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Interacting with the e-tailer's service investment in the presence of a store brand: Selling model choice

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A B S T R A C T

To alleviate the inventory pressure and improve operational performance, the e-tailer that develops a store brand (SB) may make a service investment in her self-operated stores. However, the existing literature rarely considers such a service investment strategy and its impact on national brand (NB) suppliers, especially on their selling model selection. We employ a theoretical model to explore the interactions of the NB supplier's selling model selection and the service investment strategy of the e-tailer developing an SB. Our findings show that under the reselling model, the e-tailer always benefits from her service investment. Interestingly, however, the e-tailer may suffer from her service investment under the agency model. Meanwhile, the likelihood of the e-tailer adopting service investment decreases as the consumer service sensitivity increases. Furthermore, we find that the service investment increases the scope wherein both firms prefer the reselling model. In addition, we show that the supplier may adopt the agency model rather than the reselling model to counteract the service investment strategy of the e-tailer. These findings provide actionable insights to help suppliers and e-tailers make strategic operational decisions.

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1. Introduction

As online retailing continues to develop rapidly, a growing number of e-tailers have not only launched store brand (SB) products but also invested in service enhancements to increase the competitiveness of their self-operated stores. For example, in 2009, Amazon launched its first SB, Amazon Basics, which mainly provided electronics and household goods. Presently, 9% of Amazon's revenues in the shoes and clothing category are attributable to its SBs (Zhang and Hou, 2022; Liu et al., 2024). According to statistics, the number of SBs worldwide will continue to grow. Meanwhile, in order to further grab profits, many e-tailers have begun to actively increase the service level of their self-operated stores (Li et al., 2023). JD.com and Amazon, for instance, have heavily invested in their self-operated channels, with enhancements evident in product listings, logistics, and after-sales services[1]. Such a service investment strategy (denoted by SIS below) of the e-tailer that operates SBs undoubtedly increases the competitiveness of her self-operated stores.

On the other hand, besides self-operated store channels (i.e., reselling model), the e-tailer can also provide a direct channel (i.e., agency model) to the national brand (NB) supplier (Cheng et al., 2022; Abhishek et al., 2016). The key distinction between the two models lies in which party decides the final retail price. More specifically, under reselling, the supplier sells his NB goods to the e-tailer that resells them to customers. Under agency selling, however, he sells his NBs directly to customers but pays a certain percentage of income to the e-tailer.

Notably, in response to the SIS of the e-tailer developing SBs, the supplier may strategically select a selling model to mitigate competitive pressures. Existing evidence suggests that, in markets where the e-tailer operates SBs, NB suppliers may adopt distinct selling models to maximize their profits and counteract the competitive effects of the e-tailer's service investments.

[1] https://help.jd.com/user/issue/list-112.html

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For instance, JD.com has developed the SBs of the home appliance (i.e., JingXuan) and lifestyle (i.e., JingZao). In this context, JD.com has implemented more aggressive SIS for its home appliance SBs (e.g., offering a 180-day replacement policy within its self-operated channels) compared to its lifestyle SBs, aiming to strengthen the competitiveness of its home appliance brands. Consequently, on JD.com, NB suppliers in the home appliance category tend to adopt the reselling model, while those in the lifestyle category predominantly opt for the agency model. However, whether such strategic responses benefit both e-tailers and suppliers remains unclear and has not been fully studied in the previous studies. Therefore, considering the above practical observations, we examine the interrelationship between the supplier's selling model choice and the SIS of the e-tailer operating the SB. Specifically, the research question follows: (1) Under different selling models, should the e-tailer that develops the SB adopt the SIS? (2) How does the choice of selling models affect such SIS? What is the interaction between the two strategies? (3) How do different strategy decision sequences influence the equilibrium outcomes?

To answer these questions, this study examines a market including one supplier (he) and one e-tailer (she) that operates an SB. The e-tailer determines whether to implement the SIS in her self-operated channel to increase her competitiveness. Considering the e-tailer's SIS, the NB supplier needs to choose one suitable selling model between the agency or reselling models to optimize his profit. By analyzing the SIS under different selling models, this study analyses the effects of selling model selection on the SIS and the interaction between them. We obtain several interesting results. First, the e-tailer's SIS does not always yield profitable outcomes. Especially in the agency model, using the service investment might hurt the e-tailer if the SB's perceived value is high. Second, the consumer service sensitivity has a significant effect on the equilibrium result --- as the service sensitivity increases, the likelihood of the e-tailer adopting SIS decreases under agency selling, but the probability that the supplier chooses reselling increases. Finally, the e-tailer's SIS can strategically guide the supplier's choice of selling model. For instance, when commission rates are moderate and the SB's perceived value is low, she may incentivize the supplier to adopt the agency model by refraining from service investments.

This paper makes several contributions. First, our work studies the interplay between selling model selection and the SIS in the presence of the SB, which, to our knowledge, has been ignored by the previous literature. Second, different from the existing literature on selling model selection, our study explains the effect of SIS on selling model choices and examines the e-tailer's preference for the SIS. We show that if the SB's perceived value is high, she might choose not to use the SIS due to the fierce competition. Finally, our research could help interpret the business practice and provide valuable guidance for making decisions regarding the SIS and selling model selection.

The remainder of this paper proceeds below. Section 2 reviews the related literature. Section 3 shows model setting. Section 4 presents the pricing and service investment level decisions. Section 5 derives the equilibrium outcome. To discuss the influence of game sequences, Section 6 considers one extended case. Section 7 concludes this work.

2. Literature review

The article is intimately linked to three branches of literature: Competition between NB and SB, selling model selection, and service investment. The early literature on competition between NB and SB investigates how traditional retailers compete with the suppliers' NBs by introducing SBs (Pauwels and Srinivasan, 2004; Choi and Fredj, 2013). In this framework, some literature examines how the retailer positions SBs in horizontally differentiated markets and analyses the SB introduction and its influence on the supply chain (Sayman & Raju, 2004; Li et al., 2021); some other literature studies the impacts of the SB's vertical differentiation (e.g., the introduction of high- and low- quality SBs) (Li et al., 2022; Liao et al., 2020; Xiao et al., 2023). Unlike the above literature, our work mainly targets the SIS of the e-tailer that operates an SB, rather than studying the introduction problem of the SB. Another branch of the literature on the brand competition is from the NB supplier perspective. To confront the retailer's SB, NB suppliers might adopt a series of countermeasures, such as promotion (Volles et al., 2023), channel encroachment (Zhang et al., 2021b), advertising (Karray and Martín-Herran, 2022), and introduction of new products (Li et al., 2024). Zhang and Hou (2022) study how the NB supplier deals with the online e-tailer's SB via the selling model selection. Liu et al. (2024) further enrich the work of Zhang and Hou (2022). They examine the interplay of sales channels and the e-tailer's information sharing when she builds her own brand. In contrast to the above literature, we mainly study how NB suppliers deal with the SIS of the e-tailer developing an SB through the selling model selection. We aim to investigate how the NB supplier interacts with the SIS when facing the e-tailer's SB competition.

The literature on selling model selection is also relevant to our study. As online retailing booms, choosing the suitable selling model between the agency and reselling models has received lots of attention from academia and industry. The prior literature addresses this problem from different aspects, such as blockchain technology (Ma and Hu, 2022; Xu et al., 2023), information sharing (Zhang and Zhang, 2020; Chen et al., 2023), promotion (Chen et al., 2020), leader-follower relationship (Wei et al., 2020), advertising (Zhang et al., 2020), slotting fee (Shen et al., 2019; Tian et al., 2018), and data-driven marketing (Liu et al., 2020). Huang et al., (2023) study the customer strategic behavior's impacts on the sales model choice and show that either the agency or reselling models might help to achieve a Pareto improvement. The above literature is performed in the case wherein there only exists suppliers' NBs. With the prosperity of the e-tailer's SBs, scholars have started looking at SB's influence on the selling model choice. Some scholars have considered the effect of SB introduction (Zhao et al., 2023; Li et al., 2021). For instance, Cheng et al. (2022) examine the impact of SB introduction and green technology improvement on

selling model selection. Several other scholars have considered this issue in the presence of the SB. For instance, Liu et al. (2022) study the influence of the SB's outsourcing strategy and show that adopting the same factory motivates the NB manufacturer to use the reselling model. Li et al. (2024) discuss channel selection and product introduction when the retailer operates her own brand. Zhang and Hou (2022) study the supplier's selling model choice on the platform that operates an SB, and found that the two firms may have a consistent preference for the selling model choice. In reality, the e-tailer that operates an SB may adopt the SIS in his self-operated stores to increase his competitiveness, which might affect the NB supplier's selling model selection. But, how the SIS influences the selling model selection in the presence of the SB, to our knowledge, has been neglected. Differing from the above papers, we complement the selling model literature by examining the impact of the e-tailer's SIS on the supplier's selling model selection when the e-tailer develops her own brand. Surprisingly, we reveal that she may induce the supplier to adopt the agency model by choosing not to implement the SIS.

The third stream of literature focuses on service investment. As e-commerce rapidly evolves, e-tailers offer various services to extract more profits, which has attracted much attention. Liu et al. (2020) investigate the impact of the platform's data-driven marketing services on selling model selection, and found that with the increase of the data-driven efficiency, the e-tailer prefers the reselling model. Choi et al. (2020) study the role of product information disclosure and reveal that product information disclosure might lead to a Pareto improvement. Li and Li (2023) consider whether the platform should build logistics services and find that suppliers always prefer her to build logistics. Dan et al. (2012) discuss the impact of sales services on pricing decisions and reveal that sales services matter. Pi et al. (2019) study the joint price and service competition, and find that either of the two retailers could opt to cooperate or not in this case. In addition, some literature considers how the retailer adopts the SIS to cope with supplier encroachment. Zhang et al. (2019) study if the retailer service investment could help to deter supplier encroachment and show that the encroachment decreases retail service level. Zhang et al. (2021b) examine the role of the e-tailer's SIS on supplier encroachment and show that the SIS is not always intended to deter the encroachment. In contrast, we mainly target the SIS of the e-tailer that operates an SB by considering the supplier's endogenous selling model choice. It is shown that the e-tailer's SIS is quite different under various selling models.

3. Model setting

We consider a market in which one supplier (S, he) manufactures national brand (NB) goods and distributes them via the e-tailer (E, she) that operates a store brand (SB). Given the competition between the two brands, the supplier needs to choose a suitable channel model between agency and reselling models. Under reselling, the supplier sells his NB goods to the e-tailer at a price w_1, and the e-tailer resells them at a price p_1. Under agency selling, the supplier sells NB goods directly to customers at a price p_1 but pays a commission fee r to the e-tailer. Similar to Liu et al. (2024), r is an exogenous variable, since she charges one fixed commission rate for a large category of commodities, which cannot be affected by a single enterprise. On the other hand, to alleviate the inventory pressure of the self-operated store channel (reselling channel) and improve her competitiveness, the e-tailer can raise the sales volume of her self-operated stores by implementing the SIS. For example, JD.com provides the service of "180 days only replace and not repair" (i.e., if the product is broken due to non-human factors within 180 days, JD.com will provide the consumer with another new product rather than repairing the original product) for some goods in the reselling channel to increase her self-operated store's sales volume and market competitiveness.

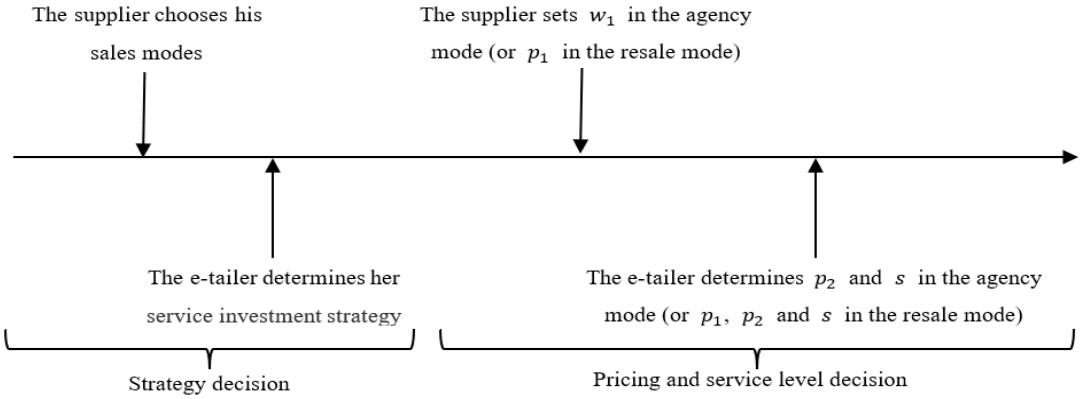


Fig. 1. Timeline of events

Based on the above assumptions, Fig. 1 shows the game sequence. (1) In the strategy decision stage, the supplier, as the leader, first chooses his agency or reselling models, and the e-tailer then decides her SIS. It is assumed that the supplier first selects his channel model. This is because in general, after viewing the sales of agency and reselling channels (that is, the supplier first chooses his selling model), the e-tailer then chooses whether to use her SIS of the reselling channel, which is common in practice. In order to study the influence of strategy decision timing, the extended section considers the case wherein the e-tailer first determines her SIS, and the supplier then chooses the selling model. Based on sales model selection and the e-tailer's SIS, this paper studies four strategy scenarios, namely, the e-tailer selects non-service investment strategy under agency selling (AN), she chooses non-service investment strategy under reselling (RN), she chooses her SIS under agency selling

(AI), and the e-tailer chooses the SIS under reselling (RI). (2) In the pricing decision stage, the supplier first sets the price w_1 under reselling, or p_1 under agency selling. Then the e-tailer determines the price p_2 and service investment level s under agency selling, or the retail prices p_1 and p_2 and service level s under reselling.

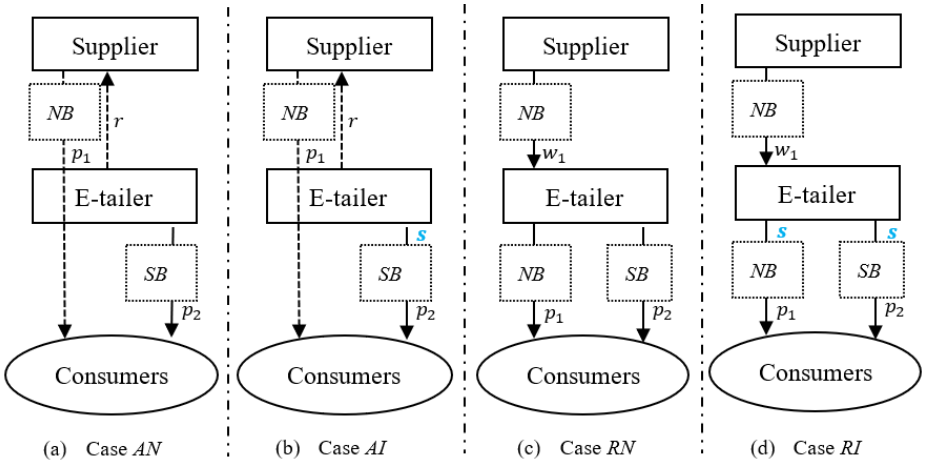


Fig. 2. Channel structures and service investment strategies

In accordance with Heese (2010) and Huang et al. (2023), the consumer's valuation (reserve price or retention utility) v of the NB is heterogeneous and evenly distributed in [0,1] for analytic simplicity. Moreover, the SB is often regarded as an inferior substitute for the NB (Chiang et al., 2003). Thus, normalizing the NB's perceived value to 1, we assume that the SB's perceived value is θ ($0 < \theta < 1$). On the other hand, we use the parameter s to represent the level of service investment, and a larger s means a higher utility that consumers can gain from the reselling channel. Meanwhile, the parameter t is used to represent the sensitivity of consumers to the service. Following Zhang et al. (2021a), it is assumed that the service investment's cost is $\frac{s^2}{2}$. Each customer buys at most one unit of SB or NB products, and makes buying decision by maximizing their utilities. Therefore, when the SIS is not adopted, the utility of the customer purchasing one unit of NB products is $u_1 = v - p_1$, and the utility of the customer purchasing one unit of SB products is $u_2 = \theta v - p_2$. At this time, their demand functions are $d_1 = 1 - \frac{p_1 - p_2}{1-\theta}$ and $d_2 = \frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}$, respectively. When the SIS is adopted under the reselling model, the utility of the customer purchasing one unit of NB products is $u_1 = v + ts - p_1$, and the utility of the customer purchasing one unit of SB products is $u_2 = \theta(v + ts) - p_2$. At this time, their demand functions are $d_1 = 1 - \frac{p_1 - p_2}{1-\theta} + ts$ and $d_2 = \frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}$, respectively. When the e-tailer uses the SIS and the supplier chooses the agency model, the utility of the customer purchasing one unit of NB products is $u_1 = v - p_1$, and the utility of the customer purchasing one unit of SB products is $u_2 = \theta(v + ts) - p_2$. At this time, their demand functions are $d_1 = 1 - \frac{p_1 - p_2 + \theta ts}{1-\theta}$ and $d_2 = \frac{p_1 - p_2 + \theta ts}{1-\theta} - \frac{p_2}{\theta} + ts$, respectively. In addition, following Zhang and How (2022) and Liu et al. (2022), the production costs for both brands are normalized to 0.

Table 1
Notations

Notation	Description
w_1	NB's wholesale price
p_1, p_2	NB's /SB's retail price
d_1, d_2	NB's /SB's demand
r	Commission rate，$1 > r > 0$
u_i	The utility the customer gets from product i (i=NB, SB)
v	The consumer's valuation of the NB
θ	SB's perceived value
s	Service investment level
t	The consumer's sensitivity to service investment
π_X^Y	The profit of the firm X under case Y，$X = (S, \mathrm{E})$ and $Y = (AN, AI, RN, RI)$

4. Pricing and service investment level decisions

According to the model assumption above, we then analyze four strategic scenarios under the interaction between channel model selection and the service investment strategy: AN, AI, RN, and RI. Specifically, we derive the optimal prices and service investment level of four scenarios. Moreover, we assume that $0 < t < \frac{\sqrt{2-2\theta}}{\sqrt{\theta}}$, which ensures that the sales of both brands under service investment are positive.

4.1. Case AN

Under Case AN, the supplier utilizes agency selling and the e-tailer chooses not to use the SIS. Thus, the profits for the enterprises are $\pi_S^{AN} = (1-r)p_1d_1$ and $\pi_E^{AN} = rp_1d_1 + p_2d_2$. Then, we can get the suitable outcomes for case AN by employing the backward induction.

Lemma 1. In case AN, the optimal results are $p_1^{AN^*} = \frac{1-\theta}{2-\theta-r\theta}$ and $p_2^{AN^*} = \frac{(1+r)(1-\theta)\theta}{2(2-\theta-r\theta)}$.

According to Lemma 1, under scenario AN, as the SB's perceived value θ rises, the NB's retail price continuously falls, but the SB's retail price first rises and then falls. This is because if θ is small, the SB's competitive advantage rises with the increase of θ, while the NB is subject to fierce competition from the SB. This results in an increase in the SB's retail price and a decrease in the NB's retail price. However, when θ is sufficiently large, as θ rises, the fierce competition between both brands might cause a price war, which causes the reduction of both brands' retail prices. Furthermore, these retail prices increase in r, because as r rises, the selling cost of the supplier accordingly goes up, and thus he would raise the NB's retail price. At this time, the e-tailer also increases SB's retail prices to improve her profits because of the brand competition.

4.2. Case AI

Under Case AI, the supplier utilizes agency selling and the e-tailer chooses to use the SIS. Thus, the profits for the enterprises are $\pi_M^{AI} = (1-r)p_1d_1$ and $\pi_E^{AI} = rp_1d_1 + p_2d_2 - \frac{s^2}{2}$. Then, we can get the suitable outcomes for case AI.

Lemma 2. In case AI, the optimal results are $p_1^{AI^*} = \frac{2\theta+t^2\theta-2}{2(\theta+r\theta+t^2\theta-2)}$, $p_2^{AI^*} = \frac{\theta(\theta-1-r+r\theta+rt^2\theta)}{2(\theta-2+r\theta+t^2\theta)}$ and $s^{AI^*} = \frac{(1-r)t\theta}{2(2-\theta-r\theta-t^2\theta)}$.

Lemma 2 shows that under scenario AI, the commission rate r, the consumer service sensitivity t, the SB's perceived value θ have important effects on the price and service investment level. Specifically, as t rises, the retail price for the NB continually falls, the service investment level continually ascends, and the SB's retail price may also decline. The reason is that the increased service sensitivity stimulates the e-tailer to increase the service investment of self-operated stores to attract more consumers, which can improve the SB's competitiveness and negatively affect the retail price of its rival brand (i.e., NB). Therefore, when t increases, the service investment level increases, whereas the NB product's retail price decreases. It is worth noting that if the SB's perceived value is sufficiently large, the increase in consumer service sensitivity will further stimulate the competition between both brands (i.e., price war), resulting in a decline in the SB's retail price. Furthermore, both brands' retail prices increase in the commission rate, whereas their retail prices may fall in the perceived value of the SB. The analysis process of this result is omitted due to the similarity to Lemma 1. In addition, we show that an increase in r could reduce the stimulating effect of service investment level on consumers, that is, the increase in r results in the reduction of service investment level. Similarly, an increase in the SB's perceived value can enhance the e-tailer‘s competitive advantage, encourage the e-tailer to enhance her brand image, and improve the profits by improving her service investment level, that is, the increase in the perceived value of the SB could motivate the e-tailer to improve her service investment level.

4.3. Case RN

Under Case RN, the supplier utilizes reselling and the e-tailer chooses not to use the SIS. Thus, the profits for the enterprises are $\pi_S^{RN} = w_1d_1$ and $\pi_E^{RN} = (p_1 - w_1)d_1 + p_2d_2$. Then, we can get the suitable outcomes for case RN by employing the backward induction.

Lemma 3. In case RN, the optimal results are $w_1^{RN^*} = \frac{1-\theta}{2}$, $p_1^{RN^*} = \frac{3-\theta}{4}$ and $p_2^{RN^*} = \frac{\theta}{2}$.

It is shown that under scenario RN, the two brands' prices are affected by the SB's perceived value θ. Specifically, as θ rises, the NB's prices gradually fall while the SB's retail price continues to rise. This is because the e-tailer has the right to determine both brands' retail prices under reselling. When θ increases, the e-tailer could use this right to raise the SB's price and accordingly reduce the NB's price to improve her overall profit. At this time, to get more orders, the supplier will decrease the NB's wholesale price, which echoes Shi and Geng (2021).

4.4. Case RI

Under Case RN, the e-tailer opts to utilize her SIS and the supplier adopts the reselling model. Thus, the profits for the enterprises are $\pi_S^{RI} = w_1d_1$ and $\pi_E^{RI} = (p_1 - w_1)d_1 + p_2d_2 - \frac{s^2}{2}$. Then, we can get the suitable outcomes for case RI by employing the backward induction.

Lemma 4. In case RI, the optimal results are $w_1^{RI^*} = \frac{1-\theta}{2-t^2\theta}$, $p_1^{RI^*} = \frac{3-t^2-\theta}{(2-t^2)(2-t^2\theta)}$, $p_2^{RI^*} = \frac{\theta(4-t^2-t^2\theta)}{2(2-t^2)(2-t^2\theta)}$, $s^{RI^*} = \frac{t(1+\theta-t^2\theta)}{(2-t^2)(2-t^2\theta)}$.

Similar to Lemma 3, when θ rises, the NB's wholesale price falls, but the SB's retail price and service investment level increase. In addition, as the consumer service sensitivity t goes up, the selling prices of both brands and the service investment level increase. These findings are similar to Lemma 2, thus we omit the analysis. Based on the above optimal results, we can obtain the influence of the SIS on the equilibrium prices, which is summarized below.

Proposition 1. （1） $p_1^{AI*} < p_1^{AN*}$； when $\theta < \frac{1}{1+r}$， $p_2^{AI*} > p_2^{AN*}$， when $\theta > \frac{1}{1+r}$， $p_2^{AI*} < p_2^{AN*}$.（2） $p_1^{RI*} > p_1^{RN*}$； $p_2^{RI*} > p_2^{RN*}$； $w_1^{RI*} > w_1^{RN*}$.

Proposition 1 indicates the effect of the SIS on the prices. Specifically, under the agency model, adopting the SIS reduces the NB's retail price and may also reduce the SB's retail price, whereas under reselling, the SIS could raise their retail prices but may decrease the NB's wholesale price. The reason is that under agency selling, the supplier and the e-tailer decide the retail prices of the NB and the SB, respectively. At this time, adopting the SIS can enhance the competitiveness of the SB and accordingly reduce the NB's competitive edge. This enhanced competitive edge of the SB may lead to a decline in the NB's retail price and an increase in the SB's retail price. However, if $\theta > \frac{1}{1+r}$, the brand competition is fierce. In this case, the service investment may intensify their competition, thereby reducing their retail prices. In addition, the e-tailer owns the pricing power for both brands under reselling. In this case, the SIS of the self-operated store could raise the demand for the two brands, which could push up their retail prices and NB's wholesale prices. The above results echo business practices. For instance, to enhance the competitiveness of self-operated stores and reduce the inventory pressure, some e-tailers opt to implement service investment in their self-operated stores (e.g., "return and exchange door-to-door pickup" and "180 days only replace and not repair") to improve consumers' shopping experience, which could raise the consumers' utility. As a result, these customers prefer to pay higher prices for the self-operated store's products, which may benefit the e-tailers. In reality, the self-operated stores of the e-tailer that provides the service often charge a higher retail price for the product (Li et al., 2023).

5. Comparative analysis

By using the equilibrium results of four cases, this section further discusses the sales model choice, the SIS, and their interaction. More specifically, according to the game sequence of the firms' strategies in Section 3, this section will first study the e-tailer's SIS under different sales models, then analyze the choice of the supplier's channel models, and finally study their interactions.

5.1. The service investment strategy in the agency model

We first discuss the optimal SIS under the agency model by comparing the e-tailer's profits.

Proposition 2. Under agency selling, if $\theta < \theta_1$, the e-tailer opts to implement her SIS; if $\theta > \theta_1$, she chooses not to implement the SIS.

Proposition 2 reveals the optimal SIS under agency selling. Specifically, in the agency model, if the SB's perceived value is low ($\theta < \theta_1$), the e-tailer adopts service investment. If the SB's perceived value is high ($\theta > \theta_1$), she opts not to employ the service investment. This is because under agency selling, the supplier and the e-tailer directly compete. In this case, the self-operated store's SIS of the e-tailer has two effects on her profit. One effect is that the service investment increases the competitive advantage of the SB, thereby raising her income from selling SB products. The other effect is that the service investment could also intensify the brand competition, which may reduce her commission income from selling NBs. If the SB's perceived value is low (i.e., the brand competition is weak), the former effect dominates, and the e-tailer prefers service investment. However, if the SB's perceived value is high, the latter effect dominates, and she opts not to implement the service investment. In practice, more and more NB suppliers are adopting the agency model to sell their products. In this context, when the SB's perceived value is high, the e-tailer should not implement the SIS, because it may lead to losses for the e-tailer. This finding implies that the e-tailer introducing SB products needs to evaluate the role of the SB's perceived value when choosing whether to implement the SIS.

5.2. The service investment strategy in the reselling model

Similar to Subsection 5.1, we could get the e-tailer's SIS in the reselling model.

Proposition 3. In the reselling model, the e-tailer always favors adopting the SIS.

It is revealed that given that the reselling model is chosen, implementing the self-operated store's SIS always benefits the e-

tailer. This is because under reselling, adopting service investment could increase both brands' attractiveness (i.e., the service investment can raise the sales price and sales volume of the two brands), which can raise her overall profit. Therefore, under reselling, the e-tailer always prefers to adopt the SIS. These results are in line with practical observation. For instance, some NB suppliers of home appliances choose the reselling model on JD.com which has developed its SB products of home appliances. In order to obtain more profits, JD.com has already launched several home appliance service investments in its self-operated stores (e.g., "180 days only replace and not repair"). In addition, by comparing the supplier's preferences of the SIS under different selling models, it is found that the SIS in the agency model negatively influences the supplier's profit, whereas under reselling, implementing the service investment could raise his profit. This conclusion is straightforward, and thus the analysis process is omitted here.

5.3. Selling model selection

To obtain the supplier's selling model selection and explore the impacts of the SIS on the selling model choice, we first examine the supplier's selling model selection under different service investment strategies, which is summarized as follows.

Proposition 4. (1) When the SIS is not used, the supplier selects agency selling if $r < r_1$; he favors reselling if $r > r_1$. (2) When the SIS is used, the supplier selects agency selling if $r < r_2$; he favors reselling if $r > r_2$.

Part (1) reveals that when if the e-tailer selects non-service investment strategy, when commission rates are low ($r < r_1$), the supplier favors the agency model; when commission rates are high ($r > r_1$), he favors the reselling model. The reason is that when $r < r_1$ (i.e., the supplier's channel cost is low), agency selling is more efficient and can help to improve his profit (Zhen et al., 2022). However, when $r > r_1$, the agency model increases the selling channel cost and then damages the supplier's profit. In this case, he prefers reselling. In addition, we show that with the increase of the SB's perceived value θ, the supplier prefers to employ agency selling ($\frac{\partial r_1}{\partial \theta} > 0$). This is because as θ rises, the competitive advantage of the SB increases. At this time, to squeeze more profit, the supplier is more inclined to own the NB's pricing power (i.e., he prefers the agency model). Therefore, the likelihood that the supplier employs agency selling rises with the increase of the SB's perceived value. This conclusion is consistent with practical observation. For example, on Amazon.com, facing the SBs with different perceived values, the possibility of the supplier adopting the agency model varies. In general, since the electronic goods of SBs have low perceived values, NB suppliers are more willing to adopt the reselling model, whereas because textile products of SBs have high perceived values, NB suppliers are more willing to employ the agency model.

Similar to Proposition 4(1), if the e-tailer uses the SIS, when commission rates are low ($r < r_2$), the agency model is preferred by the supplier; when commission rates are high ($r > r_2$), he favors the reselling model. However, it is found that with the increase of θ, the probability of the supplier choosing to employ agency selling first increases and then decreases. This means that if commission rates are low, the supplier might choose reselling rather than agency selling when the SB's perceived value is sufficiently large, which echoes business practice. For instance, due to the service investment in self-operated stores and the price competition, even if commission rates are small, some NB manufacturers still use reselling rather than agency selling. Furthermore, we show that as the consumer service sensitivity t increases, r_2 decreases, which implies that with the increase of t, NB suppliers favor using reselling. Moreover, by comparative analysis, we show that the self-operated store's service investment can improve the probability that the supplier chooses reselling.

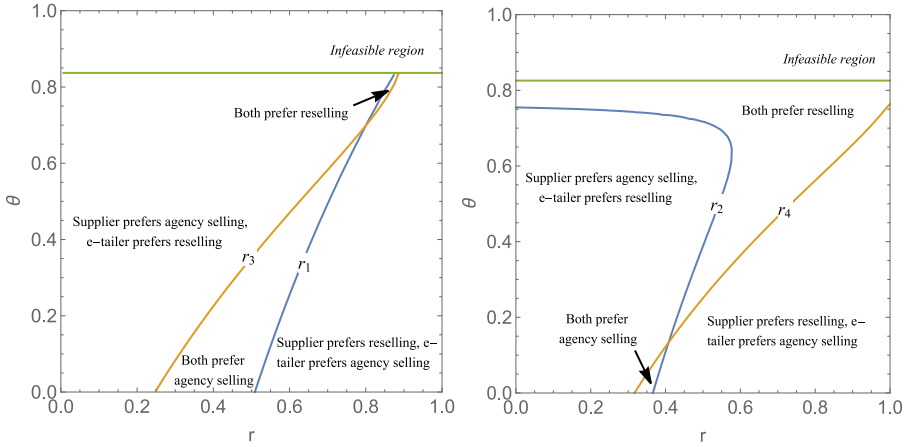
Next, we study the selling model preference of the e-tailer, which is summarized below.

Lemma 5. (1) When the e-tailer uses no SIS, the e-tailer prefers the reselling model if $r < r_3$; she prefers the agency model if $r > r_3$. (2) When the e-tailer uses the SIS, she prefers the reselling model if $r < r_4$; she prefers the agency model if $r > r_4$.

Lemma 5 shows the e-tailer's selling model preference under various service investment strategies. Specifically, when the SIS is not used by the e-tailer, if commission rates are low ($r < r_3$), she favors reselling; if commission rates are high ($r > r_3$), she favors the agency model. Similarly, when the SIS is adopted, if $r < r_4$, she favors reselling; if $r > r_4$, she favors the agency model. This is because if commission rates are low, the e-tailer's commission income under agency selling is small, thus the e-tailer likes reselling. If commission rates are high, she can obtain the commission revenue from using the agency model, and thus she prefers the agency model.

By comparing the selling model preference of the e-tailer and the supplier, we can obtain the results below.

Proposition 5. When the SIS is not used, both the e-tailer and the supplier favor agency selling if $r_3 < r < r_1$; they prefer reselling if $r_1 < r < r_3$. When the SIS is used, the two firms favor agency selling if $r_4 < r < r_2$; they prefer reselling if $r_2 < r < r_4$.



(a) Under non-service investment (b) Under service investment

Fig. 3. Both firms' selling model preferences ($t = 0.65$)

We reveal the selling model preferences for the supplier and the e-tailer (see Fig. 3). Specifically, regardless of the SIS, if commission rates are moderate, both firms favor reselling or agency selling; otherwise, they have conflicting preferences. By comparing the selling model preferences under different service investment strategies, it is shown that the e-tailer can make the two firms favor the reselling model even more by implementing the service investment. Complementing Wei et al. (2020), we reveal that implementing the SIS increases the likelihood of supply chain members' preference for reselling models.

Based on the propositions above, we could employ backward induction to derive the optimal outcome.

Proposition 6. When $r < r_2$ and $\theta < \theta_2$, the supplier chooses agency selling, then the e-tailer adopts the SIS; when $r < r_5$ and $\theta > \theta_2$, the supplier chooses agency selling, then the e-tailer adopts non-service investment; otherwise, the supplier adopts reselling, and the e-tailer uses the SIS.

Fig. 4 graphically presents the final equilibrium outcome. Specifically, when commission rates r and the SB's perceived value θ are low, the supplier chooses agency model, and the e-tailer uses the SIS. When θ is relatively high and r is relatively low, the supplier chooses agency model, and the e-tailer opts not to use the SIS. When r is high, the supplier chooses reselling, and the e-tailer chooses the SIS. In addition, it is found that case NR does not arise at equilibrium, since the SIS is the dominant strategy under reselling, that is, the e-tailer always favors the SIS if the supplier chooses reselling.

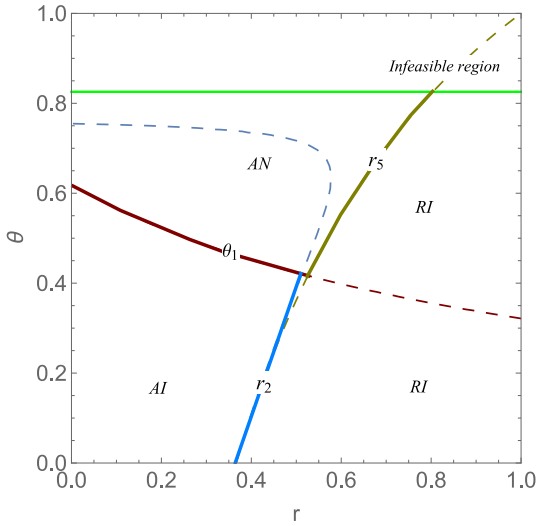


Fig. 4. The equilibrium outcome ($t = 0.65$)

Through mathematical analysis, we show that as consumer service sensitivity rises, the probability of scenario AI appearing on the equilibrium result decreases, and the likelihood of the supplier adopting the reselling model increases. This is because a higher service sensitivity leads to more consumer utility. At this time, the e-tailer's SIS could effectively enhance the SB's competitive advantage. As a result, the supplier suffers from competitive product pressures under agency selling. Therefore, when rises, the supplier tends to use the reselling model, which results in a lower probability of scenario AI appearing on the equilibrium outcome.

Proposition 6 also implies that the e-tailer's SIS is not always beneficial to herself. Specifically, when agency selling is selected and the SB's perceived value is large, she does not use the SIS. This is because, in addition to increasing her income from selling the SB, the service investment may intensify the two firms' competition and reduce her commission income (Proposition 2), which hurts herself. Especially when is high, the two brands' competition is fierce. At this time, adopting the SIS may intensify the competition and make them fall into a price war, which significantly reduces her commission income and ultimately damaged the e-tailer's profit. Therefore, when the SB's perceived value is large, the e-tailer favors no SIS under agency selling.

6. Extended model—another game sequence

To discuss the influence of game sequences, this section assumes that the e-tailer first sets her SIS, then the supplier decides his selling model. By employing backward induction, we could get the final equilibrium outcome.

Proposition 7. When $r < r_2$ and $\theta < \theta_1$, the e-tailer adopts the SIS, then the supplier uses the agency model; when（i）$r < r_2$ and $\theta > \theta_1$ or（ii）$max\{r_2, r_6\} < r < r_1$, the e-tailer adopts non-service investment strategy, then the supplier uses the agency model; otherwise, the e-tailer adopts the SIS, then the supplier uses the reselling model.

Fig. 5 graphically presents the final equilibrium outcome if the SIS is first decided. Specifically, when the SB's perceived value θ or commission rates r are low, the e-tailer implements the SIS, then the supplier uses the agency model. When (i) θ is moderate and the r is low or (ii) θ is low and r is moderate, the e-tailer uses no SIS and the supplier adopts agency model. When r or θ is sufficiently high, she adopts the SIS, then the supplier uses reselling. Similar to the base model, scenario RN does not appear at equilibrium. In addition, as the consumer service sensitivity t rises, the probability of the supplier choosing reselling goes up, and the area for the e-tailer adopting the SIS gradually expands.

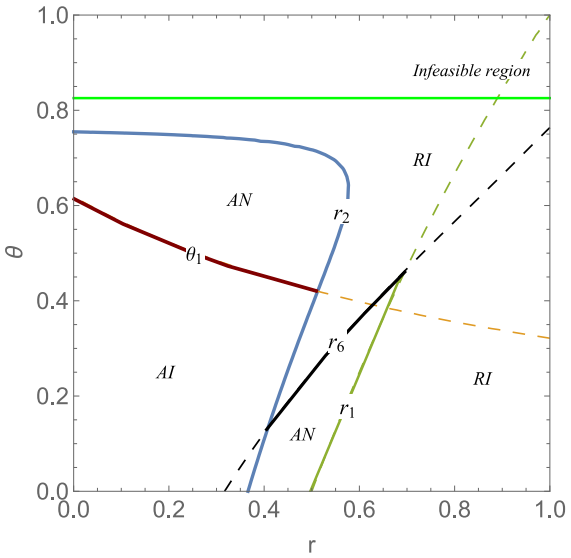


Fig. 5. The equilibrium outcome of the extended model ($t = 0.65$)

Proposition 7 also shows that the e-tailer could guide the supplier's selling model selection through the SIS, which may help the e-tailer manage channels. Specifically, when commission rates are moderate ($r_2 < r < r_1$), if she uses the SIS to increase the competitiveness of the reselling channels, the supplier favors reselling; if the SIS is not used, the supplier prefers agency selling (see Propositions 4). In this case (i.e., $r_2 < r < r_1$), the SIS affects selling model selection, that is, if commission rates are high ($r_6 < r < r_1$), the e-tailer will give up the SIS because a higher commission income from the agency model makes her better off. In other words, when $r_6 < r < r_1$, the e-tailer first chooses non-service investment, and the supplier then uses agency selling. However, if $r_2 < r < r_6$, the e-tailer will first adopt the SIS to obtain a higher payoff from the increased product demand, and then the manufacturer will choose the reselling model. This conclusion indicates that the e-tailer's SIS has an important impact on the supplier's selling model selection. Therefore, the supplier should carefully consider the e-tailer's SIS when making the selling model choice.

Compared with the baseline model, the equilibrium result of the extended model has the following two differences. When is low, the supplier always uses agency selling if the supplier first sets selling models, whereas he may use reselling if the e-tailer first determines the SIS. The decision sequence of the firms' strategy contributes to this conclusion. On the other hand, when the SB's perceived value is low and commission rates are moderate, if the selling model is first decided, the supplier will choose reselling and the e-tailer then adopts the SIS. However, if the SIS is first set, the e-tailer adopts the non-service investment strategy, then the supplier uses agency selling. This is because if commission rates are moderate, when the supplier

first decides his selling model, he will not bear a higher commission cost and adopts reselling, then the e-tailer adopts the SIS; when the SIS is first set, to obtain the high commission fee, she could choose to adopt the SIS to guide the supplier to use agency model. Therefore, when both firms are deciding on their strategies, they need to consider the timing of their strategic decisions.

7. Conclusion

By utilizing the game theory, this paper studies the interplay between selling models and the SIS in the presence of the SB, which has been ignored by previous papers. Specifically, we study a market consisting of one supplier and the e-tailer that operates an SB. The supplier sells his NB via agency or reselling models, while the e-tailer decides whether to implement the SIS in her self-operated stores. Then, we investigate the SIS under various selling models and their interplay. This paper has obtained several interesting results. First, the e-tailer's SIS raises the probability that the supplier chooses reselling. Specifically, if the SIS is not adopted by the e-tailer, only when commission rates are high will the supplier use the reselling model. However, in the case of adopting the SIS, when commission rates are moderate or high, the supplier favors reselling. In addition, the e-tailer and the supplier may have a consistent selling model preference. In particular, under non-service investment, the two firms tend to prefer agency selling if commission rates are moderate. However, under service investment, they prefer the reselling model if commission rates are moderate. Second, the SIS is quite different under different selling models. By comparing equilibrium outcomes, we show that under reselling, the e-tailer always adopts the SIS, because the service investment can increase the sales volume of both brands, thereby making herself better off. However, under agency selling, if the SB's perceived value is low, she implements the SIS; if the SB's perceived value is high, she will not adopt the SIS. This is the result of the trade-off between the increased SB's competitive advantage and the price competition in the agency model (see Proposition 2). Therefore, when deciding whether to implement the SIS, the e-tailer should carefully consider the impact of several key factors (e.g., selling models and the SB's perceived value).

Finally, we get the final equilibrium outcomes and show that when commission rates and the SB's perceived value are low, the e-tailer implements the SIS, and the supplier chooses agency selling. When the SB's perceived value is high and commission rates are low, the e-tailer adopts the non-service investment strategy, while the supplier chooses the agency model. When commission rates are sufficiently high, the e-tailer chooses the SIS and the supplier uses reselling. Furthermore, the consumer service sensitivity affects the equilibrium result, that is, as the service sensitivity increases, the possibility for the e-tailer adopting the SIS in the agency model decreases, while the possibility that the supplier uses the reselling model increases. Moreover, the extended model of this paper investigates the influence of the strategy decision sequence. Besides proving the validity of our main findings, we also find that the e-tailer's SIS can guide the supplier's sales model selection. Especially when the SB's perceived value is low and commission rates are moderate, the e-tailer might induce him to adopt agency selling by first using the non-service investment strategy.

The above conclusions have important practical significance for enterprise managers. On the one hand, the SIS for the e-tailer has an important impact on the supplier's selling model selection, especially adopting the service investment could increase the probability that the supplier chooses reselling. For example, the e-tailer can attract suppliers to join her self-operated stores by providing the service investment (e.g., logistics and payment services), which can improve her control over the brands and profit margins and enhance the shopping experience of consumers. Meanwhile, the SB product's perceived value and the consumer service sensitivity also matter. Therefore, supplier managers need to carefully evaluate the influence of several factors such as the service investment strategy, the SB's perceived value and the consumer service sensitivity when choosing the selling model. On the other hand, the SIS is not always the best choice for the e-tailer. Specifically, when the supplier chooses agency selling, if the SB's perceived value is high, the adoption of the SIS may intensify the competition between the two firms, which may pull down the e-tailer's income. Thus, she needs to carefully assess the effects of these factors (e.g., selling models and the SB's perceived value) when choosing the SIS.

Some directions deserve further examination. First, we follow some excellent papers to assume that both brands have the same production cost (Zhang and Hou, 2022). However, these costs may be different in reality. In the future, we could study the influences of production cost differentiation. Second, the demand for information asymmetry between the firms is gaining more attention. In future research, one can further discuss this study in a business environment with asymmetric information. Finally, we only employ a game model to derive the final outcome, and thus one can empirically validate and complement this study in future research.

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Appendix

Proof of optimal solutions.

Take the first- and second-order of p_2 for the equation $\pi_E^{AN} = rp_1d_1 + p_2d_2$, respectively. Note that $\frac{\partial^2\pi_E^{AN}}{\partial p_2^2} = \frac{2}{(-1+\theta)\theta} < 0$, which implies that π_E^{AN} is concave in p_2. Thus, we could get $p_2 = \frac{p_1(1+r)\theta}{2}$ from the first-order condition $\frac{\partial\pi_E^{AN}}{\partial p_2} = 0$. Then, substituting $p_2 = \frac{p_1(1+r)\theta}{2}$ into the equation $\pi_S^{AN} = (1-r)p_1d_1$, we show that π_S^{AN} is concave in p_1 due to $\frac{\partial^2\pi_S^{AN}}{\partial p_1^2} = \frac{(-1+r)(2-\theta-r\theta)}{1-\theta} < 0$. Therefore, we can obtain the optimal price $p_1^{AN^*} = \frac{1-\theta}{2-\theta-r\theta}$. Substituting $p_1^{AN^*} = \frac{1-\theta}{2-\theta-r\theta}$ into $p_2 = \frac{p_1(1+r)\theta}{2}$, we can obtain Lemma 1 (i.e., the equilibrium outcome for Scenario AN).

The poofs of optimal outcomes under Scenarios AI, RN and RI are similar to Scenario AN. Thus we omit it.

Proof of Proposition 1.

(1) $p_1^{AI*} - p_1^{AN*} = \frac{(-1+r)t^2\theta^2}{2(-2+\theta+r\theta)(-2+\theta+r\theta+t^2\theta)} < 0$. Moreover, we derive $p_2^{AI*} - p_2^{AN*} = \frac{(1-r)t^2\theta^2(1-\theta-r\theta)}{2(2-\theta-r\theta)(2-\theta-r\theta-t^2\theta)}$. The denominator of $\frac{(1-r)t^2\theta^2(1-\theta-r\theta)}{2(2-\theta-r\theta)(2-\theta-r\theta-t^2\theta)}$ is positive. Thus, we can easily have: when $\theta < \frac{1}{1+r}$, $p_2^{AI*} > p_2^{AN*}$, when $\theta > \frac{1}{1+r}$, $p_2^{AI*} < p_2^{AN*}$. (2) Similar to Part 1, We can prove this part.

Proof of Proposition 2.

In the feasible region $0 < \theta < \frac{2}{2+2t^2}$, we can obtain the only meaningful root θ_1 of the equation $\pi_E^{AI*} - \pi_E^{AN*} = 0$, where $\theta_1 = \frac{4+4r+t^2-\sqrt{4+8r+4r^2+t^4}}{(1+r)(3+3r+2t^2)}$. Proposition 2 follows from a simple analysis.

Proof of Propositions 3-6.

We omit the procedures due to the similarity. Note that $r_1 = \frac{2+\theta}{4-\theta}$; $r_2 = \frac{-4t^4\theta+t^6\theta^2+2t^2(2+\theta-\theta^2)+2(-2+\theta+\theta^2)}{-2(-4+\theta)(-1+\theta)-4t^4\theta+t^6\theta^2+4t^2(1+\theta-\theta^2)}$; $r_3 = \frac{8-4\sqrt{(-4+\theta)(-1+\theta)^3}+\theta(-10-3(-3+\theta)\theta)}{\theta(12+\theta(-11+3\theta))}$; $r_4 = (-16+8t^2+20\theta-12t^2(-1+t^2)\theta+2(-9-5t^2+3t^6)\theta^2-(1+t^2)(-6+t^6)\theta^3+\sqrt{(-2+t^2)(-2+t^2\theta)^2(2-\theta(2+t^2))^2(-2(4-\theta)(1-\theta)-4t^4\theta+t^6\theta^2+4t^2(1+\theta-\theta^2))})/(\theta(-24+22\theta-12t^4\theta-6\theta^2+3t^6\theta^2+4t^2(3+(3-2\theta)\theta)))$; $r_5 = \frac{(-1+t^2)(-2+(-1+t^2)\theta)}{4-\theta+t^4\theta-2t^2(1+\theta)}$; $r_6 = (-2t^2(-2+\theta)-t^4(-2+\theta)(-1+\theta)\theta+\theta(10+3(-3+\theta)\theta)+2(-4+\sqrt{(-2+t^2)(-1+\theta)^2(-2+t^2\theta)(4-5\theta+\theta^2+t^4\theta(1+(-1+\theta)\theta)-2t^2(1+(-1+\theta)\theta^2))}))/(\theta(-12+(11-3\theta)\theta+3t^4(-1+\theta)\theta+t^2(6-4\theta^2)))$.

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