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The calculation method of critical chain buffer based on GERT network and information entropy

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A B S T R A C T

In order to address project delays caused by uncertain factors in the construction project schedule management process, a new method for calculating the size of critical chain buffers is proposed. Taking into account process uncertainty and based on the GERT network, the correction process and duration of different logical processes are provided to transform the uncertain network into a deterministic one. By combining the critical chain management method and information entropy theory to assess the impact of increasing uncertainty in the merging process on the critical chain buffer, a calculation model for the critical chain buffer size based on the GERT network is established and analyzed using a numerical example. The feasibility and effectiveness of the model are verified through an example, demonstrating its ability to reduce project risks and shorten the construction period while ensuring a high probability of project completion.

1. Introduction

The size of critical chain buffers is a key focus in critical chain project management research. This involves the establishment of various buffer types at the beginning or end of critical and non-critical chains to absorb uncertainties related to project delays caused by analytical factors inherent in the project construction process, and to minimize their impact on project progress. In the calculation method of critical chain buffer size, Goldratt (1997) first proposed the "cut-and-paste" method, which takes half of the sum of the safety time required by each process in the critical chain as the project buffer. Newbold (1998) utilized the concept of the central limit theorem, summing the root variances as an item buffer. Considering the rationality of the calculation results of the two buffer sizes, Herroelen and Leus (2001, 2002) monitored the construction period and buffer of projects of different sizes and pointed out that the "cut-and-paste" method is simple to calculate but may lead to excessively long buffer times, making it more suitable for small-scale projects. The root variance method is better suited for buffer calculation in large-scale projects, yet the manager's experience in estimating process duration greatly influences this method). Hoel and Taylor (1999) employed Monte Carlo simulation to determine the project buffer as the time needed under the completion probability required by the owner minus the average time, reducing the impact of the manager's experience on process duration estimation in buffer calculation). Considering the different safety level requirements of different projects. Shou and Yeo (2000) studied the setting of buffer zones in terms of the uncertainty of the project manager's risk preference and other factors on the distribution of project durations. Long and Ario acknowledge the reliance of the root variance method on expert experience and historical data, apply fuzzy mathematics to the calculation of engineering project buffers and propose a fuzzy critical chain buffer calculation method that enhances the root variance approach (Long & Ario, 2008). To provide a more realistic estimation of process duration, Fallh et al. (2010) considered the various shape parameters of the time distribution in the route when determining the size of the input buffer and project buffer. Chen et al. (2016) proposed to add the remaining buffer of the non-critical chain to the correction model of the project buffer, thus reducing the impact of the non-critical chain on the project duration (Chen et al., 2015). Liu et al. (2015) used the exploratory factor analysis

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method, extracted 21 influencing factors of the project buffer setup, and established a comprehensive weight calculation model for the process. Tukel (2006) proposed a buffer calculation method considering resource tension and network complexity. Xu et al. (2017) proposed a buffer setting and adjustment control linkage model considering multiple factors. Jiang and Chen (2004) classified resource constraints into rigid resource constraints and flexible resource constraints to adjust the network model. Wang et al. constructed a mathematical model of progress-resource to make a buffer calculation method to improve it (Wang et al., 2005). Zhang et al. (2015) determined an optimized buffer zone calculation method based on the trapezoidal fuzzy number to measure the uncertain risk of the process, which was combined with the resource tension as well as the network complexity. Jiang et al. (2019) used the minimum substitutability of the resources to determine the degree of constraints of the activity resources, which was combined with the intensity of the actual need of the activity resources to give an algorithm for the integrated resource impact coefficients. Nie et al. (2021) constructed a buffer calculation model by considering the influence of multi-resource constraints and relay potential. Some scholars apply the entropy weight method to the study of critical chain buffer size. For example, Gong et al. (2022)proposed the steps and methods of human factor entropy measurement based on interval intuitionistic trapezoidal fuzzy numbers, which fully considered the influence of human behavioral factors on the project schedule. Li et al. (2020) considered information entropy as a measurement method to study uncertainty factors after comparing the connection between thermodynamic entropy and information entropy. Zhang et al. (2017) proposed an algorithm for determining buffer of the critical chain project based on entropy weight method to assess the uncertainty of the project and optimize the buffer estimation accuracy (Zhang et al., 2017; Zhang & Zhou, 2021; Lin & Zhou quantified the uncertainty of each process according to the entropy weight TOPSIS method and obtained the uncertainty coefficients of each process (Lin & Zhou, 2011). Peng et al. (2021) combined with the information entropy theory and proposed a buffer setup model based on the complexity entropy, the multi-resource entropy, and the project team entropy. In the network planning of engineering projects, the application of GERT network can enhance the flexibility of network diagrams, which is suitable for making decisions on engineering solutions and stochastic problems. Based on the theory of flow graph and related probability statistics. Huang (1979) derived a method to solve the moments' mother function and conditional moments' mother function, and expressed the equivalent function of the GERT network based on the table algorithm. Xu (1981) proposed a new solution method based on the table algorithm to better satisfy the computational needs of GERT. Zhao (1995) proposed the theory of a multi-additive element GERT network analysis method and established a mathematical planning GERT model based on nonlinear multi-objective, which was used to express the universal link between additional cost and work time reduction (Zhao, 1995; Zhao, 1990). Li and Lv used computer algorithms and GERT network together to provide hardware technology support for the calculation of GERT network in large-scale projects (Li & Lv, 1998). He et al. (2004) simulated the GERT network process criticality and change curve to assess the degree of influence of each process on the schedule. Shi and Gong (2012) introduced CCPM into the GERT network, and identified the critical chain of the GERT network by simulation through the computer. Tao et al. constructed the CF-GERT model by using the characteristic function and the transfer probability as the transfer function (Tao et al., 2018). The aforementioned research is confined to the affirmative network model, where every activity in the network must be realized without room for activity or decision, and no possibility of probability branching. However, every process in the GERT network can be realized, indicating that certain processes involve uncertainties, their nodes can be realized repeatedly, and there is a closed loop in the network, which addresses issues of circular operation or rework in the project, aligning more closely with engineering practice.

Considering that the duration of a certain process cannot be determined due to rework or uncertain process construction schemes in a project, this paper combines critical chain project management with GERT network. It establishes a critical chain buffer calculation model based on information entropy and GERT network, drawing on critical chain project management and information entropy theory. By using the Mason formula and counting moment generating function, the expected duration and execution times of operations with loop and multi-flow structures are solved, transforming uncertain operations into definite ones and further calculating the buffer size.

2. GERT network transformation

2.1 Principle of GERT network analysis method

The network logic of the GERT network represents a logical relationship between projects or processes, expressed through the fundamental elements (nodes and arrows) of a network diagram. The GERT network analysis method combines signal flow graph theory, moment generating functions, and PERT to convert uncertain GERT networks into deterministic networks. The moment generating function $M_i(s)$ of step i is:

$$M_i(s) = E(e^{t_i s}) = \begin{cases} \int_{t_i} e^{st_i} f(t_i) dt_i & t_i \text{ is a continuous variable} \\ \sum_{t_i} e^{st_i} p(t_i) & t_i \text{ is a discrete variable} \end{cases} \tag{1}$$

where, t_i is the duration of the process, $f(t_i)$ represents the probability density function of t_i, $p(t_i)$ represents the probability of implementing step i, and s is a constant.

The auxiliary function $W_i(s)$ introduced is:

$$W_i(s) = p_i M_i(s) \tag{2}$$

According to the signal flow graph theory, the transmission elements of the GERT network are simplified, and the equivalent auxiliary function $W_E(s)$ of the GERT network is obtained using the *Mason* formula as follows:

$$W_E(s) = \frac{\sum_j p_i(s)\left[1+\sum_m (-1)^m L_j(m,s)\right]}{H(s)} \tag{3}$$

$$H(s) = 1+(-1)^n \sum L_n \tag{4}$$

where $p_i(s)$ represents the product of $W_i(s)$ along the j route without loops from the initial node to the terminal node of the line, $L_j(m, s)$ represents the sum of the $W_j(s)$ function for the m-level loops not intersected by the j route, $H(s)$ denotes the determinant of the entire GERT network, L_n indicates the value of the n-order loop in the network.

2.2 The expected number of loop executions

In accordance with the theory of counting moment generating functions, when a loop in a GERT network is executed only once, it conforms to a discrete distribution with a constant value of 1, and the moment generating function is e^c. In the scenario of network implementation, the transmission element e^c is multiplied by the moment generating function e^s of the loop, illustrating the GERT network roadmap incorporating the transmission element as depicted in Fig. 1.

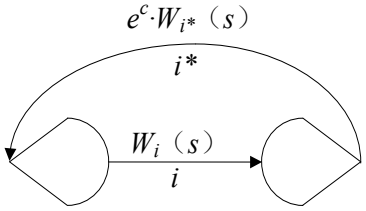

Fig. 1. GERT network roadmap incorporating transmission elements

When s=0, the time generating function simplifies to 1. The expected frequency $E(c)$ of loop realization obtained by Mason's formula is (Zheng, & Zhao, 1985):

$$M^*(c) = M_E(s,c)\big|_{s=0} = W_E(s,c)\big|_{s=0} \tag{5}$$

$$E(s) = \left.\frac{dM^*(c)}{dc}\right|_{c=0} \tag{6}$$

2.3 Expected duration of a project

2.3.1 Network diagram correction

Correction of GERT network diagrams can be categorized into loop process correction and multi-flow process correction.

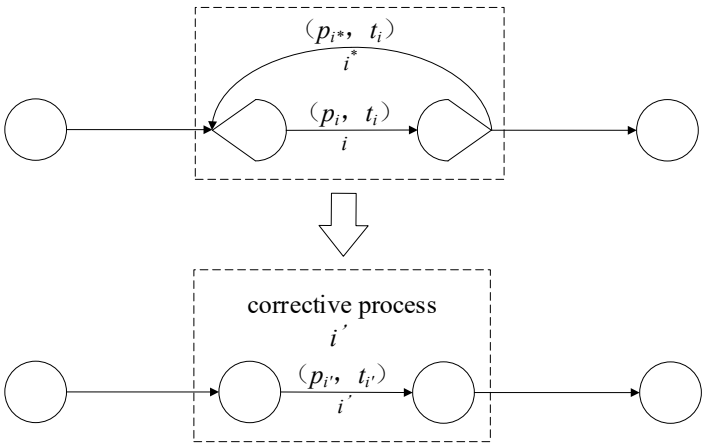

Fig. 2. Procedure correction diagram with loop

The loop procedure correction involves adjusting the expected construction period of the loop network. This is achieved by calculating the probability function and the expected frequency function of the loop implementation. Subsequently, the GERT network is transformed into a deterministic network by consolidating uncertain processes (Li, & Zuo, 2007). By merging loop

i and loop *i** into loop *i*' without a loop, the corrective step for loop *i* becomes step *i*'. The diagram illustrating the procedure correction with a loop is depicted in Fig. 2. Multi-flow process correction refers to a scenario where, due to multiple flow directions and uncertain implementation processes, it is impossible to predict in advance which process will be carried out. As a result, the critical path and construction period of the project cannot be definitively established. Thus, processes i_l and i_h, structured with multiple flows, are consolidated into the revised process *i*', denoted as the adjusted process from i_l to i_h. The diagram illustrating the multi-flow process correction is presented in Fig. 3.

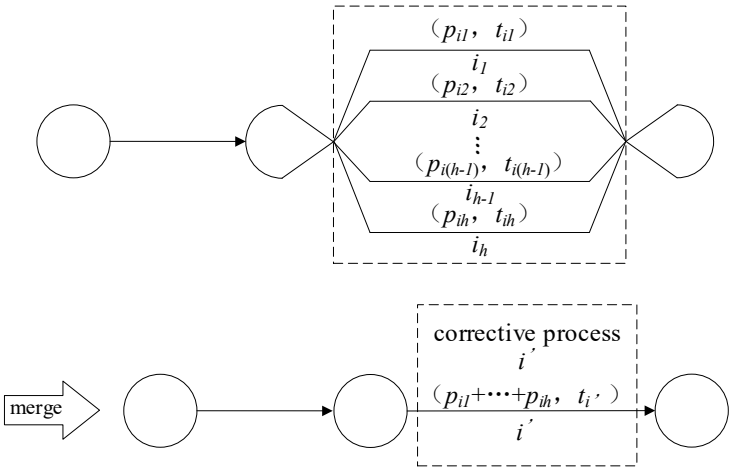

Fig. 3. Multi-flow process correction diagram

2.3.2 Expected duration calculation

(1) Single-loop project

1) Loops span a single process

The project has only one process *i* with a loop, and the diagram of the single-loop project spanning a single process is shown in Fig. 4. The duration of the correction process taking into account the number of loop repeats is:

$$t_{i'} = t_i + E\left(c_{i^*}\right)\cdot\left(t_i + t_{i^*}\right) \tag{7}$$

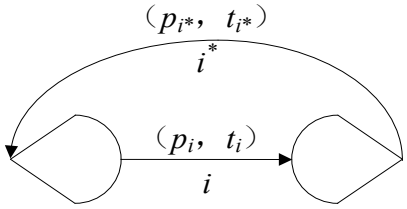

Fig. 4. Single-loop project diagram spanning a single process

where, $t_{i'}$ is the duration of the correction process *i*', t_{i*} is the duration of loop i^*, $E(c_{i*})$ is the expected number of implementations of loop i^*.

2) Loops span multiple processes

When loop ij^* spans multiple processes, the project diagram of single loop spanning multiple processes is shown in Fig. 5. The duration of correction process of loop spanning multiple processes is as follows:

$$t_{ij'} = t_i + \cdots + t_j + E\left(c_{ij^*}\right)\cdot\left(t_i + \cdots + t_j + t_{ij^*}\right) \tag{8}$$

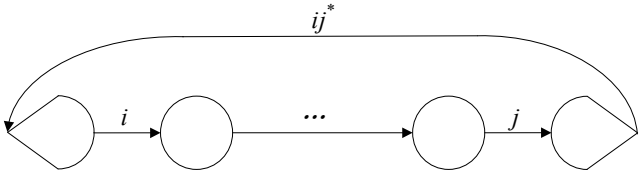

Fig. 5. A single loop project diagram spanning multiple processes

where $t_{ij'}$ is the duration of the correction process ij' in the loop spanning *j*-*i*+1 processes, t_{ij*} is the duration of the loop process ij^*, $E(c_{ij*})$ represents the expected number of times the loop procedure ij^* is implemented.

(2) Multi-loop project

1) Series structure multi-loop project

When the project contains multiple series processes from i to j with loops, the diagram of the series structure multi-loop project is shown in Fig. 6. The modified network project duration T of the series structure multi-loop project is:

$$T = t_{i'} + \cdots + t_{j'} \tag{9}$$

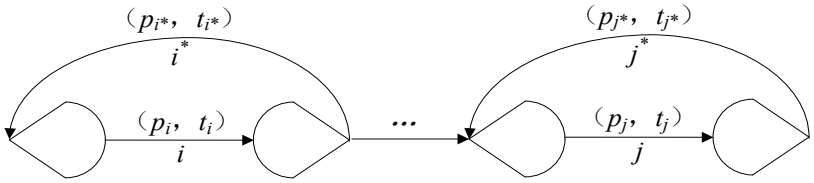

Fig. 6. Multi-loop project diagram of series structure

2) Parallel structure multi-loop project

When the project contains v parallel processes with loops, the diagram of the multi-loop project with parallel structure is shown in Fig. 7. The modified network project duration T of the parallel structure multi-loop project is:

$$T = t_{u'} = \max\left(t_{i'}, \cdots, t_{v'}\right) \tag{10}$$

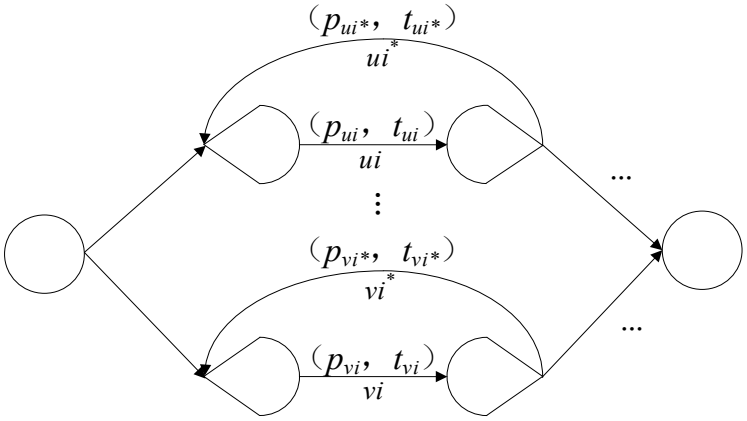

Fig. 7. Diagram of a multi-loop project with a parallel structure

where, T_u is the duration of route u, $t_{ui'}$ is the duration of correction of process i on Route u.

3) Hybrid structure multi-loop project

The hybrid structure denotes the inclusion of both series and parallel structures within the project. The project diagram for the multi-loop hybrid structure is illustrated in Fig. 8. The modified network project duration T for the hybrid structure multi-loop project is as follows:

$$T = t_{u'} + \cdots t_{j'} \tag{11}$$

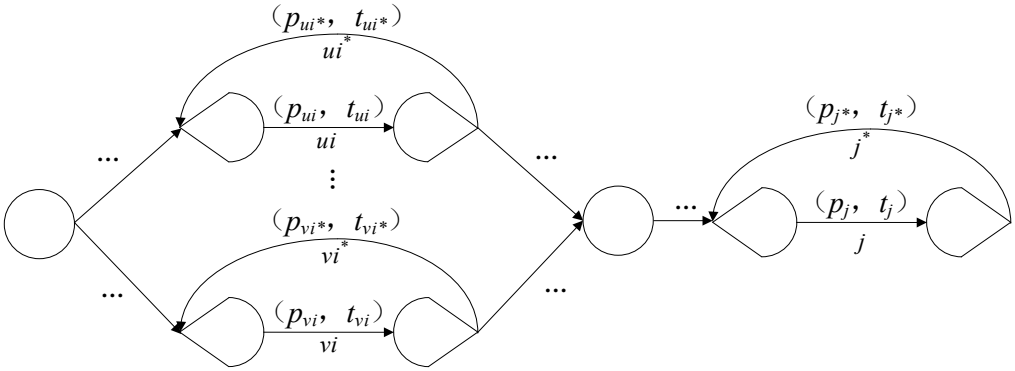

Fig. 8. Multi-loop project diagram of hybrid structure

4) Multi-flow structure project

Multi-flow structure refers to the structure implemented through one of the multiple route options in the project, where the

process combines various flows. The diagram illustrating the multi-flow structure project is presented in Fig. 9. The adjusted duration T of the network project for the multi-flow structure is as follows:

$$T = t_{i'} = \frac{\sum_{z=1}^{h}\left(p_{iz}t_{iz}\right)}{\sum_{z=1}^{h} p_{iz}} \tag{12}$$

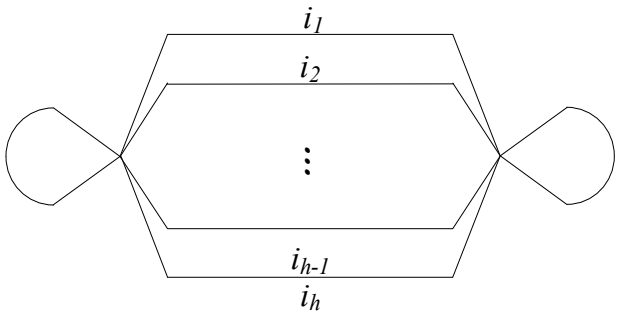

Fig. 9. Diagram of a multi-flow structure project

where h is the total number of flow processes in the multi-flow structure, and p_{iz} is the probability of implementing the iz-th flow process in the multi-flow structure.

3. Analysis of Factors Influencing Buffer Size

3.1 Information entropy

The construction period of a construction project is easily affected by a variety of uncertain factors. The purpose of setting the critical chain buffer is to eliminate the schedule risk caused by multiple uncertain factors, and the influence of uncertainty on the critical chain buffer is measured by information entropy.

$$H = -K\sum_{i=1}^{n} p_i \log p_i \tag{13}$$

where H represents information entropy, $K \geq 0$ is a constant, and $p_i (0 \leq p_i \leq 1)$ represents the probability that the system is in the i microscopic state. In engineering applications, logarithms are mostly in the form of base 2, the constant $K=1$, and the unit of information entropy is *bit*.

3.2 Process entropy

(1) Process handover complexity

The process handover complexity is utilized to assess the influence of the quantity of predecessor and successor processes on the duration of a given process. A higher count of predecessor and successor processes contributes to increased process handover complexity, thereby elevating the risk of project delays. The formula for calculating process handover complexity is as follows:

$$\alpha_i = 1 - \frac{1}{B_i + A_i} \tag{14}$$

where, α_i denotes the process relay complexity of process i, B_i represents the number of processes immediately preceding process i, A_i represents the number of successor processes for process i.

(2) Process technical complexity

Process technical complexity is employed to evaluate the impact of process duration on process technology. A smaller allowable relative deviation in the process quality feature target value indicates a higher level of process technical complexity, thereby increasing the risk of project delays (Pan & Wang, 2000). The formula for calculating process technical complexity is as follows:

$$\beta_i = 1 - \frac{U_i - L_i}{USL_i - LSL_i} \tag{15}$$

In the formula, β_i represents the Process Technical Complexity of process i, U_i represents the upper limit of the quality characteristic value of process i, L_i represents the lower limit of the quality characteristic value of process i, USL_i represents the upper limit of the quality characteristic value when rework is caused by process i, LSL_i represents the lower limit of the quality characteristic value when rework is caused by process i.
Where, β_i represents the technical complexity of process i, U_i represents the upper limit of the quality characteristic value of process i, L_i represents the lower limit of the quality characteristic value of process i, USL_i represents the upper limit of quality characteristics caused by process i at rework, LSL_i indicates the lower limit of the quality characteristic caused by process i at rework.

(3) Process entropy

The information entropy of complex events composed of independent events is additive, so the process relay complexity and process technical complexity of process i are combined into process entropy, and the calculation formula of process entropy H_{Di} is as follows (Qiu, 2021):

$$H_{D_i} = -\alpha_i \log \alpha_i - \beta_i \log \beta_i \tag{16}$$

3.3 Calculation of process complex entropy

The merging of processes and the correction of the network diagram will increase the uncertainty of processes, and the information entropy theory is combined with it to measure the uncertainty risk of this part. The risk is offset by the correction buffer.

3.3.1 Loop process complexity entropy

The loop complexity value combines the uncertainty caused by the loop, and considering the realization probability of the loop after correction, the loop complex entropy H_{MCi} is expressed as:

$$MC_i = p_{c_i} + p_{c_{i^*}} \cdot E\left(c_{i^*}\right) \tag{17}$$

$$H_{MC_i} = -MC_i \log MC_i \tag{18}$$

where, MC_i represents the loop process complexity.

3.3.2 Multi-flow structure process complexity entropy

The complexity of a multi-flow structure process pertains to the uncertainty arising from the collective processes within the multi-flow structure. Taking into consideration the probability of realization subsequent to the modification of the multi-flow structure, the complex entropy of the multi-flow structure process, denoted as HMOCi, is expressed as:

$$t_{im} = Max\left(t_{i1}, t_{i2}, \cdots, t_{ih}\right) \tag{19}$$

$$MOC_i = \frac{t_{i'}}{t_{im}} \tag{20}$$

$$H_{MOC_i} = -MOC_i \log MOC_i \tag{21}$$

where, MOC_i represents the process complexity of multi-flow structure.

3.3.3 Process complexity entropy

Based on the additive information entropy of complex events composed of independent events, process handover complexity, process technical complexity, loop process complexity, and multi-flow structure process complexity are amalgamated to form process complexity entropy. The calculation formula for process complexity entropy, denoted as H_{DCi}, is as follows:

$$H_{DC_i} = H_{D_i} + H_{MC_i} + H_{MOC_i} \tag{22}$$

3.4 Resource entropy

(1) Learning efficiency model

Flexible resources can enhance resource utilization through learning, and the learning ability will improve with the increase in learning time during the initial stages, in line with the learning efficiency model proposed by Wright. The learning ability of flexible resources is advantageous for alleviating resource constraints. Referring to the learning efficiency model, the resource substitution efficiency of flexible resources is expressed as (Chen et al., 2016):

$$\omega = \frac{P_{i0}}{t_i^s} \tag{23}$$

$$s = \frac{\ln(Le)}{\ln 2} \tag{24}$$

where ω represents the substitution efficiency of flexible resources, P_{i0} represents the initial working capability value of flexible resources for process i, t_i represents the duration of process i, s stands for the learning factor, and Le represents the learning rate, where $0 \le Le \le 1$. In practical applications, Le is typically around 75%-95%.

(2) Resource tension

Resource tension is used to measure the impact of resource constraints on the duration of a process. The formula for calculating resource tension, taking into account the substitution efficiency of flexible resources, is as follows (Guo et al., 2018):

$$\delta_i = \max_{K \in R_i} \left(\frac{\sum_{i=1}^{m} r_{ik} - \sum_{i=1}^{m} (\omega - 1) r_{iq}}{R_k} \cdot \frac{t_i}{T} \right) \tag{25}$$

where, δ_i represents the resource tension of the i process, r_{ik} represents the amount of the k resource used by the i process, m represents the number of processes that use the same resource simultaneously, r_{iq} represents the amount of the q flexible resource used by the i process, R_k represents the total amount of the k-th resource. R_i represents the set of resources used by the i process. T represents the project duration.

(3) Resource entropy

The formula for calculating the resource entropy $H_{\delta i}$ of process i is:

$$H_{\delta_i} = -\delta_i \log \delta_i \tag{26}$$

3.5 Artificial entropy

Human complexity is a measure of the impact of human factors on the duration of a process. During a project, the uncertainty associated with human factors, such as habitual delays, transient efficiency, false redundant time, changes in thoughts, and burnout, is difficult to measure. Human factors can be categorized into positive factors (e.g., transient efficiency, changes in thoughts) that affect the duration of the process, and negative factors (e.g., habitual delays, false redundant time, burnout state). Considering the impact of positive human factors on project progress, the calculation steps for anthropic complexity, based on the probabilistic hesitant fuzzy entropy multi-attribute decision-making method, are as follows (Liu et al., 2019):

Step 1: Identify the attributes of human factors. Human factor attributes include benefit attributes and cost attributes. Since the attribute values of the two cannot be directly subjected to mathematical operations, they need to be processed according to the following rules:

1) The value of the beneficial attribute remains unchanged

2) The value of the cost attribute is transformed into a beneficial attribute value, namely

$$h_{ij}(p_x) \Rightarrow h_{ij}^*(p_x) = \left\{ 1 - \gamma^{\lambda} \middle| p^{\lambda} \middle| x = 1, 2, \cdots, l_{ij} \right\} \tag{27}$$

$$P\left(\mathrm{x}=\gamma^{\lambda}\right) = \sum_{\lambda=1}^{n} P\left(x = h\left(p_x\right)\right) P\left(x = \gamma^{\lambda} / x = h\left(p_x\right)\right) \tag{28}$$

$$D=\left(h_{ij}(p)\right)_{m\times n} \tag{29}$$

Among them, $h(p_x)=\left\{\gamma^{\lambda}\left|p^{\lambda}\right|\lambda=1,2,\cdots,l\right\}$ is a probabilistic fuzzy element, γ^{λ} is the membership degree of an element x in the probabilistic hesitant fuzzy set $H_p=\left\{\left\langle x,h\left(p_x\right)\right\rangle\middle|x=1,2,\ldots,n\right\}$ in the set X, $\gamma^{\lambda}\in[0,1]$, and p^{λ} is the probability of membership degree γ^{λ}, $p^{\lambda}\in[0,1],\sum_{\lambda=1}^{l}p^{\lambda}=1$. D represents the probabilistic hesitant fuzzy decision matrix.

Step 2: Calculate the fuzzy entropy $E_{Fp}\left(h_{ij}(p)\right)$ of probabilistic hesitant fuzzy element $h_{ij}(p)$. The fuzzy entropy of the following probabilistic hesitant fuzzy element is obtained by changing the expression of $E_F\left(\gamma^{\lambda}\right)$ as follows:

$$E_{Fp1}(h(p))=-\frac{1}{\ln 2}\sum_{\lambda=1}^{l}p^{\lambda}\left[\gamma^{\lambda}\ln\left(\gamma^{\lambda}\right)+\left(1-\gamma^{\lambda}\right)\ln\left(1-\gamma^{\lambda}\right)\right] \tag{30}$$

$$E_{Fp2}(h(p))=\frac{1}{\sqrt{e}-1}\sum_{\lambda=1}^{l}p^{\lambda}\left[\gamma^{\lambda}e^{1-\gamma^{\lambda}}+\left(1-\gamma^{\lambda}\right)e^{\gamma^{\lambda}}-1\right] \tag{31}$$

$$E_{Fp3}(h(p))=\frac{1}{(1-q)\ln 2}\sum_{\lambda=1}^{l}p^{\lambda}\ln\left[\left(\gamma^{\lambda}\right)^{q}+\left(1-\gamma^{\lambda}\right)^{q}\right] \tag{32}$$

$$E_{Fp4}(h(p))=4^{t}\sum_{\lambda=1}^{l}p^{\lambda}\left(\gamma^{\lambda}\right)^{t}+\left(1-\gamma^{\lambda}\right)^{t} \tag{33}$$

If $\overline{E}_{Fpi}(h(p))\left(i=1,2,3,4\right)$ is the fuzzy entropy of $h(p)$, then:

$$\overline{E}_{Fp}(h(p))=\Phi_{Fp}(\mathrm{E}_{Fp1}(h(p)),\mathrm{E}_{Fp2}(h(p)),\mathrm{E}_{Fp3}(h(p)),\mathrm{E}_{Fp4}(h(p))) \tag{34}$$

In the formula, $q>0,q\neq 1$, $0<t\leq 1$, $\Phi_{Fp}(x_1,x_2,\cdots,x_n)=\sum_{i=1}^{n}a_ix_i$, $a_i\in[0,1]$ and $\sum_{i=1}^{n}a_i=1$.

Step 3: Calculate the hesitation entropy $E_{Hp}\left(h_{ij}(p)\right)$.

When l =1,

$$E_{Hp}\left(h_{ij}(p)\right)=0 \tag{35}$$

When l >1, the hesitation entropy of the following probabilistic hesitant fuzzy element is obtained by changing the expression of $E_H\left(\gamma^{\lambda}\right)$:

$$E_{Hp1}(h(p))=\sum_{i=1}^{l}\sum_{j=i+1}^{l}4p_1^ip_1^j\left(u^{ij}\right)^{r},r>0 \tag{36}$$

$$E_{Hp2}(h(p))=\sum_{i=1}^{l}\sum_{j=i+1}^{l}4p_1^ip_1^j\sin\left(0.5u^{ij}\pi\right) \tag{37}$$

$$E_{Hp3}(h(p))=\sum_{i=1}^{l}\sum_{j=i+1}^{l}4p_1^ip_1^j\left(1-\cos\left(0.5u^{ij}\pi\right)\right) \tag{38}$$

$$E_{Hp4}(h(p))=\sum_{i=1}^{l}\sum_{j=i+1}^{l}4p_1^ip_1^j\left(\frac{2u^{ij}}{u^{ij}+1}\right) \tag{39}$$

If $\overline{E}_{Hpi}(h(p))\left(i=1,2,3,4\right)$ is the hesitation entropy of $h(p)$, then:

$$\overline{E}_{Hp}(h(p))=\Phi_{Hp}(\mathrm{E}_{Hp1}(h(p)),\mathrm{E}_{Hp2}(h(p)),\mathrm{E}_{Hp3}(h(p)),\mathrm{E}_{Hp4}(h(p))) \tag{40}$$

In the formula, $u^{ij}=\left|\gamma^{i}-\gamma^{j}\right|$, $\Phi_{Hp}(x_1,x_2,\cdots,x_n)=\sum_{i=1}^{n}b_ix_i$, $b_i\in[0,1]$ and $\sum_{i=1}^{n}b_i=1$.

Step 4: Determine the total entropy $E_{Tp}\left(h_{ij}(p)\right)$. The total entropy of the following probabilistic hesitant fuzzy element is obtained by changing the expression of $E_T\left(\gamma^{\lambda}\right)$:

$$E_{Tp1}\left(h(p)\right)=\max\left(\overline{E}_{Fp}\left(h(p)\right),\overline{E}_{Tp}\left(h(p)\right)\right) \tag{41}$$

$$E_{Tp1}\left(h(p)\right)=\min\left(\overline{E}_{Fp}\left(h(p)\right)+\overline{E}_{Tp}\left(h(p)\right),1\right) \tag{42}$$

$$E_{Tp3}\left(h(p)\right)=\overline{E}_{Fp}\left(h(p)\right)+\overline{E}_{Tp}\left(h(p)\right)-\overline{E}_{Fp}\left(h(p)\right)\times\overline{E}_{Tp}\left(h(p)\right) \tag{43}$$

If $E_{Tpi}(h(p))(i=1,2,3,4)$ is the total entropy of $\boldsymbol{h(p)}$, then:

$$\overline{E}_{Tp}(h(p))=\Phi_{Tp}(\mathrm{E}_{Tp1}(h(p)),\mathrm{E}_{Tp2}(h(p)),\mathrm{E}_{Tp3}(h(p))) \tag{44}$$

In the formula, $\Phi_{Tp}(x_1,x_2,\cdots,x_n)=\sum_{i=1}^{n}c_i x_i, c_i\in[0,1]$ and $\sum_{i=1}^{n}c_i=1$.

Step 5: Calculate the entropy value and attribute weight of attribute M_j.

$$\overline{E}_p(M_j)=\frac{1}{m}\sum_{i=1}^{m}\overline{E}_{Tp}\left(h_{ij}(p)\right) \tag{45}$$

$$w_j=\frac{1-\overline{E}_p\left(a_j\right)}{n-\sum_{j=1}^{n}\overline{E}_p\left(a_j\right)} \tag{46}$$

Step 6: Calculate the weighted positive ideal distance D^{+}_{pi} between process $X_i\left(i=1,2,\ldots,m\right)$ and the positive ideal probabilistic hesitant fuzzy set $X^{+}=\left\{\tilde{1}_i(p)\middle|i=1,2,\cdots,m\right\}$, and also calculate the weighted negative ideal distance D^{-}_{pi} between process $X_i\left(i=1,2,\ldots,m\right)$ and the negative ideal probabilistic hesitant fuzzy set $X^{-}=\left\{\tilde{0}_i(p)\middle|i=1,2,\cdots,m\right\}$.

$$D^{+}_{pi}=\sum_{j=1}^{n}w_j d_p\left(h_{ij}(p),\tilde{1}_j(p)\right) \tag{47}$$

$$D^{-}_{pi}=\sum_{j=1}^{n}w_j d_p\left(h_{ij}(p),\tilde{0}_j(p)\right) \tag{48}$$

$$d_p\left(h_1(p),h_2(p)\right)=\sum_{\lambda=1}^{l}\left|\gamma_1^{\lambda}p_1^{\lambda}-\gamma_2^{\lambda}p_2^{\lambda}\right| \tag{49}$$

In the formula, the number of elements in $\tilde{1}_j(p)=\left\{1\middle|p^{\lambda}\middle|\lambda=1,2,\cdots,l_{ij}\right\}$ and $\tilde{0}_j(p)=\left\{0\middle|p^{\lambda}\middle|\lambda=1,2,\cdots,l_{ij}\right\}$ is determined by the number of elements in probabilistic hesitant fuzzy element $h_{ij}(p)$, the membership probabilities in probabilistic hesitant fuzzy element $p^{\lambda}\left(\lambda=1,2,\cdots,l_{ij}\right)$ and probabilistic hesitant fuzzy element $h_{ij}(p)$ are equal.

Step 7: Calculate the total closeness degree CI_{pi} between each process and the ideal solution.

$$CI_{pi}=\frac{D^{-}_{pi}}{D^{-}_{pi}+D^{+}_{pi}} \tag{50}$$

Step 8: Calculate the subjective entropy. Since the total closeness degree represents the comprehensive impact of subjective factors on the progress of each process, that is, $P_i=CI_i$, the formula for calculating subjective entropy is represented as (Gong et al., 2022):

$$H_{P_i}=-P_i\log P_i \tag{51}$$

3.6 External entropy

The degree of influence of external factors is used to measure the impact of uncertainty from external factors on the project duration. An analysis of the macro environment was conducted to quantify the uncertainty of external factors based on four aspects: policy, economy, society, and technical environment. Four levels were established as ‘favorable, good, average, and unfavorable’, with corresponding scores of ‘1, 0.75, 0.5, and 0.25’. The calculation formula for the impact of external causes (E) is as follows:

$$E_v=\begin{cases}0.25, & \sum_{j=1}^{n}\mu_{vj}\geq 3\\ 0.5, & 2.5<\sum_{j=1}^{n}\mu_{vj}<3\\ 0.75, & 2<\sum_{j=1}^{n}\mu_{vj}\leq 2.5\\ 1, & \sum_{j=1}^{n}\mu_{vj}\leq 2\end{cases} \tag{52}$$

$$E=\frac{\sum_{v=1}^{z}E_v}{z} \tag{53}$$

In the formula, $\mu_{vj}\left(j=1,2,3,4\right)$ represents the evaluation of the jth factor by the vth expert, and $E_v\left(v=1,2,\cdots,z\right)$ represents the evaluation score of exogenous entropy by the vth expert.

The formula for calculating exogenous entropy H_E is expressed as:

$$H_E=-E\log E \tag{54}$$

4. Calculate the size of the buffer

4.1 Initial buffer

The time of the process was estimated based on Beta distribution and simulation was performed using Crystal Ball software to determine the predicted values of process duration T95% and T50% corresponding to confidence levels of 95% and 50% respectively (Hu et al., 2019; Tianming et al., 2020; Tianming et al., 2019). Calculate the safety time for the process:

$$ST_i=T_{95\%}-T_{50\%} \tag{55}$$

Taking into account the complexity of the process, the initial $buffer_l$ for route l is:

$$buffer_l=\sqrt{\sum_{i\in l}\left[\left(1+H_{MC_i}\right)\left(1+H_{MOC_i}\right)ST_i\right]^2} \tag{56}$$

4.2 Import buffer

In order to mitigate the impact of non-critical chain buffering on the critical chain, the buffer value FB_l allocated to non-critical chain l is determined as the minimum value between the free time difference of the last step in the non-critical chain and the calculated value of the initial buffer for the non-critical chain (Xu & Han, 2007).

$$FB_l=\min\left(FF_i,buffer_l\right) \tag{57}$$

The operation to calculate the free slack of the last operation i in the non-critical chain is:

$$FF_i=\min_{j\in s_i}\left(ES_j,EF_i\right) \tag{58}$$

where, FF_i is the free time difference of process i after adding resource constraints, ES_j is the earliest start time of process j after process i, S_i is the set of all the compact work of procedure i.

4.3 Remaining buffer

When the merge buffer of the non-critical chain exceeds the free slack of the operation, the excess portion of the buffer will have an impact on the execution of operations on the critical chain. By extracting the portion of the merge buffer that exceeds the free slack, this portion of the buffer is added to the project buffer to absorb the associated risk. The remaining buffer is denoted as KB, and the remaining buffer of the lth non-critical path is represented as KB_l.

$$KB_l=\begin{cases}buffer_l-FF_i, & buffer>FF_i\\ 0, & buffer<FF_i\end{cases} \tag{59}$$

When the procedure on the non-critical path of Article l and the procedure on the non-critical path of Article l' are imported to the same node, the remaining buffer of the node is:

$$KB^*=\max\left(KB_l,KB_{l'}\right) \tag{60}$$

4.4 Project buffer

Taking into account the influence of safety time, resource constraints, relay potential, and network structural entropy, the calculation formula for the project buffer *PB* is:

$$PB = \left(1 + H_Q\right)\sqrt{\sum_{i \in l}\left[\left(1 + H_{MC_i}\right)\left(1 + H_{MOC_i}\right)ST_i\right]^2} + \sum KB^* \tag{61}$$

4.5 Calculation steps of critical chain buffer size based on GERT network and information entropy

The steps for calculating the critical chain buffer size based on GERT network and information entropy are as follows:

Step 1: Process Correction. The crux of calculating the duration of a GERT network project using the critical chain management methodology lies in converting the uncertain network diagram into a definitive one. This involves amalgamating multi-flow processes with uncertain outcomes into those with known outcomes, and refining loop-containing processes into loop-free ones. These processes, post-correction and with definite realizations, are termed as modified processes.

Step 2: Determining the Duration of the Correction Process involves calculating the amended durations of processes featuring loops and multi-flow structures individually, as per formulas (1) to (12), hence establishing the duration for the refined process.

Step3: Derive the Corrected Network Diagram and Determine the Critical Path. Based on the preceding two steps, the original GERT network diagram is transformed into a diagram devoid of uncertain implementation processes, known as the modified network diagram. With the removal of uncertain implementation steps in the modified network diagram, the critical path of the project can be determined using the PERT/CPM method.

Step4: Calculation of uncertainty in modified activities. Calculate the increased uncertainty of activities in the modified network diagram according to equations (17) to (22). Measure the magnitude of uncertainty using information entropy.

Step 5: With reference to equations (55) to (61), the initial buffer, merge buffer, remaining buffer, project buffer, and project duration for each activity within the modified network diagram can be determined.

5. Case analysis

5.1 Example introduction

The GERT network plan diagram of a water conservancy construction project is shown in Fig. 10, with 9 nodes (1-9) and 12 activities (A-J). The durations of activities follow a normal distribution, and the planned cost of the project is 237614.31 yuan. The basic parameters of the project are shown in Table 1 and Table 2:

Table 1
Basic project parameters

process	predecessor process	successor process	probability of	Mean m	Variance σ	moment generating function	equivalent function W(s)
A	–	D	1	4	2	e^{4s+s^2}	e^{4s+s^2}
B	–	E_1orE_2	1	3	1	$e^{3s+0.5s^2}$	$e^{3s+0.5s^2}$
C	–	F	1	2	1	$e^{2s+0.5s^2}$	$e^{2s+0.5s^2}$
D	AorD*	G	1	4	2	e^{4s+s^2}	e^{4s+s^2}
D*	D	D	0.3	2	1	$e^{2s+0.5s^2}$	$0.3\,e^{2s+0.5s^2}$
E_1	B	I	0.1	2	1	$e^{2s+0.5s^2}$	$0.1\,e^{2s+0.5s^2}$
E_2	B	I	0.9	3	1	$e^{3s+0.5s^2}$	$0.9\,e^{3s+0.5s^2}$
F	C	H	1	6	3	$e^{6s+1.5s^2}$	$e^{6s+1.5s^2}$
G	D	J	0.7	3	1	$e^{3s+0.5s^2}$	$0.7\,e^{3s+0.5s^2}$
H	F	J	1	5	2	e^{5s+s^2}	e^{5s+s^2}
I	E_1orE_2	J	1	3	2	e^{3s+s^2}	e^{3s+s^2}
J	G,H,I	–	1	6	2	e^{6s+s^2}	e^{6s+s^2}

Note: "-" in the table indicates that there is no data available for that entry.

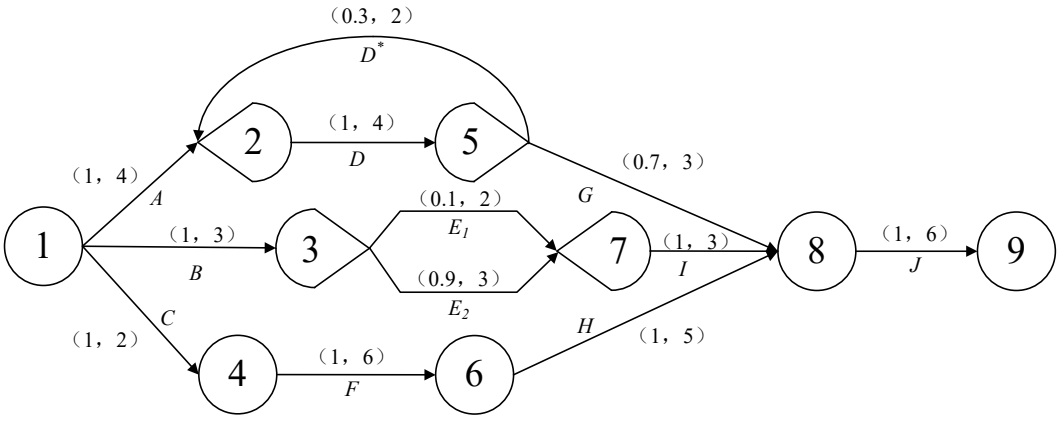

Fig. 10. GERT network plan diagram

5.2 Modified activities

According to the GERT network diagram of the project, it can be seen that there is one activity with a cyclic structure and one activity with a multi-flow structure that need to be modified.

(1) Modify the activity with a cyclic structure

The modified network diagram for the cyclic activity in the project is shown in Fig. 11.

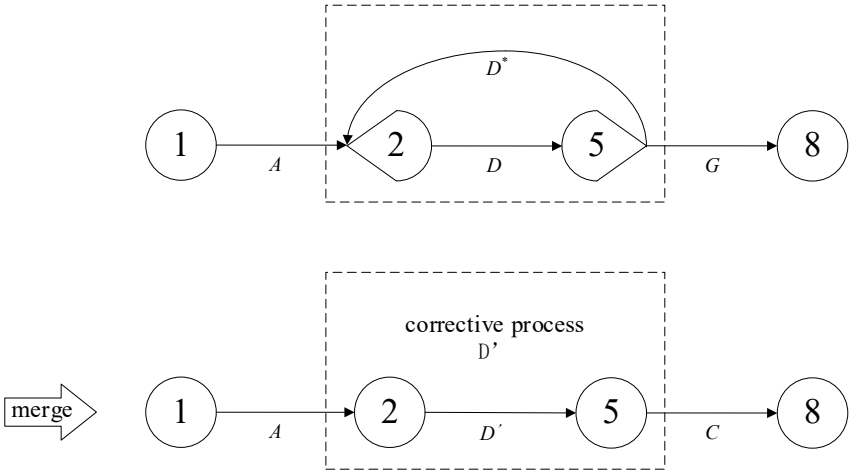

Fig. 11. Modified network diagram for the cyclic activity

(2) Modify the activity with a multi-flow structure

The modified network diagram for the activity with a multi-flow structure in the project is shown in Figure 12.

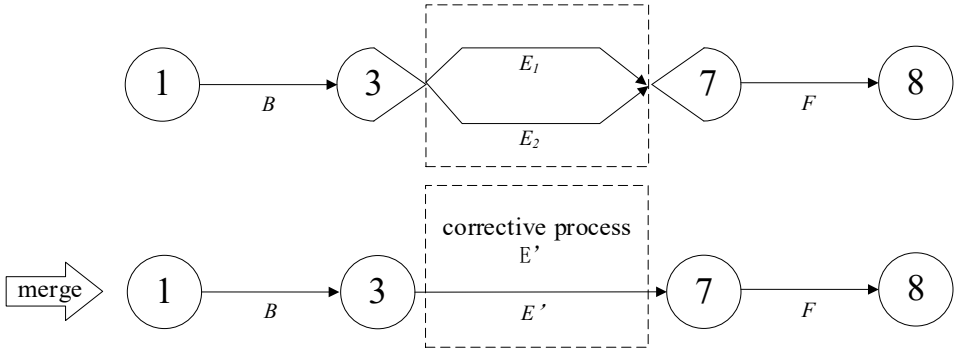

Fig. 12. Modified network diagram for the activity with a multi-flow structure

5.3 Determination of the modified activity durations

According to Eqs. (1-4), the equivalent function of the network diagram for this project is obtained as follows:

$$W_E(s)=\frac{0.7e^{17s+3.5s^2}+\left(e^{19s+4s^2}+0.9e^{15s+3s^2}+0.1e^{14s+3s^2}\right)\left(1-0.3e^{2s+0.5s^2}\right)}{1-0.3e^{2s+0.5s^2}}$$

Introducing the transmission element e^{c}, the expected number of times $E_{D^*}(c)$ that loop D^* is repeated is calculated as 0.4286 according to Eqs. (5-6). By using Eqs. (7-12), the durations of the modified activities are calculated. The results of the modified activity durations calculation are shown in Table 2.

Table 2
Calculation results of expected durations for modified activities

process	D	D*	E_1	E_2
modified subsequent process	D’		E’	
expected duration	6.57		2.90	

5.4 Export the modified route map and determine the critical paths

Using Crystal Ball to perform 10,000 simulations with a normal distribution for each activity of the project, let $T_{50\%}$ be the normal duration time for activities, and $T_{95\%}$ be the safe completion time. The modified network diagram after simulation is shown in Fig.13, therefore, the critical path is A-D'-G-J.

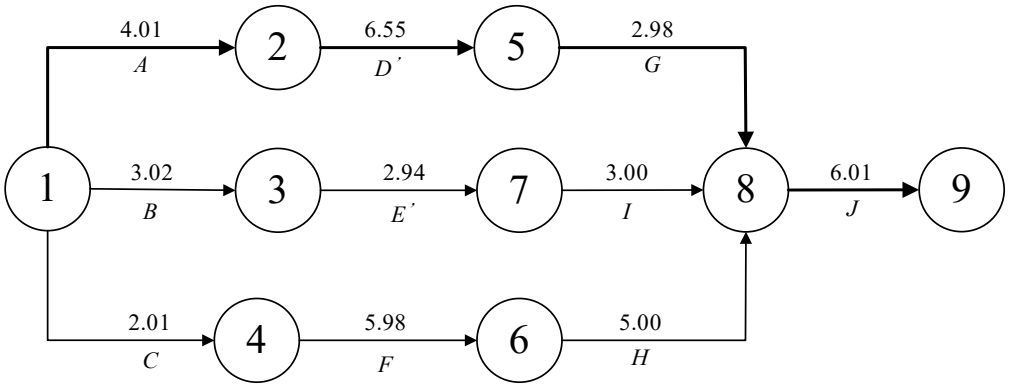

Fig. 13. Modified network diagram after simulation

5.5 Calculation of information entropy

(1) Process complexity entropy

Calculate the relay complexity α_i and the technical complexity β_i of each activity, calculate the cyclic activity complexity entropy $H_{MCD'}$ and the multi-flow structure activity complexity entropy $H_{MCOE'}$ using equations (17) and (21), and then calculate the activity complexity entropy. Taking activity G as an example, with B_G=1, N_G=1, we get α_G=0.5, L_G=97.5 (cm), U_G=101.6 (cm), LSL_G=83.5 (cm), USL_G=123.36 (cm), we get β_G=0.1, H_{DG}=0.16, H_{DCG}=0.25. The activity entropy calculation table is shown in Table 3.

Table 3
Calculation table for activity complexity entropy

process	*L*	*U*	*LSL*	*USL*	α_i	β_i	H_{MCi}	H_{MOCi}	H_{Di}	H_{CDi}
A	3.42	3.50	1.73	8.33	0.00	0.01	0	0	0.02	0.02
B	8.00	10.00	5.38	17.60	0.00	0.16	0	0	0.13	0.13
C	10.60	13.20	7.50	15.80	0.00	0.31	0	0	0.16	0.16
D’	0.63	0.73	0.44	1.92	0.50	0.07	0.2248	0	0.23	0.45
E’	11.76	15.80	9.31	20.38	0.50	0.36	0	0.0286	0.31	0.34
F	97.50	101.60	83.50	123.36	0.50	0.10	0	0	0.25	0.25
G	47.60	50.36	39.30	59.30	0.50	0.14	0	0	0.27	0.27
H	30.58	37.86	17.65	42.96	0.50	0.29	0	0	0.16	0.16
I	1.51	1.93	1.20	4.83	0.75	0.12	0	0	0.20	0.20
J	–	–	–	–	0.67	0.00	0	0	0.12	0.12

Note: "-" in the table indicates that there is no data available for that entry.

(2) Resource Entropy

Calculate the resource tension δ_i for each activity and then calculate the resource entropy. Taking activity G as an example, with Le_G=0.79 and s_G=-0.34, we get ω_G=1.84, δ_G=0.24, and $H_{\delta G}$=0.15. The resource entropy calculation table is shown in

Table 4.
Table 4
Resource entropy calculation table

process	*Le*	*s*	*ω*	R_1	R_2	R_3	R_4	δ_1	δ_2	δ_3	δ_4	δ	$H_{\delta i}$
A	0.85	-0.23	1.38	4	1	1	0	0.08	0.16	0.16	0.04	0.16	0.13
B	0.87	-0.20	1.25	5	0	1	1	0.08	0.00	0.12	0.06	0.12	0.11
C	0.94	-0.09	1.06	2	0	0	1	0.07	0.00	0.04	0.04	0.07	0.08
D`	0.82	-0.29	1.71	2	1	1	0	0.04	0.26	0.26	0.07	0.26	0.15
E`	0.90	-0.15	1.18	5	1	1	0	0.08	0.12	0.12	0.00	0.12	0.11
F	0.79	-0.34	1.84	3	0	1	0	0.01	0.24	0.24	0.00	0.24	0.15
G	0.92	-0.12	1.14	3	0	1	1	0.09	0.12	0.12	0.06	0.12	0.11
H	0.91	-0.14	1.24	4	0	1	0	0.13	0.20	0.10	0.05	0.20	0.14
I	0.78	-0.36	1.48	3	1	0	1	0.05	0.12	0.06	0.03	0.12	0.11
J	0.80	-0.32	1.78	7	2	3	4	0.02	0.24	0.12	0.06	0.24	0.15

(3) Artificial entropy

To calculate the human-made entropy of each activity, the owner invites 4 experts Pi (i=1,2,3,4) to assess the impact of five factors on activities: Benefit-oriented attributes (Transience efficiency M1, Ideological transformation M2) and Cost-oriented attributes (Habitual procrastination M3, Pseudo-redundant time M4, Fatigue state M5), with equal weights assigned to the experts. Calculate the total probability value of hesitant fuzzy set membership. Among the five attributes, M1 and M2 belong to benefit-oriented attributes with unchanged attribute values, while M3, M4, and M5 belong to cost-oriented attributes that need to be converted into benefit-oriented attributes for calculation. The obtained probability hesitant fuzzy decision matrix $D=\left(h_{ij}\left(p\right)\right)_{10\times5}$ is as follows:

$$
D=\begin{bmatrix}
M_1 \\
\{0.5|0.25,0.6|0.25,0.7|0.25,0.8|0.25\} \\
\{0.3|0.25,0.5|0.5,0.6|0.25\} \\
\{0.5|0.25,0.6|0.175,0.65|0.325,0.7|0.25\} \\
\{0.3|0.1,0.4|0.325,0.45|0.15,0.5|0.2,0.6|0.225\} \\
\{0.5|0.25,0.6|0.5,0.7|0.25\} \\
\{0.65|0.5,0.7|0.5\} \\
\{0.5|0.1,0.55|0.15,0.6|0.2,0.7|0.4,0.8|0.15\} \\
\{0.4|0.125,0.45|0.125,0.55|0.15,0.6|0.35,0.8|0.25\} \\
\{0.5|0.25,0.65|0.175,0.7|0.075,0.75|0.25,0.8|0.25\} \\
\{0.5|0.125,0.55|0.1,0.6|0.525,0.75|0.25\}
\end{bmatrix}
\rightarrow
\begin{matrix}
M_2 \\
\{0.5|0.4,0.6|0.4,0.63|0.075,0.7|0.125\} \\
\{0.3|0.4,0.4|0.1,0.67|0.25,0.85|0.25\} \\
\{0.2|0.125,0.3|0.125,0.6|0.25,0.73|0.25,0.85|0.25\} \\
\{0.2|0.25,0.35|0.25,0.55|0.15,0.6|0.1,0.8|0.25\} \\
\{0.3|0.125,0.4|0.1,0.55|0.15,0.6|0.15,0.7|0.35,0.75|0.125\} \\
\{0.4|0.25,0.7|0.25,0.8|0.25,0.85|0.25\} \\
\{0.5|0.25,0.7|0.1,0.75|0.075,0.8|0.575\} \\
\{0.3|0.25,0.4|0.25,0.5|0.1,0.55|0.15,0.65|0.25\} \\
\{0.5|0.2125,0.6|0.1375,0.65|0.25,0.7|0.15,0.73|0.25\} \\
\{0.3|0.25,0.6|0.1,0.65|0.125,0.7|0.15,0.78|0.375\}
\end{matrix}
\rightarrow
$$

$$
\rightarrow
\begin{matrix}
M_3 \\
\{0.25|0.25,0.3|0.25,0.6|0.25,0.7|0.25\} \\
\{0.15|0.375,0.25|0.25,0.3|0.25,0.35|0.125\} \\
\{0.35|0.25,0.4|0.05,0.5|0.325,0.55|0.075,0.6|0.3\} \\
\{0.15|0.25,0.25|0.25,0.3|0.125,0.35|0.275,0.4|0.1\} \\
\{0.25|0.15,0.3|0.1,0.4|0.625,0.5|0.125\} \\
\{0.15|0.5,0.2|0.25,0.63|0.25\} \\
\{0.25|0.25,0.35|0.275,0.4|0.475\} \\
\{0.3|0.225,0.4|0.6625,0.6|0.1125\} \\
\{0.3|0.25,0.4|0.2,0.45|0.3,0.5|0.25\} \\
\{0.2|0.65,0.25|0.1,0.3|0.25\}
\end{matrix}
\rightarrow
\begin{matrix}
M_4 \\
\{0.3|0.4375,0.4|0.2375,0.5|0.325\} \\
\{0.15|0.25,0.3|0.25,0.4|0.5\} \\
\{0.3|0.275,0.35|0.225,0.4|0.25,0.5|0.15,0.6|0.1\} \\
\{0.3|0.25,0.4|0.425,0.45|0.075,0.5|0.25\} \\
\{0.2|0.25,0.4|0.15,0.5|0.6\} \\
\{0.2|0.25,0.3|0.5,0.6|0.125,0.7|0.125\} \\
\{0.2|0.325,0.25|0.25,0.3|0.175,0.35|0.25\} \\
\{0.25|0.15,0.35|0.1,0.4|0.175,0.45|0.075,0.5|0.5\} \\
\{0.3|0.25,0.35|0.25,0.5|0.25,0.6|0.25\} \\
\{0.25|0.25,0.3|0.325,0.35|0.1,0.4|0.275,0.5|0.05\}
\end{matrix}
$$

$$
\rightarrow
\begin{bmatrix}
M_5 \\
\{0.35|0.5,0.5|0.325,0.6|0.175\} \\
\{0.2|0.25,0.35|0.2,0.4|0.275,0.45|0.275\} \\
\{0.2|0.25,0.3|0.1,0.35|0.075,0.4|0.575\} \\
\{0.25|0.1125,0.3|0.2375,0.5|0.15,0.7|0.125,0.8|0.375\} \\
\{0.25|0.35,0.3|0.4,0.4|0.25\} \\
\{0.3|0.25,0.4|0.25,0.5|0.25,0.6|0.25\} \\
\{0.35|0.25,0.4|0.3,0.5|0.45\} \\
\{0.25|0.375,0.35|0.125,0.5|0.5\} \\
\{0.2|0.25,0.4|0.425,0.5|0.225,0.6|0.1\} \\
\{0.35|0.075,0.1|0.675,0.5|0.25\}
\end{bmatrix}
$$

Based on the above matrix, calculate the fuzzy entropy of probability hesitant fuzzy elements, where q=0.5, t=1, and a_1=a_2=a_3=a_4=0.25. The results of the fuzzy entropy calculation are shown in Table 5.

Table 5
Calculation results of fuzzy entropy

	process	M_1	M_2	M_3	M_4	M_5
$\overline{E}_{Fp}$	A	0.8903	0.9685	0.8819	0.8321	0.9602
	B	0.6615	0.8266	0.7636	0.8551	0.9045
	C	0.9414	0.8015	0.9715	0.9399	0.8938
	D'	0.9698	0.8362	0.8150	0.9556	0.8265
	E'	0.9771	0.9082	0.9394	0.9242	0.8747
	F	0.9038	0.7915	0.7183	0.8478	0.9539
	G	0.9007	0.8093	0.9178	0.8199	0.9736
	H	0.8592	0.9431	0.9486	0.9580	0.9183
	I	0.8587	0.9190	0.9608	0.9443	0.9128
	J	0.9345	0.8465	0.7645	0.8959	0.6367

Calculate the hesitancy entropy of probability hesitant fuzzy elements, where r=0.5 and b1=b2=b3=b4=0.25. The results of the hesitancy entropy calculation are shown in Table 6.

Table 6
Calculation results of hesitancy entropy

	process	M_1	M_2	M_3	M_4	M_5
$\overline{E}_{Hp}$	A	0.2787	0.1342	0.4531	0.1705	0.2318
	B	0.1499	0.5465	0.1824	0.2369	0.2255
	C	0.1300	0.5334	0.2351	0.2245	0.1894
	D'	0.2276	0.5550	0.2127	0.1755	0.5414
	E'	0.1959	0.3508	0.1617	0.2727	0.1356
	F	0.0567	0.3990	0.4061	0.3779	0.2787
	G	0.2298	0.2709	0.1404	0.1467	0.1534
	H	0.3272	0.3310	0.1671	0.2094	0.2710
	I	0.2772	0.2049	0.1818	0.2921	0.3041
	J	0.0671	0.4339	0.0936	0.1677	0.3641

Calculate the total entropy of probability hesitant fuzzy elements, where c_1=c_2=c_3=1/3. The results of the total entropy calculation are shown in Table 7.

Table 7
Results of total entropy calculation

	process	M_1	M_2	M_3	M_4	M_5
$\overline{E}_{Tp}$	A	0.9371	0.9804	0.9391	0.8976	0.9765
	B	0.7284	0.9160	0.8387	0.9148	0.9435
	C	0.9635	0.9029	0.9833	0.9644	0.9359
	D'	0.9821	0.9211	0.8898	0.9730	0.9156
	E'	0.9863	0.9495	0.9628	0.9564	0.9222
	F	0.9245	0.8887	0.8503	0.9177	0.9735
	G	0.9414	0.8901	0.9491	0.8775	0.9838
	H	0.9215	0.9684	0.9686	0.9749	0.9529
	I	0.9189	0.9515	0.9762	0.9683	0.9507
	J	0.9578	0.9199	0.8031	0.9364	0.8019

Based on the above calculation results, the attribute weight set of activities is obtained

$$w = \left(0.2080, 0.2003, 0.2362, 0.1743, 0.1812\right)^T$$

Calculate the positive ideal distance D_{pi}^{+}, weighted negative ideal distance D_{pi}^{-}, overall closeness CI_i, and human-made entropy for each activity. The results of the human-made entropy calculation are shown in Table 8.

Table 8
Results of human-made entropy calculation

process	D_i^+	D_i^-	CI_i	H_i
A	0.4810	0.4998	0.5096	0.1492
B	0.5872	0.3504	0.3737	0.1597
C	0.5041	0.4959	0.4959	0.1511
D'	0.5708	0.4292	0.4292	0.1577
E'	0.5559	0.2811	0.3359	0.1591
F	0.5104	0.4896	0.4896	0.1519
G	0.5144	0.4856	0.4856	0.1523
H	0.5423	0.4577	0.4577	0.1554
I	0.4855	0.5145	0.5145	0.1485

J	0.1723	0.2982	0.6338	0.1255

(4) External factor entropy

The external factor entropy is calculated as H_E=0.1571, based on the expert rating of E=0.4375.

5.6 Calculation of buffer size

Taking activity A as an example, the safe time ST_A can be calculated as 3.27 days according to equation (55). Considering the complexity entropy of the activity, $buffer_I$ and PB can be calculated as 11.72 days and 22.18 days, respectively, using equations (56) to (61). The total project duration T is 41.73 days. The calculation results for the buffer size of the critical chain based on information entropy and GERT network are shown in Table 9.

Table 9
Calculation results for critical chain buffer size based on information entropy and GERT network

process	$T_{50\%}$	$T_{95\%}$	ST_i	H_{CDi}	$H_{\delta i}$	H_{Pi}	H_E	H_Q	FF_i	FB	KB	$buffer_I$	PB
A	4.01	7.28	3.27	0.02	0.13	0.1492	0.1571	0.1035	0	–	–	11.72	22.18
D’	6.55	11.42	4.87	0.45	0.15	0.1577			0	–	–		
G	2.98	4.64	1.66	0.27	0.11	0.1523			0	–	–		
J	6.01	9.31	3.3	0.12	0.15	0.1255			0	–	–		
B	3.02	4.68	1.66	0.13	0.11	0.1597			0				
E’	2.94	6.26	3.32	0.34	0.11	0.1591			0	2.58	5.43	8.01	
I	3.00	6.31	3.31	0.20	0.11	0.1485			2.58				
C	2.01	3.63	1.62	0.16	0.08	0.1511			0				
F	5.98	10.86	4.88	0.25	0.15	0.1519			0	2.55	7.22	9.77	
H	5.00	8.25	3.25	0.16	0.14	0.1554			2.55				

Note: "-" in the table indicates that there is no data available for that entry.

5.7 Analysis of completion rate

To validate the practicality of the critical chain buffer calculation model for GERT networks, 10,000 simulations were conducted using a normal distribution for the project duration through Crystal Ball. The completion status was statistically analyzed. From the analysis of on-time completion probability for the project, compared with the GERT analytical method, the critical chain buffer calculation model based on information entropy and GERT network ensures a completion probability of over 99% by considering activity entropy, resource entropy, human-made entropy, external factor entropy, and quality entropy, while reducing the total project duration. The simulation graph of completion rate is shown in Fig. 14.

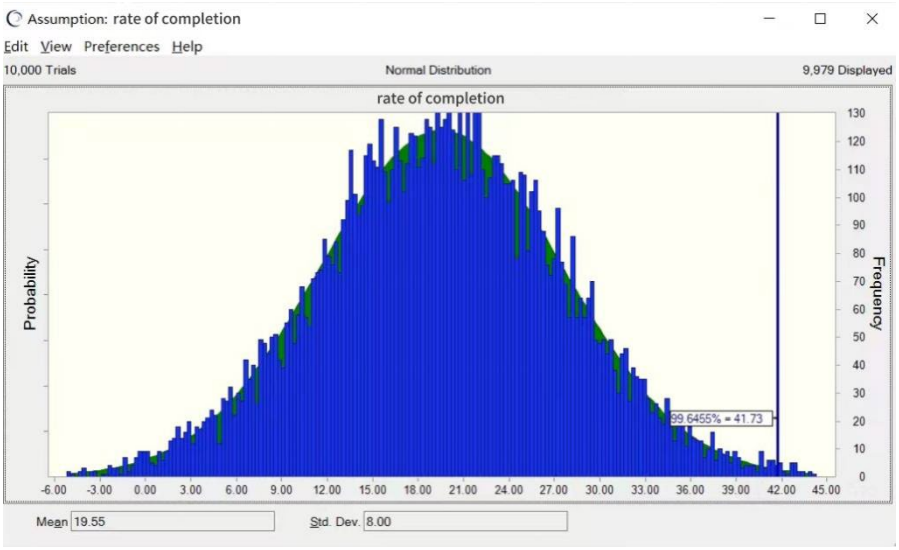

Fig. 14. Simulation graph of completion rate

6. Conclusion

Considering the existence of different types of processes in GERT network, the processes with uncertain realization are divided into loop processes and multi-flow structure processes, which are combined and modified to definite realization processes. According to the moment generating function and Mersenne formula, the expected realization times of the loop are calculated, and the correction process duration considering the repeated realization times of the loop is proposed. The calculating method of the duration of GERT network correction process is given. Considering that the combined network will increase the uncertainty of the process duration, the loop process complexity entropy and multi-flow structure process complexity entropy are measured. Considering the complexity entropy of process, resource entropy, human entropy and external factor entropy, a calculation model of key chain buffer size based on information entropy and GERT network is proposed. The result shows that the project risk is reduced and the construction period is shortened under the condition of ensuring a high completion

probability of the project.

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