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Ordering and financing strategies in electronic business platform financing with a loss averse retailer

Liandi Zhang[a], Shenglin Ma[b], Na Hao[a], Wenping Li[a] and Wenguang Tang[a*]

[a]*College of Science, Tianjin University of Commerce, China*
[b]*School of Economics and Management, North University of China, China*

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A B S T R A C T

With the rapid growth of e-commerce, platform-based financing in electronic business (EB) has emerged as an innovative solution for online retailers facing capital constraints. This study develops a Stackelberg game-theoretic framework to analyze strategic financing decisions in a two-tier e-commerce supply chain, where an electronic business platform (EBP) , as the leader, assumes leadership by setting financing interest rates, while a capital-constrained, loss-averse online retailer (LOR), as the follower, optimizes order quantities and financing participation under behavioral risk preferences. A hierarchical game-theoretic framework is established to examine strategic interactions between an EBP and a LOR, and the equilibrium outcomes are given. The model derives optimal decisions for both financing rates and ordering strategies. Results demonstrate that when the retailer's initial capital grows, their necessity for external financing diminishes correspondingly, leading to smaller order quantities due to reduced bankruptcy risk. Moreover, higher levels of loss aversion cause retailers to order less and avoid financing, reflecting risk-sensitive behavior. The study also presents comprehensive numerical analyses to explore additional managerial implications, offering insights into how capital availability and behavioral factors like loss aversion shape decision-making in EB financing environments.

1. Introduction

Given the surge in information technology, electronic business (EB) continues growing into various industries and sectors. While the large electronic business platform (EBP) has begun to cooperate with financial institutions or establish small loan subsidiaries to provide loan services for SMEs, many online retailers (usually SMEs) still face limited operating capital for typical supply chain functions, such as procurement, production, etc. A major challenge for SMEs is that financing to facilitate e-commerce operations is limited by a lack of collateral and credit scoring. This is where the practice of EBP financing assumes critical importance, offering both distribution channels and financial services online to SMEs, thereby mitigating their capital limitations. Additionally, by engaging in lending activities, these platforms may extend trade credit to online retailers, further supporting their operational liquidity (Lin et al., 2024).

In China, prominent EBPs such as Tmall and Jingdong Mall have expanded their operations to include lending services, building upon their initial leasing business ventures. Jingdong Mall provides financial services including credit loans, order loans, business owner loans, and movable property financing, among others. Tmall is a new B2C shopping business based on Taobao, which integrates thousands of brands and manufacturers to provide one-stop services for merchants and consumers. When Tmall merchants apply for loans, their qualifications are approved by Tmall, and loans are issued by third-party financial institutions. Tmall merchants outside Zhejiang Province can apply for support loans from Alibaba Microfinance Co LTD, which is known for its lower eligibility requirements, faster payment speeds and flexible assets compared to banks. Vipshop, a large company in the EBP industry, has more than 100 million registered members and more than 20,000 cooperating brands with multiple types of online financial services through its EBP. Vipshop's suppliers only need to provide their procurement business information, sales business information and enterprise financial data to obtain a maximum loan

* Corresponding author
E-mail lxytwg@tjcu.edu.cn (W. Tang)
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amount of 50 million, without collateral.

The key to this type of financing, which offers several competitive advantages, is its ability to utilize behavioral footprints aggregated from continuous consumer engagements across EBPs. These include the capability to secure loans without the need for collateral or guarantees, a reduced risk of default, and expedited loan disbursement processes (Gavrila Gavrila & de Lucas Ancillo, 2021). Therefore, EBPs provide financing support based on their understanding of the sales of products, which not only alleviates the difficulties of online retailers in accessing capital, but also boosts the platform's transaction volume and increases its operating income. However, efficient operations of an EBP require addressing the financing decision in conjunction with the ordering decision. Specifically, the EBPs determine a loan interest rate while anticipating the order quantity from an online retailer (Liu & Lu, 2025). Optimization of such decisions becomes more complex with uncertain demand, which is ubiquitous in e-commerce. These have motivated our study in this paper.

We examine financing an online retailer through an EBP. To capture the contracting and transaction operations, we use a Stackelberg game. This reflects real-world EB financing, where platforms like Tmall and Jingdong hold dominant market power. We also explicitly consider uncertain demand in the decision problem to provide reliable decision-support under risks. Our analytical approach simultaneously addresses the interest rate and order quantity decisions to better alight with an economic performance metric, e.g., profit maximization or cost minimization (Asian & Nie, 2014). Surveys and experimental research have consistently demonstrated that firms exhibit a stronger aversion to losses compared to their attraction to gains of equivalent value, a phenomenon known as loss aversion (Kai-Ineman & Tversky, 1979). This behavioral pattern is widely documented in the scholarly works of economics, finance, marketing, organizational behavior, and business disciplines (Costa et al., 2019). Our study addresses loss aversion under explicit demand-side risks, which has not been sufficiently studied in prior scholarly work.

This research makes three pivotal contributions to the field. First, this paper developed a formulation of a supply chain finance game model to analyse the reciprocal effects between platform-determined financing interest rates and online retailers' ordering decisions. The model captured the strategic leadership of electronic business platforms in setting financial terms and derived the optimal ordering and financing strategies within the platform financing context, thereby extending the analytical framework of supply chain financing. Second, By incorporating the behavioural feature of loss aversion, the study investigated how risk preferences shaped retailers' financing and ordering behaviours under demand uncertainty. The results indicated that an elevated sensitivity to potential losses was found to induce a pronounced shift toward risk-averse inventory strategies among decision-makers, and it also reduced the likelihood of seeking external financing, highlighting the behavioural dimensions of financing choices in e-commerce supply chains. Third, The research examined the non-linear effect of initial capital on optimal decision-making, offering new insights into capital sensitivity within platform-based financing mechanisms. It was found that as initial capital increased, the LOR's order quantity declined, reflecting the effect of bankruptcy protection incentives on risk-taking behaviour and providing implications for platform-level financial service design.

The subsequent sections of this study are systematically structured as follows: Section 2 conducts a comprehensive review of the extant literature, synthesizing key findings from supply chain finance and platform economics. Section 3 establishes the theoretical framework by formalizing the event sequence, defining notational conventions, and articulating foundational assumptions. Section 4 introduces a conceptual model of EBP financing, deriving the equilibrium conditions for SCF under EBP-mediated transactions. Section 5 reports a series of computational experiments examining the sensitivity of optimal decisions to parameter variations. Section 6 concludes with a synthesis of major contributions and outlines avenues for future research, while supplementary mathematical proofs are provided in the Appendix

2. Literature Review

In light of the broad scope of the topic under consideration, we have organized our literature review into two distinct sections: the first focusing on supply chain finance, while the second exploring loss and risk aversion behaviors.

2.1 Supply chain finance

The conceptualization of SCF gained formal academic recognition during the early 21st century, shifting focus from firm-level financing to optimizing financial flows across entire supply networks (Li et al., 2023). Supply chain finance, a pivotal financial mechanism, adeptly addresses the challenge of capital scarcity by enhancing the availability of capital and mitigating financing costs within supply chains (Kucukaltan et al., 2024). Typically, supply chain financing encompasses two main modalities: external financing, which involves obtaining financial resources from external third-party entities such as banks, and internal financing, wherein a supply chain member extends financial services to another, as illustrated by the provision of trade credit (Lin et al., 2018). Within the realm of external financing, SCF stands out as a paradigm-shifting financial architecture in the banking sector, representing a quantum leap in trade financing efficiency. Khan and Vipin (2025) considered the effect of bank credit , which showed that suppliers sell goods to underfunded retailers through wholesale price contracts in a supply chain model. Zhuo et al. (2021) constructed a capital-constrained supply chain model wherein a manufacturer supplies a risk-averse retailer adopting a newsvendor framework with loss-averse preferences, with a bank offering financing to them. They elucidated how the ordering strategies and optimal pricing for them are shaped by their initial capital positions as well as the financing decisions made by the bank.

Cai and Yan (2023) examined the bank financing strategies for retailers in EBP supply chains, assuming deterministic demand. Li et al. (2025) developed a behavioral operations framework to analyze how financing requirements shape the ordering decisions of risk-averse retailers. Tang and Yang (2020) analyze how channel leadership shapes the financing strategy selection of a capital-constrained upstream supplier, who has the option to seek financing from either a bank or a well-capitalized retailer. Li et al. (2023) shifted their research focus to an agri-supply chain framework, within the context of three distinct power structures. They analyzed the applicability and function of agricultural insurance across three channel configurations. Among internal financing strategies, trade credit financing (TCF) is widely utilized, allowing borrowers to increase their purchasing power (Kaur et al., 2023). A multitude of studies have underscored the effectiveness of trade credit financing. Chen and Wang (2012) analyzed how trade credit influences retailer behavior, ultimately finding that a financially constrained retailer tends to increase order quantities when a trade credit arrangement is established. Lee et al. (2018) observed that suppliers holding a smaller market share tend to favor trade credit financing, potentially employing it as a strategic tool to secure a competitive edge. Cao and Yu (2018) examined trade credit within the context of an emission-sensitive supply chain architecture and noted that the smaller-scale operator with limited capital will tend to increase its order quantity when supported by trade credit. Jin and Zhang (2021) investigated a two-echelon supply chain with a loss-averse upstream firm and a financially distressed downstream partner, where financing is accessible through either formal financial credit or commercial operational credit. They analyzed and determined the equilibrium trade credit agreement under asymmetric information within the frameworks of non-cooperative games.

In recent years, the swift expansion of EB practices has introduced a novel financing channel for online retailers: the EBP. Recognized as a crucial supply chain partner for online retailers, these platforms are instrumental in driving sales and market share success. Li and Luo (2025)highlighted that EBPs can mitigate the risks faced by partners and improve liquidity efficiency through structured receivables financing mechanisms. Other works address optimal solutions of innovative platform financing schemes. Wang et al. (2019) studied the financing of EBPs of online retailers and found that active EBP financing could achieve the coordination of supply chain financing, while conservative platforms could not.

Zhen et al. (2020) addressed three different financing options for a manufacturer with limited capital to sell products through retailers and platforms in a hybrid supply chain, namely platform financing, retailer financing and bank financing. This research results show that for manufacturers, digital platform financing exhibits comparative advantages over traditional bank financing, yet its dominance relative to retailer-led financing schemes remains context-dependent. Liu et al. (2021) based on Gupta and Chen (2020), by incorporating overconfident behavioral biases of EB retailers, investigated how such biases shape platform-financed capital-constrained supply chains. Their analysis revealed that mild overconfidence levels attenuate the platform's bargaining leverage. Sun et al. (2023) further constructed an online dual-channel financing model based on EBP for manufacturers with financial constraints to adopt internal financing (EBP) or external financing (bank), and studied the supply chain pricing strategy and optimal interest rate decision under different financing modes.

2.2 Loss averse behavior

The aforementioned works assume that decision makers have no risk preference, that is, assuming risk-neutral preferences, decision-makers aim to determine the optimal order quantity that maximizes expected profit or equivalently minimizes expected cost under certainty equivalence. However, in reality, enterprises in a supply chain often have certain risk avoidance behavior, which has been rigorously documented through theoretical models and empirical validations in the extant literature. Xie et al. (2011) conducted research on a supply chain model that features both a supplier and a manufacturer facing financial risks. Their study aimed to understand the implications of these financial risks on the overall supply chain performance and the strategies that can be adopted to mitigate them. The researchers integrated the concept of loss aversion into three distinct supply chain structures: complete vertical structure, manufacturer-led and supplier-led Stackelberg structure. The findings revealed that risk aversion and the adoption of different strategic approaches both significantly influence pricing and decisions related to quality investments within the supply chain. Applying the prospect theory, Li et al. (2016) embedded prospect theory's loss aversion parameter into a newsvendor framework, integrating supplier-provided trade credit insurance as a risk-sharing mechanism to mitigate consumer default risk under demand uncertainty. In a complementary study, Cui et al. (2016) investigated the risk-averse optimal decision strategies under prospect theory framework by introducing a private-label product. They found that the retailer’s financial constraints significantly influence its risk management strategies.

Yan et al. (2019) conducted a comparative analysis of two different financing options available to cash-constrained retailers: supplier financing and supplier investment, considering the retailers’ loss aversion preferences. The results demonstrate that under severe liquidity constraints, both retailers and suppliers exhibit a stronger preference for supplier investment mechanisms over supplier financing options. Zhuo et al. (2021) analyzed the ordering decision-making of a loss-averse retailer facing liquidity constraints, demonstrating that both equity capital infusion and increasing loss aversion coefficients synergistically drive upward adjustments in optimal order quantities. Yan et al. (2020) compared and explored two kinds of financing arrangements offered by retailers to liquidity-constrained suppliers: debt financing and equity investment. Their analysis demonstrates that under heightened retailer loss aversion and severe supplier liquidity constraints, both parties exhibit a stronger preference for investment over loan options. Xie et al. (2011) studied the influence of financing structure, out-ofstock loss and random demand on the ordering options of retailers, and obtained that under certain conditions of market demand distribution, the best ordering quantity is inverse-proportional to the ratio of debt financing, debt financing interest rate and loss aversion. Zhen et al. (2020) explored a supply chain context where the retailer faces liquidity constraints, while the supplier exhibits prospect theory-consistent risk-averse preferences. The supplier's degree of risk aversion was measured using the CVaR measure. The researchers investigated two kinds of financing schemes: partial credit guarantee and trade credit financing. They derived equilibrium outcomes and elucidated the preferences for each of the financing strategies under consideration.

These studies all show that decision makers' loss aversion substantially influences supply chain financing and operations. Thus, when the market demand is uncertain, there is a critical need to investigate how loss aversion among decision-makers influences financing structure selection and inventory management strategies in supply chain contexts. Such exploration is essential for understanding behavioral biases in capital allocation and order optimization.

2.3 Summary of literature review

Although supply chain finance has been extensively investigated in academic discourse, the topic of financing online retailers with risk aversion and limited capital on EBPs has not been sufficiently addressed. Table 1 summarizes a list of related articles to our work in this paper and identifies whether each of them addresses the issue of EBP financing, uncertain demand and loss averse preference. Our research provides three principal innovations beyond current literature. (1) The financing of EBPs as leaders is studied, which is a topic of significant practical relevance. In the current EB ecosystem, EBPs often play the role of core and leader. They not only provide trading platforms and services, but also actively participate in each stage of the supply chain. Therefore, studying the financing problems of EBPs, especially when they are leaders, can shed light on the optimal financing strategy for EB. (2) Uncertain demand in the market is explicitly addressed in the decision problem. In practice, the market is full of randomness and risk, which will have a direct impact on the financing and operational decisions. (3) Our work considers online retailers the loss aversion preference, which will directly affect their financing demand, the financing efficiency and stability. Through in-depth analysis of the financing decision of online retailers with limited capital and loss aversion, our work obtains optimal financing and operational strategies while considering such risk behavior.

Table 1

Literature comparison

Related works	Market structure	EBP financing	Uncertain demand	Loss averse
Chen and Wang (2012)	Supplier-led	✗	✓	✗
Jing et al. (2012)	Manufacturer-led	✗	✓	✗
Kouvelis and Zhao (2011)	Supplier-led	✗	✓	✗
Zhen et al. (2020)	Manufacturer-led	✗	✓	✓
Jin and Zhang (2021)	Retailer-led supplier-led	✗	✓	✓
Li et al. (2023)	Bank-led, insurer-led, vertical Nash	✗	✓	✓
Li et al. (2018)	Supplier-led	✗	✓	✓
Yan et al. (2020)	Retailer-led	✗	✓	✓
Zhang et al. (2016)	Bank-led	✗	✓	✓
Lin et al. (2024)	Manufacturer-led	✓	✗	✗
Wang et al. (2019)	EBP-led	✓	✗	✗
Liu et al. (2021)	EBP-led	✓	✓	✗
Wang et al. (2019)	Bank-led, EBP-led	✓	✓	✗
This paper	EBP-led	✓	✓	✓

3. Problem Formulation and Assumptions

3.1 Problem formulation

We consider a two-timer supply chain with an online retailer that avoids losses and financial constraints and a loan-qualified EBP. The online retailer operates within the EBP, who charges the online retailer a usage fee at the rate of θ_f (θ_f is a constant) for every unit of product sold.

The detailed order of events and the established decisions are visually depicted in Fig. 1, allowing for an intuitive understanding of the process flow and the decision-making framework. Prior to the sales season, EBP (referred to as "he"), assuming the role of the leader in the strategic interaction, discloses its proposed loan interest rate, denoted as r_p. Subsequently, the online retailer (referred to as "she"), functioning as the follower in this scenario, computes her initial optimal order quantity in light of the market's demand distribution and the disclosed interest rate. She also evaluates the adequacy of her initial working capital, B, for the procurement process. In the event that the retailer is short of working capital, she has the option to opt for financing through the EBP or to forgo financing altogether. Next, in the event that the online retailer elects to pursue financing through the EBP, she proceeds to calculate the revised optimal order quantity, denoted as q_p, and the corresponding loan amount, which is calculated as cq_p-B, in accordance with the financing terms offered by EBP. Here, c represents the unit cost of the product. Subsequently, the retailer submits a loan application to the EBP for the determined loan amount. Upon receipt of the loan funds, the online retailer engages in a full payment transaction with the supplier to acquire the products. All of these actions are completed before the peak selling season, ensuring that the retailer is adequately stocked and financially prepared for the upcoming market demands. During the sales season, as time progresses within the interval $t \in (0,T)$, the online retailer engages in sales activities on the EBP. For each unit dispatched, the retailer incurs a usage fee, which is a fixed percentage θ_f of the sales revenue, payable to the EBP. As the sales season culminates at the point $t = T$, the actual level of market demand is revealed, marking the conclusion of the sales period. She bears a legal duty to repay the generated loan debt. Subject to the condition that the post-deduction sales revenue (net of the EBP-imposed usage fee) meets the minimum repayment threshold, the loan obligations will be fully extinguished. However, the online retailer faces bankruptcy if the remaining earning is insufficient to pay off the loan. In such an event, she is required to remit all remaining sales revenue to the EBP.

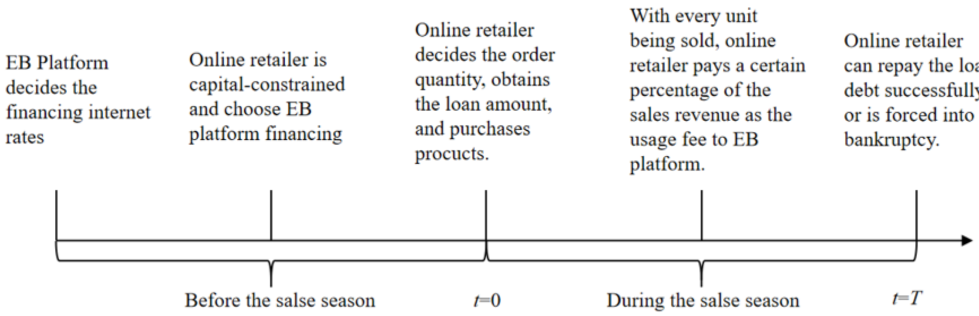

Fig. 1. Order of events and decisions.

3.2 Assumptions

We postulate that the LOR resells to the market under conditions of demand uncertainty, which is represented by a random parameter x. This parameter captures the variability and unpredictability of consumer demand for the product. During peak sales seasons, the distribution of market demand x is assumed to follow a probability density function (PDF) $f(x)$. Following Jing et al. (2012) and Kouvelis and Zhao (2011), we define the failure rate as

$$h(x) = \frac{f(x)}{\overline{F}(x)},$$

where $\overline{F}(x)$ represents the complementary cumulative distribution function of the market demand x. Table 2 summarizes the remaining other symbols of the model.

Table 2
Index of Symbols and Notations

c	The unit product cost
p	Retail price of the product
θ_f	Platform usage fee rate, $\theta_f \in (0,1)$
r_f	Risk-free rate
B	Initial capital of LOR, $B \geq 0$
r_p	Loan interest rate of the EBP, $r_p \geq r_f$
q_i	LOR's order quantity, $q_i \geq 0$, $i = 0, p$
λ	Loss aversion indicator, $\lambda \geq 1$
$\Delta\pi_i^r$	LOR's profit, $i = 0, p$
$E[U_i^r(q_i; x)]$	Expected utility of LOR, $i = 0, p$
$E[\pi_p^p(r_p)]$	Expected profit of the EBP

We assume that when an online retailer facing capital limitations seeks financing from an EBP, she is able to secure the full loan amount requested (i.e., the actual loan disbursed matches the loan amount applied for by the online retailer). The unit purchase cost for online retailers is $c(1+r_f)$, which includes loan interest, and the unit sales revenue after deducting platform usage fees is $(1-\theta_f)p$, thus, $(1-\theta_f)p > c(1+r_f)$. The order quantity q_i ($i = 0, p$) is the online retailer's decision variable when she is not limited by capital ($i = 0$) or uses EBP financing ($i = p$). The EBP's decision variable is r_p, $r_p \geq r_f$. Additionally, we use the superscript * to indicate the scenarios where the participants have selected the optimal strategies within the Stackelberg equilibrium framework.

4. Analysis and Equilibrium Solutions

In accordance with the findings of these papers (Schweitzer & Cachon, 2000; Wang & Webster, 2009), we posit that a gain or loss is recognized based on whether the final wealth of the LOR at the conclusion of the selling season surpasses or falls short of the initial funding. To capture her loss aversion behavior, we adopt a piece-wise linear utility function, which provides a structured representation of the retailer's preference for avoiding losses.

$$U(\pi)=\begin{cases}\lambda(\pi-\pi_0) & \text{if} \quad \pi<\pi_0\\ \pi-\pi_0 & \text{if} \quad \pi\geq\pi_0\end{cases} \tag{1}$$

where π_0 represents the LOR's reference level before sales, π denotes her final funding upon the conclusion of the selling season, and λ is the ratio that quantifies the punishment incurred for not achieving the benchmark target.

If $\lambda = 1$, the LOR is risk neutral, as there is no additional penalty for losses beyond the proportional decrease in wealth. However, if $\lambda > 1$, there exists a change in slope coefficient at the reference level, indicating that the retailer experiences a stronger emotional impact from losses than from equivalent gains. Consequently, higher values of λ signify a greater loss aversion, with the retailer being more sensitive to declines in funding relative to the reference point.

To comparatively study the influence of loss aversion on retailers' optimal ordering decisions under capital constraint and no capital constraint, we consider two situations: (1) ordering decision of LOR without capital constraint, and (2) the supply chain financing and ordering strategy under EBP financing.

4.1 Loss-averse retailer's ordering strategy without capitalconstraint

As a benchmark, consider a loss averse online retailer without capital constraint (denoted by subscript 0). In this scenario, devoid of financing expenses, the retailer's initial working capital corresponds to its initial investment. When the retailer deposits the cost of product orders into a bank, she accrues interest income at the risk-free rate. This interest is considered as the opportunity cost associated with the initial investment. The reference level of the LOR is the ordering cost and its opportunity cost, i.e.,

$\pi_0 = cq_0(1+r_f)$,

where $c(1+r_f)$ is the product cost and its opportunity cost per unit. The final wealth π represents retailer's sales revenue after removing the platform usage fee, i.e.,

$\pi = (1\text{-}\theta_f)p \min[x,q_0]$,

where $(1\text{-}\theta_f)p$ is the proportion of the sales price that the retailer gets to keep after the platform usage fee is deducted. Thus, the profit of the LOR is as follows:

$\Delta\pi_0^r = \pi\text{-}\pi_0 = (1\text{-}\theta_f)p \min[x,q_0]\text{-}cq_0(1+r_f)$.

Consequently, we can draw the following conclusion.

(1) When $x \geq q_0$, $\Delta\pi_0^r = (1\text{-}\theta_f)pq_0\text{-}cq_0(1+r_f)$;
(2) When $x < q_0$, $\Delta\pi_0^r = (1\text{-}\theta_f)px\text{-}cq_0(1+r_f)$;

Moreover, if $\Delta\pi_0^r < 0$,

$$x < \frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0;$$

otherwise, $\Delta\pi_0^r > 0$,

$$q_0 > x > \frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0.$$

So $\Delta\pi_0^r$ can be given as:

$$\Delta\pi_0^r = \begin{cases}(1\text{-}\theta_f)px\text{-}cq_0(1+r_f) & \textit{if} \;\; x < \dfrac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0\\ (1\text{-}\theta_f)px\text{-}cq_0(1+r_f) & \textit{if} \;\; \dfrac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0 \leq x < q_0\\ (1\text{-}\theta_f)pq_0\text{-}cq_0(1+r_f) & \textit{if} \;\; x \geq q_0\end{cases}$$

According to Eq. (1), the LOR's expected utility can be expressed as:

$$E[U_0^r(q_0;x)] = \lambda \int_0^{\frac{c(1+r_f)}{(1-\theta_f)p}q_0} [(1-\theta_f)px-cq_0(1+r_f)]f(x)dx + \int_{\frac{c(1+r_f)}{(1-\theta_f)p}q_0}^{q_0} [(1-\theta_f)px-cq_0(1+r_f)]f(x)dx + \int_{q_0}^{+\infty} [(1-\theta_f)pq_0-cq_0(1+r_f)]f(x)dx$$

which can be expressed as

$$E\left[U_0^r(q_0;x)\right] = \left[\left(1-\theta_f\right)p - c\left(1+r_f\right)\right]q_0 - \left(1-\theta_f\right)p\int_0^{q_0} F(x)dx - \left(\lambda-1\right)\left(1-\theta_f\right)p\int_0^{\frac{c(1+r_f)}{(1-\theta_f)p}q_0} F(x)dx \quad (2)$$

Proposition 1, derived from solving the maximization problem (2), specifies the optimal order quantity.

Proposition 1. In the absence of capital constraints, the LOR's expected utility function exhibits concavity about the initial order quantity q_0. Consequently, a unique optimal order quantity is obtained, denoted as q_0^*, which satisfies the necessary conditions for maximizing her expected utility

$$\overline{F}(q_0^*) = \frac{c(1+r_f)}{(1-\theta_f)p}[\lambda+(1-\lambda)\overline{F}(\frac{c(1+r_f)}{(1-\theta_f)p}q_0^*)]$$

Proof See Appendix.

Corollary 1. In the absence of capital constraints, when the retailer exhibits a neutral attitude towards loss, corresponding to a loss aversion parameter of $\lambda = 1$, q_0^{1*} satisfies

$$\overline{F}(q_0^{1*}) = \frac{c(1+r_f)}{(1-\theta_f)p}$$

and the optimal ordering quantity for a loss-averse online retailer exhibits a significant downward deviation relative to that of a loss-neutral counterpart, i.e., $q_0^* < q_0^{1*}$.

Proof See Appendix.

Corollary 1 concludes that in the case without capital constraint, online retailers with loss aversion preference will be more conservative in ordering than loss neutral online retailer. This discrepancy is attributable to the loss-averse retailer's increased sensitivity to potential losses, which influences their ordering strategy to be more conservative.

Corollary 2. In scenarios where there are no capital constraints, the optimal order quantity for an online retailer exhibits a negative monotonic relationship with the loss aversion coefficient, i.e., $\frac{dq_0^*}{d\lambda} < 0$.

Proof See Appendix.

Corollary 2 shows that in the case without capital constraint, loss aversion preference has a negative impact on the quantity of the retailer, that is, the higher the degree of loss aversion of an online retailer. Specifically, as the loss aversion indicator intensifies, she tends to order a smaller quantity to minimize the potential for losses. This proposition underscores the inverse relationship between the loss aversion and the order quantity decision in a capital-unconstrained environment. Corollary 1 can also be derived from this corollary.

4.2 LOR's ordering strategy without capital constraint

We construct a two-stage Stackelberg game framework to analyze the strategic interaction dynamics between the EBP and LOR, where the EBP (leader), establishes the optimal loan interest rate, r_p^*, to achieve the goal of maximizing expected profits. Subsequently, the LOR (follower), selects the EBP's financing option and determines her optimal order quantity, q_p^*, by maximizing her expected utility.

4.2.1 LOR's problem

When the LOR faces a capital constraint, meaning her available funds are abundant to cover the cost of order quantity, i.e., $B < cq_0^*$, she must adjust her ordering strategy. In such cases, the retailer may opt to order the quantity $q_c^* = B/c$ using her entire capital or chooses to borrow from the EBP to augment her order quantity (indicated by subscript p). If the LOR prefers to borrow, the borrowed amount is cq_p-B, and which will be paid back to the EBP at the end of the selling season.

Given that the LOR's initial capital B represents her initial investments, she has the option to deposit these funds in a bank to earn interest at a risk-free rate. Therefore, when evaluating the retailer's decision-making under capital constraints, it is essential to consider the foregone interest as part of the opportunity cost of B. Thus, we suppose that capital-constrained online retailer's reference level is the initial investments and the opportunity costs associated with B, i.e. $\pi_0^r = B(1+r_f)$. The total profit π_p^r represents the LOR's sales revenue after subtracting the platform usage fee subtract financing cost and its interest income, i.e.,

$$\pi_p^r = [(1\text{-}\theta_f)p \min[x,q_p]\text{-}(cq_p\text{-}B)(1+r_p), 0]^+,$$

where $(1\text{-}\theta_f)p$ is the net sales revenue per unit, accounting for the deduction of the platform usage fee, $(cq_p\text{-}B)(1+r_p)$ represents financing cost and its interest income. We may obtain the following observations.

(1) When $x < q_p$, the revenue of the online retailer is

$$\pi_p^r = [(1\text{-}\theta_f)px\text{-}(cq_p\text{-}B)(1+r_p), 0]^+.$$

If the sales revenue falls short of the financing expenses, i.e.

$$(1\text{-}\theta_f)px \le (cq_p\text{-}B)(1+r_p)$$

that is,

$$x \le x_1(q_p) = \frac{(cq_p\text{-}B)(1+r_p)}{(1\text{-}\theta_f)p},$$

the online retailer declares bankruptcy to seek legal protection, resulting in the EBP receiving the entirety of her sales revenue. Consequently, the LOR's revenue is effectively reduced to zero, i.e., $\pi_p^r = 0$. Moreover, when

$$(cq_p\text{-}B)(1+r_p) < (1\text{-}\theta_f)px < (cq_p\text{-}B)(1+r_p)+B(1+r_f)$$

i.e.,

$$x_1(q_p) < x < x_2(q_p),$$

where

$$x_2(q_p) = \frac{(cq_p\text{-}B)(1+r_p)+B(1+r_f)}{(1\text{-}\theta_f)p},$$

the online retailer sustained financial losses but avoided insolvency. In addition, since $c(1+r_p) < (1\text{-}\theta_f)p$, it is straightforward to verify $x_1(q_p) < x_2(q_p) < q_p$. When $x \ge x_2(q_p)$, online retailer will consider their operating results to be profitable.

(2) When $x \ge q_p$,

$$\pi_p^r = (1\text{-}\theta_f)pq_p\text{-}(cq_p\text{-}B)(1+r_p),$$

and the operating result of the online retailer is profitable. The above observations show that, based on the relationship between real demand and order quantities, the profit and loss status of the LOR can be decompose into the following segments, as shown in Figure 2. Thus the profit of LOR can be described as follows:

$$\Delta\pi_p^r = \pi_p^r - \pi_0^r = \begin{cases} -B\left(1+r_f\right) & \text{if} \quad x < x_1\left(q_p\right) \\ \left(1-\theta_f\right)px - \left(cq_p - B\right)\left(1+r_p\right) - B\left(1+r_f\right) & \text{if} \quad x_1\left(q_p\right) < x < x_2\left(q_p\right) \\ \left(1-\theta_f\right)px - \left(cq_p - B\right)\left(1+r_p\right) - B\left(1+r_f\right) & \text{if} \quad x \ge q_0 \\ \left(1-\theta_f\right)pq_p - \left(cq_p - B\right)\left(1+r_p\right) - B\left(1+r_f\right) & \text{if} \quad x \ge q_p \end{cases}$$

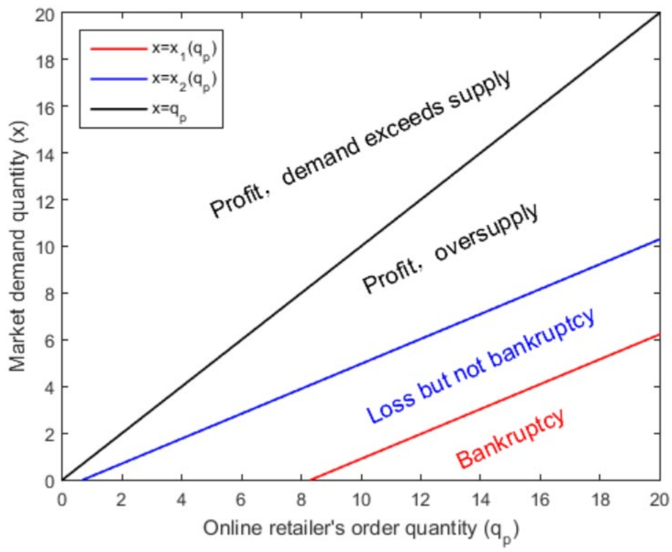

Fig. 2. Market demand with different online retailer's order quantities

Drawing from the utility function outlined in Eq. (1), the expected utility for the LOR can be expressed as follows:

$$E[U_p^r(q_p;x)] = \lambda\int_0^{x_1(q_p)} \text{-}B(1+r_f)f(x)dx + \lambda\int_{x_1(q_p)}^{x_2(q_p)} [(1\text{-}\theta_f)px\text{-}(cq_p\text{-}B)(1+r_p)\text{-}B(1+r_f)]f(x)dx$$
$$+\int_{x_2(q_p)}^{q_p} [(1\text{-}\theta_f)px\text{-}(cq_p\text{-}B)(1+r_p)\text{-}B(1+r_f)]f(x)dx + \int_{q_p}^{+\infty} [(1\text{-}\theta_f)pq_p\text{-}(cq_p\text{-}B)(1+r_p)\text{-}B(1+r_f)]f(x)dx$$

which is alternatively written as

$$E\left[U_p^r(q_p;x)\right] = (1\text{-}\theta_f)pq_p\text{-}(cq_p\text{-}B)(1+r_p)\text{-}B(1+r_f)\text{-}(1\text{-}\theta_f)\int_0^{q_p} F(x)dx\text{-}(\lambda\text{-}1)(1\text{-}\theta_f)p\int_0^{x_2(q_p)} F(x)dx + \lambda(1\text{-}\theta_f)p\int_0^{x_1(q_p)} F(x)dx \quad (3)$$

By maximizing Eq. (3), we derive proposition 2 which delineates the optimal order quantity.

Proposition 2. As the LOR operates with an initial capital B and loans from the EBP at an interest rate of r_p, the expected utility function with respect to the order quantity q_p is concave. Consequently, a unique optimal order quantity q_p^* is obtained, which exhibits as:

$$\overline{F}(q_p^*) = \frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\lambda\overline{F}(x_1(q_p^*))+(1\text{-}\lambda)\overline{F}(x_2(q_p^*))].$$

Proof See Appendix.

Proposition 2 concludes that the LOR's optimal order quantity is dependent not only upon her loss aversion and initial capital level, but also the loan interest rate, and other factors such as costs and wholesale prices such as costs and wholesale prices.

Corollary 3. As the LOR operates under a capital constraint with an initial budget of zero, i.e., $B = 0$, her optimal order quantity q_{pz}^* is determined by

$$\overline{F}(q_{pz}^*)\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}\overline{F}(\frac{c(1+r_p)}{(1\text{-}\theta_f)p}q_{pz}^*) = 0$$

which is unaffected by her loss aversion.

Corollary 3 demonstrates that when the LOR operates under a capital constraint with an initial budget of zero and finances the entire order through borrowing from the EBP, her ordering decision becomes independent of her loss aversion. This is unsurprising, as the retailer incurs no personal loss regardless of the order quantity or the actual realization of stochastic demand, due to the principle of limited liability and the absence of initial capital.

Corollary 4. When the online retailer exhibits a neutral attitude towards loss, corresponding to a loss aversion parameter of $\lambda = 1$, the optimal order quantity q_{p1}^* satisfies

$$\overline{F}(q_{p1}^*)-\frac{c(1+r_p)}{(1-\theta_f)p}\overline{F}(x_1(q_{p1}^*)) = 0,$$

and given p, θ_f and c, the optimal order quantity for a loss-averse online retailer is lower compared to that of a loss-neutral online retailer, i.e., $q_p^* < q_{p1}^*$.

Proof See Appendix.

Corollary 4 reveals that a loss averse retailer would tend to order less, which is consistent with the previous studies (WANG and WEBSTER, 2009).

Corollary 5. The LOR's optimal order quantity for the online retailer diminishes as the loss aversion index increases, i.e., $\frac{dq_p^*}{d\lambda} < 0$; It decreases with the loan interest rate of the EBP, i.e., $\frac{dq_p^*}{dr_p} < 0$.

Proof See Appendix.

Corollary 5 also demonstrates that the loss aversion and the loan interest rate of the EBP adversely affect the ordering level of the LOR. In the extreme case of loss aversion, the LOR may opt not to order any inventory. This behavior is particularly pronounced when she is capital constrained, as there is a preference for ordering a lower quantity to secure a modest profit with minimal loss exposure, rather than ordering a larger quantity that exposes the retailer to higher loss risks.

4.2.2 EBP's problem

In the Stackelberg game, the EBP receives platform usage fees from the online retailer, and also receives financing interest from the retailers, while bearing the risks coming from the LOR. The EBP's expected profit can be expressed as:

$$E[\pi_p^P(r_p)] = \theta_f\, pE[\min(x,q_p)] + E\{\min[(1-\theta_f)p\min(x,q_p), (cq_p-B)(1+r_p)]\}-(cq_p-B)(1+r_f),$$

where $\theta_f\, pE[\min(x,q_p)]$ is the usage fee to the EBP, $(1-\theta_f)p\min(x,q_p)$ is the net sales revenue of the LOR, accounting for the deduction of the EBP's usage fees, $(cq_p-B)(1+r_p)$ is the online retailer's agreed repayment. And the LOR's actual repayment obligation is determined by whichever is smaller between the preset repayment amount and the residual net sales revenue after platform usage fee deduction, $(cq_p-B)(1+r_f)$ is the operating cost and opportunity cost of financing.

Consequently, the online retailer's actual repayment at the post-season period is

$$\min[(1-\theta_f)p\min(x,q_p^*), (cq_p^*-B)(1+r_p)] = (1-\theta_f)p\min[\min(x,q_p^*),\frac{(cq_p^*-B)(1+r_p)}{(1-\theta_f)p}] \tag{4}$$

Since

$$c(1+r_p) \le (1-\theta_f)p$$

we obtain that

$$\frac{(cq_p^*-B)(1+r_p)}{(1-\theta_f)p} \le q_p^*,$$

i.e., $x_1(q_p^*) \le q_p^*$, thus, the formula (4) is

$$(1-\theta_f)p\min[\min(x,q_p^*),\frac{(cq_p^*-B)(1+r_p)}{(1-\theta_f)p}] = (1-\theta_f)p\min(x,x_1(q_p^*)).$$

This implies that when the sales revenue x exceeds a certain threshold $x_1(q_p^*)$, the online retailer's actual repayment may be the full loan amount $(cq_p^*-B)(1+r_p)$, allowing her to settle the entire debt; Conversely, if the sales revenue x falls below $x_1(q_p^*)$, having failed to meet the full loan obligation, she is mandated to forward all proceeds generated from sales transactions $(1-\theta_f)px$ to the EBP.

Hence, the online retailer's expected actual repayment is

$$(1-\theta_f)pE[\min(x,x_1(q_p^*)]$$

and the EBP's expected profit is

$$E[\pi_p^P(r_p)]=\theta_f\, pE[\min(x,q_p^*)] + (1-\theta_f)pE[\min(x,x_1(q_p^*))]-(cq_p^*-B)(1+r_f). \quad (5)$$

By maximizing Eq. (5), we have proposition 3.

Proposition 3. With the optimal order quantity established for the LOR, the EBP's expected profit is concave in r_p and r_p^* exists, which satisfies

$$[\theta_f\, p\overline{F}(q_p^*) + c(1+r_p^*)\overline{F}(x_1(q_p^*))-c(1+r_f)]\frac{dq_p^*}{dr_p} + (cq_p^*-B)\overline{F}(x_1(q_p^*)) = 0$$

The proof is provided in the Appendix

The proposition above shows that the optimal interest rate strategy of the EBP can be more complex, and its determination is related to the order strategy of the LOR, the real market situation, and the cost of the product. The other properties of the optimal strategy for the platform will be described using numerical models in the next chapter.

5. Numerical Results

Due to the complexity of the addressed problem, the analytical equilibrium solutions are limited in the insights they can generate. The objective of the computational experiment is to garner supplementary managerial insights that may not be readily discernible from the analytical solutions alone. Furthermore, it aims to investigate the influence of select critical input parameters on the derivation of optimal solutions. We consider three types of strategies: EBP financing (Decision "1"), all self-owned investment (Decision "2"), and partial self-owned investment (Decision "3"). Decision "1" means that the retailer raises funds based on the financing interest rate provided by the EBP, and returns the financing's principal and accrued interest to the EBP are realized at the end of the sales. In Decision 2, the retailer makes full use of their initial capital, neither financing for ordering nor having any remaining capital. Decision 3 represents that her initial capital is sufficient without the need for financing, and there is capital surplus after ordering. We also study the influence of online retailers' start-up Funding, loss aversion on the optimal order quantity, optimal financing interest rate, optimal utilities and optimal decision choice. We post that x follows a uniform distribution $x \sim U[0, 100]$, and the numerical simulation is carried out with reference to the parameter settings of Wang et al. (2019), $p = 3.6$, $c = 2$, $\theta_f = 0.25$, $r_f = 0.1$.

5.1 The effect of initial capital on decision-making optimization

5.1.1 The optimal order quantities for the LOR across various initial capital level

With the above three strategies, Figure 3 depicts the optimal order quantities across different initial capital levels at $\lambda = 2$. From Fig. 3, we can see that as the initial capital of retailers changes, the ordering strategies of the retailer also change accordingly.

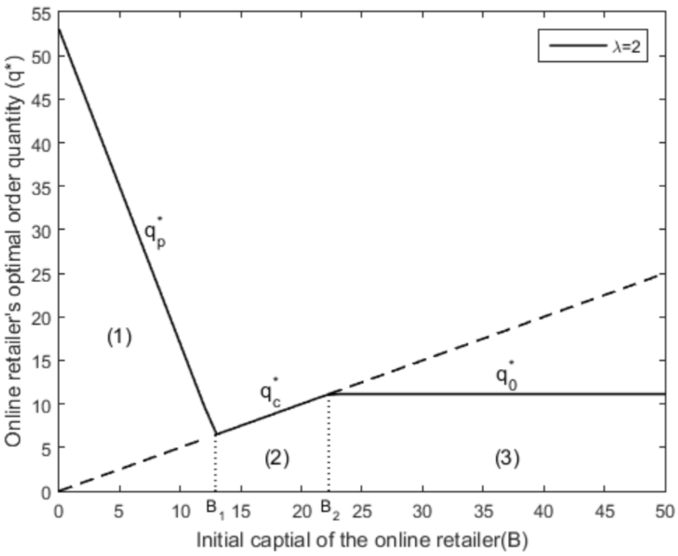

Fig. 3. LOR's optimal order quantities across various initial capital levels

Insight 1 Under the EBP financing decision, the LOR's optimal order quantities decrease with the increase of start-up funding;

Under all the self-owned investment decision, her optimal order quantities are positively correlated with the initial capital; Under the partial self-owned investment decision, her optimal order quantity remains constant regardless of the initial capital. Insight 1 has the following managerial implications.

(1) Under the EBP financing decision, q_p^* demonstrates a negative correlation with initial capital growth. When the initial capital of a retailer is less than B_1, the optimal ordering capital required by the retailer under platform financing decisions exceeds B. At this time, the LOR faces two strategies: either financing or fully utilizing her own capital, which is determined by the EBP's financing interest rate and her expected utilities;

(2) Under all the self-owned investment decision, the optimal order quantity q_c^* demonstrates a positive correlation with the initial capital obviously. As the initial capital of the retailer is between B_1 and B_2, due to platform financing decisions, the capital required for retailer ordering is less than the initial capital, and there is no need for financing at this time; At the same time, due to the initial capital being less than B_2 and the inability to meet the condition of no capital constraint, the LOR is restricted to use her own capital for executing procurement;

(3) Under the partial self-owned investment decision, the retailer's optimal order quantity q_0^* is unaffected by working capital limitations, the required ordering capital is B_2. Only when the initial capital of a retailer exceeds B_2, will her capital be sufficient to make such a decision.

Intuitively, one might expect that an LOR with a higher initial capital would to order a larger quantity. However, this assumption is valid only when the initial capital surpasses the critical value, as demonstrated in Figure 3. Specifically, under EBP financing, the LOR's optimal order quantity decreases in B with $\lambda > 1$. Under the EBP financing decision, as B increases, the tendency of the retailer to avoid or reduce potential risks and lower order quantities in the face of uncertainty due to her loss aversion preference is indeed a reasonable explanation. For retailers, increasing order volume may mean taking on greater market risk, which reflects their emphasis on risk management and financial stability after improving their financial situation. Retailers with lower initial capital often have lower risk-taking ability, but due to bankruptcy protection providing a certain degree of risk buffer for retailers, they may be more inclined to increase their order volume through borrowing in the hope of expanding sales scale, increasing market share, and improving their financial situation. And the EBP may also observe the strong demand of retailers for funds, asking for higher financing interest rates (as can be seen from the analysis in the next section), hedging the risks brought by retailer's lending operations, and thus promoting the formation of a community of interests with retailers.

5.1.2 The EBP's optimal financing interest rate with different initial capital

The financing interest rate of the EBP play an important role in financial activities, and they interact with each other. A high or a low financing interest rate directly affects the cost for retailers to obtain financing from the platform, which in turn affects their business operations and profitability. Figure 4 shows r_p^* of the EBP across various initial capital levels when $\lambda = 2$.

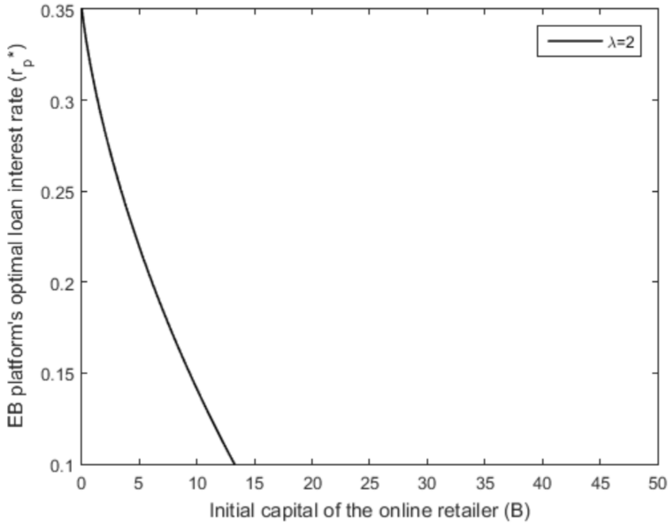

Fig. 4. The EBP's optimal financing interest rate with different initial capital amounts.

Insight 2 The optimal financing interest rate of the EBP demonstrates an inverse relationship with the LOR's working capital.

The retailer with more initial capital may be more likely to receive financing support from the EBP by offering more favorable financing rates to attract them as high-quality customers, as she can provide better guarantees or collateral to reduce platform risks. The retailer with less initial capital may face greater challenges in financing, and the platform may set higher financing rates to balance risk and return, as she may find it difficult to meet the platform's financing requirements or bear higher financing costs. In financial markets, risk and return are usually proportional. For retailers with lower risk (i.e. those with higher initial capital), the platform's required return (i.e. financing interest rate) will also decrease accordingly. This is because the platform believes that these retailers are more likely to repay on time, so there is no need to compensate for potential risks through high interest rates. For retailers, a reasonable allocation of their initial capital can help them better obtain financing support and promote business development. For the EBP, by formulating reasonable financing interest rate policies, they can better balance risk and return, attract high-quality customers, and promote the stable development of the platform.

5.1.3 The LOR's optimal expected utility with different initial capital

We conduct a comparative analysis of the three ordering decisions of the LOR, especially under financing decision, where the financing conditions provided by the platform are an important factor affecting the expected returns of retailers. If platforms or financial institutions can provide retailers with favorable financing rates, even if the initial capital is low, retailers may expand their business scale and enhance market competitiveness through financing, thereby achieving higher expected utilities. Fig. 5 shows the optimal expected utility of the retailer for a range of initial capital amounts, given $\lambda = 2$. When $B < B_1$, the optimal expected utility of the LOR's financing decision (Decision 1) exhibits a positive correlation with the initial capital. When $B > B_1$, the platform will terminate financing arrangements to the retailer due to its optimal financing rate being lower than the risk-free rate. Under all self-owned investment (Decision 2), U^* initially rises before declining as B increases, and the boundary point B_2 happens to be the optimal capital required by the retailer under partial self-owned investment (Decision 3). Decision 3 can only be adopted when $B > B_2$, that is, when her initial capital is sufficient. In this case, U^* is independent of its initial capital.

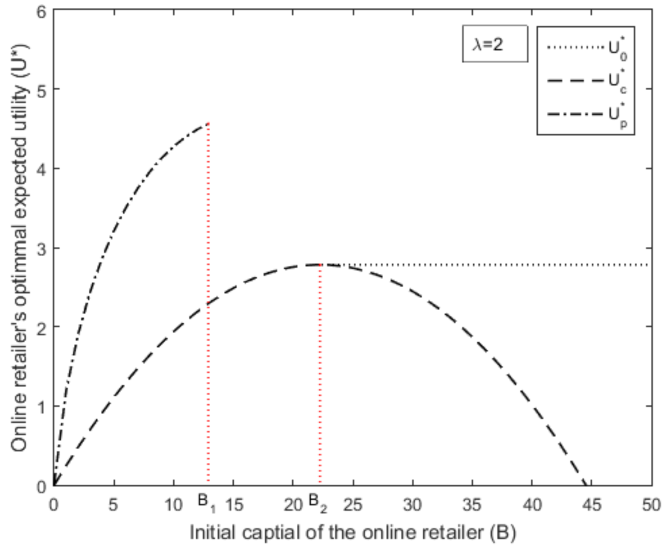

Fig. 5. The LOR's optimal expected utility with different initial capital amounts.

Insight 3 When the retailer faces liquidity constraints, financing arrangements are executed at the EBP-prescribed interest rate, compared to situations with abundant capital and no financing demand, it may achieve greater expected utility.

It can be observed from Figure 5 that the retailer with limited capital will achieve greater expected utility under financing decisions, especially when she has a certain capital scale and the platform provides lower financing rates. This is because the retailer transfers risks in situations of uncertain demand through financing, reducing operational risks, and forming a risk sharing community with the EBP. The following insights can be obtained:

(1) When $B < B_1$, she faces two decision choices: EBP financing (Decision 1) and all self-owned investment (Decision 2). It can be clearly seen from the graph that the expected utility under Decision 1 is higher than that under Decision 2. In this case, the optimal choice for the retailer is platform financing.

(2) When the initial capital of retailers is between B_1 and B_2, on the one hand, the EBP no longer provides financing opportunities for the LOR, and on the other hand, the initial capital of the LOR is insufficient to support the capital demand under Decision 3. At this time, the retailer can only choose Decision 2.

(3) When $B > B_2$, the LOR has sufficient funds and does not need to raise funds from the platform. Comparing the optimal expected utility under Decision 2 and Decision 3, Decision 3 is her optimal choice.

5.1.4 The LOR's optimal decision choice with different initial capital

The subsequent numerical example, with a parameter value of $\lambda = 4$, is presented to elucidate the influence of initial capital on the LOR's financial status, order quantity, and decision- making process.

Table 3
Comparison of online retailers with different initial capitals

Conditions and decision	Notation	Online retailers		
		Retailer 1	Retailer 2	Retailer 3
Initial capital	B	5	10	15
Loss-aversion coefficient	λ	4	4	4
Optimal order quantity	q_p^*	17.6471	*	*
	q_c^*	2.5000	5.0000	7.5000
	q_0^*	6.1898	6.1898	6.1898
Financial status	$B\text{-}cq_p^*$	−30.2942	*	*
	$B\text{-}cq_c^*$	0.0000	0.0000	0.0000
	$B\text{-}cq_0^*$	−7.3796(×)	−2.3796(×)	2.6204
Optimal interest rate	r	0.1505	*	*
Optimal expected utility	U_p^*	**1.6476**	*	*
	U_c^*	0.9976	**1.4903**	1.4781
	U_0^*	1.5475(×)	1.5475(×)	**1.5475**
Decision		Decision 1	Decision 2	Decision 3

From Table 3, it is evident that despite having identical loss aversion levels apart from the initial capital, the three online retailers exhibit entirely distinct optimal decisions. The optimal expected utility for the retailer under three different decisions is 1.6476, 0.9976 and 1.5475, respectively. Due to her initial capital of 5, it cannot meet the requirement of a capital of 12.3796 under decision 3, so the retailer can only adopt the first two decisions. According to the optimal financing interest rate given by the EBP, compare her optimal expected return under the first two decisions, and her optimal decision is platform financing. The final financing requirement of the retailer is 30.2942. When B of the LOR increases to $B = 10$, the optimal ordering capital for retailers under EBP financing decisions is less than the initial capital, and financing is not required. At the same time, the capital required for ordering under the partial self-owned investment decision is higher than the initial capital. Therefore, the optimal decision for retailers at this time is decision 2, and the optimal expected utility obtained is 1.4903. When $B = 15$, financing is also not required. The retailer has two decision choices, namely all self-owned investment and partial self-owned investment. The optimal order quantity is 7.500 and 6.1898, and the optimal expected utility is 1.4781 and 1.5475, respectively. Comparing the optimal expected utility in the two decisions, the optimal decision for retailers is Decision 3, and she still has a capital surplus of 2.6204 after placing an order.

The level of initial capital does indeed affect the financing decisions of the LOR, and the higher the initial capital, the less inclined retailers are towards financing. Retailers with higher initial capital may have more abundant funds, and they may be more inclined to use their own funds rather than obtaining funds through financing. This is because using self-owned funds can avoid risks such as the interest burden that may arise from financing.

5.1.5 EBP's optimal expected profit with different initial capital in EBP financing decision

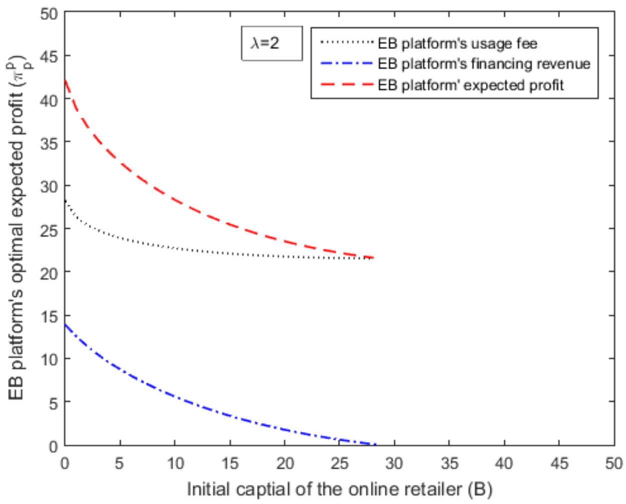

Fig. 6. EBP's optimal expected utility with different initial capital amounts.

Fig. 6 illustrates the EBP's optimal expected profit across various initial capital amounts, with $\lambda = 2$, under the context of the EBP's financing decision.

Insight 4 Under platform financing decisions, the expected profit of EBP demonstrates an inverse relationship with the initial capital of the LOR Among them, the platform usage fee and financing revenue decrease concomitantly.

The expected profit of the platform includes two parts: platform usage fees and financing revenue. Retailers with loss aversion may be more inclined to adopt conservative business strategies when their initial capital increases. Next, we will discuss and explain the revenue from the above two parts of the platform separately.

First, a conservative ordering strategy means that retailers reduce the number of orders to mitigate potential risks. This behavior directly leads to a decrease in platform usage fees, as the platform charges fees based on the transaction volume or order size of retailers, which is a direct reason for the decrease in platform profits. Notwithstanding the initial trend, once the LOR's capital endowment surpasses a threshold value, the rate of decrease will begin to slow down. This is because even if retailers have more funds, they still need a certain amount of products to be sold on the platform to maintain their profits.

Secondly, the increase in initial capital reduces the demand for external financing from retailers. This means that retailers may no longer rely so much on platform provided financing services, or at least reduce their dependence on financing. In order to attract and retain these retailers, the platform may need to provide more favorable financing rates to maintain cooperative relationships. The favorable financing interest rate reduces the interest income obtained by the platform through financing services, further compressing the platform’s profit margin. The platform financing income will be more affected by the initial capital increase of retailers. The rate of decrease in platform financing revenue will be greater than the rate of decrease in platform usage fees, which poses a significant challenge to the overall profit of the platform.

5.2 The effect of loss averse on optimal choices

5.2.1 Optimal order quantities vary with different levels of loss aversion

Corollary 5 posits that loss aversion exerts a dampening effect on the ordering behavior of online retailers. Specifically, it indicates that as an online retailer’s degree of loss aversion intensifies, they would to order inventory diminishes. This dynamic is particularly intriguing when considering a capital-constrained online retailer, who might favor ordering a smaller quantity to secure a modest profit with reduced risk of loss, as opposed to ordering a larger quantity that exposes them to a higher risk of loss. This behavior underscores the trade-off between the potential for higher profits and the desire to mitigate loss in the context of limited financial resources. To gain a comprehensive understanding of how λ and B jointly influence the optimal order quantity and the financial status of the online retailer, we present Fig. 7. The figure illustrates the interplay of two contrasting forces that shape the LOR’s procurement strategies. On one hand, the pursuit of profit motivates the LOR to raise order quantities in an attempt to maximize earnings. On the other hand, loss aversion behavior exerts a contrary pressure, compelling the retailer to reduce order quantities to enhance the probability of survival. The synergy of these opposing effects is manifest in the phenomena depicted in Fig. 7, providing insight into the complex decision-making process of the online retailer.

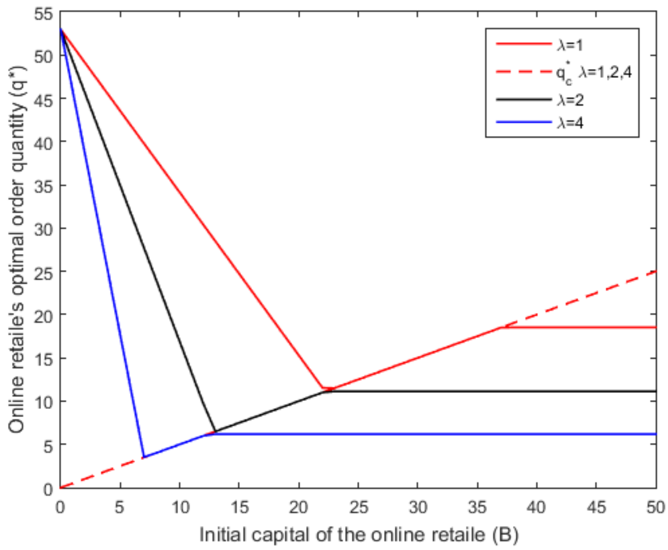

Fig. 7. Optimal order quantities across various degrees of loss aversion.

The parameter λexhibited by the online retailer inversely affects the quantity ordered, regardless of the initial capital owned or the financial state. As depicted in Fig. 7, all three optimal reaction functions, corresponding to different λ, initiate from a common value. This observation indicates that q_{pz}^* is independent of coefficient λ, in alignment with the findings of Corollary 3. Intuitively, given that the LOR operates under a zero initial capital endowment, the risk of loss is effectively nullified owing to the principle of l liability limitation. Consequently, the quantity ordered by the retailer does not impact the potential for financial loss, as there is no capital to be lost.

5.2.2 EBP's optimal financing interest rate with different loss aversion

Insight 5 The equilibrium financing rate offered by the EBP exhibits a downward trend as the LOR's loss aversion increases. Financing platforms appear to be more inclined to extend financial support to retailers who exhibit a heightened loss aversion. In Fig. 8, the EBP's optimal financing rate is related to both λ and B of the LOR. The more loss averse the LOR is, the fewer orders of the product, the lower the required loan amount, and the financing business platform also needs to lower interest rates. The reason why the EBP prioritizes to provide financing support to retailers with a higher degree of λ, is because that EBP usually believe that such retailers are more stable and reliable in their business processes, thereby reducing the risk of default and the possibility of non-performing loans.

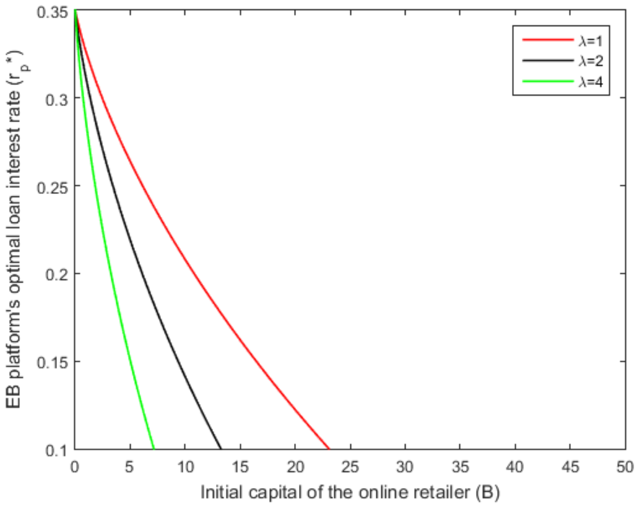

Fig. 8. EBP's optimal loan interest rate with different loss aversion.

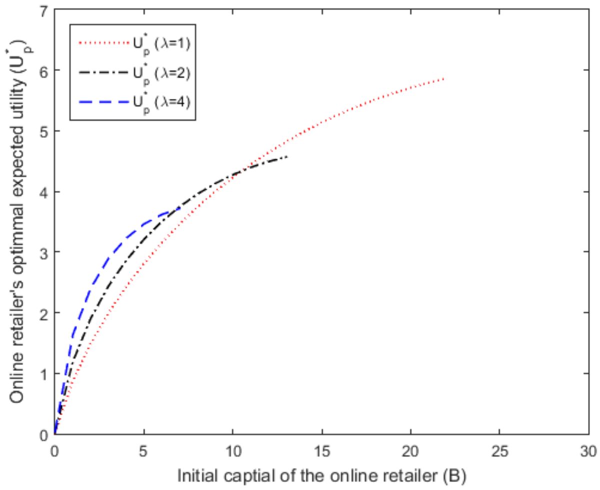

Fig. 9. The LOR's optimal expected utility with different loss aversion.

5.2.3 The LOR's optimal expected utility with various loss aversion

Insight 6 Under EBP financing decisions, the higher the coefficient λ of the LOR, the more obvious the phenomenon of diminishing marginal expected utility. For retailers with severe financial constraints, the more sensitive they are to potential risks (the higher their loss aversion), the greater their expected utilities may be. From Fig. 9, it can be seen that the expected utility of retailers has a significant decrease in marginal utility for initial capital, i.e. the increment of expected utility obtained by the retailer decreases for every unit of initial capital increase. In addition, the more retailers are averse to losses, the more significant the decrease in marginal utility they gain. When the initial capital of retailers is low, retailers with higher loss aversion can achieve higher expected utility due to the platform providing more favorable financing rates. However, as the initial capital increases, the utility of retailers with lower loss aversion will surpass that of retailers with higher aversion coefficient due to the impact of diminishing marginal utility. From this, it can be seen that on the one hand, small and medium-sized retailers with severe capital constraints will benefit more from platform financing if their loss aversion is higher. On the other hand, as the initial capital increases, retailers with higher loss aversion actually see a decrease in profits from platform financing decisions. When the initial capital is low, retailers with high loss aversion will be more cautious in decision-making, and financing platforms prefer to offer financing support for retailers with high loss aversion, so that retailers can gain more profits from financing decisions. When the initial capital is high, retailers have the ability to bear higher risks. At this time, retailers with lower loss aversion tend to adopt more aggressive ordering strategies in order to obtain higher profits. This adventurous spirit may bring greater profits, but it may also be accompanied by higher risks.

5.2.4 Online retailer's optimal decision choice with different loss averse

When loss aversion increases, retailers are more inclined to use their own capital for investment rather than financing. The subsequent numerical example, with a parameter value of $B = 15$, is provided to elucidate the effect of loss aversion in the financial status, the optimal order quantity, and the decision choice of the online retailer.

Insight 7 Loss aversion behavior significantly alters the fiscal position of the LOR. As loss aversion intensifies, retailers tend to prioritize the use of their own capital for investment over seeking external financing options.

From Table 4, it is evident that despite having identical initial capital conditions, the three online retailers exhibit distinct loss aversion attitudes, leading to entirely different optimal decisions among them. The optimal expected utility for the retailer under three different decisions is 3.8294, 2.9906 and 4.6296, respectively. Due to her initial capital of 15, it cannot meet the requirement of a capital of 37.0370 under Decision 3, so the retailer can only adopt the first two decisions. According to the optimal financing interest rate given by the EBP, compare her optimal expected utility under the first two decisions, and her optimal decision is platform financing. The final financing requirement of the retailer is 34.4118.

Table 4
Analyzing online retailers with varied loss aversion coefficients

Conditions and decision	Notation	Online retailers		
		Retailer 1	Retailer 2	Retailer 3
Initial capital	B	15	15	15
Loss-aversion coefficient	λ	1	2	4
Optimal order quantity	q_p^*	24.7059	*	*
	q_c^*	7.5000	7.5000	7.5000
	q_0^*	18.5185	11.1294	6.1898
Financial status	$B\text{-}cq_p^*$	−34.4118	*	*
	$B\text{-}cq_c^*$	0.0000	0.0000	0.0000
	$B\text{-}cq_0^*$	−22.0370(×)	−7.2588(×)	2.6204
Optimal interest rate	r	0.1621	*	*
Optimal expected utility	U_p^*	**3.8294**	*	*
	U_c^*	2.9906	**2.4865**	1.4781
	U_0^*	4.6296(×)	2.7824(×)	**1.5475**
Decision		Decision 1	Decision 2	Decision 3

When the loss averse coefficient of the retailer is $\lambda = 2$, the optimal ordering capital for retailers under EBP financing decisions is less than the initial capital, and financing is not required. At the same time, the capital required for ordering under the partial self-owned investment decision is higher than the initial capital. Therefore, the optimal decision for retailers at this time is Decision 2, and the optimal expected utility obtained is 2.4865.

When $\lambda = 4$, financing is also not required. The retailer has two decision choices, namely all self-owned investment and partial self-owned investment. The optimal order quantity is 7.500 and 6.1898, and the optimal expected utility is 1.4781 and 1.5475, respectively. Comparing the optimal expected utility in the two decisions, the optimal decision for retailers is Decision 3, and she still has a capital surplus of 2.6204 after placing an order.

It can be seen that loss aversion indeed affects the financing decisions of retailers to a certain extent. High loss averse retailers may be more cautious and conservative when facing financing. They may be concerned about the potential risks and uncertainties brought by financing, so they are more inclined to maintain their own and stable funds. In this case, they may indeed be more inclined to avoid financing.

5.2.5 EBP's optimal expected profit with different loss aversion

The parameter λ exhibited by the LOR will not merely directly affect her optimal decision and profit, but also pass on to the EBP through the supply chain, affecting the profit of the EBP. Fig. 10 shows the optimal expected profit of the EBP for $\lambda = 1$, $\lambda = 2$, $\lambda = 4$.

Insight 8 Under platform financing decisions, the platform usage fees, financing revenue and total expected profit decrease as loss aversion increases.

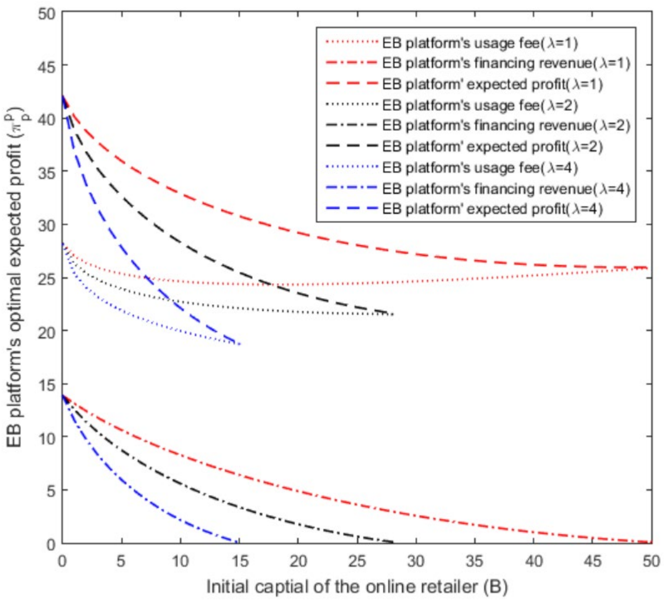

Fig. 10. The platform's optimal expected profit with different loss averse.

As can be seen from Fig. 10, when facing potential losses, retailers with high loss aversion adopt more conservative and prudent orders, resulting in reduced commodity supply and transaction volume on the platform, which will affect the revenue of platform usage fees on EBPs. In addition, retailers with high loss aversion have taken a more cautious attitude towards the financing services provided by EBPs according to the previous analysis, reducing financing demand and choosing a more conservative financing scheme. This will reduce the scale of EBP financing business and affect financing interest income.

Due to the interdependence between retailers and EBPs, retailers' loss aversion will be transmitted to EBPs through the supply chain, and potentially exert systemic impacts on the entire EB industry through the market signal transmission mechanism. Therefore, EBPs need to pay close attention to retailers' loss aversion tendency, and reduce the negative impact of this transmission effect by optimizing supply chain management, strengthening cooperation and communication, providing risk management and market analysis support. At the same time, EBPs should also strengthen their own risk management capabilities, improve the stability and operational effectiveness of the supply chain, and cope with possible market changes caused by retailers' loss aversion.

6. Conclusion

In the present research, we examine a supply chain financing issue involving a capital-constrained LOR that sells products via an EBP. The EBP assumes a leadership role by providing financing to the retailer, setting the financing interest rate. Concurrently, the LOR, acting as the follower, identifies the optimal order quantity in the uncertain demand market. We arrive at the optimal equilibrium financing decisions for the platform. Our analysis reveals that the LOR's optimal order quantity is intricately linked to both their initial capital and level of loss aversion. The optimal order quantity for the LOR is inversely interrelated to their initial capital when financing is obtained from the EBP. A notable finding is that bankruptcy protection incentivizes capital-constrained LORs to implement a proactive ordering approach; consequently, retailers with lower initial capital tend to order larger quantities. Furthermore, the LOR's optimal order quantity is also negatively affected by an increase in loss aversion. As loss aversion intensifies, the LOR's order quantity diminishes, which subsequently lowers their financing requirements. This reduction in financing needs may transition the retailer from a capital-constrained to a capital-sufficient position.

Through a comprehensive numerical study, we have obtained additional observations on the link between the optimal financing decision-making and initial capital. We found that under different decisions, the order quantity of retailers has different trends with the increase of initial capital, especially under the financing model, and the increase of initial capital leads to a decrease in the order quantity of retailers. It is possible for an online retailer to decrease her order quantity while increasing the expected utility. In addition, our study showed that with lower initial capital the retailer can achieve higher expected utility through financing compared to those with sufficient capital under financing models. This again demonstrates the value of platform financing for small and medium-sized retailers. Moreover, online retailers, based on their initial capital, will choose different ordering strategies based on their optimal expected utility. The growth of retailers' initial capital results in a reduction of platform's expected profits . This is because, on the one hand, loss averse retailers adopt more conservative strategies to reduce platform usage rates; Conversely, the growth of retailers' initial capital compels the platform to offer reduced financing rates, thereby resulting in diminished financing returns.

We also discussed the influence of loss aversion on the optimal decision processes of supply chain members through numerical analysis. Retailers with high loss aversion will adopt more conservative ordering strategies, and platforms will offer more favorable financing rates. Retailers with high loss aversion may obtain higher returns through more favorable financing rates supplied by the platform in scenarios where initial capital is insufficient. Moreover, the expected utility of retailers with high loss aversion exhibits a more significant diminishing marginal utility effect on the increase of initial capital. The conclusion is that retailers with high loss aversion tend to use their own capital for ordering rather than making financing decisions. In addition, the loss aversion of retailers is transmitted to the platform through the supply chain, and the greater loss aversion, the smaller expected profit of the platform, whether it is platform usage fees or financing returns.

Our work sheds light for understanding the effect of initial capital and loss aversion on financing and operation under the new platform financing paradigm, and opens the door to an avenue of future research opportunities. First, the financing business of the EBP studied in this paper is limited in that it is independent of its leasing business. Future study may consider the usage fee rate as a decision of the platform as well. Secondly, this paper only considers the impact of loss aversion, whereas in real life operations, other behavioral issues such as anchoring effect, fairness concern, and overconfidence, can be studied. In addition, our current model considers only one financing platform, it is interesting to address the more general setting of multiple competing EBPs.

Declaration of Interest statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Research Data Policy and Data Availability Statements

The datasets generated during and analyzed during the current study are available from the corresponding author on reasonable request.

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Appendix:

A. Proof of Proposition 1

By computing the first derivative of $E[U_0^r(q_0;x)]$ concerning q_0, we obtain

$$\frac{dE[U_0^r(q_0;x)]}{dq_0} = (1\text{-}\theta_f)p\text{-}c(1+r_f)\text{-}[(1\text{-}\theta_f)p]F(q_0)\text{-}[(\lambda\text{-}1)c(1+r_f)]F(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0) = 0.$$

By taking the second derivative of $E[U_0^r(q_0;x)]$ with respect to q_0, we obtain

$$\frac{d^2E[U_0^r(q_0;x)]}{dq_0^2} = (1\text{-}\theta_f)p\{[\frac{c(1+r_f)}{(1\text{-}\theta_f)p}]^2(1\text{-}\lambda)f(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0)\text{-}f(q_0)\}.$$

Considering $0 < \theta_f < 1$, $f(x) > 0$ and $\lambda \geq 1$, we have

$$\frac{d^2E[U_0^r(q_0;x)]}{dq_0^2} < 0,$$

so $E[U_0^r(q_0;x)]$ is concave in q_0. It is clear that there exists a singular optimal ordering quantity that meets the required conditions

$$\overline{F}(q_0^*) = \frac{c(1+r_f)}{(1\text{-}\theta_f)p}[\lambda+(1\text{-}\lambda)\overline{F}(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)].$$

B. Proof of Corollary 1

When $\lambda = 1$, the optimal order quantity q_0^{1*} satisfies

$$\overline{F}(q_0^{1*}) = \frac{c(1+r_f)}{(1\text{-}\theta_f)p},$$

then

$$\overline{F}(q_0^{1*})\text{-}\overline{F}(q_0^*) = \frac{c(1+r_f)}{(1\text{-}\theta_f)p}\text{-}\frac{c(1+r_f)}{(1\text{-}\theta_f)p}[\lambda+(1\text{-}\lambda)\overline{F}(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)] = \frac{c(1+r_f)(1\text{-}\lambda)}{(1\text{-}\theta_f)p}[F(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)].$$

Since $\lambda > 1$, we have $\overline{F}(q_0^{1*}) < \overline{F}(q_0^*)$. So we can obtain $q_0^{1*} > q_0^*$

C. Proof of Corollary 2

By computing the first derivative of q_0^* concerning λ derives

$$\text{-}f(q_0^*)\frac{dq_0^*}{d\lambda} = \frac{c(1+r_f)}{(1\text{-}\theta_f)p}[1\text{-}\overline{F}(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)\text{-}(1\text{-}\lambda)f(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)\frac{c(1+r_f)}{(1\text{-}\theta_f)p}\frac{dq_0^*}{d\lambda}],$$

which can be rewritten as

$$\frac{dq_0^*}{d\lambda} = \frac{\frac{c(1+r_f)}{(1\text{-}\theta_f)p}F(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)}{[\frac{c(1+r_f)}{(1\text{-}\theta_f)p}]^2(1\text{-}\lambda)f(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)\text{-}f(q_0^*)},$$

from the proof of Proposition 1, we have

$$[\frac{c(1+r_f)}{(1\text{-}\theta_f)p}]^2(1\text{-}\lambda)f(\frac{c(1+r_f)}{(1\text{-}\theta_f)p}q_0^*)\text{-}f(q_0^*) < 0.$$

Thus

$$\frac{dq_0^*}{d\lambda} < 0.$$

D. Proof of Proposition 2

By computing the first derivative of $E[U_p^r(q_p;x)]$concerning q_p derives

$$\frac{dE[U_p^r(q_p;x)]}{dq_p} = (1\text{-}\theta_f)p\overline{F}(q_p)\text{-}c(1+r_p)[\lambda\overline{F}(x_1(q_p))+(1\text{-}\lambda)\overline{F}(x_2(q_p))].$$

Taking

$$\frac{dE[U_p^r(q_p;x)]}{dq_p} = 0,$$

we have

$$\overline{F}(q_p)\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\lambda\overline{F}(x_1(q_p))+(1\text{-}\lambda)\overline{F}(x_2(q_p))] = 0.$$

By computing the second derivative of $E[U_p^r(q_p;x)]$ concerning q_p derives

$$\frac{d^2E[U_p^r(q_p;x)]}{dq_p^2} = \text{-}f(q_p)\text{-}[\frac{c(1+r_p)}{(1\text{-}\theta_f)p}]^2[(\lambda\text{-}1)f(x_2(q_p))\text{-}\lambda f(x_1(q_p))] = \text{-}\overline{F}(q_p)[h(q_p)\text{-}$$
$$\frac{c(1+r_p)}{(1\text{-}\theta_f)p}h(x_1(q_p))\text{-}(\lambda\text{-}1)[\frac{c(1+r_p)}{(1\text{-}\theta_f)p}]^2\overline{F}(x_2(q_p))[h(x_2(q_p))\text{-}h(x_1(q_p))] + \frac{c(1+r_p)}{(1\text{-}\theta_f)p}h(x_1(q_p))\{\frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\lambda\overline{F}(x_1(q_p))+(1\text{-}\lambda)\overline{F}(x_2(q_p))]\text{-}$$
$$\overline{F}(q_p)\}.$$

Because of the strictly increasing failure rate (IFR) characteristic, $h(q_p) > h(x_2(q_p)) > h(x_1(q_p))$ is obtained.

Considering

$(1\text{-}\theta_f)p > c(1+r_p)$, $\lambda \geq 1$

and $\overline{F}(x_2(q_p)) > \overline{F}(q_p) > 0$, we obtain

$$\frac{d^2E[U_p^r(q_p;x)]}{dq_p^2} < 0,$$

and $E[U_p^r(q_p;x)]$ is concave in q_p. It is clear that there exists a singular optimal ordering quantity that meets the required conditions

$$\overline{F}(q_p^*)\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\lambda\overline{F}(x_1(q_p^*))+(1\text{-}\lambda)\overline{F}(x_2(q_p^*))] = 0.$$

E. Proof of Corollary 4

When $\lambda = 1$, from Proposition 2, it's easy to get

$$\overline{F}(q_{p1}^*)\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}\overline{F}(x_1(q_p^*)) = 0.$$

We define

$$G(q_p) = \overline{F}(q_p)\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\lambda\overline{F}(x_1(q_p))+(1\text{-}\lambda)\overline{F}(x_2(q_p))],$$

where $G(q_p^*) = 0$. Due to

$$\frac{dG(q_p)}{dq_p} = \text{-}f(q_p)\text{-}[\frac{c(1+r_p)}{(1\text{-}\theta_f)p}]^2[(\lambda\text{-}1)f(x_2(q_p))\text{-}\lambda f(x_1(q_p))].$$

Then

$$\frac{dG(q_p)}{dq_p} < 0.$$

Thus, $G(q_p)$ decreasing in q_p. Considering

$$G(q_{p1}^*) = \text{-}\frac{c(1+r_p)(\lambda\text{-}1)}{(1\text{-}\theta_f)p}[\overline{F}(x_1(q_{p1}^*))\text{-}\overline{F}(x_2(q_{p1}^*))],$$

due to $\lambda > 1$, $\overline{F}(x_1(q_{p1}^*)) > \overline{F}(x_2(q_{p1}^*))$, we obtain

$$G(q_{p1}^*) < 0 = G(q_p^*)$$

thus $q_p^* < q_{p1}^*$.

F. Proof of Corollary 5

(1) By calculating the partial derivative of q_p^* concerning λ, we can obtain

$$\text{-}f(q_p^*)\frac{dq_p^*}{d\lambda}\text{-}\frac{c(1+r_p)}{(1\text{-}\theta_f)p}[\overline{F}(x_1(q_p^*))\text{-}\overline{F}(x_2(q_p^*))\text{-}\lambda f(x_1(q_p^*))\frac{\partial x_1(q_p^*)}{\partial\lambda}\text{-}(1\text{-}\lambda)f(x_2(q_p^*))\frac{\partial x_2(q_p^*)}{\partial\lambda}] = 0$$

where

$$\frac{\partial x_1(q_p^*)}{\partial\lambda} = \frac{\partial x_2(q_p^*)}{\partial\lambda} = \frac{c(1+r_p)}{(1\text{-}\theta_f)p}\frac{dq_p^*}{d\lambda}$$

then we obtain

$$\frac{dq_p^*}{d\lambda} = \frac{\frac{c(1+r_p)}{(1-\theta_f)p}[\overline{F}(x_1(q_p^*))-\overline{F}(x_2(q_p^*))]}{[\frac{c(1+r_p)}{(1-\theta_f)p}]^2[\lambda f(x_1(q_p^*))+(1-\lambda)f(x_2(q_p^*))]-f(q_p^*)}.$$

Due to $\overline{F}(x_1(q_p^*)) > \overline{F}(x_2(q_p^*))$, from the proof of Proposition 2, we obtain

$$[\frac{c(1+r_p)}{(1-\theta_f)p}]^2[\lambda f(x_1(q_p^*))+(1-\lambda)f(x_2(q_p^*))]-f(q_p^*) < 0.$$

Therefore, we have

$$\frac{dq_p^*}{d\lambda} < 0.$$

(2) By calculating the derivative of q_p^* concerning r_p, we can obtain

$$-f(q_p^*)\frac{dq_p^*}{dr_p} = \frac{c}{(1-\theta_f)p}[\lambda\overline{F}(x_1(q_p^*))+(1-\lambda)\overline{F}(x_2(q_p^*))]+\Omega[-\lambda f(x_1(q_p^*))\frac{\partial x_1(q_p^*)}{\partial r_p}-(1-\lambda)f(x_2(q_p^*))\frac{\partial x_2(q_p^*)}{\partial r_p}],$$

where

$$\frac{\partial x_1(q_p^*)}{\partial r_p} = \frac{\partial x_2(q_p^*)}{\partial r_p} = \frac{c(1+r_p)}{(1-\theta_f)p}\frac{dq_p^*}{dr_p}+\frac{(cq_p^*-B)}{(1-\theta_f)p}, \quad \Omega = \frac{c(1+r_p)}{(1-\theta_f)p} < 1,$$

then, we have

$$\frac{dq_p^*}{dr_p} = -\frac{\lambda\overline{F}(x_1(q_p^*))[1-x_1(q_p^*)h(x_1(q_p^*))]-(\lambda-1)\overline{F}(x_2(q_p^*))[1-x_1(q_p^*)h(x_2(q_p^*))]}{(1+r_p)\{\lambda\overline{F}(x_1(q_p^*))[h(q_p^*)-\Omega h(x_1(q_p^*))]-(\lambda-1)\overline{F}(x_2(q_p^*))[h(q_p^*)-\Omega h(x_2(q_p^*))]\}}$$

Owing to the strictly IFR property, we get $h(q_p^*) > h(x_2(q_p^*)) > h(x_1(q_p^*))$, considering $\overline{F}(x_1(q_p^*)) > \overline{F}(x_2(q_p^*))$, thus we can get

$$\frac{dq_p^*}{dr_p} < 0.$$

G. Proof of Proposition 3

Calculating the derivative of $E[\pi_p^p(r_p)]$ concerning r_p gives

$$\frac{dE[\pi_p^p(r_p)]}{dr_p} = [\theta_f p\overline{F}(q_p^*)+c(1+r_p)\overline{F}(x_1(q_p^*))-c(1+r_f)]\frac{dq_p^*}{dr_p}+(cq_p^*-B)\overline{F}(x_1(q_p^*)) = 0.$$

Calculating the second-order derivation of $E[\pi_p^p(r_p)]$ concerning r_p gives

$$\frac{d^2E[\pi_p^p(r_p)]}{dr_p^2} = -\frac{(cq_p^*-B)^2 f(x_1(q_p^*))}{(1-\theta_f)p}-\{\theta_f pf(q_p^*)+\frac{[c(1+r_p)]^2 f(x_1(q_p^*))}{(1-\theta_f)p}\}(\frac{dq_p^*}{dr_p})^2$$
$$+[2c\overline{F}(x_1(q_p^*))-2cx_1(q_p^*)f(x_1(q_p^*))]\frac{dq_p^*}{dr_p}+[\theta_f p\overline{F}(q_p^*)+c(r_p-r_f)-c(1+r_p)F(x_1(q_p^*))]\frac{d^2q_p^*}{dr_p^2}$$

From the proof of Corollary 5, we have

$$\frac{dq_p^*}{dr_p} < 0.$$

Due to

$$cq_p^*-B > (1-\theta_f)p$$

and according to the derivation of $E[\pi_p^p(r_p)]$ concerning r_p,

$$\theta_f p\overline{F}(q_p^*)+c(r_p\text{-}r_f)\text{-}c(1+r_p)F(x_1(q_p^*)) = \theta_f p\overline{F}(q_p^*)+c(1+r_p)\overline{F}(x_1(q_p^*))\text{-}c(1+r_f) > 0,$$

when

$$\frac{d^2 q_p^*}{dr_p^2} < 0,$$

we can get

$$\frac{d^2 E[\pi_p^p(r_p)]}{dr_p^2} < 0.$$

Then, the optimal ordering quantity fulfills the conditions:

$$[\theta_f p\overline{F}(q_p^*) + c(1+r_p^*)\overline{F}(x_1(q_p^*))\text{-}c(1+r_f)]\frac{dq_p^*}{dr_p} + (cq_p^*\text{-}B)\overline{F}(x_1(q_p^*)) = 0.$$