta^2 t_1 (bw\theta - a\theta\beta - G - 2bt_2) + (1 - \beta) \left[\begin{matrix} 2ab\beta\lambda + 2b^2 \lambda (w - C_K) + b\beta\theta^2 (C_K - 2w) + \beta\theta(G + 2bt_2) \end{matrix} \right] (\beta H + C_K I - wI + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2)}{4b\beta^2 (2b\lambda - \beta\theta^2)} \tag{A.35}$$

The retailer's profit function is

$$\Pi_r^I = \frac{\left[(\beta H + C_K I - wI + \beta J + \beta t_2 L) - 2bw(2b\lambda - \beta\theta^2) \right] (\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta (2b\lambda - \beta\theta^2)} - C_K \frac{2\beta (bw\theta - a\theta\beta - G - 2bt_2) - (2b\lambda - \beta\theta^2) (\beta H + C_K I - wI + \beta J + \beta t_2 L)}{2\beta (2b\lambda - \beta\theta^2)} - \frac{2t_2 (bw\theta - a\theta\beta - G - 2bt_2) (2b\lambda - \beta\theta^2) - \lambda (bw\theta - a\theta\beta - G - 2bt_2)^2}{2(2b\lambda - \beta\theta^2)^2} \tag{A.36}$$

$$\frac{\partial \Pi_m^I}{\partial \beta} = \frac{\left[2ab\beta\lambda + 2b^2 \lambda (w - C_K) + b\beta\theta^2 (C_K - 2w) + \beta\theta(G + 2bt_2) \right] (\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta^2 (2b\lambda - \beta\theta^2)} < 0$$

$$\frac{\partial \Pi_r^I}{\partial \beta} = \frac{\left[2ab\beta\lambda + 2b^2 \lambda (w - C_K) + b\beta\theta^2 (C_K - 2w) + \beta\theta(G + 2bt_2) \right] (\beta H + C_K I - wI + \beta J + \beta t_2 L)}{4b\beta^2 (2b\lambda - \beta\theta^2)} > 0$$

Proof of corollary 3

$$\frac{\partial w}{\partial t_1} = \frac{\left[\begin{matrix} -4b\beta^2 (a\beta\theta + G + 2bt_2) + 4b^2 \beta^2 \theta w \left[\begin{matrix} 4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2 \beta^2 \theta t_1 \end{matrix} \right] \\ + 8b^2 \beta^2 \theta \left\{ \begin{matrix} 2b\beta(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) + 2bc\beta I(2b\lambda - \beta\theta^2) - 4b\beta^2 t_1 (a\theta\beta + G + 2bt_2) \\ + 4b^2 \beta^2 \theta t_1 w + (1 - \beta)(I - 2b\beta\theta^2)(\beta H + C_K I + \beta J + \beta t_2 L)(2b\lambda - \beta\theta^2) - (1 - \beta)I \left[\begin{matrix} 2ab\beta\lambda - IC_K + b\beta\theta^2 C_K + \beta\theta(G + 2bt_2) \end{matrix} \right] \end{matrix} \right\} \end{matrix} \right]}{\left[4b\beta I(2b\lambda - \beta\theta^2) + 2I(1 - \beta)(I - 2b\beta\theta^2) - 8b^2 \beta^2 \theta t_1 \right]^2} \tag{A.37}$$

$$\begin{aligned}
& 16b^2\beta^2\theta[-4b\beta^2(a\theta\beta+G+2bt_2)+4b^2\beta^2\theta w][4b\beta I(2b\lambda-\beta\theta^2)+2I(1-\beta)(I-2b\beta\theta^2)-8b^2\beta^2\theta t_1] \\
& +8b^2\beta^2\theta \left\{ \begin{aligned} & 2b\beta(\beta H+C_k I+\beta J+\beta t_2 L)(2b\lambda-\beta\theta^2)+2bc\beta I(2b\lambda-\beta\theta^2)-4b\beta^2 t_1(a\theta\beta+G+2bt_2) \\ & +4b^2\beta^2\theta t_1 w+(1-\beta)(I-2b\beta\theta^2)(\beta H+C_k I+\beta J+\beta t_2 L)(2b\lambda-\beta\theta^2) \end{aligned} \right\} 4b\beta I(2b\lambda-\beta\theta^2) \\
& \left. \begin{aligned} & -(1-\beta)I \left[\begin{aligned} & 2ab\beta\lambda-IC_k \\ & +b\beta\theta^2 C_k+\beta\theta(G+2bt_2) \end{aligned} \right] \end{aligned} \right\} \\
\frac{\partial^2 w}{\partial t_1^2} &= \frac{+2I(1-\beta)(I-2b\beta\theta^2)-8b^2\beta^2\theta t_1}{[4b\beta I(2b\lambda-\beta\theta^2)+2I(1-\beta)(I-2b\beta\theta^2)-8b^2\beta^2\theta t_1]^2} > 0 \\
\frac{\partial w}{\partial t_2} &= \frac{2b\beta^2 LX_1-8b^2\beta^2 t_1+X_1 X_2 X_3 \beta L-2b\beta\theta IX_2}{4b\beta IX_1+2IX_2 X_3-8b^2\beta^2\theta t_1} < 0
\end{aligned} \tag{A.38}$$

where, $X_1 = 2b\lambda - \beta\theta^2$, $X_2 = 1 - \beta$, $X_3 = I - 2b\beta\theta^2$.

Proof of Proposition 4

The proportion of income sharing is determined by excess income. The profit function of retailers and suppliers is as follows,

$$\Pi_m^c = (w_1 - c)D_1 + t_1 w_1 K_1 + (D_1 - D)P_1(1-\beta) \tag{A.39}$$

$$\Pi_r^c = P_1 D_1 - w_1 D_1 - C_K(K_1 - D_1) - t_2 K_1 - \frac{1}{2} \lambda K_1^2 - (D_1 - D)P_1(1-\beta) \tag{A.40}$$

$$\frac{\partial \Pi_r^c}{\partial P_1} = \beta(a - \theta K_1) - 2b\beta P_1 + b(w_1 - C_K) + D(1-\beta) \tag{A.41}$$

$$\frac{\partial^2 \Pi_r^c}{\partial P_1^2} = -2b\beta < 0 \tag{A.42}$$

$$\frac{\partial \Pi_r^c}{\partial K_1} = w_1\theta - C_K - \theta C_K - t_2 - \lambda K_1 - \beta\theta P_1 \tag{A.43}$$

$$\frac{\partial^2 \Pi_r^c}{\partial K_1^2} = -\lambda < 0. \tag{A.44}$$

The retailer has a unique optimal commodity pricing and inventory of

$$P_c = \frac{A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2}{2b\beta(2b\beta - \beta^2\theta^2)} \tag{A.45}$$

$$K_c = \frac{A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1-\beta)}{2b\beta - \beta^2\theta^2} \tag{A.46}$$

Among them, $Y = C_K(1 + \theta)$, $A_3 = b\theta C_K - 2bY$,

$$X = 2a - C_K\theta^2 + 2\theta Y, A_1 = bX + \theta + b\theta^2 C_K - D\theta^2 - 2bD, A_2 = 2bD - \theta - 2b^2 C_K$$

Substituting Eq.(A.45) and Eq.(A.46) into Eq.(A.39) gives the profit function of the supplier as follows

$$\begin{aligned}
& 2\beta(w_1 - c)(2b\beta - \beta^2\theta^2)(\beta^2 A_3 - \beta A_4 - 2b^2\beta w_1 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4\beta^4 t_1 w_1(2b\beta - \beta^2\theta^2)[A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1-\beta)] \\
& + (1-\beta)[\beta^2 A_3 - \beta A_4 - 2b^2\beta w_1 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)][A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2] \\
\Pi_m^c &= \frac{4\beta^2(2b\beta - \beta^2\theta^2)^2}{}
\end{aligned} \tag{A.47}$$

$$\begin{aligned}
& 2\beta(2b\beta - \beta^2\theta^2)(\beta^2 A_3 - \beta A_4 - 2b^2\beta w_1 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) - 4b^2\beta^2(w_1 - c)(2b\beta - \beta^2\theta^2) \\
& + 4\beta^4 t_1(2b\beta - \beta^2\theta^2)[A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1-\beta)] \\
& + 4b\beta^3\theta t_1 w_1(2b\beta - \beta^2\theta^2) - 2b^2\beta(1-\beta)[A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2] \\
\frac{\partial \Pi_m^c}{\partial w_1} &= \frac{+(1-\beta)[\beta^2 A_3 - \beta A_4 - 2b^2\beta w_1 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)](-2\beta^2 b\theta^2 + 2b^2\beta)}{4\beta^2(2b\beta - \beta^2\theta^2)^2}
\end{aligned} \tag{A.48}$$

$$\frac{\partial^2 \Pi_m^c}{\partial w_1^2} = - \frac{(2b\beta - \beta^2\theta^2)(8b\beta^3\theta t_1 - 8b^2\beta^2) - 4b^2\beta(1-\beta)(2b^2\beta - 2b\beta^2\theta^2)}{4\beta^2(2b\beta - \beta^2\theta^2)^2} < 0 \tag{A.49}$$

So the supplier has a unique optimal wholesale price at this time.

$$w_1 = \frac{4\beta^2(2b\beta - \beta^2\theta^2)^3 \left\{ \begin{aligned} &2a\beta - b\beta^2(X - 2t_2\theta + U) - \beta\theta(1-\beta) \\ &- [-bC_k + D(1-\beta)](2b\beta - \beta^2\theta^2) + 2a\beta^3\theta^2(1-\beta) + 2b^2c\beta + 2\beta t_1 \left[\begin{aligned} &b\beta\theta C_k - 2b\beta(Y + t_2) \\ &- a\beta^2\theta(1-\beta) + D(1-\beta) \end{aligned} \right] \end{aligned} \right\} + (1-\beta)(2b^2\beta - 2b\beta^2\theta^2) \left\{ \begin{aligned} &2a\beta - b\beta^2(X - 2t_2\theta + U) - \beta\theta(1-\beta) \\ &- [-bC_k + D(1-\beta)](2b\beta - \beta^2\theta^2) + 2a\beta^3\theta^2(1-\beta) - 2\beta(2b\beta - \beta^2\theta^2)D \end{aligned} \right\} 2\beta(2b\beta - \beta^2\theta^2)^2 - 2b^2\beta(1-\beta) \{ b\beta^2(X + 2\theta t_2) - \beta\theta(1-\beta) + [-bC_k + D(1-\beta)](2b\beta - \beta^2\theta^2) \} 2\beta(2b\beta - \beta^2\theta^2)^2}{4b\beta^2(2b\beta - \beta^2\theta^2)^2 [(2b\beta - \beta^2\theta^2)(4b\beta^2\theta t_1 - 2b\beta^2\theta^2 - 4b^2\beta) - 2b^2\beta^2\theta^2(1-\beta)]} \tag{A.50}$$

Among them, $A_4 = A_2 + D\theta$, $A_5 = 4ab - A_1 - D\theta^2 - 2A_3\theta + 2D\theta$. So the retailer's optimal pricing and inventory are

$$P_c = \frac{A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2}{2\beta(2b\beta - \beta^2\theta^2)} (\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4b^2\beta^2 c(2b\beta - \beta^2\theta^2) + 4\beta^2 t_1 (2b\beta - \beta^2\theta^2) [A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1-\beta)] + 4b\beta^3\theta t_1 (2b\beta - \beta^2\theta^2) - 2b^2\beta(1-\beta) [A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2] + (1-\beta) [\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)] (-2\beta^2 b\theta^2 + 2b^2\beta) + (2b^2\beta - 2\beta^2 b\theta^2) \frac{4b^2\beta(1-\beta)(-2\beta^2 b\theta^2 + 2b^2\beta) - (2b\beta - \beta^2\theta^2)(8b\beta^3\theta t_1 - 8b^2\beta^2)}{2b\beta(2b\beta - \beta^2\theta^2)} \tag{A.51}$$

$$K_c = \frac{A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1-\beta)}{2\beta(2b\beta - \beta^2\theta^2)} (\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2) + 4b^2\beta^2 c(2b\beta - \beta^2\theta^2) + 4\beta^2 t_1 (2b\beta - \beta^2\theta^2) [A_3\beta - 2t_2 b\beta + (D - a\beta^2\theta)(1-\beta)] + 4b\beta^3\theta t_1 (2b\beta - \beta^2\theta^2) - 2b^2\beta(1-\beta) [A_1\beta^2 + A_2\beta + \beta^3\theta^2 D + 2b\theta t_2\beta^2] + (1-\beta) [\beta^2 A_5 - \beta A_4 + 2b\beta^2\theta t_2 - 2a\beta^4\theta^2 - 2\beta D(2b\beta - \beta^2\theta^2)] (-2\beta^2 b\theta^2 + 2b^2\beta) + b\beta\theta \frac{4b^2\beta(1-\beta)(-2\beta^2 b\theta^2 + 2b^2\beta) - (2b\beta - \beta^2\theta^2)(8b\beta^3\theta t_1 - 8b^2\beta^2)}{2b\beta - \beta^2\theta^2} \tag{A.52}$$

$$P_c - P_1 = \frac{A_1\beta^2 + A_2\beta - 2\beta^2 w_1 b\theta^2 + 2b^2\beta w_1 + \beta^3\theta^2 D + 2b\theta t_2\beta^2}{2b\beta(2b\beta - \beta^2\theta^2)} - \frac{2ab\beta\lambda + 2b^2\lambda(w - C_k) + b\beta\theta^2(C_k - 2w) + \beta\theta(G + 2bt_2)}{2b\beta(2b\lambda - \beta\theta^2)} < 0 \tag{A.53}$$

$$K_c - K_l = \frac{A_3\beta + b\beta(w_1\theta - 2t_2) + (D - a\beta^2\theta)(1-\beta)}{2b\beta - \beta^2\theta^2} - \frac{bw\theta - a\beta\theta - G - 2bt_2}{2b\lambda - \beta\theta^2} > 0 \tag{A.54}$$



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