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Mixed traffic optimization considering road reservation travel based on the bi-level programming model

Shiyu Zheng[a], Jianjun Wang[a*] and Xiaojuan Lu[a]

[a]*College of Transportation Engineering，Chang＇an University, China*

C H R O N I C L E

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A B S T R A C T

Reservation travel has gradually become an important strategy to alleviate urban traffic congestion by finely matching traffic supply and demand. In order to quantify the effectiveness of the reservation travel strategy, ordinary vehicles and reservation vehicles are considered, The reservation travel problem is formulated as a Bi-level Programming (BP), where the upper-level objective is to maximize traffic demand, while the lower-level model considers the System Optimal-Stochastic User Equilibrium (SO-SUE) mixed traffic equilibrium. The equivalence of the mixed equilibrium problem and the existence of the solution are proved. On the basis of the WOA algorithm, combined with the lower-level Partan Frank-Wolfe algorithm, the traffic assignment solution process is connected to the upper-level as a function. A Whale Optimization Algorithm (WOA) nested Partan Frank-Wolfe algorithm is proposed to solve the model. Finally, the Sioux-Falls network numerical experiment proves that the reservation travel has a gratifying benefit in alleviating traffic congestion. By comparing the Vehicle-to-Capacity (V/C) ratio and cost before and after the implementation of reservation, it shows the effectiveness of reservation travel for urban traffic congestion management, and discusses the impact of the number of reserved roads and reservation trends on road network capacity, service level and travel cost under the implementation of reservation travel.

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1. Introduction

With the rapid advancement of urbanization, major cities around the world are generally facing increasingly serious traffic congestion problems. Traditional measures such as building new roads and expanding transportation infrastructure can alleviate traffic congestion in the short term, but it is difficult to meet the rapid growth of travel demand due to the long construction period, high cost and limited space. Reservation travel has increasingly been recognized as a crucial strategy for reducing urban traffic congestion, owing to its ability to offer both flexibility and efficiency. The concept of reservation travel was first proposed by Akahane et al. (2000), aiming to manage the traffic demand of Tokyo Expressway in Japan. Subsequently, Wong (1997) further clarified the concept of reservation travel, the function and basic framework of the reservation system.The advancement of intelligent transportation systems and the evolution of Internet communication technologies have made the implementation of road reservation travel systems more precise and practical (Vasirani, 2012). Dobre (2012) and Liu et al. (2013) proposed a new type of highway intelligent lane reservation system based on various simulation methods. Zhao et al. (2010) first studied urban road reservations and proposed a downtown space reservation system that requires drivers to make reservations before entering the city. The idea of the reservation system is similar to the work of Teodorovic et al. (2005). Integer programming model is used to allocate travel quotas, and artificial neural networks are used to determine reservation requests. Chen et al. (2021) improved Zhao 's research and established an urban road area reservation system based on the warning line. He required that all non-public transport vehicles must reserve the right of way in advance, and non-reservation vehicles must not enter the warning line. The system's effectiveness was validated through a combination of Vissim simulation experiments and analytical modeling techniques.

In terms of reservation travel strategy formulation, Menelaou et al. (2017) introduced a route reservation method with low complexity, aimed at efficiently managing urban road traffic and assisting bookers in making optimal route choices. Menelaou et al. (2017, 2018) proposed a reservation mechanism for route reservation on non-congested roads and constructed a

* Corresponding author
E-mail 2021034018@chd.edu.cn (J. Wang)
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continuous-time route reservation framework. Cheng et al. (2022) considered the differing traffic demands of both general and special task vehicles, developed a discrete bi-level programming model, and analyzed the discontinuous lane reservation problem under conditions of traffic oversaturation.In addition, relevant scholars have pointed out that the setting of reserved lanes should also incorporate operating costs into actual decision-making (Wu et al.,2018), and fully consider the fairness and flexibility of individual selection in reservation demand management (Li et al.,2023). Simultaneously, in enhancing road reservation services, attention must be given to optimizing both the network structure and road attributes (Ye et al., 2024).

The reservation travel strategy enhances the efficiency of reservation vehicles, but it will also bring negative effects such as driving delays to ordinary vehicles. In general, previous studies mainly emphasize the reservation strategy formulation framework, the reservation lane selection strategy, and the reservation vehicle scheduling model and algorithm, aiming to optimize the benefits of reservation vehicles. However, the consideration of ordinary vehicles is insufficient, so it is difficult to maximize the traffic demand and ensure the fairness of travel.This paper presents the reservation travel problem as a bi-level programming model, considering the trade-offs between policymakers and road users. The upper-level model seeks to maximize traffic demand, while the lower-level model addresses the mixed traffic equilibrium of reservation and ordinary vehicles through the System Optimal-Stochastic User Equilibrium (SO-SUE) framework. The equivalence and existence of solutions to the mixed equilibrium problem are established. To solve this model, a Whale Optimization Algorithm (WOA) integrated with the Partan Frank-Wolfe algorithm is proposed.Finally, a numerical experiment using the Sioux-Falls network is performed to evaluate the effects of implementing the reservation strategy. This analysis provides important insights for managers and decision-makers in tackling traffic congestion and identifying key parameters essential for the effective implementation of reservation-based travel.

The remainder of this study is structured as follows. Section 2 develops a bi-level programming model; Section 3 uses the Sioux-Falls network numerical data to conduct a case study to analyze the effectiveness of reservation travel; Section 4 discusses the influence of factors and summarizes this paper.

2. Model construction

2.1 Problem description

Examine a densely interconnected transportation network $G(N, A)$,with N as the set of nodes, A as the set of links, a is an element in A , namely $a \in A$, and W representing the collection of all OD pairs. and w represents an element in W , which is $w \in W$. Table 1 summarizes the indices, sets, parameters, and decision variables utilized in the formulation.

The model assumptions are as follows:

(1) The road network accommodates two types of vehicles: standard vehicles and reservation-based vehicles. The reservation vehicles can only drive in the reservation lane, and the ordinary vehicles cannot drive in the reservation lane.
(2) Lane is divided into reservation lanes and non-reservation lanes, and the traffic flow between them is independent.
(3) Throughout the study duration, traffic demand for every origin-destination (OD) pair in the road network is presumed to stay constant.
(4) Ordinary vehicles and reservation vehicles follow different path selection principles. Compared with non-reservation vehicles, travelers using reservation vehicles can accurately obtain traffic travel information through the reservation platform. Therefore, this paper assumes that the reservation vehicle is completely rational, and fully and accurately grasps all the information of the traffic network, they will use the least cost path to reach the destination, that is, obey the SO principle. The ordinary vehicle chooses the travel path according to the perceived travel cost, that is, obeys the SUE principle.
(5) The travel costs of ordinary vehicles and reservation vehicles only consider travel time.
(6) The vehicle's route can consist of segments with reservation lanes as well as those without reservation lanes.

Table 1
Notations

Parameter	Description
A	Set of links
A_1	Set of reservation road links, $A_1 \subseteq A$
A_2	Set of ordinary road links, $A_2 \subseteq A$
K	The set of all paths, the corresponding $\lvert K \rvert$ is the total number of paths.
K_w	The set of all paths connecting OD pairs $w \in W$, $\lvert K_w \rvert$ is the total number of paths connecting OD pairs.
$\mathbf{q}$	Travel demand vector, $\mathbf{q} = (q_w, w \in W)^T$

q_w	The original travel demand of travelers between OD pairs $w \in W$
η_w	The travel demand multiplier of OD pairs $w \in W$
λ	The proportional coefficient of the reserved vehicle, $\lambda \in [0,1]$
q_w^1	Travel demand of ordinary vehicles on OD pairs $w \in W$
$\mathbf{q}^1$	Travel demand vector of ordinary vehicles, $\mathbf{q}^1 = \left(q_w^1, w \in W\right)^T$
q_w^2	The travel demand of reservation vehicles on OD pairs $w \in W$, $q_w^2 = (1-\lambda) q_w$
$\mathbf{q}^2$	Travel demand vector of reservation vehicle, $\mathbf{q}^2 = \left(q_w^2, w \in W\right)^T$
f_{kw}	Traffic of travelers on path, $k \in K_w$, $w \in W$
$\mathbf{f}$	Path flow vector, $\mathbf{f} = \left(f_{kw}, k \in K_w, w \in W\right)^T$
f_{kw}^1	The flow of ordinary vehicles on path $k \in K_w$, $w \in W$
f_{kw}^2	The flow of reservation vehicles on path $k \in K_w$, $w \in W$
$\mathbf{f}^1$	The path flow vector of ordinary vehicles, $\mathbf{f}^1 = \left(f_{kw}^1, k \in K_w, w \in W\right)^T$
$\mathbf{f}^2$	The path flow vector of the reserved vehicle, $\mathbf{f}^2 = \left(f_{kw}^2, k \in K_w, w \in W\right)^T$
x_a	Represents the total flow of travelers on link a
x_a^{res}	Represents the optimal reservation travel volume on link a
res_a^j	Vehicle j Enters the acceptable status of the request for link a
x_a^1	Traffic flow of ordinary vehicles on link a
x_a^2	Traffic flow of reservation vehicles on Link a
$\mathbf{x}^1$	The traffic flow vector of ordinary vehicles, $\mathbf{x}^1 = \left(x_a^1, a \in A\right)^T$
$\mathbf{x}^2$	The traffic flow vector of reservation vehicle, $\mathbf{x}^2 = \left(x_a^2, a \in A\right)^T$
$t_a\left(x_a\right)$	The separable travel time function on link a is a differentiable function that increases with the total flow of link x_a
$c_a\left(x_a\right)$	The travel cost for the traveler along link a is a rising function relative to the cumulative traffic on road link x_a
C_{kw}	Traveler 's real impedance on path, $k \in K_w$, $w \in W$
$\mathbf{C}$	The vector of the real impedance of the path, $\mathbf{C} = \left(C_{kw}, k \in K_w, w \in W\right)^T$
U_{kw}	Traveler 's perceived impedance on path, $k \in K_w$, $w \in W$
$\Delta = \left[\delta_{kw}^a\right]$	The incidence matrix of link and path,If the link a is on the k path of the OD pair w, $\delta_{kw}^a = 1$; Otherwise, $\delta_{kw}^a = 0$
$\Lambda = \left[\lambda_{kw}\right]$	The correlation matrix of OD pairs , if path k connects OD pairs w, $\lambda_{kw} = 1$; Otherwise, $\lambda_{kw} = 0$
ε_k^w	The relationship between the reservation part and the path,If there is a reservation link on path k between OD pairs w, $\varepsilon_k^w = 1$; Otherwise, $\varepsilon_k^w = 0$
c_a	Link travel cost
$\mathbf{c}$	Link cost vector, $\mathbf{c} = \left(c_a, a \in A\right)^T$
θ_w	The non-negative correction coefficient represents the size of the random disturbance
p_{kw}	The probability of ordinary vehicle travel choice path

2.2 Bi-level programming model

In this study, vehicles are categorized into two groups: reservation vehicles and ordinary vehicles Reservation vehicles must be booked in advance through the reservation platform and are permitted to use the reserved lanes upon successful booking.

In contrast, ordinary vehicles are not allowed to access these reserved roads. The reservation vehicles minimize its own perceived travel cost according to SO, and the ordinary vehicles minimize its own understood travel cost according to the SUE. The two types of heterogeneous users choose their own path according to their own path selection principles, and the final traffic network will be in the SUE-SO mixed traffic equilibrium state. A Bi-level Programming (BP) model is designed with an emphasis on reserved capacity. The primary goal at the upper level is to maximize traffic demand while adhering to the limitations imposed by the capacity of the designated road. Meanwhile, the lower level seeks to minimize travel costs for both reserved and ordinary vehicles.

The acceptable condition for vehicle j to enter link a within a specified time frame is denoted by res_a^j. If vehicle j's request is approved, then $res_a^j = 1$; otherwise, $res_a^j = 0$. Consequently, this can be computed as shown in Eq.(1):

$$res_a^j = \begin{cases} 1, & if \sum_{j=1}^{J} res_a^j \le x_a^{res}, a \in A_1 \\ 0, & otherwise \end{cases} \tag{1}$$

(1) Upper Model: Maximize Traffic Demand

The upper-level model is formulated as follows:

$$\max Z_{\text{up}} = \sum_w \eta_w q_w \tag{2}$$

subject to

$$x_a \le x_a^{\text{res}}, a \in A_1 \tag{3}$$

Eq.(2) characterizes the objective function of the upper-level model. w denotes a collection of OD pairs, $w \in W$; whereas q_w reflects the initial travel demand for OD pair w. η_w indicates whether there is available capacity in the current network. If $\eta_w > 1$, the network is considered to have surplus capacity; if not, it is deemed overloaded. $\sum_w \eta_w q_w$ denotes the comprehensive capability of the road system. Eq.(3) outlines the capacity limitation for the reserved road, with x_a indicating the traffic volume on link a.

(2) Lower Model: Minimize the Travel Costs

Through the above definition, we can get the relationship between the following road flow, road flow and travel demand:

$$\Omega^m \triangleq \left\{ \left\{ \mathbf{f}^m, \mathbf{x}^m \right\} \middle| \mathbf{x}^m = \Delta \mathbf{f}^m, \mathbf{q}^m = \Lambda \mathbf{f}^m, \mathbf{f}^m \ge 0 \right\}, m = 1,2 \tag{4}$$

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2, \mathbf{f} = \mathbf{f}^1 + \mathbf{f}^2, \mathbf{x} = \mathbf{x}^1 + \mathbf{x}^2 \tag{5}$$

In order to describe traffic congestion, the following widely used assumptions are used (Patriksson,2015):

Hypothesis 1: The impedance c_a on path $a \in A$ is a convex function that is continuously differentiable with respect to the total link flow x_a; Additionally, the impedance of each link depends solely on its individual flow.

As in previous models, the impedance of a path is the cumulative sum of the impedances of all the road links it traverses. This can be expressed in matrix form as $\mathbf{C} = \Delta \mathbf{c}$.

$$C_{kw} = \sum_{a \in A} \delta_{kw}^a c_a \tag{6}$$

Considering the influence of chance elements and information imbalances, ordinary vehicles choose a path to minimize their perceived travel costs. This perceived expense is delineated in Eq. (7):

$$U_{kw} = -\frac{C_{kw}}{\theta_w} + \varepsilon_{kw} \tag{7}$$

where θ_w is a non-negative correction coefficient, which represents the size of the random disturbance; ε_{kw} follows an independent and identically distributed Gumbel distribution, which models the perception error. According to Sheffi 's (1985) derivation , the probability of ordinary vehicles choosing path $k \in K_w$ is Eq. (8):

$$p_{kw} = \frac{\exp\left(-C_{kw}/\theta_w\right)}{\sum_{s\in K_w}\exp\left(-C_{sw}/\theta_w\right)}, k \in K_w, w \in W \tag{8}$$

Each type of user considers the traffic distribution of other types of users as known when making their own path selection, and optimizes their own objective function through path flows. However, it is not aware of how other users will react to their path decision-making, that is, assuming that the path selected by the reservation vehicle is given, The allocation of ordinary vehicles across the road network can be established by addressing the subsequent minimization problem:

$$\min_{x^1\in\Omega^1} Z_{down}^{sue} = \sum_{a\in A}\int_0^{x_a^1} c_a\left(x_a^2+\tau\right)d\tau + \sum_{w\in W}\sum_{k\in K_w}\theta_w f_{kw}^1 \ln\left(f_{kw}^1\right) \tag{9}$$

subject to

$$q_w^1 = \left(1-\lambda\right)\eta_w q_w, \forall w \in W \tag{10}$$

$$q_w^1 = \sum_{k\in K_w} f_{kw}^1 \varepsilon_k^w, \forall w \in W, k \in K_w \tag{11}$$

$$x_a^1 = \sum_{w\in W}\sum_{k\in K_w} f_{kw}^1 \delta_{k\mathrm{w}}^a, \forall w \in W, k \in K_w, a \in A_2 \tag{12}$$

$$x_a^1 \geq 0, f_{kw}^1 \geq 0, \forall w \in W, k \in K_w, a \in A_2 \tag{13}$$

Eq. (9) specifies the objective function of the stochastic user equilibrium model, while the constraints are detailed in Eq. (10-13).

Constraint 10 reflects the travel demand for ordinary vehicles.

Constraint 11 guarantees that the total traffic assigned to each path matches the travel demand. Here, f_{kw}^1 represents the traffic on path k for the ordinary vehicle OD pair w. ε_k^w represents the relationship between reserved links and paths, ensuring that ordinary vehicles are prohibited from using the reserved roads. If there is a reservation link on path k, then $\varepsilon_k^w = 1$; otherwise $\varepsilon_k^w = 0$.

Constraint 12 guarantees that the total flow of all paths passing through link a is equal to $f_{kw}^1 \delta_{k\mathrm{w}}^a$. $\delta_{k\mathrm{w}}^a$ represents the relationship between paths and links; if link a is part of path k, then $\delta_{k\mathrm{w}}^a = 1$; otherwise, $\delta_{k\mathrm{w}}^a = 0$.

Constraint 13 indicates that the variable is positive.

It is assumed that all reservation vehicles are controlled by a company or government, and the reservation user can only specify the starting point and destination, and can not choose to take a specific path.The entity in charge, whether it's a corporation or a governmental body, manages the reserved vehicles to help travelers map out their itineraries efficiently, cutting down on overall system resistance. As a result, the placement of these reservation vehicles throughout the roadways can be pinpointed by tackling the following minimization issue:

$$\min_{x^2\in\Omega^2} Z_{down}^{so} = \sum_{a\in A} c_a\left(x_a^1+x_a^2\right)\left(x_a^1+x_a^2\right) \tag{14}$$

subject to

$$q_w^2 = \lambda\eta_w q_w, \forall w \in W \tag{15}$$

$$q_w^2 = \sum_{k\in K_w} f_{kw}^2, \forall w \in W, k \in K_w \tag{16}$$

$$x_a^2 = \sum_{w\in W}\sum_{k\in K_w} f_{kw}^2 \delta_{kw}^a, \forall w \in W, k \in K_w, a \in A \tag{17}$$

$$x_a^2 \geq 0, f_{kw}^2 \geq 0, \forall w \in W, k \in K_w, a \in A \tag{18}$$

Eq. (14) defines the objective function for the optimal system model, which must satisfy the constraints provided in Eq.(15-18).

Constraint 15 aligns with the flow conservation rule, modified via the OD matrix coefficient η_w. In the upper model, η_w is a variable that denotes the travel demand multiplier for reservation vehicles, while λ indicates the proportional coefficientof

the reservation vehicle.

Constraint 16 guarantees that the total traffic assigned to each path equals the travel demand. Here, K_w refers to the set of paths, while f_{kw}^2 denotes the traffic of reserved vehicles on path k for the OD pair w.

Constraint 17 ensures that the total flow across all paths passing through link a equals f_{kw}^2. $\delta_{k\mathbf{w}}^a$ represents the path-link relationship; if link a is included in path k, then $\delta_{k\mathbf{w}}^a = 1$; otherwise, $\delta_{k\mathbf{w}}^a = 0$.

Constraint 18 indicates that the variable is positive.

2.3 Equivalence and existence of solutions for mixed equilibrium problems

Under **Hypothesis 1,** with reference to Sheffi and Fisk (1980), the above two minimization problems (Eq.9and Eq.14) have a unique link flow solution.Given that Ω^1 and Ω^2 denote separate viable domains, the mixed equilibrium challenge is formulated as this variational inequality:

$$\sum_{a\in A}\left(c_a^1\left(x_a^{1,*},x_a^{2,*}\right)\left(x_a^1-x_a^{1,*}\right)+c_a^2\left(x_a^{1,*},x_a^{2,*}\right)\left(x_a^2-x_a^{2,*}\right)\right)\geq 0,\forall \mathbf{x}^k\in\Omega^k,k=1,2 \tag{19}$$

where c_a^1 and c_a^2 are defined as Eq.(20):

$$\begin{cases} c_a^1\left(x_a^1,x_a^2\right)=\dfrac{\partial F^1}{\partial x_a^1}=c_a\left(x_a^1,x_a^2\right)+\left.\dfrac{\partial f}{\partial x_a^1}\right|_{\left(x_a^1,x_a^2\right)} \\ c_a^2\left(x_a^1,x_a^2\right)=\dfrac{\partial F^2}{\partial x_a^2}=c_a\left(x_a^1,x_a^2\right)+\left(x_a^1+x_a^2\right)c_a^{'}\left(x_a^1,x_a^2\right) \end{cases} \tag{20}$$

Lemma 1: Under **Hypothesis 1**, the mixed equilibrium solution $\left(x^{1,*},x^{2,*}\right)$ of the variational problem (Eq. 19) exists.

Proof: This proof closely follows the approach outlined in Yang et al. (2007), Property 2, and Theorem 3.1(Kinderlehrer and Stampacchia, 1980). In order to highlight the distinctiveness of the proposed solution for this problem of variation, it is essential to impose stronger assumptions on the road impedance function (Zhang & Nie, 2018).

Hypothesis 2: The link impedance function is in polynomial form as shown in Eq. (21):

$$c_a\left(x_a\right)=t_a^0+\beta_a\left(x_a\right)\gamma \tag{21}$$

where β_a and γ are related parameters.

It is easy to see that the widely used BPR impedance function is a special case of polynomial form. Using **Hypothesis 2**, the variational problem (Eq.19) can be rewritten as an optimization problem.

Theorem 1: Under **Hypothesis 2**, the variational problem (Eq.19) is equivalent to the following optimal problem:

$$\min\sum_{a\in A}\int_0^{x_a}\beta_a w\gamma dw+\theta_w\sum_{w\in W}\sum_{k\in K_w}f_{kw}^1\ln\left(f_{kw}^1\right)+\sum_{a\in A}\left(x_a^1t_a^1+x_a^1\frac{t_a^1}{\gamma+1}\right) \tag{22}$$

subject to

$$\left\{\mathbf{f}^m,\mathbf{x}^m\right\}\in\Omega^m,m=1,2 \tag{23}$$

Proof: To demonstrate the parity between the variational issue (Eq.19) and the minimization problem (Eq.22), it is crucial to demonstrate that the solution to the minimization problem corresponds to a mixed equilibrium point. The given optimization issue's Lagrangian expression is detailed as follows:

$$L=F-\sum_{w\in W}\lambda_w\left(\sum_{k\in K_w}f_{kw}^1-q_w^1\right)-\sum_{w\in W}\mu_w\left(\sum_{k\in K_w}f_{kw}^2-q_w^2\right) \tag{24}$$

Because the travel path selection of ordinary vehicles follows the stochastic user equilibrium, $f_{kw}^1>0$ can be obtained.

Through the partial derivative of the path flow f_{kw}^1 by Lagrange function L, Eq.(25) can be obtained:

$$\frac{\partial L}{\partial f_{kw}^1}=\sum_{a\in A}\left(t_a^1+\beta_a\left(x_a\right)^\gamma\right)\delta_{kw}^a+\theta_w\left(1+\ln\left(f_{kw}^1\right)\right)-\lambda_w=C_{kw}+\theta_w\left(1+\ln\left(f_{kw}^1\right)\right)-\lambda_w=0 \tag{25}$$

Combining the Eq. (8) and $\sum_{k\in K_w} f_{kw}^1=q_w^1$, it is easy to derive:

$$f_{kw}^1=q_w^1\frac{\exp\left(-C_{kw}/\theta_w\right)}{\sum_{s\in K_w}\exp\left(-C_{sw}/\theta_w\right)} \tag{26}$$

For the traffic flow composed of reserved travel vehicles, $f_{kw}^2\geq 0$ is obviously established. Through the first-order optimal condition, Eq. (27) can be obtained:

$$\frac{\partial L}{\partial f_{kw}^2}\geq 0,\ f_{kw}^2\frac{\partial L}{\partial f_{kw}^2}=0\ , \tag{27}$$

where:

$$\frac{\partial L}{\partial f_{kw}^2}=\sum_{a\in A}\left(\frac{t_a^1}{\gamma+1}+\beta_a\left(x_a\right)^\gamma\right)\delta_{kw}^a-\mu_w=\sum_{a\in A}\frac{1}{\gamma+1}\left(t_a^1+\beta_a\left(x_a\right)^\gamma+\gamma\beta_a\left(x_a\right)^\gamma\right)\delta_{kw}^a-\mu_w=\frac{1}{\gamma+1}\sum_{a\in A}\left(c_a+x_a\frac{dc_a}{dx_a}\right)\delta_{kw}^a-\mu_w \tag{28}$$

Combining Eq. (27) and Eq. (28), Eq. (29) can be obtained:

$$\sum_{a\in A}\left(c_a+x_a\frac{dc_a}{dx_a}\right)\delta_{kw}^a\begin{cases}=(\gamma+1)\mu_w, f_{kw}^*>0\\ \geq(\gamma+1)\mu_w, f_{kw}^*=0\end{cases}\quad \forall k\in K_w, w\in W \tag{29}$$

Based on Eq.(25) and Eq.(29), the optimal solution to the minimization problem can be demonstrated as a mixed equilibrium point.

Lemma 2: Under **Hypothesis 2**, the link flow $x_a^{0.*}$ composed of ordinary vehicles in the mixed equilibrium state and the link flow $x_a^{1.*}$ composed of reserved users are unique.

Proof: Let $n=|A|$ denote the overall count of road link in the network. In order to establish the uniqueness of the solution, it's enough to show that the Hessian matrix, pertaining to the link flow $\left[x_a^1,x_a^2\right]$, is positively definite throughout. The dimension of the Hessian matrix is $2n\times 2n$, and it can be expressed as follows:

$$\begin{bmatrix}
\beta_1\left(x_1\right)^{\gamma-1}+\frac{\partial^2 f}{\partial x_1^1\partial x_1^1} & \frac{\partial^2 f}{\partial x_2^1\partial x_1^1} & \cdots & \frac{\partial^2 f}{\partial x_n^1\partial x_1^1} & \beta_1\left(x_1\right)^{\gamma-1} & 0 & \cdots & 0\\
\frac{\partial^2 f}{\partial x_1^1\partial x_2^1} & \beta_2\left(x_2\right)^{\gamma-1}+\frac{\partial^2 f}{\partial x_2^1\partial x_2^1} & \cdots & \frac{\partial^2 f}{\partial x_n^1\partial x_2^1} & 0 & \beta_2\left(x_2\right)^{\gamma-1} & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
\frac{\partial^2 f}{\partial x_1^1\partial x_n^1} & \frac{\partial^2 f}{\partial x_2^1\partial x_n^1} & \cdots & \beta_n\left(x_n\right)^{\gamma-1}+\frac{\partial^2 f}{\partial x_n^1\partial x_n^1} & 0 & \cdots & \cdots & \beta_n\left(x_n\right)^{\gamma-1}\\
\beta_1\left(x_1\right)^{\gamma-1} & 0 & \cdots & 0 & \beta_1\left(x_1\right)^{\gamma-1} & 0 & \cdots & 0\\
0 & \beta_2\left(x_2\right)^{\gamma-1} & \cdots & 0 & 0 & \beta_2\left(x_2\right)^{\gamma-1} & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & \cdots & \beta_n\left(x_n\right)^{\gamma-1} & 0 & 0 & \cdots & \beta_n\left(x_n\right)^{\gamma-1}
\end{bmatrix} \tag{30}$$

Here $f=\sum_{w\in W}\sum_{k\in K_w}\theta_w f_{kw}^1\ln\left(f_{kw}^1\right)$, in order to simplify the expression, let $\Gamma_1=\begin{bmatrix}\Gamma_2 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}\Gamma_3 & \Gamma_3\\ \Gamma_3 & \Gamma_3\end{bmatrix}$, where Γ_2 and Γ_3 are the $n\times n$ matrices expressed as follows:

$$\Gamma_2 = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^1 \partial x_1^1} & \frac{\partial^2 f}{\partial x_2^1 \partial x_1^1} & \cdots & \frac{\partial^2 f}{\partial x_n^1 \partial x_1^1} \\ \frac{\partial^2 f}{\partial x_1^1 \partial x_2^1} & \frac{\partial^2 f}{\partial x_2^1 \partial x_2^1} & \cdots & \frac{\partial^2 f}{\partial x_n^1 \partial x_2^1} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 f}{\partial x_1^1 \partial x_n^1} & \frac{\partial^2 f}{\partial x_2^1 \partial x_n^1} & \cdots & \frac{\partial^2 f}{\partial x_n^1 \partial x_n^1} \end{bmatrix} \tag{31}$$

$$\Gamma_3 = \begin{bmatrix} \beta_1 (x_1)^{\gamma-1} & 0 & \cdots & 0 \\ 0 & \beta_2 (x_2)^{\gamma-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \beta_n (x_n)^{\gamma-1} \end{bmatrix} \tag{32}$$

In the case of any column vectors $\mathbf{z} = (z_i) \in R^n$ and $\mathbf{y} = (y_i) \in R^n$, Eq.(33) is derived as:

$$\begin{bmatrix} \mathbf{z}^T & \mathbf{y}^T \end{bmatrix} \Gamma_1 \begin{bmatrix} \mathbf{z} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{z}^T & \mathbf{y}^T \end{bmatrix} \left(\begin{bmatrix} \Gamma_2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \Gamma_3 & \Gamma_3 \\ \Gamma_3 & \Gamma_3 \end{bmatrix} \right) \begin{bmatrix} \mathbf{z} \\ \mathbf{y} \end{bmatrix} = \mathbf{z}^T \Gamma_2 \mathbf{z} + \mathbf{z}^T \Gamma_3 \mathbf{z} + \mathbf{y}^T \Gamma_3 \mathbf{z} + \mathbf{z}^T \Gamma_3 \mathbf{y} + \mathbf{z}^T \Gamma_3 \mathbf{y} \tag{33}$$

According to Theorem 1 in Evans (1973), function $f = \sum_{w \in W} \sum_{k \in K_w} \theta_w f_{kw}^1 \ln\left(f_{kw}^1\right)$ is a strictly convex function with respect to path flow $\mathbf{f}^1$ in feasible region Ω^1 .At the same time, according to the mapping $\mathbf{x}^1 = \Delta \mathbf{f}^1$, it can be obtained that Γ_2 is also strictly convex about the link flow in the feasible region Ω^1. Therefore, it is expressed as follows:

$$\mathbf{z}^T \Gamma_2 \mathbf{z} > 0 \tag{34}$$

And because Γ_3 is a symmetric matrix:

$$\mathbf{z}^T \Gamma_3 \mathbf{z} + \mathbf{y}^T \Gamma_3 \mathbf{z} + \mathbf{z}^T \Gamma_3 \mathbf{y} + \mathbf{y}^T \Gamma_3 \mathbf{y} = \mathbf{z}^T \Gamma_3 \mathbf{z} + 2\mathbf{y}^T \Gamma_3 \mathbf{z} + \mathbf{y}^T \Gamma_3 \mathbf{y} = \sum_{i=1}^{n} \beta_i \gamma (x_i)^{\gamma-1} (z_i + y_i)^2 \geq 0 \tag{35}$$

Combining Eq. (34) and Eq. (35):

$$\begin{bmatrix} \mathbf{z}^T & \mathbf{y}^T \end{bmatrix} \Gamma_1 \begin{bmatrix} \mathbf{z} \\ \mathbf{y} \end{bmatrix} > 0 \tag{36}$$

From the Eq.(35), it can be concluded that the Hessian matrix Γ_1 is positive definite, so the road traffic flow $x_a^{1,*}$ composed of ordinary vehicles under the mixed equilibrium state and the road traffic flow $x_a^{2,*}$ composed of reservation vehicles are unique.

2.4 Solution algorithms

The bi-level programming problem is classified as a non-deterministic polynomial-time hard problem (NP-hard) (Ben-Ayed, 1993). Due to the complexity of this problem, solving it using exact algorithms is challenging and computationally intensive, thus heuristic algorithms are commonly employed. Compared to optimization methods such as genetic algorithms and artificial bee colony algorithms, the Whale Optimization Algorithm (WOA) offers significant advantages in both optimization performance and computational efficiency, abstracts the hunting behaviors of humpback whales, including encircling prey, bubble-net feeding, and searching for prey (Mirjalili & Lewis, 2016). In each iteration, the algorithm updates decision variables, with the position of each whale representing a feasible solution. The whale moves closer to the optimal solution by continuously adjusting its position. Based on WOA, and combined with the lower-level Partan Frank-Wolfe algorithm, the traffic assignment process is integrated into the upper-level model. The detailed procedures of the WOA nested Partan Frank-Wolfe algorithm are outlined as follows:

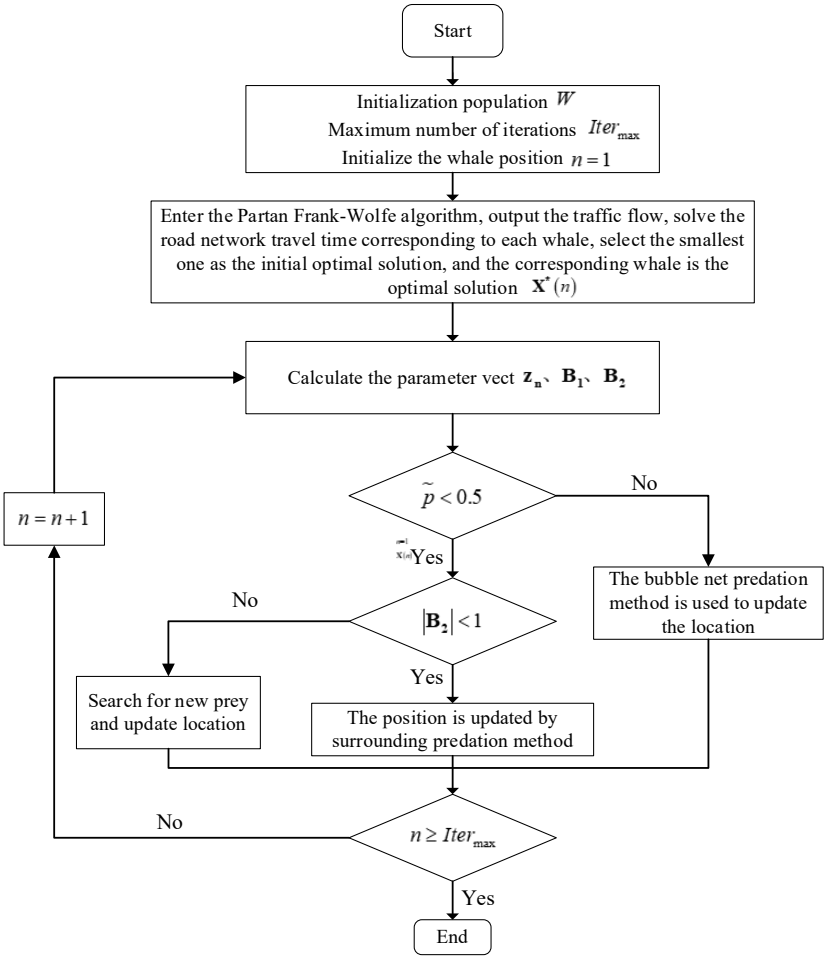

Fig. 1. Solving algorithm flow chart

3. Numericai experiment

This section showcases a case study designed to demonstrate the proposed method's efficacy and to establish the critical parameter values essential for rolling out reservation-based travel within particular settings. The research not only lays down a solid theoretical framework but also offers practical insights for the advancement of this travel approach.

3.1 Data description

The current section utilizes the model that was previously developed for the Sioux Falls network, sourced from the "Transportation Networks for Research" GitHub repository.

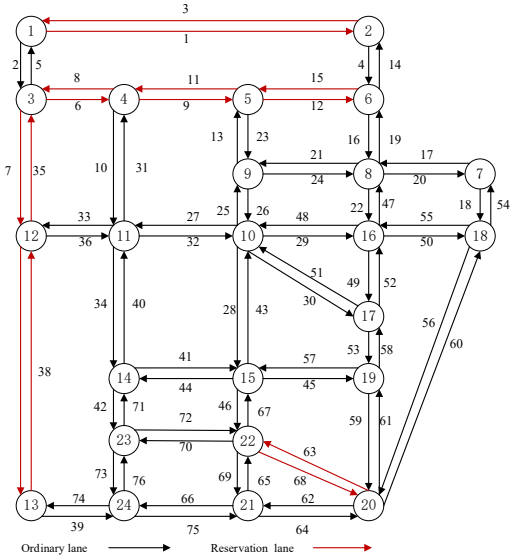

Fig. 2. Sioux-Falls network

As depicted in Fig.2, the layout of the network and the designated lanes for exclusive use are illustrated. The specific

parameters of the network are detailed in Table 2. The designated reservation time spans the morning rush, from 8:00 to 9:00, and during this period, access to the reserved lanes is restricted solely to vehicles with reservations.

Table 2
Basic Parameters of Road Network

Link Number	Length of Link (km)	Impedance at Zero Flow (min)	Traffic Capacity (veh/h)	Link Number	Length of Link (km)	Impedance at Zero Flow (min)	Traffic Capacity (veh/h)
1	6	6	25900	39	4	4	5091
2	4	4	23403	40	4	4	4877
3	6	6	25900	41	5	5	5128
4	5	5	4958	42	4	4	4925
5	4	4	23403	43	6	6	13512
6	4	4	17111	44	5	5	5128
7	4	4	23403	45	3	3	14565
8	4	4	17111	46	3	3	9599
9	2	2	17783	47	5	5	5046
10	6	6	4909	48	4	4	4855
11	2	2	17783	49	2	2	5230
12	4	4	4948	50	3	3	19680
13	5	5	10000	51	8	8	4994
14	5	5	4958	52	2	2	5230
15	4	4	4948	53	2	2	4824
16	2	2	4899	54	2	2	23403
17	3	3	7842	55	3	3	14565
18	2	2	23403	56	4	4	4824
19	2	2	4899	57	3	3	5003
20	3	3	7842	58	2	2	23403
21	10	10	5050	59	4	4	5003
22	5	5	5046	60	4	4	23403
23	5	5	10000	61	4	4	5003
24	10	10	5050	62	6	6	5060
25	3	3	13916	63	5	5	5076
26	3	3	13916	64	6	6	5060
27	5	5	10000	65	2	2	5230
28	6	6	13512	66	3	3	4885
29	4	4	4855	67	3	3	9599
30	8	8	4994	68	5	5	5076
31	6	6	4909	69	2	2	5230
32	5	5	10000	70	4	4	5000
33	6	6	4909	71	4	4	4925
34	4	4	4877	72	4	4	5000
35	4	4	23403	73	2	2	5079
36	6	6	4909	74	4	4	5091
37	3	3	25900	75	3	3	4885
38	3	3	25900	76	2	2	5079

3.2 Computational results

This paper outlines a bi-level programming model, which has been coded in Python 2.7 and executed on a revised Sioux-Falls network. This research posits that the traffic demand within the roadway system remains unchanged both prior to and following the implementation of reservation-based travel. By comparing the results from the original traffic assignment based on user equilibrium (with no reserved lanes) and the results of the SO-SUE mixed traffic equilibrium subsequent to the implementation of the reservation travel strategy, we can confirm the efficacy of the reservation approach.The findings are succinctly presented in Table 3. Prior to implementing the strategy, the average V/C ratio of the roads was 0.83, indicating significant traffic congestion. After the strategy was implemented, congestion was reduced, and the overall travel costs decreased. The findings demonstrate that this strategy effectively reduces road network congestion and helps to alleviate the imbalance in traffic flow distribution across the network.

Table 3
V/C Ratio and Cost Comparative Analysis

	V/C of All Links	Reservation Link V/C	Travel Cost
Ordinary travel	0.83	0.87	8796
Reservation travel	0.78	0.75	6580

4. Discussion and conclusion

This section explores the impact of elements like the reservation ratio and the quantity of reserved roads on the capacity of

the road network, the average travel time for vehicles, and the general patterns of vehicle reservations.

4.1 Discussions

4.1.1 Road network capacity

Fig.3 illustrates the computed capacity of the road network based on varying numbers of reserved roads and their corresponding reservation ratios. The dotted line reflects the capacity of the network with the initial traffic distribution. The findings reveal that, under consistent conditions, an increase in the number of reserved roads correlates with a decline in the overall network capacity. Moreover, when the number of reserved roads remains constant, an uptick in the reservation ratio λ leads to a further reduction in capacity, which eventually levels off. In the end, the capacity of the road network stabilizes at a point below the original capacity, suggesting that during the morning rush hour, heightened traffic levels result in congestion, leaving no additional capacity available. Consequently, some commuters will need to modify their departure times. Notably, when the reservation ratio λ hits 0.4, the road network capacity achieves a stable condition.

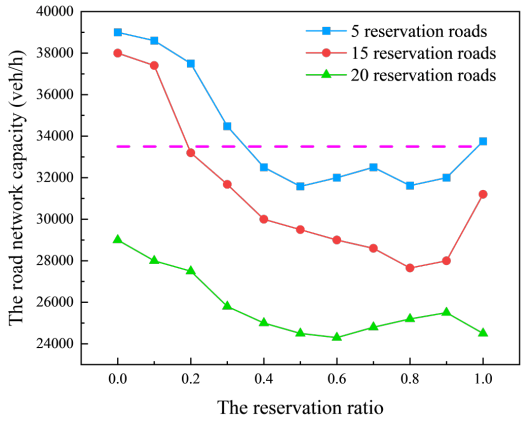

Fig. 3. Road network capacity under different scenarios

4.1.2 The average travel time of vehicles

Fig. 4 shows the travel time and improvement effect of reservation vehicles under different proportions.As the share of reserved vehicles rises, the average travel time for these vehicles drops from 615s to 475s, and the benefit ratio of reservation vehicles continues to rise and the potential benefit ratio decreases.

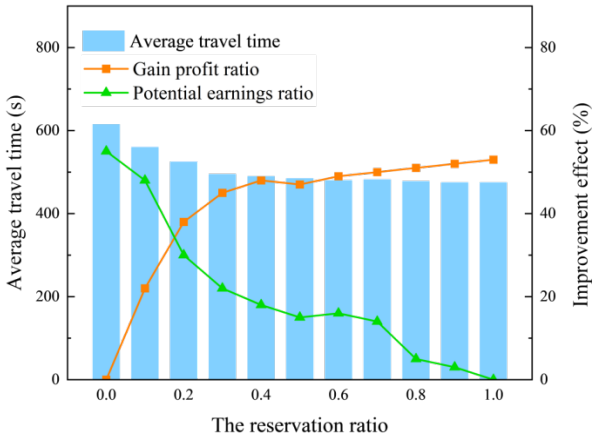

Fig. 4. Travel time and improvement effect of reservation vehicles with different proportions

Note:
Gain benefit ratio = Gain benefit / Congestion loss;
Potential profit ratio = Potential profit / Congestion loss;
Congestion loss = Average travel time - Shortest travel time;
Earnings = Average travel time without reservation - Average travel time;
Potential revenue = Average travel time - Average travel time for all vehicles to participate in reservation

The improvement effect is obvious. When the reservation ratio is 0% ~ 40%, increasing the reservation ratio can significantly reduce the average travel time, obtain greater benefits, and have high marginal benefits. When the reservation ratio is 40%, the reservation gain ratio is 48%. As the proportion of reserved vehicles continues to rise, the number of reservation vehicles increases, but the travel time for each path no longer decreases. The average travel time of reservation vehicles rises slightly

due to the allocation of longer travel time paths, and then decreases slightly due to the increase of adjustable allocation paths of vehicles. The benefits brought by increasing the proportion of reservations are low.

4.1.3 The overall reservation of vehicles

It is anticipated that the anticipated count of reserved vehicles increases linearly with the rise in the reservation ratio. The overall distribution of reservation vehicles is shown in Fig. 5. As the reservation proportion reaches 40%, the number of vehicles expected to be reservationed is very close to the number of reservations. The proportion of unsuccessful reservations decreased from 1 to 19%, and the proportion of unsuccessful reservations decreased from 2% to 18%. The restriction that hinders the successful reservation of vehicles is insufficient reservations. As the proportion of reservations continues to rise,the number of reservations increases rapidly, and the proportion of unsuccessful reservations decreases slowly. When all vehicles are reservation vehicles, it is 13%; the proportion 2 decreased faster, and the reservation time of all vehicles decreased to 2%, which reflected that the restrictions affecting the successful reservation of vehicles shifted from the reservation quota limit to the reservation demand limit.

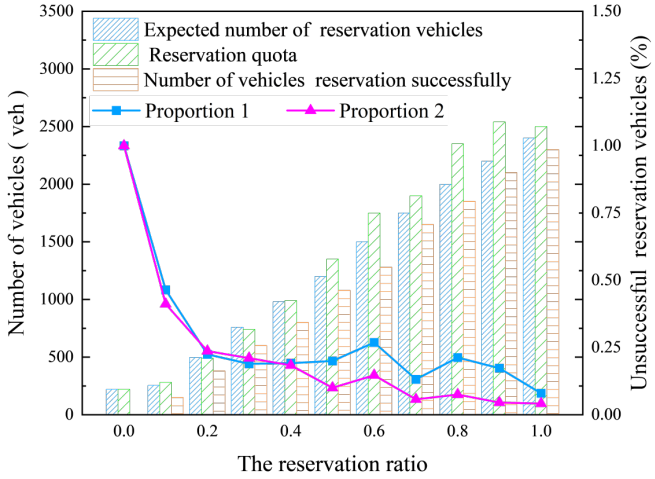

Fig. 5. The change of reservation quota of reservation vehicles under different proportions

Note:
Proportion 1 = (Reservation quota-number of vehicles successfully reserved) / Reservation quota;
Proportion 2 = (Expected number of reservation vehicles-successful number of reservation vehicles) / Expected number of reservation vehicles

4.2 Conclusion

This research develops the reservation travel issue as a bi-level programming framework that reflects the competing interests of policymakers and road users. It is validated through numerical simulations using the Sioux-Falls network. By analyzing the differences in the V/C ratio and travel costs before and after the introduction of reservation travel, the study demonstrates the effectiveness of reservation travel in alleviating urban traffic congestion.Furthermore, the research explores the impact of variations in the number of reserved roads and the reservation ratio on road network capacity, service level, and travel costs during the implementation of reservation strategies. The study concludes the following:

(1) The more reserved roads there are, the less the capacity of the road network. Even with a constant number of reserved roads, an increase in the reservation ratio leads to a reduction in network capacity, which eventually levels off.
(2) With the increase of the reservation ratio, the average travel time of the reservation vehicle decreases from 615s to 475s, the profit ratio of the reservation vehicle continues to rise and the potential profit ratio decreases, and the improvement effect is obvious.
(3) In the process of increasing the reservation ratio to 40%, the expected number of reservation vehicles is very close to the reservation quota. When the reservation ratio continues to increase, the number of reservation quotas increases rapidly, and the proportion of unsuccessful reservations decreases slowly.

The results and conclusions of this research offer both theoretical and practical value for the implementation of reservation travel. This approach helps to balance supply and demand, reduce traffic congestion, and achieve an optimal shift from user-based to system-based traffic management. However, while reservation travel has the potential to improve the matching of supply and demand in urban networks, it does not account for the dynamic fluctuations in road capacity or the alterations in residents' daily commuting behaviors post-implementation. This oversight could impact the strategy's overall effectiveness. Future research will focus on capturing these dynamic behaviors more comprehensively.

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