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Insight into bilateral efforts in green supply chain driven by manufacturers: A new dimension of coordination mechanisms

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A B S T R A C T

Under the rapid development of the global economy, environmental pollution has intensified significantly. The evolving external environment presents both opportunities and challenges to traditional supply chains. As an innovative management philosophy and practical approach, the green supply chain has rapidly become an integral component of corporate sustainable development strategies. However, the transition from traditional supply chain to green supply chain necessitates effective collaboration and coordination among supply chain members. Contractual coordination has therefore emerged as an effective methodology to enhance operational efficiency and profitability across supply chain participants. To investigate the impact of various coordination mechanisms on the optimal decision-making of green supply chains, we construct a Stackelberg game model involving bilateral green investments by both manufacturer and retailer within a two-echelon green supply chain system. Considering the bilateral green efforts from both manufacturer and retailer, we comparatively analyze game equilibrium solutions under three scenarios: non-coordination, cost-sharing contract coordination, and two-part contract coordination. Specifically，we examine how these coordination mechanisms influence pricing strategies, green investment levels, and profit distributions within the supply chain network. Finally, the results are validated and illustrated using numerical simulation. It is discovered that 1) the cost-sharing contract cannot simultaneously increase the Pareto improvement of producers' and retailers' revenues; 2) the cost-sharing contract cannot increase the social utility and additive greenness of products; however, it can improve the marketing effort; and 3) When retailers maintain an optimistic stance toward manufacturers' green initiatives, the two-part tariff contract enables concurrent Pareto improvements in both parties' profits while simultaneously enhancing product greenness, marketing efforts, and social welfare, thereby achieving efficient supply chain coordination.

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1. Introduction

Resolving the conflict between economic growth and environmental deterioration has become a prevalent issue facing human society as a result of the global economy's rapid expansion and the escalating pollution crisis (Juan, 2021). In order to realize the win-win situation of "environment-society-economy"(Amin et al., 2015), green development has been widely concerned. One of the most crucial strategies for advancing environmentally friendly economic growth and environmental preservation is the greening of the supply chain (Hoggett et al., 2014). Enterprises integrate manufacturers' and retailers' environmental behavior and green technology into every link of production and sales, forming a new supply chain model that meets enterprises' economic interests while considering environmental interests, i.e., a green supply chain. Faced with the growing environmental awareness of consumers and government's vigorous promotion of green development policies to meet

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consumers' growing desire for eco-friendly goods, enterprises in the manufacturer-led supply chain are actively carrying out various green supply chain management cooperation and have achieved good results. Many manufacturers and retailers are actively carrying out various green supply chain cooperation practices, only a few manufacturing companies have begun their journey toward green technology-based competitiveness. For instance, research on Gree's ecologically friendly inverter air conditioner and Nike's eco-friendly footwear. In 2012, FAW-Volkswagen publicly introduced the "Green Partner" initiatives, that is, to impose environmental management on suppliers and retailers; to strengthen the awareness of energy saving and environmental protection of the retailers in the sales process; to decrease the amount consumed of non-renewable resources, and to enhance the environmental management of the retailers. To further maximize the efficient utilization of capital and to ensure the coordinated development between manufacturers' and retailers' business activities and environmental protection, it is also necessary to coordinate the collaboration methods, green production, green marketing inputs, and profit distribution strategies as a way to enhance the supply chain's overall economic and ecological efficiency to gain new competitive advantages. In management practice, the contradiction between the cost of green inputs and the maximization of their respective benefits inevitably triggers a dynamic game of continuous competition and cooperation among enterprises since the strategic alliance constituted by core manufacturing enterprises (manufacturers) and retailers usually represents two different interests. This problem, in turn, inevitably has a negative impact on the effective collaboration and operation of green supply chains. In addition, core manufacturing companies are often in a monopolistic position and dominate the fulfillment of corporate environmental responsibility. However, the lack of consideration of retailers' profits may reduce their motivation to participate in ecological responsibility, leading to a series of problems, including a lack of active participation in green inputs and promotion of environmentally friendly sales, which in turn undermines the effectiveness of core manufacturing enterprises in green production inputs. The phenomenon known as the "double marginalization effect" plays a significant role in the dynamics of supply chain management. In many real-world scenarios, enterprises operating within a supply chain often focus solely on their individual interests, demonstrating a reluctance to engage in cooperative efforts with their partners. This self-centered approach frequently leads to a suboptimal performance in supply chain operations, as decentralized decision-making creates inefficiencies and hinders the overall effectiveness of the supply chain. Currently, a significant portion of related studies focuses on various types of contracts, such as two-part contracts, revenue-sharing contracts, and cost-sharing contracts, aimed at maximizing the gains from the delivery system. These contracts are designed to align the incentives of different stakeholders within the supply chain, thereby fostering cooperation and ultimately leading to improved performance and enhanced profitability for all parties involved. Still, few studies consider improving the agreement's effect in green supply chains for better coordination and operational efficiency between supply chains. Most of the scholars usually focus on manufacturers and research on manufacturers' green production inputs in the design of the contractual coordination mechanism. Studying the impact of manufacturers' green production inputs and other effects on coordination effects, and few studies take into account how retailers' actions affect how green supply chains coordinate, and even fewer studies combine the two. However, retailers have a better understanding of market demand and consumer preferences, and the degree of retailers' marketing efforts will also directly affect manufacturers' green inputs and pricing decisions, thus impacting the overall green supply chain's operational effectiveness. In our study, we build a Stackelberg game model that takes into account bilateral efforts between retailers and producers, study the impacts of manufacturers' and retailers' game behavior on their respective economic and environmental benefits, and analyze and compare the advantages and disadvantages of various contractual coordination mechanisms under the impacts of manufacturers' green production inputs and retailers' green marketing inputs.

Our study focuses on analyzing game-theoretic decision-making and contractual coordination between manufacturers, who are the leading decision-makers in green supply chains, and other supply chain members, while they pursue economic and environmental benefits. Our study also provides enterprises with guidance on the market impact. The ability of contractual coordination to create a win-win situation for profit and the environment among supply chains is of great theoretical and practical significance for promoting the green transformation of enterprises. In this study, we consider the dynamic changes in product greenness and marketing efforts. We establish a two-echelon supply chain game model consisting of a single manufacturer and retailer. We use Stackelberg game theory to study the game among green supply chain members and find the pure strategy equilibrium solution. This simple setup allows the main theoretical results to take center stage rather than the technical complexity inherent in more general models. The key issues and innovations addressed in this study are as follows:

(1) Unlike previous studies, we enhance the effectiveness of contracts to better coordinate the operational efficiency within the supply chain. When designing the contract mechanism, our focus is not solely on the manufacturer. Moreover, in addition to considering the impact of the manufacturer's green practices on the coordination of the green supply chain, we also take into account the influence of the retailer's actions on the coordination effect.

(2) Considering that retailers have a better understanding of market demand and consumer preferences, when designing the contract-coordination mechanism, we take into account the impacts of two indicators, namely the manufacturer's green production investment and the retailer's green marketing investment, on the effectiveness of the contract-coordination mechanism.

The rest of this paper is structured as follows. Section II reviews the literature related to this study. In Section III, a

manufacturer-driven Stackelberg game model is developed under consideration of the bilateral efforts of manufacturers and retailers, and the related coordination mechanism is designed to achieve the effect of centralized decision-making, and the related parameters are given. Section IV draws relevant managerial lessons and insights through a comparison study of the design and the equilibrium solutions of the game under four different situations. In Section V, these findings are simulated through numerical example analysis, further analyzing and illuminating the model's and the coordinated mechanism's validity is the comparison of the simulation results, which shows that, in some circumstances, the sharing of expenses contract and the two-part contract may more effectively coordinate and maximize the decision-making between the manufacturer and the retailer. Section VI concludes with several suggestions for further study.

2. Literature Review

Nowadays, the topic of optimal decision-making within the context of green supply chains has garnered considerable attention from scholars. They have conducted extensive research by developing a Stackelberg game model specifically tailored for green supply chains. For instance, Ghosh and Shah (2012) undertook an investigation into how various channel structures impact critical factors such as product greenness, pricing strategies, a retailer and so on. Further expanding on this topic, Giri et al. (2018) constructed a comprehensive model to systematically compare the impacts of various factors, including selling prices, product warranty periods, and levels of product greenness, on market demand for these goods. Their findings revealed a significant conclusion: under conditions of centralized decision-making, both the product greenness level and warranty period yielded more favorable outcomes than when decisions were made in a decentralized manner. Chaudhuri et al. (2023) studied how to enhance the output of the organization, the study results showed that under certain contextual factors, green supply chain technology can significantly enhance enterprise value. Wang et al. (2022) constructed a green supply chain game model when retailers are fair-neutral and fair-concerned, respectively, and examined the effects of fair-concerned retail practices on green product prices, green quality of goods, member earnings etc. Furthermore, Chen et al. (2022) looked into how customers' level of green awareness affected the price of green products and the best supply chain profitability in a heterogeneity duo-oligopoly product supply chain with various channel authority structures. Han et al. (2022) investigated the impacts on a green heterogeneous goods supply chain under other channel power structures, considering the competition between green products and ordinary products regarding selling price, greenness, and service. Pal et al. (2023) investigated a two-channel closed-loop supply chain consisting of a manufacturer, a retailer, and a recycler under an uncertain market environment, using Vertical Nash and Stackel. Vertical Nash and Stackelberg game methods were used to model the interactions between participants in a decentralized condition to portray the profits. Lai et al. (2023) conducted a case study of six manufacturers operating the digital transformation of the green supply chain to explore the adoption of the digital transformation of the green supply chain in practice.

Green production means organizing the production process with principles conducive to ecological and environmental protection and creating green products to meet green consumption. The gradual increase in consumers' green preferences has led manufacturers to consider green production using green technologies to improve the competitive advantage of the supply chain. Regarding the inclination of customers towards sustainability, Zhang et al. (2020) established a two-part tariff pact to increase firms' green investment. Sarkar et al.(2020) found that consumer environmental awareness primarily determines whether a manufacturer chooses to produce green products. More extensive research has been carried out by numerous academics in the area of green supply chains. In general, green marketing includes various activities at the critical point of exchange of a product or service. Other similar terms are environmental marketing and eco-marketing, which can also be referred to as the process of selling a product or service based on its environmental benefits. Yang et al. (2022) findings suggest that in the case of manufacturer-retailer alliance cooperation, the more effective the manufacturer's green R&D is, the more effectively it can support the green marketing initiatives of the retailer. Chang et al. (2019) argued that the ability of retailers to benefit from green marketing mainly depends on the level of consumer environmental awareness. When consumers' environmental awareness level is higher, retailers will invest in higher green marketing costs to stimulate the expansion of market demand. Shang et al. (2020) found that retailers' dominant position in the supply chain is crucial to their profitability. Especially when the market shows high sensitivity to the promotion of green marketing, the dominant position of retailers further optimizes supply chain decisions. In addition, firms can significantly enhance their competitive advantages by actively adopting green marketing strategies. Ji et al. (2017b) investigated manufacturers' low-carbon innovations and retailers' green promotions under a cap-and-trade system. He et al. (2019) established corresponding centralized and decentralized models, proposed promotional strategies to make the retailer more competitive, and analyzed the supply chain performance under the promotional strategies found that appropriate sales efforts are beneficial to the whole supply chain, while too much or too little sales efforts have no effect the supply chain, and could lower the chain's efficiency.

Unquestionably, the creation of a green economy is on the rise. By fostering collaboration among supply chain participants, green innovation can both successfully ease financial strain and, through a synergistic impact, advance the growth of a green economy. Nonetheless, creating a functional coordination framework is a requirement for supply chain participants to engage in cooperative green innovation. By establishing suitable agreements within the green supply chain, enterprises may further boost their ability to carry out sustainability and operational efficiency. Numerous similar studies have also been conducted by both domestic and foreign researchers. By means of demonstrating analysis, Min et al. (2021) intended a contract that coordinates green R&D and publicity cost sharing in each direction. In 2020, Glock et al. (2020) examined a supply chain

that included an organization that began production upon receiving an order from a retailer and a retailer of short-life products with stochastic customer demand. The model they proposed could be used to determine the ideal amount of orders and production ability in both concentrated and distributed environments, as well as to coordinate an autonomous supply chain. Profitability of the supply chain facilitates utilizing a buyback agreement. Li and Gong (2020) developed a comprehensive closed-loop supply chain model that features two upstream suppliers who possess competitive advantages, along with a downstream remanufacturer. The primary objective of this research was to examine how the incorporation of dual-source supply strategies and low-carbon manufacturing practices could impact the overall efficiency and sustainability of the closed-loop supply chain system. To achieve effective coordination within the supply chain, the researchers implemented a cost-sharing agreement, which serves as an innovative approach to align the interests of different participants within the supply chain framework. Cheng et al. (2022) made significant contributions to the understanding of green supply chains by formulating a differential game model specifically aimed at facilitating research and development efforts in green technologies. Liu et al. (2021) study of the logistics service supply chain's coordination mechanism from the standpoint of the client experience level demonstrated the constraints of quality control and repurchase contracts in supply chain coordination through modeling. restrictions on supply chain coordination imposed by regulations and two innovatively suggested workable coordination methods for LSSC. While this type of scholarship examines the impacts of government subsidies, market environmental consciousness, and information exchange, and greenness on manufacturers' green production and retailers' green marketing decisions, most scholars pay more attention to suppliers' and manufacturers' green production practices and seldom consider the role of retailers as a driving force for the development of green supply chains, and even fewer literature study the impacts of manufacturers' and retailers' bilateral efforts (green production efforts, green marketing efforts) of manufacturers and retailers on green supply chain decisions. However, retailers have the best understanding of market demand and consumer preferences and can stimulate consumers' purchasing desire and increase market demand through green marketing promotional behaviors. This will also directly impact manufacturers' green inputs and pricing decisions.

In light of this, this paper creates a two-level supply chain game model with just one manufacturer and an alone retailer, and then uses Stackelberg's game concept and approach to investigate the game issue among the green supply chain members and determine the equilibrium solution. It also methodically thinks about the dynamic shifts of product greenness and marketing effort. Matlab is used to validate the arithmetic examples based on this.

3. Problem description and underlying assumptions

This essay discusses a secondary supply chain that includes a retailer and a producer. Since collaboration between manufacturers and retailers may more accurately represent the total worth of the supply chain system, because of the existence of the "double marginalization effect." it is difficult to actualize the overall cooperation choice between manufacturers and retailers spontaneously. Therefore, in order to recognize the collaboration between producers and merchants, this study takes contractual coordination into consideration. This research divides the contractual coordination under study into two scenarios. The first is the cost-sharing contract, which entails examining and assessing how the supply chain would be coordinated if the manufacturer paid for the retailer's green marketing initiatives directly. The other is the two-part system contract, which entails assessing how well the supply chain is coordinated in the event that the retailer provides the manufacturer with a discount on wholesale prices and the manufacturer reimburses the retailer. This paper is predicated on the following assumptions:

Assumption 1: In the Stackelberg game, the manufacturer plays the role of a leader and the retailer plays the role of a follower in the supply chain examined in this study, which is a secondary supply chain made up of a single manufacturer and one retailer, and the information between the node members is entirely symmetric, and in the secondary supply chain, the market demand is a linear function consisting of the selling price p , and the greenness of the product ξ (Shang et al., 2020; Lou et al., 2020; Zhou et al., 2020).This paper adds the retailer's marketing effort based on the previous research τ .Let the increment of product greenness after manufacturer's green production effort and retailer's green marketing effort be $\Delta\xi$ and the increment of marketing effort be $\Delta\tau$, then the demand function is:

$$D = Q - ap + b(\xi_0 + \triangle\xi) + d(\tau_0 + \triangle\tau) \tag{1}$$

where Q denotes the market capacity, the demand in the market decreases when the product's selling price increases; rather, as the product becomes more environmentally friendly, so does consumer demand, which also rises in response to retailer marketing initiatives.

Assumption 2: The manufacturer makes a one-time investment in green production. The functional relationship between the incremental greenness level of the product $\Delta\xi$ and the new green production cost c_1 is $c_1 = \sigma\Delta\xi^2 / 2$ (Zhang et al., 2021), $\partial c_1 / \partial\Delta\xi > 0$, $\partial^2 c_1 / \partial\Delta\xi^2 > 0$, i.e., the green production cost is increasing rapidly with the level of product greenness.

Assumption 3: A retailer makes a one-time investment in green marketing. The functional relationship between the

incremental marketing effort $\Delta\tau$ and the new green marketing cost c_2 is $c_2 = \nu\Delta\tau^2 / 2$ (Zhang et al., 2021), $\partial c_2 / \partial\Delta\tau > 0$, $\partial^2 c_2 / \partial\Delta\tau^2 > 0$, i.e., green marketing costs are increasing rapidly with the level of marketing effort.

Assumption 4: Decision makers consider economic and environmental goals and social goals, such as social welfare, when formulating strategies. In this paper, the expression for social welfare is expressed as the sum of supply chain system profits and consumer surplus(Zhou et al., 2020), without considering other factors (e.g., factors such as government carbon tax revenues and corporate carbon emissions, etc.), i.e., $sw = \pi_{sc} + cs$, where

$$\pi_{SC} = \pi_m + \pi_r \tag{2}$$

$$cs = \int_{p}^{p_{\max}} [Q - ap + b(\xi_0 + \triangle\xi) + d(\tau_0 + \triangle\tau)]dp = \frac{D^2}{2a} \tag{3}$$

4. Game modeling and analysis

In this section, the model will be examined from three perspectives. The first is the no-contract coordination condition. The second is the cost-sharing contract coordination condition, and the third is the two-part contract coordination condition. The model is solved using backward regression for the three cases. Finally, the equilibrium solutions for the three cases are compared.

4.1 No contractual coordination

(1) Decentralized situation (YD situation)

In a scenario where a manufacturer incorporates environmentally friendly production inputs and a retailer adopts green marketing strategies, a sequence of decisions unfolds that significantly impacts the overall supply chain dynamics. Initially, the manufacturer makes a critical decision regarding the wholesale price for product w. Alongside this, the manufacturer assesses the degree of incremental environmental sustainability that the product achieves—referred to here as $\triangle\xi$ —following the integration of the additional green production resources. Once the manufacturer has established these parameters, the retailer is then positioned to determine the retail price of product p. This decision is influenced by the manufacturer's previous choices, especially the chosen wholesale price and the level of greenness attained. Furthermore, the retailer also decides on the extent of additional marketing efforts—deemed $\triangle\tau$ —aimed at promoting the product after incorporating other green marketing initiatives. Right now, the value chain system, retailer, and manufacturer's profit functions are as follows:

$$\pi_m^{YD} = (w - c)(Q - ap + b(\xi_0 + \Delta\xi) + d(\tau_0 + \Delta\tau)) - \frac{1}{2}\sigma\Delta\xi^2 \tag{4}$$

$$\pi_r^{YD} = (p - w)(Q - ap + b(\xi_0 + \Delta\xi) + d(\tau_0 + \Delta\tau)) - \frac{1}{2}\nu\Delta\tau^2 \tag{5}$$

$$\pi_{sc}^{YD} = (p - c)(Q - ap + b(\xi_0 + \Delta\xi) + d(\tau_0 + \Delta\tau)) - \frac{1}{2}\sigma\Delta\xi^2 - \frac{1}{2}\nu\Delta\tau^2 \tag{6}$$

The Hessian matrix is obtained by solving Eq. (5):

$$H_1 = \begin{bmatrix} -2a & d \\ d & -v \end{bmatrix} \tag{7}$$

From Eq. (7), the first order sequential principal subequation of H_1 is $\Delta r^1 < 0$. When the second order sequential principal subequation satisfies $\Delta r^2 = 2av - d^2 > 0$, $H_1(p, \Delta\tau)$ is negatively determined with respect to p and $\Delta\tau$, then π_r^{YD} is a joint concave function of the selling price p and the marketing effort $\Delta\tau$. Taking the first order partial derivatives of π_r^{YD} with respect to p and $\Delta\tau$ respectively and making them equal to zero gives:

$$p = \frac{Qv + awv - d^2 w + bv(\xi_0 + \Delta\xi) + dv\tau_0}{2av - d^2} \tag{8}$$

$$\Delta\tau = \frac{d(Q - aw + b(\xi_0 + \Delta\xi) + d\tau_0)}{2av - d^2} \tag{9}$$

Substituting p and $\Delta\tau$ into Eq. (4) and solving for the Hessian matrix for Eq. (4) yields,

$$H_2 = \begin{bmatrix} -\frac{-2a^2 v}{2av - d^2} & \frac{abv}{2av - d^2} \\ \frac{abv}{2av - d^2} & -\sigma \end{bmatrix} \tag{10}$$

Observing Eq. (10), we can see that the first order principal subequation of H_2 is $\Delta m^1 < 0$, and when the second order principal subequation of $\Delta m^2 = 4a\sigma v - b^2 v - 2d^2\sigma > 0$ is used, $H_2(w, \Delta\xi)$ is negatively determined with respect to w and $\Delta\xi$, and π_m^{YD} is the joint concave function of wholesale price w and $\triangle\xi$. Taking the first order partial derivatives of π_m with respect to w and $\Delta\xi$ respectively and making them equal to zero, the optimal product wholesale price w^{YD*} and the optimal product greenness increment $\Delta\xi^{YD*}$ are obtained as follows:

$$w^{YD*} = c + \frac{(2a\sigma v - d^2\sigma)(Q - ac + b\xi_0 + d\tau_0)}{a(4a\sigma v - b^2 v - 2d^2\sigma)} \tag{11}$$

$$\Delta\xi^{YD*} = \frac{bv(Q - ac + b\xi_0 + d\tau_0)}{4a\sigma v - b^2 v - 2d^2\sigma} \tag{12}$$

Substituting w^{YD*} and $\Delta\xi^{YD*}$ into the expressions for p and $\Delta\tau$ yields the optimal green product selling price p^{YD*} and the optimal marketing effort increment $\Delta\tau^{YD*}$, respectively:

$$p^{YD*} = c + \frac{(3a\sigma v - d^2\sigma)(Q - ac + b\xi_0 + d\tau_0)}{a(4a\sigma v - b^2 v - 2d^2\sigma)} \tag{13}$$

$$\Delta\tau^{YD*} = \frac{d\sigma(Q - ac + b\xi_0 + d\tau_0)}{4a\sigma v - b^2 v - 2d^2\sigma} \tag{14}$$

Substituting w^{YD*}, $\Delta\xi^{YD*}$, p^{YD*}, and $\Delta\tau^{YD*}$ into Eqs. (4-6) respectively, we can obtain the optimal profit, optimal consumer surplus, and optimal total social welfare of the manufacturer, retailer, and supply chain system, respectively:

$$\pi_m^{YD*} = \frac{\sigma v(Q - ac + b\xi_0 + d\tau_0)^2}{2(4a\sigma v - b^2 v - 2d^2\sigma)} \tag{15}$$

$$\pi_r^{YD*} = \frac{\sigma^2 v(2av - d^2)(Q - ac + b\xi_0 + d\tau_0)^2}{2(4a\sigma v - b^2 v - 2d^2\sigma)^2} \tag{16}$$

$$\pi_{sc}^{YD*} = \frac{\sigma v(6a\sigma v - b^2 v - 3d^2\sigma)(Q - ac + b\xi_0 + d\tau_0)^2}{2(4a\sigma v - b^2 v - 2d^2\sigma)^2} \tag{17}$$

$$cs^{YD*} = \frac{a\sigma^2 v^2(Q - ac + b\xi_0 + d\tau_0)^2}{2(4a\sigma v - b^2 v - 2d^2\sigma)^2} \tag{18}$$

$$sw^{YD*} = \frac{\sigma v(7a\sigma v - b^2 v - 3d^2\sigma)(Q - ac + b\xi_0 + d\tau_0)^2}{2(4a\sigma v - b^2 v - 2d^2\sigma)^2} \tag{19}$$

(2) Concentration condition (YC condition)

Consider a condition in which manufacturers and retailers make centralized decisions when manufacturers add green production inputs and retailers add green marketing inputs. The manufacturer and the retailer jointly decide on the selling price of the product p , the incremental greenness of the product $\Delta\xi$ and the incremental marketing effort $\Delta\tau$ .In this case, the profit function of the supply chain system is expressed as:

$$\pi_{sc}^{YC} = (p-c)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-\frac{1}{2}\sigma\Delta\xi^2-\frac{1}{2}v\Delta\tau^2 \quad (20)$$

The Hessian matrix is obtained by solving Eq. (20):

$$H_3=\begin{bmatrix} -2a & b & d \\ b & -\sigma & 0 \\ d & 0 & -v \end{bmatrix} \quad (21)$$

From Eq. (21), we get the first-order sequential principal subequation $\Delta_{SC}^1<0$ of H_3 . When $\Delta_{SC}^2=2a\sigma-b^2>0$ and $\Delta_{SC}^3=-(2a\sigma v-b^2v-d^2\sigma)<0$ are satisfied, the Hessian matrix is negative definite, i.e., it can be known that π_{SC}^{YC} is the joint concave function of the sales price p ,the incremental product greenness $\Delta\xi$ and the incremental marketing effort $\Delta\tau$. Take π_{SC}^{YC} as the first order partial derivative of sales price p , product greenness increment $\Delta\xi$ and marketing effort increment $\Delta\tau$ respectively, and make it equal to 0, and then we can get the joint solution:

$$p^{YC*}=c+\frac{\sigma v(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \quad (22)$$

$$\Delta\xi^{YC*}=\frac{bv(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \quad (23)$$

$$\Delta\tau^{YC*}=\frac{d\sigma(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \quad (24)$$

Substituting Eqs. (22-24) into Eq. (21), respectively, yields the optimal profit, consumer surplus, and optimal total social welfare of the supply chain system, respectively:

$$\pi_{sc}^{YC*}=\frac{\sigma v(Q-ac+b\xi_0+d\tau_0)^2}{2(2a\sigma v-b^2v-d^2\sigma)} \quad (25)$$

$$cs^{YC*}=\frac{\sigma^2v^2a(Q-ac+b\xi_0+d\tau_0)^2}{2(2a\sigma v-b^2v-d^2\sigma)^2} \quad (26)$$

$$sw^{YC*}=\frac{\sigma v(3a\sigma v-b^2v-d^2\sigma)(Q-ac+b\xi_0+d\tau_0)^2}{2(2a\sigma v-b^2v-d^2\sigma)^2} \quad (27)$$

4.2 Cost Sharing Compact Harmonization situation (CC situation)

In the context of a cost-sharing contract coordination model, it is important to examine a specific arrangement in which the manufacturer takes on a portion of the green marketing expenses incurred by the retailer. Within this framework, we define the portion of green marketing costs that the retailer is responsible for as t , where this value is constrained within the range of 0 to 1 ($0\le t\le 1$). Conversely, the portion that is allocated to be borne by the manufacturer is designated as $1-t$. As the first step in this coordinated effort, the manufacturer must determine several key factors. Specifically, the manufacturer establishes the wholesale price of the product, denoted as *w*, and simultaneously evaluates the incremental level of environmental sustainability achieved by the product, referred to as $\triangle\xi$, once green production inputs have been implemented. Furthermore, the manufacturer is tasked with deciding the extent of green marketing costs that it is willing to share with the retailer, which is represented as $1-t$. Once the manufacturer has completed these critical decisions, the process moves on to

the retailer. The retailer then takes on the responsibility of determining the retail selling price of the product, labeled as p, while also deciding on the level of incremental marketing effort that will be applied, denoted as $\triangle\tau$, in order to enhance the product's market presence and consumer appeal.

$$\pi_m^{CC} = (w-c)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-\frac{1}{2}\sigma\Delta\xi^2-(1-t)\frac{1}{2}v\Delta\tau^2 \tag{28}$$

$$\pi_r^{CC} = (p-w)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-t\frac{1}{2}v\Delta\tau^2 \tag{29}$$

Solving the Hessian matrix for π_r^{CC} yields

$$H_4 = \begin{bmatrix} -2a & d \\ d & -tv \end{bmatrix} \tag{30}$$

From the Hessian matrix, we know that the first order sequential principal subequation -2a<0,and when $t > \frac{d^2}{2av}$, at this time $\Delta_{CC}^1 = 2atv - d^2 > 0$, the second order sequential principal subequation of the Hessian matrix is greater than 0, at this time the Hessian matrix is negatively definite, which can be seen that π_r^{CC} is a strictly joint concave function on the sales price p and the incremental marketing effort $\Delta\tau$, which suggests that π_r^{CC} is maximized, so it is derived that

$$p = \frac{Qtv + atwv - d^2w + btv(\xi+\xi_0) + dtv\tau_0}{2atv - d^2} \tag{31}$$

$$\Delta\tau = \frac{d(Q-aw+b(\xi_0+\Delta\xi)+d\tau_0)}{2atv-d^2} \tag{32}$$

Substituting the sales price p and the incremental marketing effort $\Delta\tau$ into π_m^{CC} and solving the Hessian matrix for π_m^{CC} yields

$$H_5 = \begin{bmatrix} \frac{-a^2v(4at^2v+d^2(1-3t))}{(2atv-d^2)^2} & \frac{abv(2at^2v+d^2(1-2t))}{(2atv-d^2)^2} \\ \frac{abv(2at^2v+d^2(1-2t))}{(2atv-d^2)^2} & -\sigma+\frac{b^2d^2v(t-1)}{(2atv-d^2)^2} \end{bmatrix} \tag{33}$$

From the Hessian matrix, the first order sequential principal subtype $\frac{-a^2v\left(4at^2v+d^2(1-3t)\right)}{\left(2atv-d^2\right)^2}<0$, and when $t < \frac{2d^2\sigma}{3d^2\sigma+\sqrt{d^2\sigma\left(-16a\sigma v+4b^2v+9d^2\sigma\right)}}$, at this time $\triangle_{CC}^2 = \frac{a^2v(t^2v(4a\sigma-b^2)+d^2\sigma(1-3t))}{(2atv-d^2)^2}$, the second order sequential principal subtype of the Hessian matrix is greater than 0, at this time the Hessian matrix is negatively definite, which shows that π_m^{CC} is a strictly joint concave function with respect to the wholesale price w and the incremental greenness of the product $\Delta\xi$, which suggests that maximizing the manufacturer's profit is possible, which results in w^{CC*}, $\Delta\xi^{CC*}$. When the manufacturer and retailer maximize their profits, the optimal selling price p^{CC*}, the optimal wholesale price w^{CC*}, the optimal product greenness increment $\Delta\xi^{CC*}$, the optimal marketing effort increment $\Delta\tau^{CC*}$, the optimal retailer profit π_r^{CC*}, the optimal manufacturer profit π_m^{CC*}, the optimal supply chain system profit π_{sc}^{CC*}, the consumer surplus cs^{CC*},and the optimal social welfare function sw^{CC*} are respectively:

$$p^{CC*} = \frac{B+3at^2v\sigma(Q+b\xi_0+d\tau_0)}{a(t^2v(4a\sigma-b^2)+d^2\sigma(1-3t))} \tag{34}$$

$$\Delta\tau^{CC*} = \frac{dt\sigma(Q-ac+b\xi_0+d\tau_0)}{t^2v(4a\sigma-b^2)+d^2\sigma(1-3t)} \tag{35}$$

$$w^{CC*} = \frac{B^2+a^2ct^2\sigma v+2at^2\xi v(Q+b\xi_0+d\tau_0)}{a(t^2v(4a\sigma-b^2)+d^2\sigma(1-3t))} \tag{36}$$

$$\Delta\xi^{CC*} = \frac{bt^2v(Q-ac+b\xi_0+d\tau_0)}{t^2v(4a\sigma-b^2)+d^2\sigma(1-3t)} \tag{37}$$

$$\pi_r^{CC*} = \frac{t^3\sigma v(2a\sigma tv-d^2\sigma)(Q-ac+b\xi_0+d\tau_0)^2}{2(t^2(4a\sigma v-b^2v)+d^2\sigma(1-3t))^2} \tag{38}$$

$$\pi_{sc}^{CC*} = \frac{t^2\sigma v(t^2(6a\sigma v-b^2v)+d^2\sigma(1-4t))(Q-ac+b\xi_0+d\tau_0)^2}{2(t^2(4a\sigma v-b^2v)+d^2\sigma(1-3t))^2} \tag{39}$$

$$\pi_{sc}^{CC*} = \frac{t^2\sigma v(t^2(6a\sigma v-b^2v)+d^2\sigma(1-4t))(Q-ac+b\xi_0+d\tau_0)^2}{2(t^2(4a\sigma v-b^2v)+d^2\sigma(1-3t))^2} \tag{40}$$

$$cs^{CC*} = \frac{at^4\sigma^2v^2(Q-ac+b\xi_0+d\tau_0)^2}{2(t^2(4a\sigma v-b^2v)+d^2\sigma(1-3t))^2} \tag{41}$$

$$sw^{CC*} = \frac{t^2\sigma v(t^2(7a\sigma v-b^2v)+d^2\sigma(1-4t))(Q-ac+b\xi_0+d\tau_0)^2}{2(t^2(4a\sigma v-b^2v)+d^2\sigma(1-3t))^2} \tag{42}$$

Among others, $B = a^2ct^2\sigma v+d^2\sigma(1-2t)(Q+b\xi_0+d\tau_0)-act(b^2tv+d^2\sigma)$.

4.3 Two-tier compact harmonization situation (TT situation)

In the framework of a two-part contractual coordination model, a distinctive relationship is established. In this arrangement, the retailer is required to pay a fixed subsidy fee, denoted as F, to the manufacturer during each distinct planning cycle. In return for this financial commitment, the manufacturer agrees to supply the retailer with the products at a reduced wholesale price, which is strategically lower than the market average. This particular contractual structure is designed to enhance cooperation between the parties involved by aligning their respective interests. It accomplishes this by adjusting the profit functions associated with both the retailer and the manufacturer. In the process, the retailer first takes the initiative by determining the selling price of the product, which is represented as p, within the context of the two-part contract framework. This decision is crucial as it directly influences the retailer's profit margins and market competitiveness. Following the retailer's decision, the manufacturer then establishes the wholesale price of the product, denoted as w, and subsequently charges the retailer the predetermined subsidy fee F. The profit functions of the manufacturer, the retailer, and the supply chain system are respectively:

$$\pi_m^{TT} = (w-c)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-\frac{1}{2}\sigma\Delta\xi^2+F \tag{43}$$

$$\pi_r^{TT} = (p-w)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-\frac{1}{2}v\Delta\tau^2-F \tag{44}$$

$$\pi_{sc}^{TT*} = (p-c)(Q-ap+b(\xi_0+\Delta\xi)+d(\tau_0+\Delta\tau))-\frac{1}{2}\sigma\Delta\xi^2-\frac{1}{2}v\Delta\tau^2 \tag{45}$$

It can be produced by applying the prerequisites for the existence of an optimal solution, which are as follows:

$$p^{TT*} = c+\frac{\sigma v(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \tag{46}$$

$$\Delta\tau^{TT*} = \frac{d\sigma(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \tag{47}$$

$$\Delta\xi^{TT*} = \frac{bv(Q-ac+b\xi_0+d\tau_0)}{2a\sigma v-b^2v-d^2\sigma} \tag{48}$$

When we set the initial rank derivative of p in Eq. (33) to 0, we obtain:

$$p = \frac{Qv + awv - wd^2 + bv(\xi_0 + \Delta\xi) + dv\tau_0}{2av - d^2} \tag{49}$$

Let $p^{TT*} = p$ have $w^{TT*} = c$

Substituting the above equations into π_r^{TT} respectively, the following conditions need to be satisfied since the retailer's profit level cannot be lower after the coordination of the two-part contract than in the absence of the contract:

$$\pi_r^{TT*} = \frac{\sigma^2 v(2av - d^2)(Q - ac + b\xi_0 + d\tau_0)^2}{2(2a\sigma v - b^2 v - d^2\sigma)^2} - F > \pi_r^{YD*} \tag{50}$$

$$\pi_m^{TT*} = -\frac{b^2\sigma v^2(Q - ac + b\xi_0 + d\tau_0)^2}{2(2a\sigma v - b^2 v - d^2\sigma)^2} + F \geq \pi_m^{YD*} \tag{51}$$

$$\frac{\sigma v(Q - ac + b\xi_0 + d\tau_0)^2}{2}\left[\frac{b^2 v}{2a\sigma v - b^2 v - d^2\sigma} + \frac{1}{4a\sigma v - b^2 v - 2d^2\sigma}\right] = F_{\min} < F < F_{\max}$$

$$= \frac{\sigma^2 v(2av - d^2)(Q - ac + b\xi_0 + d\tau_0)^2}{2}\left[\frac{1}{(2a\sigma v - b^2 v - d^2\sigma)^2} - \frac{1}{(4a\sigma v - b^2 v - 2d^2\sigma)^2}\right]$$

The optimal supply chain system profit, consumer surplus and total social welfare can be obtained as:

$$\pi_{sc}^{TT*} = \frac{\sigma v(Q - ac + b\xi_0 + d\tau_0)^2}{2(2a\sigma v - b^2 v - d^2\sigma)} \tag{52}$$

$$cs^{TT*} = \frac{a\sigma^2 v^2(Q - ac + b\xi_0 + d\tau_0)^2}{2(2a\sigma v - b^2 v - d^2\sigma)^2} \tag{53}$$

$$sw^{TT*} = \frac{\sigma v(3a\sigma v - b^2 v - d^2\sigma)(Q - ac + b\xi_0 + d\tau_0)^2}{2(2a\sigma v - b^2 v - d^2\sigma)^2} \tag{54}$$

4.4 Analysis of the results of the study

Proposition1: In a two-tier green supply chain considering green production-marketing efforts, compare the no contractual coordination case (YD model) with the cost-sharing contractual coordination case (CC model), in which the cost-sharing contract does not coordinate well with the green coordination of manufacturers and retailers in the green supply chain.

Prove that: $\pi_m^{CC} - \pi_m^{YD} = -\frac{d^2\sigma^2 v(Q - ac + b\xi_0 + d\tau_0)^2 g(t)}{2(4a\sigma v - b^2 v - 2d^2\sigma)(t^2 v(4a\sigma - b^2) + d^2\sigma(1 - 3t))}$, and

$$\pi_r^{CC} - \pi_r^{YD} = \Phi + \frac{(-2av + d^2)(4a\sigma v - b^2 v - d^2\sigma(\frac{3}{t} - \frac{1}{t}))^2 - (-2av + \frac{d^2}{t})(4a\sigma v - b^2 v - 2d^2\sigma)^2}{(4a\sigma v - b^2 v + d^2\sigma(\frac{1}{t^2} - \frac{3}{t}))^2(4a\sigma v - b^2 v - 2d^2\sigma)^2}$$, where

$\Phi = \frac{1}{2}\sigma^2 v(Q - ac + b\xi_0 + d\tau_0)^2$.

For $g(t) = (t-1)(2t-1)$,when $\frac{1}{2} < t < 1$,there is $g(t) < 0$ .At this point, $\pi_m^{CC} - \pi_m^{YD} > 0$ is $\pi_m^{CC} > \pi_m^{YD}$.

However, when $\frac{1}{2} < t < 1$,there are $\frac{d^2}{t} > d^2$ and $\frac{3}{t} - \frac{1}{t^2} > 2$, so $(-2av + d^2) < (-2av + \frac{d^2}{t})$ and $(4a\sigma v - b^2 v - d^2\sigma(\frac{3}{t} - \frac{1}{t^2}))^2 < (4a\sigma v - b^2 v - 2d^2\sigma)^2$. So $\pi_r^{CC} - \pi_r^{YD} < 0$, i.e. $\pi_r^{CC} < \pi_r^{YD}$.

Proposition 1 posits that in the context of a green supply chain led by the manufacturer, the implementation of a cost-sharing contract fails to improve the profits of both the manufacturer and the retailer. This lack of enhancement in profitability can be

attributed to several factors. Notably, when such a contract is put into place, it tends to increase the costs associated with the retailer's green marketing efforts. As a result, these heightened input costs may negate any potential financial benefits that the retailer could gain from the coordination stipulated by the cost-sharing contract. Furthermore, the shortcomings of the cost-sharing contract can be traced to the manufacturer's approach to sharing these increased green marketing input costs. Specifically, if the manufacturer does not sufficiently distribute the costs related to the retailer's green marketing efforts, it can lead to a detrimental outcome for the retailer. Conversely, if the manufacturer overcompensates by assuming too large a portion of these costs, it may also result in unfavorable conditions for the retailer's profitability. Such imbalances can ultimately hinder the overall functionality of the cost-sharing contract.

Proposition 2: From the above solution process, if the conditions below are satisfied:

(1) The manufacturer's wholesale price satisfies $w = c$;
(2) The retailer pays the manufacturer a subsidized fee $F \in \left[F_{\min}, F_{\max}\right]$ range.

Then manufacturers and retailers can harmonize the green supply chain by signing a two-part contract for a win-win situation. Proposition2 shows that the wholesale price charged by the manufacturer is equal to the unit cost of production, which, being sufficiently low, provides an incentive for retailers to set lower selling prices, thereby increasing market demand. In order to ensure the active participation of retailers in the contract, it is necessary to establish an upper limit on the subsidy cost. This particular upper bound serves as a crucial incentive, motivating retailers to engage in the contractual agreement. On the other hand, the establishment of a lower limit for the subsidy cost is equally important as it guarantees that the costs associated with the subsidy remain non-negative. During these negotiations, both parties engage in discussions to determine the value of a specific contract parameter, referred to as F, which represents the subsidy amount that the retailer is required to transfer to the manufacturer. The size and magnitude of this subsidy cost, F, are influenced by various factors, including the relative bargaining power possessed by each party—namely the manufacturer and the retailer. This relative power dynamic directly affects how the costs and benefits of the contract are distributed.

5. Numerical case analysis

In this section, to further analyze the model and validate the conclusions, numerical simulation is used for sensitivity analysis and optimal decision comparison analysis, and the parameters in this paper are assigned as $Q = 500$, $a = 8$, $c = 20$, $\xi_0 = 2$, $\tau_0 = 2$, $\sigma = 10$, $v = 10$.

5.1 Sensitivity analysis

(1) The effect of *t* on the optimal decision of the supply chain
The values of the decision variables in the cost-sharing contract coordination condition when *t* varies in the range of 0 to 1 are shown in Table 1.

Table 1
Table of values of decision variables for each different decision model under t-invariance

t	0	0.25	0.5	0.75	1
p^{CC*}	64.50	64.50	59.26	57.43	56.65
$\Delta\xi^{CC*}$	0	3.87	5.24	5.30	5.24
$\Delta\tau^{CC*}$	0	15.48	10.47	7.07	5.24
π_r^{CC*}	0	449.21	1096.33	1219.37	1233.37
π_m^{CC*}	0	1721.96	2329.71	2360.56	2329.71
π_{sc}^{CC*}	0	2171.17	3426.04	3579.93	3563.08
sw^{CC*}	0	2545.50	4111.25	4283.41	4248.29

As shown in Table 1, as the proportion of green marketing costs *t* borne by retailers gradually increases, the selling price of green products p^{CC*} decreases, the incremental marketing effort $\Delta\tau^{CC*}$ decreases, and the manufacturer's profit π_m^{CC*}, the incremental greenness of products $\Delta\xi^{CC*}$, the retailer's profit π_r^{CC*}, and the value of social welfare sw^{CC*} first increase and then decrease.

(2) Impact of σ and v on optimal supply chain decision making

When the product greenness cost coefficient σ is not fixed, set the marketing effort cost coefficient $v = 10$, which shows that

Table 2
σ Table of values of decision variables for each different decision model under uncertainty

σ	0	5	10	15	20
p^{YD*}	20	58.94	56.65	55.94	55.6
p^{CC*}	20	61.71	59.26	58.51	58.14
p^{TT*}	20	51.79	47.81	46.70	46.18
$\Delta\xi^{YD*}$	-89	11.13	5.24	3.42	2.54
$\Delta\xi^{CC*}$	-89	11.13	5.24	3.42	2.54
$\Delta\xi^{\mathrm{TT}*}$	-89	25.43	11.13	7.12	5.24
$\Delta\tau^{YD*}$	0	5.57	5.24	5.13	5.08
$\Delta\tau^{CC*}$	0	11.14	10.48	10.26	10.16
$\Delta\tau^{\mathrm{TT}*}$	0	12.71	11.13	10.68	10.47
π_{sc}^{YD*}	0	3867.68	3563.08	3471.30	3427.04
π_{sc}^{CC*}	0	3712.97	3426.04	3339.47	3297.72
π_{sc}^{TT*}	0	5657.86	4950.63	4752.60	4659.41
sw^{YD*}	0	4641.21	4248.29	4130.40	4073.66
sw^{CC*}	0	4486.50	4111.25	3998.58	3944.33
$sw^{\mathrm{TT}*}$	0	9699.19	7766.30	7604.16	7400.24

When the marketing effort cost coefficient v is not fixed, set the product greenness cost coefficient $\sigma = 10$, which shows that

Table 3
v Table of values of decision variables for each different decision model under uncertainty

v	0	5	10	15	20
p^{YD*}	42.25	58.57	56.65	56.10	55.85
p^{CC*}	20	64.50	59.26	57.78	57.08
p^{TT*}	20	51.79	47.81	46.70	46.18
$\Delta\xi^{YD*}$	0	5.93	5.24	5.04	4.94
$\Delta\xi^{CC*}$	0	5.93	5.24	5.04	4.94
$\Delta\xi^{\mathrm{TT}*}$	0	25.43	11.13	7.12	5.24
$\Delta\tau^{YD*}$	-44.50	11.87	5.24	3.36	3.12
$\Delta\tau^{CC*}$	-89	23.73	10.47	6.72	4.94
$\Delta\tau^{\mathrm{TT}*}$	-89	25.43	11.13	7.12	5.24
π_{sc}^{YD*}	0	4048.51	3563.08	3426.13	3361.54
π_{sc}^{CC*}	0	3696.47	3426.04	3341.54	3300.42
π_{sc}^{TT*}	0	5657.86	4950.63	4752.60	4659.41
sw^{YD*}	0	4928.62	4248.29	4060.61	3972.72
sw^{CC*}	0	4576.58	4111.25	3976.01	3911.60
$sw^{\mathrm{TT}*}$	0	9699.19	7766.30	7604.16	7400.24

From the analysis presented in Table 2 and Table 3, it becomes apparent that there is a notable trend concerning the various coefficients related to product sustainability and marketing expenditures. Specifically, when the coefficients for the greenness cost of the product or the marketing effort cost are situated within the range of 5 to 20, an increase in either the product greenness sensitivity coefficient or the marketing effort sensitivity coefficient leads to a decline in several key performance

indicators. Moreover, it is important to highlight that under conditions where the two-part contracts are coordinated, the values associated with each decision variable experience a noticeable improvement. This finding underscores the positive impact of contract coordination in optimizing decision-making processes. In stark contrast, the decision variables associated with a cost-sharing contract coordination condition exhibit values that remain strikingly similar to those observed in circumstances where decentralized decision-making is applied. The cost-sharing approach does not facilitate the same level of improvement as observed in the two-part contract coordination scenario. Fig. 1 and Fig. 2 give the effect of product greenness cost coefficients and marketing effort cost coefficients on the sales price of green products.

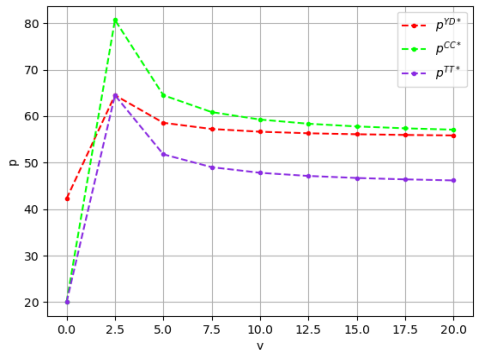

Fig. 1. Sensitivity analysis of σ to p

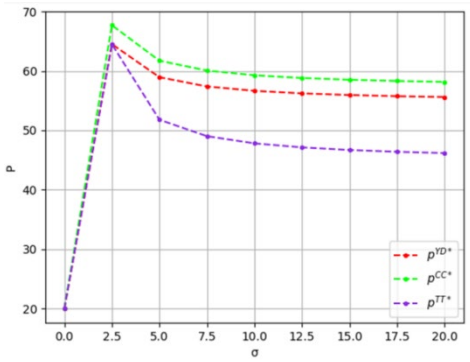

Fig. 2. Sensitivity analysis of v to p

As depicted in Fig. 1 and Fig. 2, it becomes evident that there exists a critical threshold for both the product greenness cost coefficient and the marketing effort cost coefficient, beyond which a significant shift in decision-making strategies within the supply chain occurs. Specifically, when the value of these cost coefficients surpasses 2.5, a notable pattern emerges regarding the selling prices of products under three distinct decision-making conditions: decentralized decision-making, coordinated cost-sharing contracts, and coordinated two-part contracts. Whether in the coordinated cost-sharing contract condition or the coordinated two-part contract condition, this downward trend in selling prices persists. It is particularly noteworthy that under the two-part contract coordination scenario, the rate of decline in selling prices accelerates, indicating a more pronounced sensitivity to cost increases in this contractual arrangement. Fig. 3 and Fig. 4 give the effect of the product greenness cost factor and marketing effort cost factor on product greenness increment.

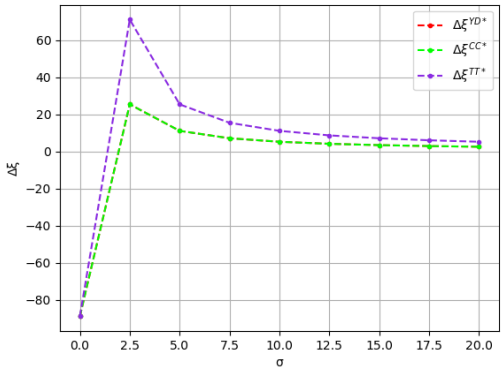

Fig. 3. Sensitivity analysis of σ on $\triangle\xi$

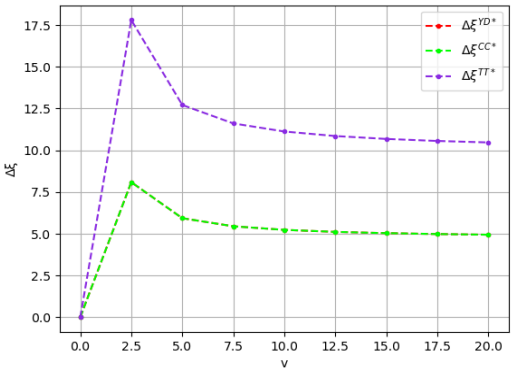

Fig. 4. Sensitivity analysis of v on $\triangle\xi$

Fig. 5 and Fig. 6 give the effect of product greenness cost coefficient and marketing effort cost coefficient on the incremental marketing effort.

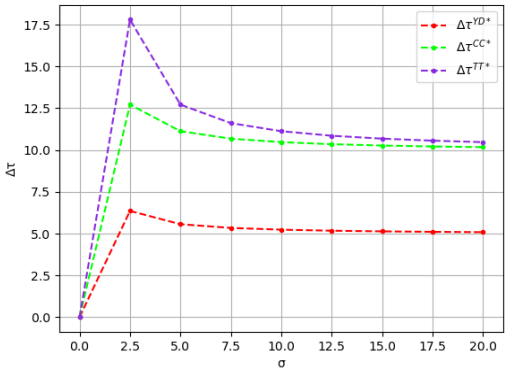

Fig. 5. Sensitivity analysis of σ to $\triangle\tau$

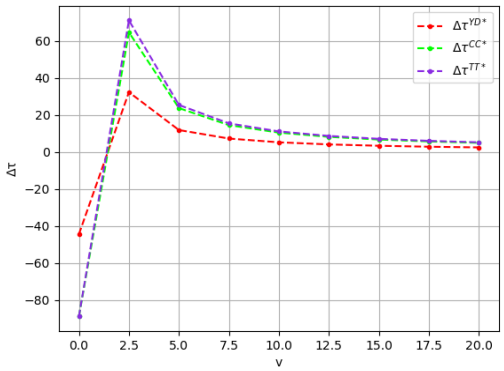

Fig. 6. Sensitivity analysis of v to $\triangle\tau$

Analyzing the data presented in Fig. 3 through Fig. 6 reveals significant insights regarding the relationship between the product greenness cost coefficient and the marketing effort cost coefficient. Specifically, it becomes apparent that when either of these coefficients exceeds the threshold value of 2.5, there are observable changes in the incremental benefits associated with both product greenness and marketing efforts. As the values of the product greenness cost coefficient or the marketing effort cost coefficient rise, a corresponding trend is noted: the incremental product greenness and the incremental marketing effort begin to exhibit a decline. This decline suggests that at higher levels of cost coefficients, the additional investments made in enhancing product sustainability or increasing marketing efforts yield diminishing returns. Such a trend indicates that as manufacturers and retailers allocate greater resources towards these areas, the effectiveness of their incremental advancements is less pronounced. This phenomenon suggests that, within a specific range, investing more resources to increase product greenness or marketing effort does lead to more significant environmental benefits. However, when the cost inputs reach a certain level, further input increases can only result in more minor ecological benefits.

Fig. 7 and Fig. 8 provide valuable insights into the influences exerted by both the product greenness cost factor and the marketing effort cost factor on the overall profitability of the supply chain system. These figures depict a detailed analysis of how variations in the cost factors are associated with product sustainability and marketing.

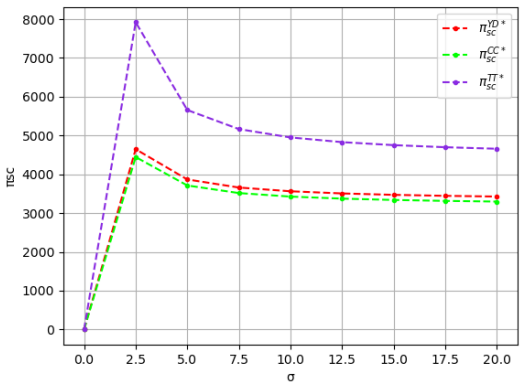

Fig. 7. Sensitivity analysis of σ to π_{sc}

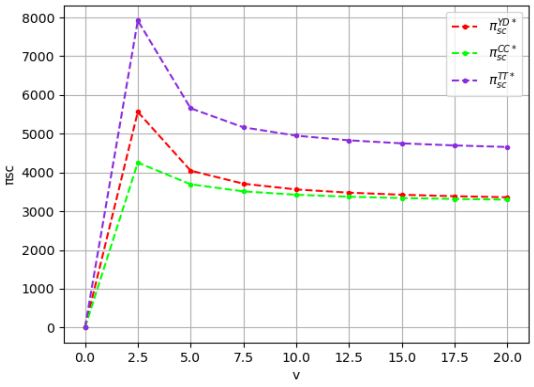

Fig. 8. Sensitivity analysis of v to π_{sc}

As can be seen from Fig. 7 and Fig. 8, when the product greenness cost factor or marketing effort cost factor exceeds 2.5, the supply chain system profit consistently decreases as the product greenness cost factor or marketing effort cost factor continues to increase and is more sensitive to changes in the product greenness cost factor or marketing effort cost factor with smaller values. In examining the various contractual frameworks impacting supply chain dynamics, it becomes clear that the profitability of the supply chain system operating under the two-part contract coordination framework consistently outperforms the profit generated under both decentralized decision-making conditions and the cost-sharing contract coordination arrangements. This indicates that the two-part contract coordination model offers significant advantages in terms of financial outcomes for the supply chain. This indicates that two-part contractual coordination is more effective from the economic efficiency perspective.This phenomenon suggests that under the double pressure of lower selling prices of green products and higher input costs of green production and green marketing, manufacturers and retailers, from the perspective of maximizing their respective profits, give up their inputs of green products to minimize the loss of profits, which leads to the loss of profits of the supply chain system. Fig. 9 and Fig. 10 give the impact of the product greenness cost factor and marketing effort cost factor on social welfare.

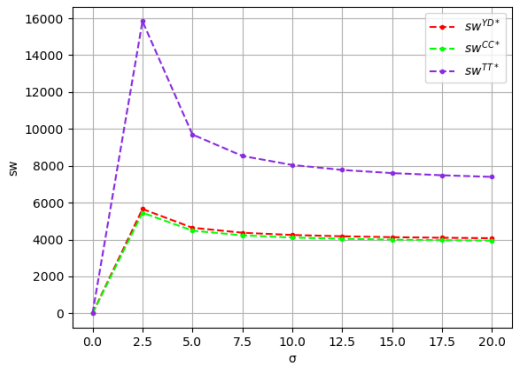

Fig. 9. Sensitivity analysis of σ to sw

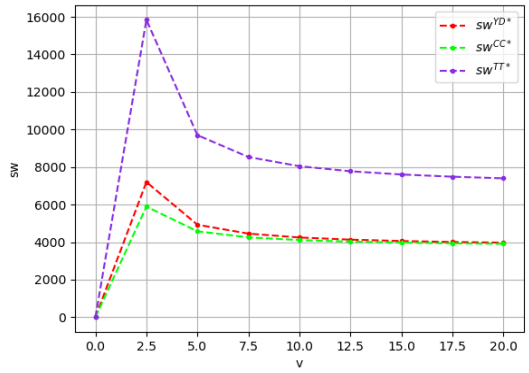

Fig. 10. Sensitivity analysis of v to sw

From the analysis presented in Fig. 9 and Fig. 10, it can be observed that there exists a significant relationship between the product greenness cost coefficient, as well as the marketing effort cost coefficient, and the corresponding value of social welfare. Specifically, when either of these coefficients exceeds the critical threshold of 2.5, the data indicates a notable trend:

the overall social welfare value begins to decline progressively as the product greenness cost coefficient or the marketing effort cost coefficient increases. On the other hand, in the two-part contract coordination condition, the social welfare value is always higher than that of the decentralized decision-making condition and the cost-sharing contract coordination condition, indicating that the two-part contract coordination has better social benefit indicators. With Fig. 1, Fig. 2, Fig. 7, and Fig. 8, it is found that when the input costs of green efforts of both manufacturers and retailers are higher than the economic profits they obtain, manufacturers and retailers will prefer to prioritize their inputs to green products from a financial perspective and reduce their social responsibilities, which leads to a reduction of social welfare, which implies that, no matter what the cost coefficients of the greenness of product or marketing effort cost coefficient take any value, the two-part contract coordination strategy always performs better in terms of social welfare optimization. Therefore, manufacturers and retailers should maximize profits by choosing the two-part contractual coordination cooperation when the input costs of bilateral green efforts are appropriate.

5.2 Comparative analysis of optimal decisions

(1). Comparison of selling price *p of* green products in YD, CC and TT situatio

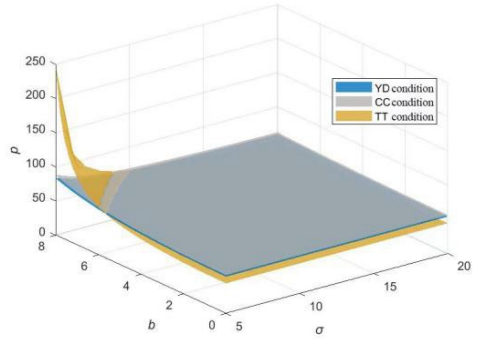

Fig. 11. Relationship between p and b, σ for three cases

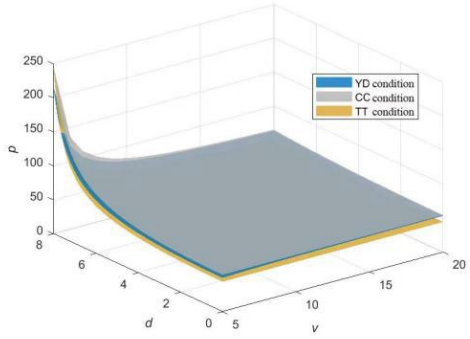

Fig. 12. Relationship between p and d, v for three cases

The analysis reflected in Figures 11 and 12 clearly illustrates a significant trend regarding the sales prices of products in three distinct scenarios: the YD (Yield Distribution), CC (Cost Coordination), and TT (Traditional Transactions) situations. It is evident from the data that, as the consumer sensitivity coefficient associated with product greenness increases, along with the marketing effort sensitivity coefficient and the cost coefficient tied to product greenness and marketing efforts, there is a corresponding rise in the sales prices across all three situations. Notably, when examining the sales prices in the context of the coordinated cost-sharing contract, it becomes apparent that these prices tend to be consistently higher than those observed under the framework of decentralized decision-making. Additionally, the analysis reveals that the prices resulting from decentralized decision-making are, in turn, greater than the sales prices negotiated under the two-part coordination condition.This suggests that retailers sell products to consumers at higher prices in the cost-sharing contractual coordination condition compared to lower prices in the two-part contractual coordination condition.

(2).Comparison of incremental product greenness in YD, CC and TT situation $\Delta\xi$

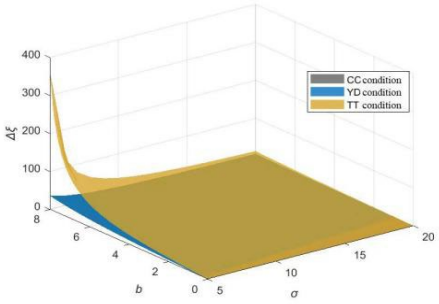

Fig. 13. Relationship between $\triangle\xi$ and b, σ for three situation

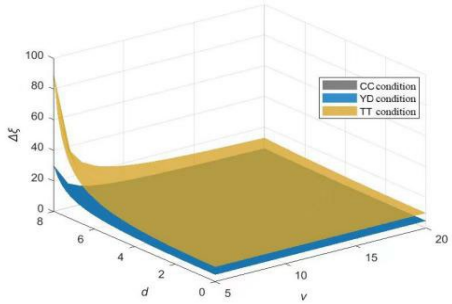

Fig. 14. Relationship between $\triangle\xi$ and d, v for three situation

The observations presented in Fig. 13 and Fig. 14 provide valuable insights into the relationship between product greenness and various influencing factors. Specifically, it becomes apparent that as the consumer's sensitivity coefficient regarding the greenness of products—often referred to as the marketing effort sensitivity coefficient—increases, there is a corresponding rise in the incremental product greenness. This suggests that consumers who are more attuned to the environmental attributes

of products tend to be more responsive to changes in product offerings, thereby enhancing the overall greenness of incremental product developments. This indicates that two-part contractual coordination significantly increases product greenness, while cost-sharing contractual coordination cannot increase product greenness.

(3).Comparison of Marketing Effort Increments in YD, CC and TT situation $\Delta\tau$

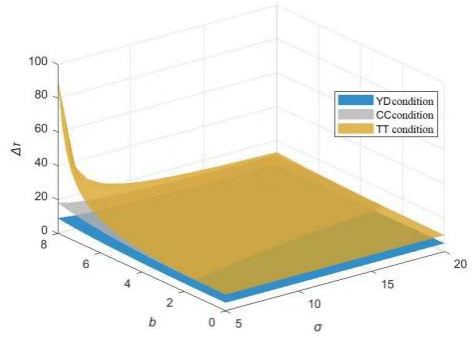

Fig. 15. Relationship between $\Delta\tau$ and b, σ for three situation

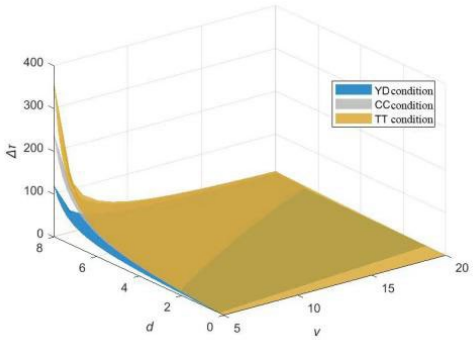

Fig. 16. Relationship between $\Delta\tau$ and d, v for three situation

The analysis illustrated in Figures 15 and 16 reveals several important trends regarding the incremental marketing efforts under different circumstances identified as YD, CC, and TT. Specifically, it is evident that the incremental marketing efforts corresponding to these situations exhibit a clear upward trajectory in response to an increase in the sensitivity coefficient of consumers toward product greenness, which is also referred to as the marketing effort sensitivity coefficient. This finding indicates that as consumers become more attentive and responsive to the environmentally friendly attributes of products, businesses feel compelled to enhance their marketing efforts accordingly to align with consumer expectations.

In contrast to this positive correlation, the incremental marketing effort demonstrates a downward trend when there is an increase in the cost coefficient associated with product greenness, commonly termed as the marketing effort cost coefficient. This suggests that as the costs related to promoting green products rise, the incremental marketing efforts tend to diminish, highlighting a potential barrier for firms aiming to effectively market their sustainable offerings. Such a trend emphasizes the delicate balance that companies must maintain between investing in marketing strategies that resonate with environmentally sensitive consumers and managing the associated costs that can limit these marketing initiatives. This indicates that both two-part contractual coordination and cost-sharing contractual coordination can increase marketing effort, and two-part contractual coordination is more effective.

(4).Comparison of Supply Chain System Profitability in YD, CC and TT situation π_{sc}

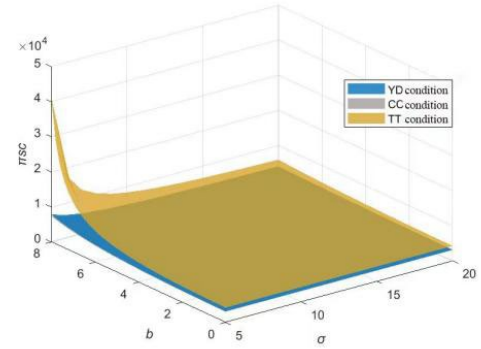

Fig. 17. Relationship between π_{sc} and b, σ for three situation

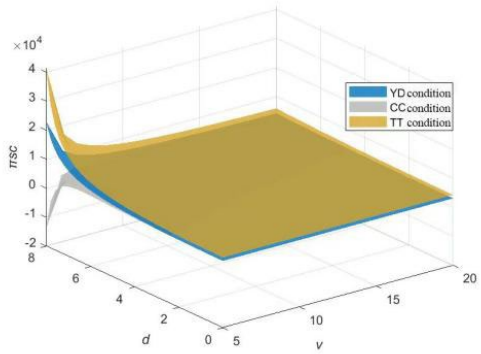

Fig. 18. Relationship between π_{sc} and d, v for three situation

(5).Comparison of social welfare sw in YD, CC and TT situation

The insights derived from Fig. 17 and Fig. 18 provide compelling evidence regarding the impact of variations in two important coefficients associated with consumer behavior and cost structures: the sensitivity coefficient of consumers to product greenness, also referred to as the marketing effort sensitivity coefficient, and the cost coefficient related to product greenness, known as the marketing effort cost coefficient. Upon careful examination of these figures, one can observe a consistent trend indicating that the profit generated by the supply chain system, as well as the overall values of social welfare in circumstances governed by a two-part contract coordination, remain consistently higher than comparable values observed in a decentralized

decision-making framework. More specifically, while analyzing the data presented in Figures 17 and 19, it becomes evident that the profit and social welfare outcomes associated with the cost-sharing contract coordination almost align with those of the decentralized decision-making scenario.

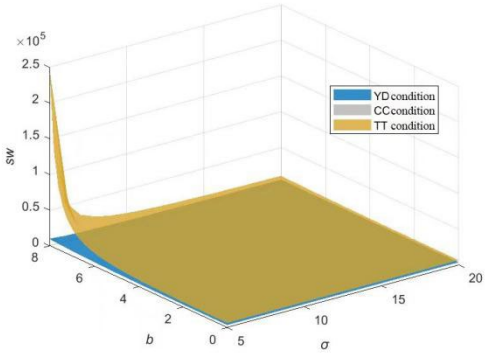

Fig. 19. Relationship between *sw* and b, σ for three cases

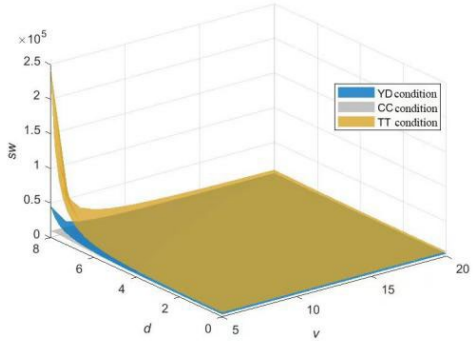

Fig. 20. Relationship between *sw* and d, v for three cases

This observation reveals that the benefits derived from the cost-sharing arrangement do not significantly exceed, and in some cases, closely resemble, the values realized through a less coordinated approach. Furthermore, in Figures 17 and 19, the graphs depicting the performance of the supply chain system illustrate that the values of profit and social welfare are nearly indistinguishable from those found in decentralized decision-making conditions, indicating a lack of substantial improvement.

6. Discussion and Conclusions

This study rigorously examines the effects of optimal decision-making strategies within green supply chains by analyzing three distinct contractual conditions: the scenario in which no contractual agreements are formed, the condition under which cost-sharing contracts are implemented, and the situation that involves two-part contract coordination. From the comprehensive investigation carried out in this research, several key conclusions have been drawn regarding the performance of these contractual approaches.

(1) Specifically, within the context of a manufacturer-dominated Stackelberg game, it has been determined that the introduction of a cost-sharing contract fails to yield Pareto improvements for both the manufacturer and retailer in terms of their profit margins. This means that neither party can enhance their financial outcomes without negatively impacting the other, and an arithmetic analysis reveals that the cost-sharing contract also fails to improve incremental product greenness and social welfare, however, it can improve marketing effort.
(2) In the manufacturer-dominated Stackelberg game, the two-part contract can achieve Pareto improvements in manufacturer and retailer profits while enhancing product greenness, marketing effort, and social welfare and effectively coordinating the supply chain. From a supply chain perspective, it is a wise decision for manufacturers, as the dominant player in the system, to invest in green technologies to improve performance. Retailers should maintain a positive attitude and work with manufacturers to achieve supply chain harmonization.

CRediT authorship contribution statement

Xiaodong Li: Methodology, Software, Writing-original draft. **Yang Xu:** Conceptualization, Methodology, Supervision, Project administration, Writing-review. **Kin-Keung Lai:** Resources, Methodology. **Zhipeng Peng**：Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability statement

Data will be made available on request.

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