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Manufacturer encroachment in refurbished product supply chains under enhanced service levels: A game-theoretic analysis of partial and full encroachment

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C H R O N I C L E

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A B S T R A C T

We examine a two-tier supply chain of manufacturers and retailers distributing new and refurbished products through conventional channels. Manufacturers encroach on the refurbished product market by establishing new direct sales channels and enhancing service levels to maximize profits. Employing a game theory model, we investigate the optimal equilibrium scenarios, including the basic, partial, and full encroachment scenarios. We analyze the impact of manufacturer encroachment on sales quantity and profits for both manufacturers and retailers. Our findings reveal that partial encroachment has a lower threshold than full encroachment, and manufacturer encroachment consistently expands the market for refurbished products. Under certain conditions, manufacturer encroachment boosts their profits and benefits retailers, thereby alleviating channel conflicts. Surprisingly, retailers help manufacturers swiftly achieve the optimal range for direct selling costs. Once this range is attained, retailers and manufacturers actively compete in the refurbished product market, maintaining their optimal profit levels.

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1. Introduction

With the advancement of the economy and the expansion of production capacity, there has been a growing influx of refurbished products into the market. Exploring efficient methods to sell refurbished products to consumers, expand the refurbished product market, and enhance manufacturers' profitability is an intriguing topic deserving of study. Manufacturer encroachment refers to a phenomenon where the supply chain is established on the foundation of the existing retail channel, and the manufacturer initiates new channels to directly sell products to end consumers without involving retailers.

Although the phenomenon of manufacturer appropriation is becoming more and more sophisticated, appropriation has also led to more intense tensions between manufacturers and retailers and to various conflicts. Research on how to mitigate these conflicts so that both parties can benefit from manufacturer capture and expand their markets is essential. New direct marketing channels often offer different products or services to the end market. Conversely, Yoon (2016) highlighted that attempting to penetrate the market with identical products or services often triggers significant channel conflicts and meets resistance from retailers. This is because manufacturers' encroachment on similar products or services tends to favor manufacturers at the expense of retailers.

Differing from encroaching on identical products or developing new ones, we have creatively introduced the feature of offering different service levels in refurbished products. In this paper, the manufacturer commits encroachment on the refurbished product by increasing its service level. Consumer willingness to pay is expected to increase by elevating the service quality. Can such a strategy of manufacturer encroachment alleviate inherent channel conflicts? In the direct sales

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channel of refurbished products, is it possible to expand the market share of refurbished products by improving the level of service, thus expanding the industry and improving the development of green supply chains? Our literature review has not uncovered any studies addressing the encroachment of service levels, specifically on refurbished products. We establish a two-tier supply chain comprising manufacturers and retailers to bridge this research gap. Manufacturers supply both new and refurbished products to retailers at varying wholesale prices. In encroachment cases, manufacturers establish new direct sales channels to vend refurbished products with upgraded service levels. Through the analysis of this game theory model, we aim to address the following questions:

1. What are the conditions for the manufacturer encroaching on refurbished products?
2. Is the outcome of encroachment mutually beneficial, and do both parties actively engage in the encroachment process?
3. What recommendations can be provided to both parties and industry managers to expand the market for refurbished products?

Based on our research findings, we observed that when manufacturers offer a higher service level, the encroachment threshold can be effectively reduced. Additionally, investing in direct sales efficiency enables retailers to edge out refurbished products potentially. However, retailers are unlikely to permit manufacturers to attain such a scenario. The situation becomes even more challenging in cases of full encroachment. Secondly, when encroachment is unavoidable, retailers should assist manufacturers in swiftly achieving optimal direct sales efficiency, as both parties stand to benefit. Thirdly, contrary to expectations, manufacturers may not always prioritize enhancing direct sales efficiency when encroaching on refurbished products, as doing so could potentially undermine their interests.

The remainder of this paper is structured as follows: In the second section, we review and synthesize pertinent literature. Section 3 outlines establishing a game theory model depicting manufacturers' encroachment on refurbished products under improved service levels. In section 4, we present the conditions conducive to encroachment and explore the equilibrium outcomes of such encroachment. Section 5 entails an analysis of the sensitivity of key parameters in the model and discussing pertinent recommendations. Finally, in the sixth section, we conclude the thesis by summarizing key findings and offering management suggestions to manufacturers, retailers, and industry managers.

2. Literature review

The literature pertinent to our research can be categorized into three main areas: manufacturers' encroachment on downstream markets, sales of refurbished products, and service levels in product supply. Numerous studies have examined manufacturers encroaching on downstream markets, encompassing various scenarios such as encroachment on the same product, encroachment on different products, and more. Li et al. (2015) examined two cases of non-identical supply chains and subsequently analyzed several identical ones. Their findings indicated that manufacturer encroachment could result in a "lose-lose" outcome for both suppliers and retailers. In contrast, we focus on when manufacturers encroach when encroachment is inevitable. What encroachment efforts should manufacturers and retailers make to achieve a win-win situation? Kittaka and Pan (2023) demonstrated that retailers opt to introduce new brands to mitigate risks and establish advantageous positions within the competitive downstream environment in response to manufacturer encroachment. Our research suggests that when manufacturers encroach, retailers are likely to work together with manufacturers to improve the efficiency of their direct marketing and create conditions favorable to them rather than choosing other avenues that intensify the conflict. The research conducted by Li et al. (2023) reveals that in instances of manufacturer encroachment, entities possessing an information advantage have the potential to optimize their profits. Xiao et al. (2023) investigated the interplay between manufacturers' encroachment strategies and retailers' information-sharing strategies. They discovered that retailers can employ information-sharing strategies to influence manufacturers' encroachment intentions. Our study points out that when a manufacturer's intent to encroach is established, the retailer influences the manufacturer's direct marketing efficiency and thus improves its own profitability by handling the direct marketing channel differently (assisting or confronting). Shi et al. (2023) demonstrate that opting for a decentralized structure may not always result in higher manufacturer profits. Conversely, selecting an integrated structure can lead to win-win outcomes for manufacturers, retailers, and the e-commerce sector, particularly when the cost of direct selling is sufficiently low. In contrast, we consider different levels of service in different direct marketing efficiencies may result in a win-win situation for manufacturers and retailers as well as for the environment. Zhang et al. (2020) investigate manufacturers' intrusion into the supply chain concerning whether manufacturers and retailers should invest in informational advertising. They demonstrate that while persuasive advertising can influence price or advertising decisions differently, retailers can always benefit from manufacturers' encroachment. We point out that at the time of manufacturer encroachment, retailers sometimes have the same goal as the manufacturer to jointly reduce the manufacturer's direct selling costs, which can benefit both parties. Huang and Chen (2023) investigate the strategic interaction between manufacturer intrusion, platform dual-purpose concerns, and retail competition. They analyze how various manufacturers' encroachment decisions affect consumer surplus. Tong et al. (2023) demonstrate that supply chain members are more inclined to establish information-sharing mechanisms when upstream manufacturers possess more information. They further illustrate that manufacturer encroachment can result in lose-lose outcomes. Unlike their studies, we focus on the profitability of manufacturers and retailers, focusing on the game process between the two parties, where retailers are more willing to assist manufacturers to familiarize themselves with the end market as soon as possible when the manufacturer's encroachment occurs, thus achieving

a win-win situation. Balasubramanian and Ponnachiyur Maruthasalam (2021) investigate manufacturer encroachment in supply chains where there is partial substitution between the retailer's store brand and the manufacturer's country brand. They demonstrate that the coexistence of manufacturer encroachment and store brand can lead to a three-win situation. Zhang and Zhang (2022) discovered that manufacturers' encroachment negatively impacted capital-constrained retailers, prompting them to seek external financing to mitigate the adverse effects. Contrary to their findings, manufacturer encroachment is not always detrimental to the retailer and sometimes increases the retailer's profitability when the retailer is more inclined to assist the manufacturer's direct sales channel. Wan et al. (2023b) research indicates whether a manufacturer encroaches depends on consumers' perception of the direct marketing channel and the associated costs. Indeed, we believe that direct marketing costs cannot be ignored as a factor in whether a manufacturer encroaches. We also show that unavoidable manufacturer encroachment causes retailers to participate actively in the encroachment process. The analysis results by Tang et al. (2022) demonstrate that in scenarios where there is no infringement, original equipment manufacturers consistently offer equity financing. However, encroachment disrupts this pattern, with financing contingent upon the original equipment manufacturer's significant shareholding and the initial capital of CM (Contract Manufacturer). Guan et al. (2023) demonstrate that direct sales channels remain consistently valuable for manufacturers. However, retailer preferences are contingent upon the information structure. Supply competition has the potential to fully convert manufacturers' disincentives into incentives for accessing information, thereby facilitating information sharing with retailers. Our research suggests that opening direct sales channels may also hurt manufacturers' profits and that manufacturers will choose to encroach only if direct sales channels are sufficiently efficient. Zhang et al. (2021) analysis suggests that manufacturer encroachment might not deter retailers from introducing competitive brands, whereas the introduction of store brands by retailers could deter manufacturer encroachment. Zheng and Yu (2021) indicate that in scenarios characterized by intense channel competition and moderate manufacturer encroachment costs, equivalent price promises enhance manufacturers' motivation to intrude. However, this action negatively impacts retailers while simultaneously augmenting consumer surplus. In contrast, to differentiate our products in the direct channel from those in the traditional channel, we offer different product services and sell them at different prices in the direct channel, which helps mitigate intense channel conflicts. In their study, Li et al. (2021) investigated the decision-making process of manufacturers regarding intrusion in a dual-channel supply chain offering alternative green products. They discovered that heightened consumer awareness about environmental sustainability favors manufacturer encroachment. However, they also noted that encroachment benefits retailers only when the associated costs are high. Overall, the intrusion was found to enhance social welfare and environmental performance. Zhao and Hou (2024) found that encroachment benefits retailers only when the manufacturer's encroachment costs are high. However, our research indicates that even when the encroachment costs are low, a win-win situation can still be achieved. In previous studies, manufacturer encroachment is always through the same product or the development of a new product, and the indirect reaction of retailers facing manufacturer encroachment is given, as well as the change in profits of each party brought by the encroachment. Instead, we creatively introduce different service levels when selling products, which mitigates the channel conflict caused by encroachment of the same product and saves the manufacturer the development cost caused by developing new products. We achieved a win-win situation after improving the service level for encroachment on refurbished products. We examine manufacturer encroachment on refurbished products and report on the conditions for expanding the market for refurbished products, which can help alleviate growing environmental problems. Our research incorporates the service level of refurbished products. We demonstrate that retailers can aid manufacturers in swiftly reducing the direct selling cost to an appropriate range, expanding the market for refurbished products, and enhancing environmental performance.

Our research is related to the sale of refurbished products. Esmaeilian et al. (2021) demonstrate that the sale of refurbished products is viewed as a sustainable business practice to enhance market image and strategy for companies. Because of this, our study was conducted through the encroachment of refurbished products by manufacturers, which, on the one hand, can increase the profitability of manufacturers and, on the other hand, can expand the market share of refurbished products and thus achieve the purpose of improving the environment as well as improving the image of the company. Chen et al. (2022) investigated the circumstances influencing retailers' willingness to offer refurbished products and the effects of such products on manufacturers' profits, encompassing local market sales, cross-market sales, and dual market sales. They analyzed the sales of refurbished products in different markets from the perspective of the profitability of refurbished products. Our study expands the market for refurbished products through the encroachment of direct sales channels implemented by manufacturers for refurbished products. Agostini et al. (2021) discovered that consumers' attention to seller reputation and distribution significantly correlates with perceived value and risk, subsequently influencing attitude and the intention to purchase refurbished products. Zhang et al. (2019) demonstrate that refurbished products can only gain traction when consumers opt for them over new products, particularly if the cost of refurbished items is sufficiently low. Unlike them, when considering consumers' choices, we are more concerned with studying the market game of new and refurbished products and expanding the refurbished product market by opening direct sales channels. Liu et al. (2018) research indicates that when consumer preference for refurbished products is low, OEMs should opt for licensing strategies, which can lead to a mutually beneficial outcome. Zhou et al. (2020) demonstrate that the secondary market benefits the original equipment manufacturer (OEM) only when licensing refurbishment proves to be sufficiently profitable. However, they also found that higher licensing fees do not always translate into benefits for the OEM. We show that the encroachment of manufacturers into the refurbished products market alongside sales of refurbished products in traditional channels helps expand the market size. Harms and Linton (2016) suggest that eco-certification is frequently advantageous as it strengthens the business case for manufacturing products using refurbished components. Niu and Xie (2020) discovered that the alignment of incentives between the brand-owner and the

remanufacturer occurs when competition is fierce, and the remanufacturer faces a substantial quality image disadvantage. Zheng et al. (2023) conducted empirical and experimental studies, consistently revealing tharefurbished products negatively impact the market value and purchase intent of corresponding brand-new products utilizing established technology standards. Conversely, introducing refurbished products does not exhibit such a detrimental effect on products employing unestablished standards. Additionally, they demonstrated that, for a particular consumer group, this green product strategy enhances their purchase intent for brand-new products. Chen and Chen (2019) discovered that the entry threshold for refurbished enterprises is consistently lower than that for remanufacturing enterprises. Additionally, original equipment manufacturers do not promote recycling or refurbishing, fearing it may negatively impact their interests. They also provided recommendations for government incentives to support refurbished enterprises. Because of this, refurbished products are used as encroachment targets in our study and provide management insights for government and industry regulators. Alyahya et al. (2023) formulated a comprehensive conceptual framework elucidating the behavioral conditions prompting consumers to buy refurbished products. They illustrated that ethical constructs, threats, coping assessment variables, and values are the primary determinants influencing consumers purchasing refurbished products. Duan and Aloysius (2019) examine the impact of sustainable supply chain transparency and information characteristics on stakeholder evaluation, addressing environmental gaps in refurbished products. These two papers examine the purchase of refurbished products from the consumer's point of view, showing the consumer factors that expand the market for refurbished products. In contrast, we give ways to expand the market for refurbished products in terms of manufacturer encroachment. Kurdhi et al. (2023) illuminate the adjustments that both competitors need to make to their refurbished product prices when there are changes in internal and cross-cannibalization coefficients. It suggests weaker firms should prioritize efforts to excel their stronger competitors in collection and refurbishing endeavors. He et al. (2020) indicate that implementing a store return policy leads to a decrease in the price of refurbished products and increased sales through online channels. Conversely, the impact is the opposite for offline channels. They suggest a return policy can create a win-win situation for offline channels. They studied the sales of refurbished products online and offline by adjusting their return policy, unlike us who introduced the service level factor to encroachment on the refurbished products market and increased the sales channels for refurbished products. Bhattacharya et al. (2018) examined the impact of government subsidies on various refurbishment models, considering different manufacturing costs, customer purchase intentions, and refurbished product quality. Liu et al. (2023) examined the optimal candidate for handling refurbished products. The findings indicate that OEMs are generally open to collaboration, although successful cooperation is not guaranteed. Refurbished products typically carry a higher environmental footprint than remanufactured products. Timoumi et al. (2021) demonstrate that in scenarios where refurbishment is practical or when refurbished products are highly appealing to consumers, retailer-led refurbishment surpasses manufacturer-led refurbishment. This holds even if manufacturers sacrifice some profit and face heightened competition. These two papers examine who would make the optimal profit from refurbished products, and we consider how to expand market share while encroaching on refurbished products in a way that protects the environment. Niu et al. (2023) indicate that retailers engaged in refurbished product sales tend to share demand information with competitors selling new products rather than manufacturers. This strategic behavior allows retailers to expand their interests. Additionally, enhancements in the quality of refurbished products are found to have adverse effects on retailers' interests. In contrast, we assume that refurbished products are of the same quality as new products but with lower consumer recognition, expanding the market share of refurbished products through manufacturer encroachment. The research conducted by Feng et al. (2021) demonstrates that as the quality of refurbished products improves, there is a reduction in the production of new and remanufactured products. Consequently, this increase in the quality of refurbished products leads to higher sales. Yoo and Kim (2016) demonstrate that maximizing first-market sales occurs when manufacturers directly sell new products, while maximizing second-market sales happens when sellers offer new and refurbished products. Furthermore, integrating supply chains can enhance overall profits. Much of the previous literature has considered expanding the market for refurbished products to protect the environment from a market or consumer perspective. In contrast, it is more common to discuss who should do the refurbishing from a market perspective. The difference between our research and the previous ones is that our research shows that manufacturers have encroached on refurbished products through direct sales channels in addition to the original sales channels, expanding their profits and increasing the number of refurbished products sold.

Our research is related to the study of service level. There is a lot of research on providing high service to consumers, and providing service to consumers brings higher emotional value, which increases their willingness to pay. Wang et al. (2015) explore the research direction of service supply chains from an operational perspective. Durgee and Agopian (2018) emphasize that services offer consumers the dual benefits of pleasure and happiness. Supply chain members can also capitalize on consumers' reliance on services to increase their power in the supply chain game. Wan et al. (2023a) demonstrate that when consumers are highly sensitive to in-store services, retailers can strategically employ in-store services as a deterrent against manufacturer encroachment. Heirati et al. (2023) show that increased competitive intensity and environmental turbulence encountered by professional service firms reduce the ability of clients and providers to collaborate to drive service performance. Salam et al. (2016) uncover the influence of inventory levels on service levels and underscore the significance of accurate and reliable information in the service supply chain. Ding and Keh (2023) argue that improved service levels increase market demand and market position, demonstrating that return behavior can stimulate manufacturers to improve service levels and increase the overall profitability of the supply chain. Chen et al. (2020) demonstrate that retailers equipped with put options tend to offer a higher standard of service and achieve superior profitability compared to those without such options. Wu et al. (2020) examined optimal recycling channel strategies, highlighting that enhancing service levels and reducing transfer prices would maximize profits. He et al. (2018) conducted research indicating that when service levels

surpass a certain threshold, suppliers' profits tend to decrease while manufacturers' profits continue to rise. Wholesale prices and production costs solely determine the magnitude of this threshold. G. Zhang et al. (2020) found that the service level in various channels correlates with the sales volume in those channels. Dan et al. (2022) suggest that value-added services will be undermined when manufacturers increase their service levels and also that the incentive for manufacturers to provide service levels increases when their bargaining power is stronger. Liu et al. (2017) analyzed the impact of service levels on optimal control power distribution and demonstrated that service levels can regulate the relationship between supply chain dominance structure and prices. Bian et al. (2023) discovered that decentralized channel service outsourcing, compared to integrated channels, results in either lower retail prices or higher service levels, but not both simultaneously. Additionally, efficient service investment by retailers can help manufacturers achieve better profits from decentralized channels. The findings of Guan et al. (2020) demonstrate that when retailers share information, manufacturers can effectively adjust wholesale prices and service levels, thus benefiting all members of the supply chain. Information sharing is more likely to occur when service investment is more efficient, and consumers show greater interest in the service. Giri and Sarker (2016) suggest that wholesale price discount schemes can alleviate service levels and price competition among retailers. Previous articles have examined more factors affecting the level of service and looked at the factors affecting the magnitude of a retailer's or manufacturer's level of service from different perspectives. It is shown that many aspects affect the level of service. To the best of our knowledge, existing literature has not explored the impact of service level enhancements on the performance of supply chain members in the context of manufacturers' intrusion. Using service level as a research condition, we explored the impact on the profitability of supply chain members under different service levels. Our study addresses this gap by investigating how varying service levels assigned to refurbished products across different channels affect competition between manufacturers and retailers. Through this analysis, we enhance our understanding of the dynamics of the refurbished product market.

In summary, this paper investigates manufacturers' encroachment on refurbished products. Unlike previous manufacturer encroachment, this paper distinguishes products in traditional channels by improving the service level of encroached products. In the encroachment of refurbished products, the market for refurbished products can be expanded under certain conditions by opening new direct sales channels to achieve further environmental protection. Through relevant studies in the literature in the field of service level, this paper investigates the changes in the profitability of each party and the number of sales of refurbished products after manufacturer encroachment under different service levels, conditional on differentiated service levels. Table 1 makes a summary of the related literature.

Table 1
Summary of partial literature.

	manufacturers encroachment	refurbished product	service level
Kittaka and Pan (2023)	√	×	×
Li et al. (2023)	√	×	×
Xiao et al. (2023)	√	×	×
Li et al. (2021)	√	√	×
Esmaeilian et al. (2021)	×	√	×
Chen et al. (2022)	×	√	×
Zheng et al. (2023)	×	√	×
Wan et al. (2023a)	√	×	√
Guan et al. (2020)	×	×	√
This paper	√	√	√

3. The model

3.1 Problem description, notation, and assumptions

We examine a two-tier supply chain comprising manufacturers and retailers, with each entity striving to maximize its profits through rational decision-making. This forms the foundation of our game theory model. Manufacturers typically distribute new and refurbished products to retailers at varying wholesale prices. To simplify the model, we stipulate that the refurbished product is the same as the new product in terms of quality, but consumers generally believe that the new product is better than the refurbished product, i.e., ceteris paribus, consumers are willing to pay a higher price for the new product. To enhance their profitability, manufacturers must decide whether to establish new direct sales channels for refurbished products and offer an elevated level of service, thus distinguishing themselves from traditional channels. This phenomenon is commonplace in real-world markets. For instance, Apple's official website, functioning as a direct sales channel, offers consumers services like customized engraving, which are unavailable through traditional channels. However, to maintain equilibrium between the two sales channels, the discount intensity on the direct sales channel is lower than that of the traditional channel. The price of similar products in direct channels is higher than in traditional channels. Suppose the price of similar products in the direct channel is the same as in the traditional channel. In that case, this will lead to a greater flow of consumers to the direct channel, which offers a higher level of service, deepening the channel conflict. Consequently, products offering enhanced services are priced higher in the direct sales channel than the traditional one.

Here are our main assumptions.

High service level. When consumers purchase a product, they acquire the physical item and the associated services. A higher level of service enhances consumer willingness to pay. Considering that the refurbished products sold in the direct channel are consistent with those in the traditional channel, for the direct channel resulting from the encroachment of manufacturers on the refurbished products, consumers have a higher willingness to pay for the refurbished products in the direct channel compared to the refurbished products in the traditional channel because of the higher level of service. Because of the higher level of service in the direct channel, consumers need to accept higher prices for refurbished products in the direct channel. For instance, consumers often prefer browsing through various bakery sections to select items that cater to their specific preferences, satisfying their individualized needs and psychological desires, ultimately boosting consumption and expanding the market size. We use the parameter "g" to denote the extent of additional services manufacturers provide to consumers through the direct sales channel. According to the basic principle of microeconomics, the influence of sales quantity on price is the main influence, and others are secondary influences. Therefore, service levels exert a lesser impact on prices compared to actual sales. Hence, it is reasonable to set $0 < g < 1$.

Direct selling cost. We assume the traditional channel is firmly established and has a retail advantage over the product. Thus, we standardize its cost per unit sold to zero. Therefore, the unit cost incurred by the manufacturer in providing a high level of service through the direct sales channel is denoted as "c" (where $c \geq 0$). Additionally, following the model proposed by Zhang et al. (2019), we normalize the production cost of new and refurbished products to zero.

3.2 Model settings

This section outlines the model setup for the three scenarios proposed in this paper. In the basic scenario, manufacturers distribute new and refurbished products to retailers at varying wholesale prices. Retailers then determine the quantity of each product based on prevailing market conditions. In the partial encroachment scenario, like the basic scenario, manufacturers wholesale new and refurbished products to retailers at different prices. Refurbished products are also retailed through direct sales channels, offering a higher level of service denoted as "g". In the full encroachment scenario, manufacturers supply new products to retailers while exclusively retailing refurbished products through direct channels and providing a higher service level "g". The setup of these scenarios is illustrated in Fig. 1.

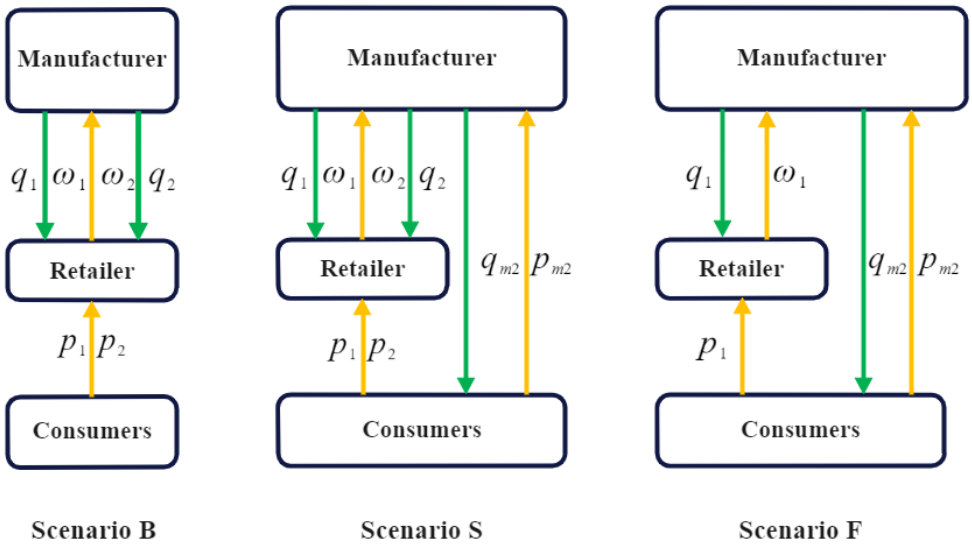


Fig 1. Model Settings

According to the principles of microeconomics, we present the inverse demand function and its corresponding profit function for each of the three models as follows.

Basic scenario (Scenario B)

$$p_{r1}^b = (1+\theta)a - q_{r1}^b - \beta q_{r2}^b \quad (1)$$

$$p_{r2}^b = a - \beta q_{r1}^b - q_{r2}^b \quad (2)$$

$$\pi_m^b = \omega_1^b q_{r1}^b + \omega_2^b q_{r2}^b \quad (3)$$

$$\pi_r^b = (p_{r1}^b - \omega_1^b) q_{r1}^b + (p_{r2}^b - \omega_2^b) q_{r2}^b \quad (4)$$

Partial encroachment scenario (Scenario S)

$$p_{r1}^s = (1+\theta)a - q_{r1}^s - \beta(q_{r2}^s + q_{m2}^s) \quad (5)$$

$$p_{r2}^{s} = a - \beta q_{r1}^{s} - q_{r2}^{s} - q_{m2}^{s} \quad (6)$$
$$p_{m2}^{s} = a - \beta q_{r1}^{s} - q_{r2}^{s} - q_{m2}^{s} + g \quad (7)$$
$$\pi_{m}^{s} = \omega_{1}^{s} q_{r1}^{s} + \omega_{2}^{s} q_{r2}^{s} + (p_{m2}^{s} - c) q_{m2}^{s} \quad (8)$$
$$\pi_{r}^{s} = (p_{r1}^{s} - \omega_{1}^{s}) q_{r1}^{s} + (p_{r2}^{s} - \omega_{2}^{s}) q_{r2}^{s} \quad (9)$$

Full encroachment scenario (Scenario F)

$$p_{r1}^{f} = (1+\theta)a - q_{r1}^{f} - \beta q_{m2}^{f} \quad (10)$$
$$p_{m2}^{f} = a - \beta q_{r1}^{f} - q_{m2}^{f} \quad (11)$$
$$\pi_{m}^{f} = \omega_{1}^{f} q_{r1}^{s} + (p_{m2}^{f} - c) q_{m2}^{f} \quad (12)$$
$$\pi_{r}^{f} = (p_{r1}^{f} - \omega_{1}^{f}) q_{r1}^{s} \quad (13)$$

where a (a > c) is positive and represents the predetermined market price of similar products (Yi et al., 2017). “g” represents the level of service. Since its effect on price is smaller than that of the quantity sold, we assume that $g \in (0,1)$ describes the level of service. In this paper's research context, we can interpret consumers' heightened willingness to pay as their desire to access these services. Alternatively, it can be viewed as expanding the market size without increasing prices. $\beta \in (0,1)$ represents the degree of interchangeability between the new product and the refurbished product, signifying varying consumer willingness to pay for each of the two products. For clarity and convenience, we employ the superscripts “b”, “s”, and “f” to denote the basic scenario, partial encroachment scenario, and full encroachment scenario, respectively. Subscripts "m" and “r” represent manufacturers and retailers, respectively, while subscripts “1” and “2” signify new and refurbished products, respectively. Table 2 outlines the decision variables and model parameters.

Table 2
Decision variables and model parameters.

Notations	Definitions
a	The market potential of refurbished product
$(1+\theta)a$	The market potential of the new product, where θ represents the popularity advantage of the new product over the refurbished product
β	The degree of substitutability between new and refurbished products
g	Service level
c	The unit sales cost of the direct sales channel
b, s, f	The superscripts b, s, and f represent scenario B, scenario S, and scenario F, respectively
$\omega_1^i (i = b, s, f)$	The wholesale price of new products under different scenarios
$\omega_2^i (i = b, s)$	The wholesale price of refurbished products under different scenarios
$q_{r1}^i (i = b, s, f)$	The sales quantity of the new product in the incumbent retail channel under different scenarios
$q_{r2}^i (i = b, s)$	The sales quantity of the refurbished product in the incumbent retail channel under different scenarios
$q_{m2}^i (i = s, f)$	The sales quantity of the refurbished products in the direct sales channel
$p_{r1}^i (i = b, s, f)$	The sales price of the new product in the incumbent retail channel under different scenarios
$p_{r2}^i (i = b, s)$	The sales price of the refurbished product in the incumbent retail channel under different scenarios
$p_{m2}^i (i = s, f)$	The sales price of the refurbished products in the direct sales channel
$\pi_m^i (i = b, s, f)$	Manufacturer's profit in scenario i
$\pi_r^i (i = b, s, f)$	Retailers' profits in scenario i

3.3 Decision Sequence

This paper adopts the Stackelberg model. In this model, the manufacturer acts as the leader and first sets the product's wholesale price. Subsequently, the retailer makes decisions regarding sales volume based on this wholesale price. Then, using the above variables, the manufacturer, independently or collaboratively with the retailer, determines the price of the refurbished product. The specific decision sequence is illustrated in Fig. 2.

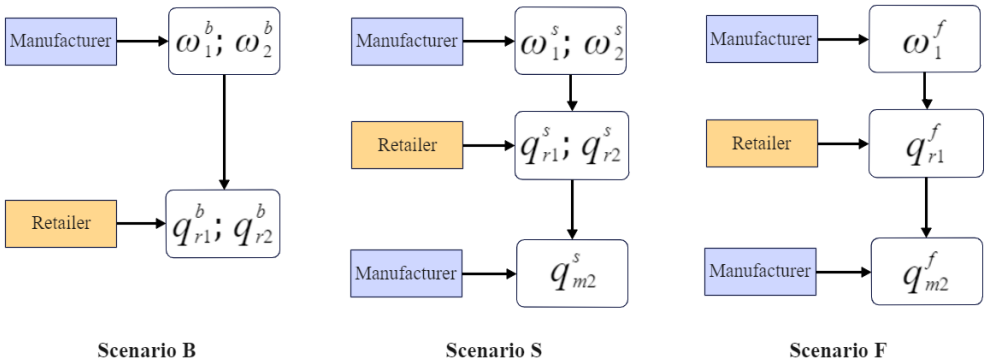


Fig 2. Decision Sequence

Remark: In scenario S, we can obtain a special case that is $q_{r2}^{s} \leq 0$. Considering the real-world context, we can infer that the retailer sells no refurbished products at this juncture, effectively being ousted from the refurbished product market entirely. This scenario aligns with the full encroachment scenario outlined in this paper, yielding identical outcomes. This observation highlights the broader applicability of our proposed model as a comprehensive examination of manufacturer encroachment within the refurbished product market.

4. Equilibrium analysis

In the preceding section, we delineated the decision sequence. Building upon this, we conduct an equilibrium analysis of the three models. The equilibrium outcomes are presented in Table 3. For conciseness, detailed calculations are provided in Appendix A.

Table 3
Optimal equilibrium outcomes

	Scenario B	Scenario S	Scenario F
ω_1^i	$\frac{1}{2}(a+a\theta)$	$\frac{1}{6}\begin{pmatrix}3a-c\beta+\\ g\beta+3a\theta\end{pmatrix}$	$\frac{(c-g)\beta^3-a\begin{bmatrix}\beta^3+8(1+\theta)\\ -6\beta^2(1+\theta)\end{bmatrix}}{2(-8+5\beta^2)}$
ω_2^i	$\frac{a}{2}$	$\frac{1}{6}(3a-c+g)$	
q_{r1}^i	$\frac{a(1-\beta+\theta)}{4-4\beta^2}$	$\frac{a(1-\beta+\theta)}{4-4\beta^2}$	$\frac{2\begin{bmatrix}(c-g)\beta+\\ a(1-\beta+\theta)\end{bmatrix}}{8-5\beta^2}$
q_{r2}^i	$\frac{a(1-\beta-\beta\theta)}{4(1-\beta^2)}$	$\frac{1}{12}\begin{bmatrix}8c-8g+\\ \frac{3a\beta(1-\beta+\theta)}{-1+\beta^2}\end{bmatrix}$	
q_{m2}^i		$\frac{1}{6}(3a-5c+5g)$	$\frac{-(c-g)(-8+3\beta^2)}{+a[-8+\beta(2+3\beta+2\theta)]}{2(-8+5\beta^2)}$
p_{r1}^i	$\frac{3}{4}a(1+\theta)$	$\frac{1}{12}\begin{bmatrix}2(c-g)\beta+\\ a(9-3\beta+9\theta)\end{bmatrix}$	$\frac{(c-g)\beta(-4+3\beta^2)-12a(1+\theta)}{+a\beta[4+\beta(8-3\beta+8\theta)]}{2(-8+5\beta^2)}$
p_{r2}^i	$\frac{3a}{4}$	$\frac{1}{6}(3a+c-g)$	
p_{m2}^i		$\frac{1}{6}(3a+c+5g)$	$\frac{-8(c+g)+(7c+3g)\beta^2}{+a[-8+\beta(2+3\beta+2\theta)]}{2(-8+5\beta^2)}$
π_m^i	$\frac{a^2\begin{bmatrix}2-2\beta(1+\theta)\\ +\theta(2+\theta)\end{bmatrix}}{8(1-\beta^2)}$	$\frac{7}{12}(c-g)^2+\frac{1}{2}a(-c+g)$ $+\frac{a^2\begin{bmatrix}-3+\beta^2+2\beta\\ (1+\theta)-\theta(2+\theta)\end{bmatrix}}{8(-1+\beta^2)}$	$\frac{(c-g)^2(-8+\beta^2)-}{2a(c-g)[-8+\beta(4+\beta+4\theta)]}{+a^2\{\beta^2+8\beta(1+\theta)-4[3+\theta(2+\theta)]\}}{4(-8+5\beta^2)}$
π_r^i	$\frac{a^2\begin{bmatrix}2-2\beta(1+\theta)\\ +\theta(2+\theta)\end{bmatrix}}{16(1-\beta^2)}$	$\frac{2}{9}(c-g)^2-\frac{a^2(1-\beta+\theta)^2}{16(-1+\beta^2)}$	$\frac{2(2-\beta^2)\begin{bmatrix}(c-g)\beta+\\ a(1-\beta+\theta)\end{bmatrix}^2}{(8-5\beta^2)^2}$

4.1 The manufacturer encroaches upon the condition of refurbished products

When manufacturers encroach on refurbished products, there are various factors to consider. While encroachment may potentially trigger channel conflicts, it can also mitigate the issue of double marginalization. Moreover, introducing direct sales channels enhances manufacturers' involvement, leading to increased sales costs for manufacturers and associated risks. However, the primary objective of encroachment remains to generate higher profits for manufacturers. Building upon the above analysis, we establish the conditions under which manufacturers usurp refurbished products in Proposition 1, with detailed proof provided in Appendix B.

Proposition 1. The manufacturer encroaches on the refurbished products (i.e., $q_{m2}^{s} \geq 0$) if and only if $c < c_1 = g + \frac{3a}{5}$.

In order to more intuitively feel the encroachment conditions of the manufacturer, we draw the region as shown in Figure 3. We set a=1, which is consistent with the relevant literature on encroachment by the manufacturer, such as Yang et al. (2023), $g \in (0,1)$.

To recap the previous setup, "c" represents the manufacturer's direct cost of goods sold, while "g" represents its service level. Proposition 1 indicates that a manufacturer will enter the refurbished product market only if its direct sales channel operates efficiently (i.e., the cost of sales "c" is sufficiently low). Furthermore, as the service level improves or the cost of direct sales decreases, the threshold for manufacturers to encroach on refurbished products gradually decreases.

As costs continue to decline, retailers will be compelled to exit the refurbished product market. In other words, if the manufacturer fully encroaches on the refurbished product market, the retailer's inverse demand function for selling refurbished

products becomes zero or negative. To demonstrate our conclusion, we set $q_{r2}^{s} \leq 0$. And get the formula, $c \leq c_2 = g - \frac{3a\beta(1-\beta+\theta)}{8(\beta^2-1)}$.

Corollary 1. Retailers' sales of refurbished products will be driven off the market entirely (i.e., $q_{r2}^{s} \leq 0$), if $c \leq c_2 = g - \frac{3a\beta(1-\beta+\theta)}{8(\beta^2-1)}$. At this point, the manufacturer has fully encroached on the refurbished product.

Here, it's essential to clarify that θ should not be excessively large or zero. A zero value would imply that consumers perceive new and refurbished products as identical, which contradicts reality. Conversely, an excessively large θ would prevent refurbished products from entering the market (i.e., $q_{r2}^{b} > 0$), we get $0 < \theta < \frac{1}{\beta} - 1$, and the proof is delegated to the Appendix B.

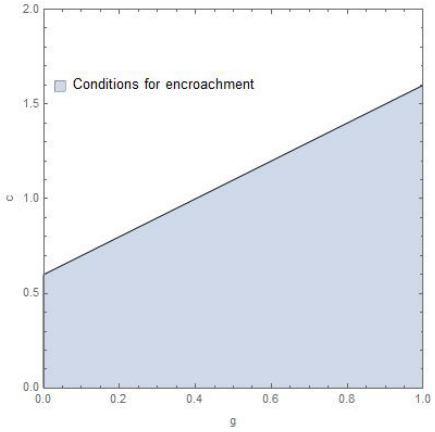
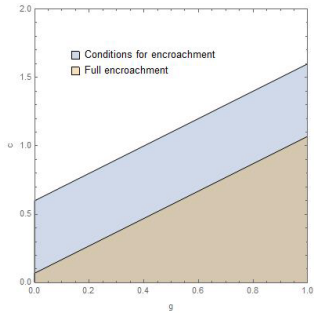
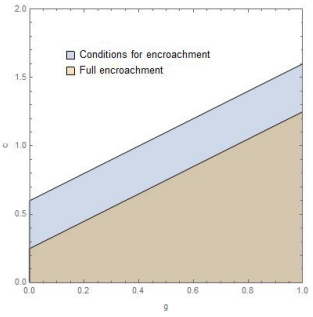


Fig. 3. Conditions of encroachment

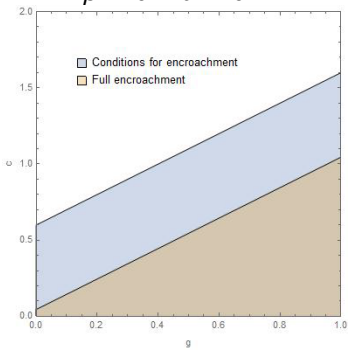
To demonstrate that Corollary 1 is a special case of Proposition 1, we combine the Figures from Proposition 1 with those from Corollary 1. As depicted in Figure 4, through the degree of overlap, it becomes apparent that the scenario described in Corollary 1 is a specific instance of Proposition 1. This outcome implies that the threshold for full encroachment is relatively high and is influenced by β. Specifically, if the direct sales channel operates efficiently enough or the service level is sufficiently high, the manufacturer will fully encroach on the refurbished product market. In our illustrations, we have chosen various values for β and θ, but the graphical representation remains consistent across different scenarios. We employ this methodology in subsequent illustrations as well.



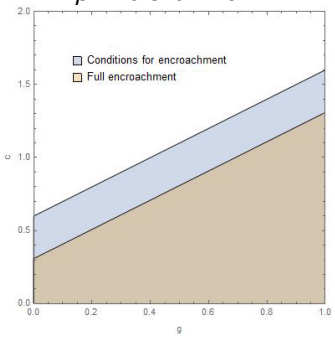
$\beta = 0.2 \;\; \theta = 0.1$



$\beta = 0.8 \;\; \theta = 0.1$



$\beta = 0.1 \;\; \theta = 0.3$



$\beta = 0.7 \;\; \theta = 0.3$

Fig 4. Partial encroachment versus full encroachment

The findings from Proposition 1 and Corollary 1 indicate that when $c > c_1$, the manufacturer will refrain from encroaching entirely on the refurbished product market. In this scenario, the equilibrium outcome aligns precisely with scenario B's.

4.2 The impact of manufacturers encroaching on refurbished products

In existing literature, numerous studies have compared new products' encroachment with identical products. In this paper, we contrast the encroachment of refurbished products with the non-encroachment scenario, focusing on their impact on the green supply chain. Furthermore, we assess how this encroachment strategy affects both supply chain members' decision-making balance and profitability. The calculation process is outlined in Appendix B. Given that manufacturers' encroachment affects retailers' sales, we initially analyze the impact of encroachment on retailers' sales volume, with the results presented in Proposition 2.

Proposition 2. The difference in sales volume to retailers between a partial and no encroachment scenario.
1. According to the equilibrium results, the retailer's sales quantity of new products does not change within the feasible range of c and g (i.e., $q_{\mathrm{r1}}^{s} = q_{\mathrm{r1}}^{b}$).
2. Retailers are selling more refurbished products than they would have without encroachment (i.e., $q_{\mathrm{r2}}^{s} > q_{\mathrm{r2}}^{b}$), if $c > c_3 = g + \frac{3a}{8}$. Conversely, when $c < c_3$, retailers sold fewer refurbished products than they did without encroachment (i.e., $q_{\mathrm{r2}}^{s} < q_{\mathrm{r2}}^{b}$). The change in the number of refurbished products retailers sell is shown in Figure 5.
3. In the feasible range of c and g, the total sales volume of refurbished products is always greater than that of the basic scenario (i.e., $q_{\mathrm{r2}}^{s} + q_{\mathrm{m2}}^{s} > q_{\mathrm{r2}}^{b}$).

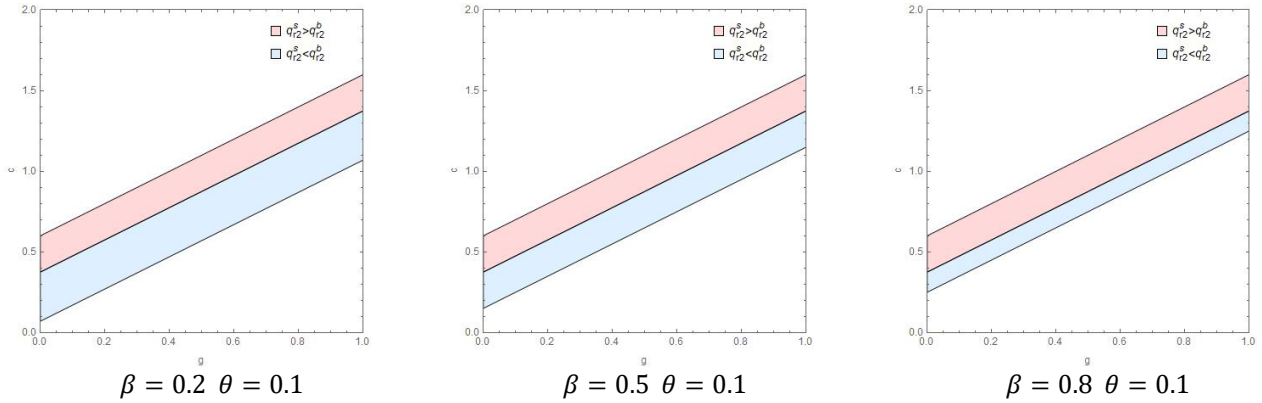


Fig 5. Comparison of sales volume of refurbished products

Likewise, in the scenario where the manufacturer fully encroaches on the refurbished product, we can set $q_{\mathrm{r2}}^{b} = 0$ to calculate the values of each parameter and obtain other necessary values to achieve equilibrium. Alternatively, we can directly utilize the equilibrium outcome from Scenario F. This leads us to Corollary 2.

Corollary 2. In the Full encroachment scenario (i.e., $0 < c \leq c_2$), the sales quantity of new products is consistently below the Basic scenario (i.e., $q_{\mathrm{r1}}^{f} < q_{\mathrm{r1}}^{b}$). The sales quantity of refurbished products is consistently higher than in the Basic scenario (i.e., $q_{\mathrm{m2}}^{f} > q_{\mathrm{r2}}^{b}$).

When a manufacturer encroaches on refurbished products, the change in the number of products sold by a retailer can be categorized into three distinct scenarios. No change in the sales of new products. 2. With higher direct selling costs, the quantity of refurbished products sold by retailers exceeds that in the basic scenario. 3. With lower direct selling costs, retailers sell fewer refurbished products compared to the base scenario. Overall, the total number of refurbished products sold is higher than in the basic scenario. Further investigation into Proposition 2 reveals that the encroachment of refurbished products instigates competition within the refurbished product market. In scenarios with minimal competition (such as the partial encroachment scenario), the sales of new products remain unaffected initially, resulting in no change in the quantity of new product sales. Moreover, as the manufacturer's direct selling costs decrease at a certain service level, the number of refurbished products retailers sell gradually decreases. This phenomenon occurs because the reduction in direct selling costs enhances the efficiency of the direct sales channel, stimulating sales of refurbished products through this channel while dampening sales through retailers. Furthermore, as depicted in Fig. 5, when the cost of the direct sales channel remains constant, an increase in the service level “g” correlates with a gradual decline in the sales volume of refurbished products through retailers. This trend arises because higher service levels enhance consumer preference for direct sales channels, reducing the number of refurbished products sold through retailers.

Next, we analyze the influence of manufacturer encroachment on refurbished products on wholesale prices.

Proposition 3. When the manufacturer partially encroaches on the refurbished product, it means $c_2 < c < c_1$, encroachment has led to a drop in wholesale prices for new and refurbished products (i.e., $\omega_1^b > \omega_1^s, \omega_2^b > \omega_2^s$).

Let's elaborate on Proposition 3. As a result of manufacturer encroachment, retailers lose their monopoly position in the downstream market. Refurbished products from manufacturers' direct channels compete directly with those from retailers. In this scenario, manufacturers can either raise wholesale prices to bolster their own competitiveness or lower wholesale prices to incentivize retailers. Proposition 3 concludes that manufacturers consistently prefer to enhance retailers' motivation to expand the scale of classic channels.

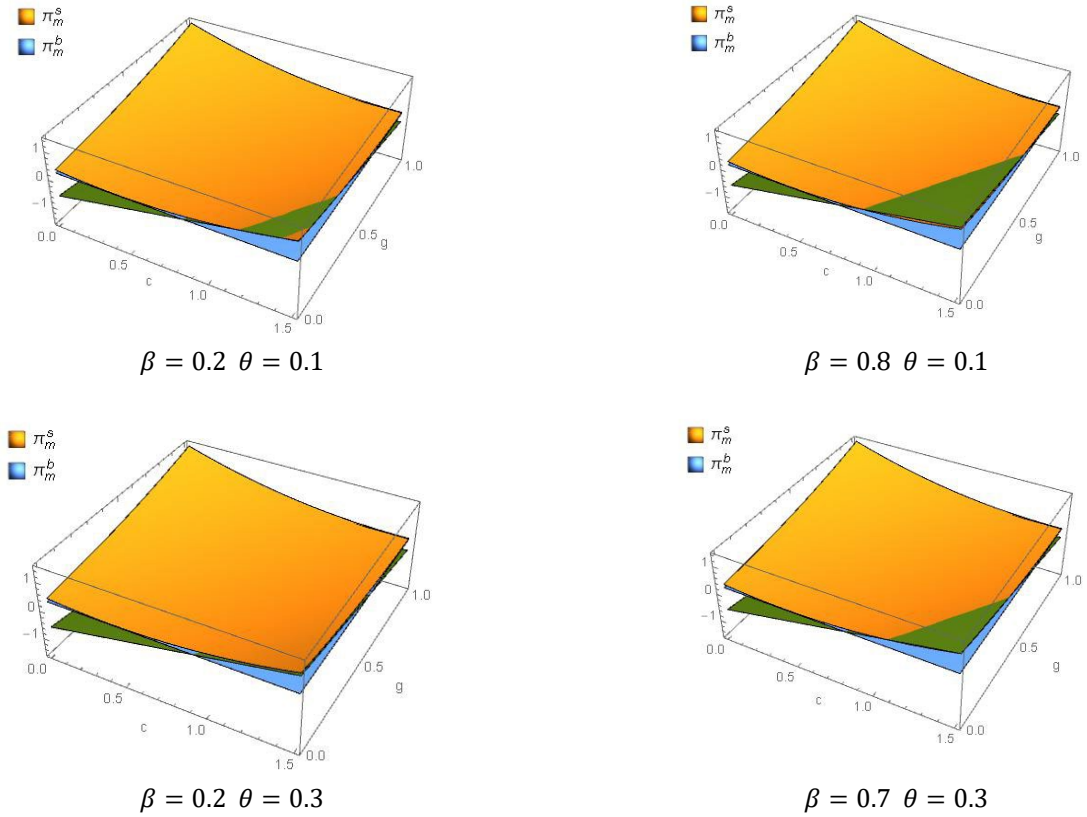
Building upon Proposition 3, we analyze the impact of the manufacturer's partial appropriation of refurbished products on the retailer's marginal profit, as detailed in Proposition 4.

Proposition 4. Manufacturers' partial encroachment has led to a decline in retailers' marginal profit (i.e., $p_{r1}^b - \omega_1^b > p_{r1}^s - \omega_1^s; p_{r2}^b - \omega_2^b > p_{r2}^s - \omega_2^s$).

Lower wholesale prices compel retailers to lower their retail prices in response to manufacturer competition, reducing retailers' marginal profit due to the impact of manufacturer competition.

Furthermore, we compared the effects of partial encroachment on the profits of manufacturers and retailers, as depicted above the green plane in Figure 6. The results are presented in Proposition 5.

Proposition 5. In the Partial encroachment scenario, manufacturers' profits are always higher than in the basic scenario (i.e., $\pi_m^s > \pi_m^b$).



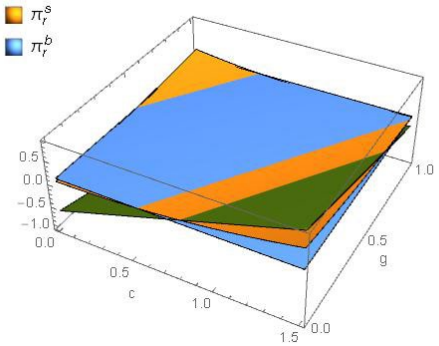
$\beta = 0.2\ \ \theta = 0.1$ $\beta = 0.8\ \ \theta = 0.1$

$\beta = 0.2\ \ \theta = 0.3$ $\beta = 0.7\ \ \theta = 0.3$

Fig 6. Manufacturer profit relationship

In the absence of manufacturer encroachment, specifically in the Basic scenario, the manufacturer's profit solely relies on wholesaling the product to the retailer. However, when conditions become favorable for manufacturer encroachment into refurbished products (i.e., when $c < c_1$), manufacturers and retailers actively compete within the refurbished product market. Manufacturers gain a stronger competitive edge by effectively balancing revenue from wholesale and direct sales channels, leading to improved profits. Proposition 5 further validates that encroachment results in enhanced profitability for manufacturers. Additionally, as depicted in Fig. 6, reducing the direct selling cost (c) or enhancing the service level (g) contributes to increased manufacturer profits.

Proposition 6. In the Partial encroachment scenario, as is shown in Figure 7, retailers' profits are higher than in the Basic scenario (i.e., $\pi_r^s > \pi_r^b$), if $c_2 < c < c_4$ or $c_5 < c < c_1$, where $c_4 = g - \frac{3a\sqrt{2}}{8}$, $c_5 = g + \frac{3a\sqrt{2}}{8}$.



$\beta = 0.2\ \ \theta = 0.1$

Fig 7. Retailer profit relationship

When the manufacturer encroaches on the refurbished product, Proposition 2 assesses whether the quantity of refurbished products sold may surpass or fall below the quantity sold in the Basic scenario. Meanwhile, Proposition 4 indicates that the retailer's marginal profit consistently falls short of the Basic scenario. Therefore, the outcome presented in Proposition 6 is to be expected.

When the increased sales volume compensates for the reduced margin, the retailer's profit surpasses the basic scenario's. Coupled with Proposition 5, this scenario fosters a win-win situation. As direct selling costs decrease, retailers' profits decline before rebounding. Therefore, if manufacturer encroachment on refurbished products is inevitable, retailers can enhance their profitability by collaborating with manufacturers to invest in reducing direct sales costs. Furthermore, Figure 7 illustrates that enhancing the service level g also initially reduces and then increases the retailer's profit.

5. Sensitivity analysis

When the manufacturer encroaches on the refurbished product, a higher service level g can increase product pricing or attract more consumers. Similarly, reducing direct selling costs can enhance the competitiveness of direct sales channels. Proposition 7 below delves into how changes in these two crucial parameters impact the profits of manufacturers and retailers.

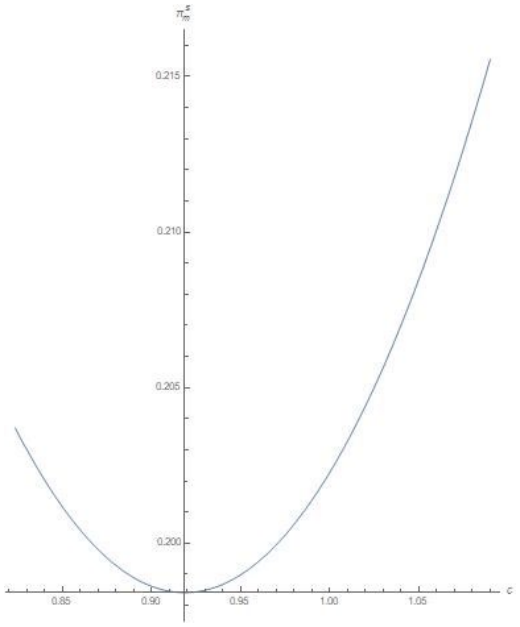
Proposition 7. Under the Partial encroachment scenario ($c_2 < c < c_1$), we have:

1. The manufacturer's profit increases in c (i.e., $\frac{\partial \pi_m^s}{\partial c} > 0$), if $c_6 < c < c_1$. The manufacturer's profit decreases in c (i.e., $\frac{\partial \pi_m^s}{\partial c} < 0$), if $c_2 < c < c_6$, where $c_6 = g + \frac{3a}{7}$.
2. The manufacturer's profit increases in g (i.e., $\frac{\partial \pi_m^s}{\partial g} > 0$), if $g_1 < g < g_2$. The manufacturer's profit decreases in g (i.e., $\frac{\partial \pi_m^s}{\partial g} < 0$), if $g_3 < g < g_1$, where $g_1 = c - \frac{3a}{7}$, $g_2 = c + \frac{3a\beta(1-\beta+\theta)}{8(\beta^2-1)}$ and $g_3 = c - \frac{3a}{5}$.
3. The retailer's profit increases in c (i.e., $\frac{\partial \pi_r^s}{\partial c} > 0$), and the retailer's profit decreases in g (i.e., $\frac{\partial \pi_r^s}{\partial g} < 0$), if $c_2 < c < c_1$.

Proposition 7 reveals that investing in enhancing the efficiency of direct sales doesn't consistently benefit manufacturers and sometimes even proves detrimental. However, when the direct selling cost is already low ($c_2 < c < c_6$), reducing it can boost the manufacturer's profit. Similarly, improving service capabilities doesn't always benefit manufacturers; it's only when the service level reaches a certain threshold ($g > g_1$) that it contributes to increased profits. This is because poor efficiency in direct sales cost reduction fails to deliver significant performance improvements and adds to the manufacturer's burden, resulting in profit decline. Likewise, when manufacturers invest in enhancing the service level, marginal changes might not significantly pique consumer interest. However, surpassing a certain threshold ($g_1 = c - \frac{3a}{7}$) can rapidly amplify the manufacturer's profit. To illustrate this, we set parameters $a = 1, \beta = 0.8, \theta = 0.2$, and depict how changes in direct selling costs affect manufacturers' profit in Fig. 8.

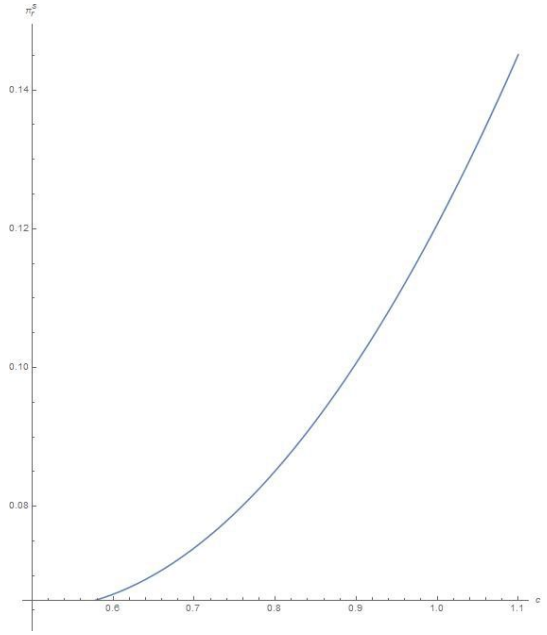
For retailers, reducing manufacturers' direct selling costs consistently diminishes their profits. Likewise, enhancements in manufacturers' service levels invariably impede retailers' profits. This occurs because improvements in manufacturers' direct sales efficiency or service levels bolster their competitiveness while eroding retailers' channel advantage, resulting in decreased profits for retailers.

To illustrate this, we set parameters $a = 1, \beta = 0.2, \theta = 0.2$, and employ a graph to depict how direct selling costs affect retailer profit, as shown in Fig. 9.



$\beta = 0.8\ \theta = 0.2$

Fig. 8. Manufacturer's profit curve under partial encroachment



$\beta = 0.2\ \theta = 0.2$

Fig. 9. Retailer's profit curve under partial encroachment

Proposition 8. The sales quantities of refurbished products in the manufacturer's direct sales channel decrease in direct sales cost c and increases in service level g (i.e., $\frac{\partial q_{m2}^S}{\partial c} = \frac{-5}{6} < 0; \frac{\partial q_{m2}^S}{\partial g} = \frac{5}{6} > 0$). The sales quantities of refurbished products by retailers increase in manufacturer direct sales cost c and decreases in manufacturer service level g (i.e., $\frac{\partial q_{r2}^S}{\partial c} = \frac{2}{3} > 0; \frac{\partial q_{r2}^S}{\partial g} = \frac{-2}{3} < 0$). We graphically show the change of the sales quantity of refurbished products in c, as shown in Fig. 10.

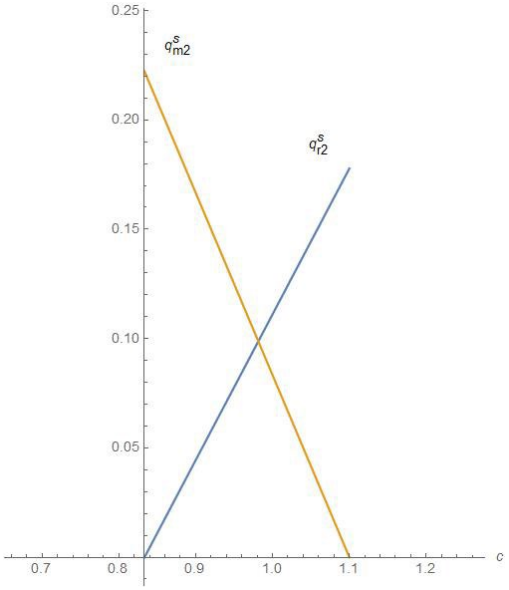


Fig 10. The sales quantities of refurbished products

In further examining Proposition 8, we observe that an increase in direct selling costs signifies a decrease in the efficiency of the direct sales channel, aligning with practical observations. Similarly, this decline in efficiency also correlates with a reduction in the competitiveness of direct sales channels, thereby enhancing retailers' competitiveness and expanding the number of refurbished products retailers sell. However, the impact of direct selling costs directly affects the sales of manufacturers' refurbished products, while its effect on retailers' refurbished products is indirect. As a result, "c" exerts a more pronounced impact on the sales volume of manufacturers' refurbished products ($\left|\frac{\partial q_{m2}^S}{\partial c}\right| > \left|\frac{\partial q_{r2}^S}{\partial c}\right|$). Furthermore, with improved service level g, manufacturers' competitiveness is augmented while retailers' competitiveness is weakened. Consequently, manufacturers' refurbished products increase while retailers' sales decrease. The managerial recommendations for manufacturers are as follows: Upon meeting the conditions for encroaching on refurbished products, manufacturers should prioritize investing in enhancing the efficiency of their direct sales channels. Despite the potential profit decline during cost

reduction efforts, once the direct selling cost reaches a certain threshold ($c = c_6$), manufacturers stand to benefit from increased profits due to cost reductions. Moreover, encroachment consistently yields better outcomes for manufacturers than alternative strategies, underscoring the importance of investing in enhancing direct channels' efficiency.

It is important to note that when encroachment becomes unavoidable, retailers should collaborate with manufacturers to maintain minimal direct selling costs. Doing so enables manufacturers to enhance their service levels, increasing their profits. Conversely, if service levels remain low, retailers can focus on improving their cost of goods sold to advantage, placing manufacturers at a disadvantage. By aligning with Proposition 6, a mutually beneficial outcome can be achieved when manufacturers encroach on refurbished products. This entails suppliers investing in more efficient direct sales and higher service levels, a goal that retailers can actively contribute to.

6. Conclusion and implications

6.1 Conclusion

With the increasing acceptance of refurbished products in the market and the growing emphasis on green supply chain practices, it has become imperative for manufacturers to explore new avenues for distributing refurbished products beyond traditional retail channels. However, the question of how manufacturers should undertake the usurpation of refurbished products to differentiate them from conventional channels and mitigate potential channel conflicts remains intriguing and underexplored. To address this issue comprehensively, we introduce a model of a two-tier supply chain comprising manufacturers and retailers. In this model, manufacturers encroach upon refurbished products by enhancing the service level g. Through a game theory analysis, we investigate the strategies preferred by manufacturers and retailers during the encroachment process when encroachment becomes inevitable, aiming to establish a consensus on the post-encroachment scenario. This paper contributes to the discourse on refurbished products by presenting the following key findings: First, where the direct channel is sufficiently efficient ($c < c_1$), the manufacturer chooses to encroach on the refurbished product because encroachment makes the manufacturer better off, whereas full encroachment is more demanding, i.e., it requires a more efficient direct channel ($c \le c_2$); and second, under certain conditions ($g - \frac{3a\beta\ (1-\beta+\theta)}{8(\beta^2-1)} < c < g - \frac{3a\sqrt{2}}{8}$ or $g + \frac{3a\sqrt{2}}{8} < c < g + \frac{3a}{5}$), encroachment by the manufacturer mitigates channel conflict because the retailer will get by better than without encroachment, and under certain conditions ($c > c_3$), and the number of sales of refurbished products by retailers will increase; finally, contrary to intuition, manufacturers investing in improving the efficiency of their direct sales doesn't necessarily improve profits and can sometimes hurt themselves.

6.2. Managerial implications

6.2.1. Implications for the manufacturer

When manufacturers venture into the refurbished products market, the threshold for partial encroachment is lower than that for complete encroachment. Moreover, enhancing service levels further decreases this threshold. However, complete encroachment would be unwise, hurting retailers' profits and intensifying channel conflicts. When the conditions are ripe for a manufacturer to encroach on a refurbished product, the choice of encroachment can result in additional profits for the manufacturer. Although investing in reducing direct selling costs and improving service levels may initially harm manufacturers' interests within certain limits, in the long term, such investments can mitigate channel conflicts and lead to a mutually beneficial outcome. When the direct sales channel is at profit-optimizing efficiency, continued investment hurts the manufacturer, so the manufacturer should keep the profit-optimizing efficiency unchanged after the encroachment.

6.2.2 Implications for the retailer

In cases where the manufacturer has encroached on refurbished products, the retailer needs to be proactive in assisting the manufacturer to quickly reduce the cost of direct sales to a specific band, as this is when a situation occurs where the retailer's margins improve. Once the direct selling cost falls within this specific range, retailers should actively enhance retail efficiency to impede further indirect reduction of the direct selling channel cost. This strategy aims to prevent manufacturers from dominating the refurbished product market and safeguard the retailer's interests.

6.2.3 Industry management

As a channel manager or government department, it is imperative to devise subsidy measures for direct sales channels, facilitating manufacturers in meeting the conditions necessary for encroachment. By enabling manufacturers to encroach on the market of refurbished products partially, it becomes possible to expand the market further, enhance resource utilization efficiency, promote the development of green supply chains, and ultimately serve the goal of environmental protection.

6.3 Theoretical implications and practical contributions

6.3.1. Theoretical implications

We study the encroachment of manufacturers on refurbished products and give the optimal equilibrium solution for manufacturers and retailers from a game-theoretic point of view through a rigorous computational process. We introduce the service level as an influencing factor and describe the conditions for manufacturers and retailers to realize a win-win situation after manufacturer encroachment, which fills the gap in research on the encroachment of refurbished products from the perspective of changing the service level. Our research provides new perspectives for manufacturers to open direct sales channels to distribute refurbished products, gives theoretical justifications for expanding the market for refurbished products, and further deepens the research in the field of manufacturer encroachment.

6.3.2. Practical contributions

This study examines the results of decision-making to mitigate channel conflict in manufacturer encroachment from this perspective by introducing improved service levels. This study gives solutions to mitigate the conflict problem from the perspective of manufacturers and retailers, as well as government and industry regulators, to solve the real problem from the perspective of the intractable problems faced in reality. This study is industry-generalized, and this model applies to different industry studies. By bringing in different parameters in different industries, the optimal service level and pricing strategy for manufacturer encroachment of refurbished products in that industry can be precisely calculated, thus mitigating the channel conflict inherent in manufacturer encroachment and benefiting both manufacturers and retailers as well as the industry and the environment.

6.4 Limitation

This study assumes that the direct selling cost represents the unit selling cost of refurbished products in the direct selling channel but overlooks other hidden costs associated with channel establishment. Furthermore, enhancing the service level will also indirectly impact the production cost of refurbished products. Regarding cost, a more comprehensive study could be undertaken.

6.5 Future research opportunities

In this study, we examined the influence of manufacturers' enhanced service levels on encroachment into the refurbished product market. While our findings indicate that encroachment facilitates the expansion of this market and can lead to mutually beneficial outcomes under specific conditions, there remains a gap in understanding the systematic conditions under which manufacturers might opt to delegate service provision to retailers, thus warranting further investigation.

Data disclosure statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Disclosure statement

The authors report there are no competing interests to declare.

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Appendix

Appendix A Equilibrium analysis

Appendix A1. The equilibrium process of the Basic scenario

In the basic scenario without encroachment, the manufacturer is responsible for production, and the retailer is responsible for selling the product. Retailers wholesale new and refurbished products from manufacturers at different wholesale prices of ω_1^b and ω_2^b and sell them directly to consumers at different prices. According to the decision order, we use backward induction to get the optimal equilibrium result. We first determine the first derivative of the retailer profit function with respect to $q_{\mathrm{r}1}^b$ and $q_{\mathrm{r}2}^b$:

$$\frac{\partial \pi_r^b}{\partial q_{\mathrm{r}1}^b} = a(1+\theta) - 2q_{\mathrm{r}1}^b - 2\beta q_{\mathrm{r}2}^b - \omega_1^b \tag{A.1}$$

$$\frac{\partial \pi_r^b}{\partial q_{\mathrm{r}2}^b} = a - 2\beta q_{\mathrm{r}1} - 2q_{\mathrm{r}2} - \omega_2^b \tag{A.2}$$

By setting the two first derivatives to zero, we get:

$$q_{\mathrm{r}1}^b = -\frac{a - a\beta + a\theta - \omega_1^b + \beta\omega_2^b}{2(-1+\beta^2)} \tag{A.3}$$

$$q_{\mathrm{r}2}^b = -\frac{a - a\beta - a\beta\theta + \beta\omega_1^b - \omega_2^b}{2(-1+\beta^2)} \tag{A.4}$$

By incorporating the two formulas into the supplier's profit function and differentiating the wholesale price separately, we have:

$$\frac{\partial \pi_m^b}{\partial \omega_1^b} = \frac{\omega_1^b}{2(-1+\beta^2)} - \frac{\beta\omega_2^b}{2(-1+\beta^2)} - \frac{a - a\beta + a\theta - \omega_1^b + \beta\omega_2^b}{2(-1+\beta^2)} \tag{A.5}$$

$$\frac{\partial \pi_m^b}{\partial \omega_2^b} = -\frac{\beta\omega_1^b}{2(-1+\beta^2)} - \frac{a - a\beta - a\beta\theta + \beta\omega_1^b - \omega_2^b}{2(-1+\beta^2)} + \frac{\omega_2^b}{2(-1+\beta^2)} \tag{A.6}$$

By setting the values of the two first derivatives to zero, we can obtain the optimal wholesale price:

$$\omega_1^b = \frac{1}{2}(a + a\theta) \tag{A.7}$$

$$\omega_2^b = \frac{a}{2} \tag{A.8}$$

Bringing this into the original function, we get:

$$q_{\mathrm{r}1}^b = \frac{a(1 - \beta + \theta)}{4 - 4\beta^2} \tag{A.9}$$

$$q_{\mathrm{r}2}^b = \frac{a(-1 + \beta + \beta\theta)}{4(-1+\beta^2)} \tag{A.10}$$

$$p_{\mathrm{r}1}^b = \frac{3}{4}a(1 + \theta) \tag{A.11}$$

$$p_{\mathrm{r}2}^b = \frac{3a}{4} \tag{A.12}$$

$$\pi_r^b = \frac{a^2(2 - 2\beta(1+\theta) + \theta(2+\theta))}{16(1-\beta^2)} \tag{A.13}$$

$$\pi_m^b = \frac{a^2(2 - 2\beta(1+\theta) + \theta(2+\theta))}{8(1-\beta^2)} \tag{A.14}$$

Appendix A2. The equilibrium process of the Partial encroachment scenario

We first determine the first derivative of the manufacturer's profit function with respect to the quantity of refurbished products sold in the direct sales channel $q_{\mathrm{m}2}^s$:

$$\frac{\partial \pi_m^s}{\partial q_{\mathrm{m}2}^s} = a - c + g - 2q_{\mathrm{m}2}^s - \beta q_{\mathrm{r}1}^s - q_{\mathrm{r}2}^s \tag{A.15}$$

And set it to zero, we can get:

$$q_{\mathrm{m}2}^s = \frac{1}{2}(a - c + g - \beta q_{\mathrm{r}1}^s - q_{\mathrm{r}2}^s) \tag{A.16}$$

By inserting the value of $q_{\mathrm{m}2}^s$ into the retailer profit function and taking the derivative of $q_{\mathrm{r}1}^s$ and $q_{\mathrm{r}2}^s$ respectively, we can get:

$$\frac{\partial \pi_r^s}{\partial q_{\mathrm{r}1}^s} = -\frac{a\beta}{2} + \frac{1}{2}(c - g)\beta + a(1+\theta) + (-2+\beta^2)q_{\mathrm{r}1}^s - \beta q_{\mathrm{r}2}^s - \omega_1^s \tag{A.17}$$

$$\frac{\partial \pi_r^s}{\partial q_{\mathrm{r}2}^s} = \frac{1}{2}(a + c - g - 2\beta q_{\mathrm{r}1}^s - 2q_{\mathrm{r}2}^s - 2\omega_2^s) \tag{A.18}$$

Let the values of the two first derivatives be equal to zero, we get

$$q_{\mathrm{r1}}^{s} = -\frac{a - a\beta + a\theta - \omega_1^s + \beta\omega_2^s}{2(-1+\beta^2)} \tag{A.19}$$

$$q_{\mathrm{r2}} = -\frac{a + c - g - a\beta - c\beta^2 + g\beta^2 - a\beta\theta + \beta\omega_1^s - 2\omega_2^s + \beta^2\omega_2^s}{2(-1+\beta^2)} \tag{A.20}$$

Substituting the values of the three sales quantities into the manufacturer's profit function and differentiating the two wholesale prices separately, we can get:

$$\frac{\partial \pi_m^s}{\partial \omega_1^s} = \frac{a - a\beta + a\theta - 2\omega_1 + 2\beta\omega_2}{2 - 2\beta^2} \tag{A.21}$$

$$\frac{\partial \pi_m^s}{\partial \omega_2^s} = \frac{1}{4(-1+\beta^2)}\begin{Bmatrix}(-1+\beta)[(-c+g)(1+\beta)+a(3+\beta)] \\ +2a\beta\theta - 4\beta\omega_1 - 2(-3+\beta^2)\omega_2\end{Bmatrix} \tag{A.22}$$

By setting the values of the two first derivatives to zero and solving them in parallel, we can conclude that the optimal wholesale price is:

$$\omega_1^s = \frac{1}{6}(3a - c\beta + g\beta + 3a\theta) \tag{A.23}$$

$$\omega_2^s = \frac{1}{6}(3a - c + g) \tag{A.24}$$

Bringing the optimal wholesale price back to the original path, we can get:

$$q_{\mathrm{r1}}^{s} = \frac{a(1-\beta+\theta)}{4-4\beta^2} \tag{A.25}$$

$$q_{\mathrm{r2}}^{s} = \frac{1}{12}\left[8c - 8g + \frac{3a\beta(1-\beta+\theta)}{-1+\beta^2}\right] \tag{A.26}$$

$$q_{\mathrm{m2}}^{s} = \frac{1}{6}(3a - 5c + 5g) \tag{A.27}$$

$$p_{\mathrm{r1}}^{s} = \frac{1}{12}[2(c-g)\beta + a(9 - 3\beta + 9\theta)] \tag{A.28}$$

$$p_{\mathrm{r2}}^{s} = \frac{1}{6}(3a + c - g) \tag{A.29}$$

$$p_{\mathrm{m2}}^{s} = \frac{1}{6}(3a + c + 5g) \tag{A.30}$$

$$\pi_m^s = \frac{7}{12}(c-g)^2 + \frac{1}{2}a(-c+g) + \frac{a^2[-3+\beta^2+2\beta(1+\theta)-\theta(2+\theta)]}{8(-1+\beta^2)} \tag{A.31}$$

$$\pi_r^s = \frac{2}{9}(c-g)^2 - \frac{a^2(1-\beta+\theta)^2}{16(-1+\beta^2)} \tag{A.32}$$

Appendix A3. The equilibrium process of the Full encroachment scenario

We first determine the first derivative of the manufacturer's profit function with respect to the quantity of refurbished products sold in the direct sales channel q_{m2}^{f}:

$$\frac{\partial \pi_m^f}{\partial q_{\mathrm{m2}}^f} = a - c + g - 2q_{\mathrm{m2}}^f - \beta q_{\mathrm{r1}}^f \tag{A.33}$$

and set it to 0, we get:

$$q_{\mathrm{m2}}^{f} = \frac{1}{2}(a - c + g - \beta q_{\mathrm{r1}}^f) \tag{A.34}$$

Put the value of q_{m2}^{f} into the retailer profit function and take the derivative of q_{r1}^{f} to get:

$$\frac{\partial \pi_r^f}{\partial q_{\mathrm{r1}}^f} = (-2+\beta^2)q_{\mathrm{r1}}^f + \frac{1}{2}\left[(c-g)\beta + a(2-\beta+2\theta) - 2\omega_1^f\right] \tag{A.35}$$

Making it zero, we get:

$$q_{r1}^{f} = \frac{-2a + a\beta - c\beta + g\beta - 2a\theta + 2\omega_1^f}{2(-2+\beta^2)} \tag{A.36}$$

By bringing the value of q_{r1}^{f} into the manufacturer's profit function and differentiating the wholesale price of the new product ω_1^f, we get:

$$\frac{\partial \pi_m^f}{\partial \omega_1^f} = \frac{1}{4(-2+\beta^2)^2}\begin{Bmatrix}(-c+g)\beta^3 + 8a(1+\theta) + \\ a\beta^2[\beta - 6(1+\theta)] + 2(-8+5\beta^2)\omega_1\end{Bmatrix} \tag{A.37}$$

Let the above formula be 0, we get:

$$\omega_1^f = \frac{(c-g)\beta^3 - a[\beta^3 + 8(1+\theta) - 6\beta^2(1+\theta)]}{2(-8+5\beta^2)} \tag{A.38}$$

By bringing back the highest wholesale price of the new product, we can get:

$$q_{r1}^{f} = \frac{2[(c-g)\beta + a(1-\beta+\theta)]}{8-5\beta^2} \tag{A.39}$$

$$q_{m2}^{f} = \frac{-(c-g)(-8+3\beta^2) + a[-8+\beta(2+3\beta+2\theta)]}{2(-8+5\beta^2)} \tag{A.40}$$

$$p_{r1}^{f} = \frac{(c-g)\beta(-4+3\beta^2) - 12a(1+\theta) + a\beta[4+\beta(8-3\beta+8\theta)]}{2(-8+5\beta^2)} \tag{A.41}$$

$$p_{m2}^{f} = \frac{-8(c+g) + (7c+3g)\beta^2 + a[-8+\beta(2+3\beta+2\theta)]}{2(-8+5\beta^2)} \tag{A.42}$$

$$\pi_r^f = \frac{2(2-\beta^2)[(c-g)\beta + a(1-\beta+\theta)]^2}{(8-5\beta^2)^2} \tag{A.43}$$

$$\pi_m^f = \frac{(c-g)^2(-8+\beta^2) - 2a(c-g)\begin{bmatrix}-8+\beta \\ (4+\beta+4\theta)\end{bmatrix} + a^2\begin{Bmatrix}\beta^2 + 8\beta(1+\theta) \\ -4[3+\theta(2+\theta)]\end{Bmatrix}}{4(-8+5\beta^2)} \tag{A.44}$$

Appendix B Proof of Propositions and Corollaries

Appendix B1 Proof of Proposition 1 and Corollary 1

As a result of manufacturer encroachment, the number of refurbished products sold in the direct sales channel is positive, so $q_{m2}^{s} > 0$, which is $\frac{1}{6}(3a - 5c + 5g) > 0$. Solve $c < g + \frac{3a}{5}$. For convenience, let's say $c_1 = g + \frac{3a}{5}$. In the same way, if you want to achieve the conditions of complete occupation, the number of sales of classic channel refurbished products is non-positive, that is, $q_{r2}^{s} \le 0$, which is $\frac{1}{12}(8c - 8g + \frac{3a\beta(1-\beta+\theta)}{-1+\beta^2}) \le 0$. Solve $c \le g - \frac{3a\beta(1-\beta+\theta)}{8(\beta^2-1)}$. We set $c_2 = g - \frac{3a\beta(1-\beta+\theta)}{8(\beta^2-1)}$.
In the basic scenario, we want to ensure that the refurbished product can enter the market normally, otherwise, the subsequent research will be meaningless, so $q_{r2}^{b} > 0$, we get $\theta < \frac{1}{\beta} - 1$.

Appendix B2 Proof of Proposition 2 and Corollary 2

We set $q_{r2}^{s} > q_{r2}^{b}$, bring in the equilibrium result and solve it, we get $c > g + \frac{3a}{8}$. For simplicity, let's say $c_3 = g + \frac{3a}{8}$, and conversely, if $q_{r2}^{s} < q_{r2}^{b}$, then $c < c_3$.
Adding the number of refurbished products by retailers and the number of refurbished products by manufacturers after partial encroachment, we get a consistently greater result than the Basic scenario.

Appendix B3 Proof of Proposition 3 and Proposition 4

In $c_2 < c < c_1$, we respectively $\omega_1^b - \omega_1^s, \omega_2^b - \omega_2^s$ values, the results are: $\frac{1}{6}(c-g)\beta$, $\frac{c-g}{6}$, the combination of c value range, we can conclude that $\omega_1^b - \omega_1^s$ and $\omega_2^b - \omega_2^s$ result is positive.
By the same token, $(p_{r1}^{b} - \omega_1^b) - (p_{r1}^{s} - \omega_1^s)$ and $(p_{r2}^{b} - \omega_2^b) - (p_{r2}^{s} - \omega_2^s)$ are positive numbers.

Appendix B4 Proof of Proposition 7

By differentiating the profit function in the partial encroachment scenario with respect to c and g, we can obtain:

$$\frac{\partial \pi_m^S}{\partial g} = \frac{a}{2} - \frac{7(c-g)}{6} \tag{B.1}$$

$$\frac{\partial \pi_m^S}{\partial c} = -\frac{a}{2} + \frac{7(c-g)}{6} \tag{B.2}$$

$$\frac{\partial \pi_r^S}{\partial c} = -\frac{4}{9}(c-g) \tag{B.3}$$

$$\frac{\partial \pi_r^S}{\partial c} = \frac{4(c-g)}{9} \tag{B.4}$$

According to the positive and negative of the first derivative of the manufacturer's profit, we can get $c_6 = g + \frac{3a}{7}$ and $g_3 = c - \frac{3a}{5}$.

In the interval of $c_2 < c < c_1$, it is determined that the first derivative of retailer profit is less than 0.

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