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# A novel multi-attribute decision making method based on Partitioned Hamy Mean Aggregator and its applications in material selection problems

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**A B S T R A C T**

In many multi-attribute decision-making (MADM) scenarios, decision makers face the challenge of evaluating both tangible and intangible attributes. These attributes often differ in nature and importance, making the decision process complex. Furthermore, the interrelationships among criteria must be carefully assessed, especially when the criteria are grouped or partitioned based on their inherent characteristics. To tackle these challenges, this paper introduces a novel multi-attribute ranking and selection approach under crisp environments called PHARSA (Partitioned Hamy Aggregator-based Ranking and Selection Approach). PHARSA is capable of addressing multi-attribute ranking and selection problems with non-ordered, ordered, ordinal, cardinal, or mixed attribute assessments. We introduce novel double-weighted partitioned Hamy mean (PHM) operators that apply weighting to both the attribute and the partitions. A real-world on-water pipe material selection is presented to demonstrate the practical applicability of the proposed method. Comparative analyses with existing methods prove that PHARSA is an effective tool for solving multi-attribute ranking and selection problems in management and engineering contexts.

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## 1. Introduction

Multi-attribute decision-making (MADM) is a common human activity. It involves selecting the best alternative(s) and ranking them from a set of possibilities. This process utilizes an aggregation technique that considers both qualitative and quantitative decision information for each alternative across various performance attributes. There are generally two categories of MADM methods.

**The conventional methods:** This category includes well-known techniques such as the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), VIsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Opricovic & Tzeng, 2004), and the Preference Ranking Organization METhod for Enrichment Evaluation (PROMETHEE) (Mareschal & Brans, 1994).

**The aggregation operator-based methods:** Such as the Weighted Aggregated Sum Product Assessment (WASPAS) (Zavadskas et al., 2012), the Double Normalization-Based Multi-Aggregation (DNMA) (Liao & Wu, 2018), the Multi-Attribute Net Inferiority and Superiority-based Ranking Approach (MANISRA) (Hidouri & Rebaï, 2019), and the Mixed Aggregation by Comprehensive Normalization Technique (MACONT) (Wen et al., 2020).

The aggregation operator-based method is a widely adopted tool for evaluating the comprehensive value of alternatives by synthesizing the associated information through an aggregation process. Unlike the conventional methods, which yield only the ranking of alternatives, the aggregation operator-based methods facilitate an integrated evaluation that accounts for the underlying data structure and criteria.

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In real-world decision-making situations, input arguments typically involve certain interrelationships. Capturing these interrelationships is crucial for effectively addressing complex decision-making problems. Nonlinear aggregation operators are particularly adequate for this purpose as they can model interactions among input arguments. Examples of nonlinear operators include Bonferroni mean (BM) operators, Hamy mean (HM) operators, Heronian mean (HeM) operators, Maclaurin symmetric mean (MSM) operators, and Muirhead mean (MM) operators. The BM and HeM operators are two powerful aggregation functions that capture pairwise interrelationships between attributes. However, in most scenarios, such interrelationships exist among multiple attributes. The MM, MSM, and HM operators generalize these interactions to multiple attributes while also computing fused attributes. Among these, the HM operator offers outstanding advantages. In 1998, Hara et al. (1998) presented a key refinement of the arithmetic mean (AM) and geometric mean (GM) inequality, showing that the HM operators and their dual forms monotonically lie between AM and GM. Compared to MM and MSM operators, the HM provides more precise normalization of correlated values (Rawat et al., 2024; Punetha et al., 2024), yielding a more accurate representation of attribute interrelationships. Still more, HM operators are particularly suitable for practical applications as they exhibit greater flexibility and provide smoother approximations during the decision analysis process (Zhao et al., 2024).

The robustness and versatility of the HM operators prompted a number of scholars to explore their properties to enhance various MADM approaches and methodologies. For instance, Wu et al. (2018) introduced several HM operators with 2-tuple linguistic neutrosophic numbers (2TLNNs), such as the 2-tuple linguistic neutrosophic HM (2TLNHM) operators, the 2-tuple linguistic neutrosophic weighted HM (2TLNWHM) operators, the 2-tuple linguistic neutrosophic dual HM (2TLNDHM) operators, and the 2-tuple linguistic neutrosophic weighted dual HM (2TLNWDHM) operators, and developed a number of MADM methods utilizing these operators. Deng et al. (2018) have also introduced some HM operators grounded in the 2-tuple linguistic Pythagorean fuzzy numbers (2TLPFNs). Another study by Stević et al. (2018) employed a rough Best-Worst Method (BWM) and rough WASPAS approach based on a novel rough Hamy aggregator to select potential locations for roundabout construction. Liu et al. (2019) devised the intuitionistic uncertain linguistic HM operators with linguistic scale functions and applied them to the selection of health-care waste treatment technologies. Building on the distinctive features of the power average (PA) and HM operators, Liu et al. (2020) proposed a new method based on two new aggregation operators: the normal wiggly hesitant fuzzy linguistic power HM (NWHFLPHAM) operators and their weighted counterparts (NWHFLPWHAM).

While the aforementioned variations of HM operators have proven effective in various applications, they all assume that attributes are interconnected. In fact, in real-world MADM problems, the attributes can be both homogeneous and heterogeneous. This implies that they can be grouped into coalitions. Within the same MADM problem, the characteristics belonging to the same coalition are interdependent, whereas those that belong to different coalitions are autonomous. Consider the selection of a vehicle. We might evaluate attributes such as Bank requirement ($C_1$), physical characteristic ($C_2$), brand influence ($C_3$), and customer evaluations ($C_4$). These four attributes can be divided into two coalitions: ($P_1$) and ($P_2$), where $P_1 = \{C_1, C_2\}$ and $P_2 = \{C_3, C_4\}$. In essence, within the same coalition the attributes interact with one another while attributes of different coalitions are independent.

For this reason, the partitioning concept was integrated with various AGOPs. Liu et al. (2020) introduced the partitioned Hamy mean (PHM) operators to address a similar scenario. These operators assume that all attributes are divided into multiple coalitions where attributes within the same coalition are interrelated, but independent of those in other coalitions. If only one coalition exists, the PHM operators reduce to the HM operators, indicating that the HM operators are a special case of the PHM operators. Due to this advantage, PHM operators have garnered significant attention and led to new achievements. For example, Rawat et al. (2024) extended PHM to PWHM operators in a q-rung fuzzy environment, while Wang et al. (2023) adapted PHM operators for neutrosophic sets. These operators are capable of handling decision-making problems comprising partitioned attributes. However, the PHM operators do not account for the self-importance of coalitions. In real-world MADM scenarios, attribute partitions may have unequal weights.

This critical pitfall has not been sufficiently addressed in prior research. To fill this gap, we propose double weighting for partitioned Hamy mean (PHM) operators by weighting both the attribute and the partitions. We introduce and define double weighted arithmetic partitioned Hamy mean (DWAPHM) operators and double weighted geometric partitioned Hamy mean (DWGPHM) operators. Those two aggregation operators lay the foundation of a completely new ranking/selection approach under crisp environments called the Partitioned Hamy Aggregator-based Ranking and Selection Approach (PHARSA). Please kindly note that this newly developed approach can tackle multi-attribute ranking/selection problems with non-ordered, ordered, ordinal, cardinal, or mixed attribute assessments. Lastly, the potential of the proposed method is demonstrated by means of a real-world water pipe material selection problem. Comparisons with some rival methods show that the proposed approach is an effective tool for solving multi-attribute ranking/selection problems in management and engineering.

The remainder of this paper is organized as follows: Section 2 provides the basic definitions. Section 3 introduces the Double Weighted Arithmetic Partitioned Hamy Mean (DWAPHM) operators and Double Weighted Geometric Partitioned

Hamy Mean (DWGPHM), along with their desirable properties and special cases. The proposed PHARSA method is detailed in Section 4. Section 5 presents a case study in water pipe material selection, including sensitivity and comparative analyses to validate the efficiency and stability of the approach. Finally, Section 6 concludes the paper with key findings and implications.

### *1.1 Hamy mean and Geometric Hamy mean operators*

**DEFINITION 1.** (Hara et al., 1998)**.** Let $x_i$ ($i = 1$ to $n$) be a set of nonnegative real numbers, and let $k$ be an integer such that $1 \leq k \leq n$. The HM operator is defined as:

$$HM^{(k)}(x_1, x_2, \dots, x_n) = \frac{\sum_{1 \leq i_1 < \dots < i_k \leq n} \left(\prod_{j=1}^{k} x_{i_j}\right)^{\frac{1}{k}}}{C_n^k} \tag{1}$$

where:
$(i_1, i_2, \dots, i_k)$ denotes a k-tuple combination of $\{1, 2, \dots, n\}$,

$C_n^k$ is the number of combinations of $n$ objects taken $k$ at a time, i.e., $C_n^k = \frac{n!}{k!\,(n-k)!}$.

The HM operator satisfies the properties of idempotency, monotonicity and boundedness.

**DEFINITION 2.** Let $x_i$ ($i = 1$ to $n$) be a collection of nonnegative real numbers, and let $k$ be an integer such that $1 \leq k \leq n$. The geometric Hamy mean (GHM) operator is defined as follows:

$$GHM^{(k)}(x_1, x_2, \dots, x_n) = \left\{ \prod_{1 \leq i_1 < \dots < i_k \leq n} \left(\frac{\sum_{j=1}^{k} x_{i_j}}{k}\right) \right\}^{\frac{1}{C_n^k}} \tag{2}$$

where:

$(i_1, i_2, \dots, i_k)$ denotes a *k-tuple* combination of $\{1, 2, \dots, n\}$,

$C_n^k$ is the number of combinations of $n$ objects taken $k$ at a time, i.e., $C_n^k = \frac{n!}{k!\,(n-k)!}$.

In most cases, it is crucial comprehensively consider input arguments. Therefore, it is natural to incorporate the weights of these arguments into the HM operator.

**DEFINITION 3.** Let $x_i$ ($i = 1$ to $n$) be a collection of nonnegative real numbers and $W = (w_1, w_2, \dots, w_n)$ be their associated vector, satisfying $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0$. The Weighted Arithmetic Hamy Mean (WAHM) operator is defined as follows:

$$WAHM^{(k)}(x, W) = \frac{\sum_{1 \leq i_1 < \dots < i_k \leq n} \left(\prod_{j=1}^{k} \left(x_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k} w_{i_j}}}}{C_n^k} \tag{3}$$

where: $x = (x_1, x_2, \dots, x_n)$.

**DEFINITION 4.** A weighted Geometric Hamy mean (WGHM) operator of dimension $n$ and rank $k$ is a mapping $WGHM^{(k)}$ : $\mathbb{R}^n \to \mathbb{R}$ that has an associated HIWV, $W = (W_1, W_2, \dots, W_n)$, such that:

$$WGHM^{(k)}(x, W) = \left\{ \prod_{1 \leq i_1 < \dots < i_k \leq n} \left(\frac{\sum_{j=1}^{k} W_{i_j}\, x_{i_j}}{\sum_{j=1}^{k} W_{i_j}}\right) \right\}^{\frac{1}{C_n^k}} \tag{4}$$

where $x = (x_1, x_2, \dots, x_n)$.

### *1.2 Partitioned Hamy mean operators*

As previously discussed, while the HM and GHM operators are capable of capturing the interrelationships among numerous, real-world scenarios often involve both heterogeneous and homogenous attributes simultaneously. For greater accuracy it

is more beneficial to partition attributes into several coalitions. With coalition, attributes are assumed interrelated. No correlation is assumed between attributes belonging to different coalitions. These scenarios are mathematically described below in detail:

Let $T = \{x_1, x_2, \cdots, x_n\}$ be a set of non-negative real numbers, where each $x_i$ represents the performance value of a corresponding attribute. Based on the aforementioned interrelationship pattern, we assume that the elements (arguments) $x_i (i = 1,2 \cdots, n)$ are partitioned into $d$ distinct coalitions $P_1, P_2, \cdots, P_d$, such that $P_i \cap P_j = \emptyset$ and $\cup_{k=1}^{d} P_h = T$. Furthermore, suppose that there exists a interrelationship among any $k_h$ elements in each coalition $P_h (h = 1,2, \cdots, d)$ while no interrelationship exists elements belonging to different coalitions $P_i$ and $P_j$. To effectively model this scenario, an extension of HM, known as PHM operator, in introduced and defined as follows.

**DEFINITION 5. Partitioned Hamy mean operators (Rawat et al., 2024)** Let $x_i$ ($i = 1$ to $n$) be a set of nonnegative real numbers, which are partitioned into $d$ distinct parts $d = \{P_1, P_2, \cdots, P_d\}$ where $P_i \cap P_j = \emptyset$ and $\cup_{k=1}^{d} P_h = T$, The PHM operator is defined as follows:

$$PHM^{(k_t)}(x) = \frac{1}{d}\sum_{t=1}^{d}\left(\frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t}\left(\prod_{j=1}^{k_t}\left(x_{i_j}\right)\right)^{\frac{1}{k_t}}}{C_{p_t}^{k_t}}\right) \tag{5}$$

where:

$|P_t|$ the cardinality of the partition
$P_t; k_t = 1,2, \cdots, |P_t|$ is the granularity of the partition
$P_t; i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers taken from the set $\{1, 2, \cdots, |P_t|\}$

$C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t| - k_t)!}$ Is binomial coefficient.

**DEFINITION 6. Partitioned Geometric Hamy mean operators (Rawat et al., 2024).** Suppose, a set $x_i$ (i = 1 to n) a collection of nonnegative real numbers, which are partitioned into $d$ distinct coalitions $d = \{P_1, P_2, \cdots, P_d\}$, Then, the PGHM operator is defined as follows:

$$PGHM^{(k_t)}(x) = = \prod_{t=1}^{d}\left\{\left\{\prod_{1 \le i_1 < \ldots < i_k \le p_t}\left(\frac{\sum_{j=1}^{k_t} x_{i_j}}{k_t}\right)^{1/C_{p_t}^{k_t}}\right\}^{\frac{1}{d}}\right\} \tag{6}$$

where:

$|P_t|$ the cardinality of the partition

$P_t; k_t = 1,2, \cdots, |P_t|$ is the granularity of the partition

$P_t; i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers taken from the set $\{1, 2, \cdots, |P_t|\}$

$C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t| - k_t)!}$ is binomial coefficient.

**DEFINITION 7. Partitioned weighted Hamy mean operator (Rawat et al., 2024).** Let $x_i$ (i = 1 to n) be a set of nonnegative real numbers, which are partitioned into $d$ distinct coalitions $d = \{P_1, P_2, \cdots, P_d\}$. Let $w_i \in [0,1]$ represent the weight (importance), of the $i^{th}$ attribute, with the normalization condition $\sum_{i=1}^{n} w_i = 1$. The PWHM operator is defined as follows:

$$PWHM^{(kt)}(x, W) = \frac{1}{d}\sum_{t=1}^{d}\left(\frac{\sum_{1 \le i_1 < \ldots < i_{kt} \le pt}\left(\prod_{j=1}^{kt}\left(x_{i_j}\right)^{W_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{kt} W_{i_j}}}}{C_{pt}^{kt}}\right) \tag{7}$$

where $|P_t|$ the cardinality of the partition $P_t$;

$k_t = 1,2, \cdots, |P_t|$ is the granularity of the partition $P_t$;

$i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers taken from the set $\{1, 2, \cdots, |P_t|\}$

$C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t|-k_t)!}$ is the binomial coefficient.

**DEFINITION 8. Partitioned weighted Geometric Hamy mean operator (Rawat et al., 2024).** Let $x_i$ (i = 1 to n) be a set of nonnegative real numbers, which are partitioned into *d* distinct coalitions $d = \{P_1, P_2, \cdots, P_d\}$. Let $w_i \in [0,1]$ represent the weight, of the $i^{th}$ attribute, with the normalization condition $\sum_{i=1}^{n} w_i = 1$. The PWHM operator is defined as follows:

$$PWGHM^{(k_t)}(x,W) \prod_{t=1}^{d} \left\{ \left\{ \prod_{1 \le i_1 < \ldots < i_k \le p_t} \left( \frac{\sum_{j=1}^{k_t} W_{i_j} x_{i_j}}{\sum_{j=1}^{k_t} W_{i_j}} \right)^{1/C_{p_t}^{k_t}} \right\}^{\frac{1}{d}} \right\} \tag{8}$$

where:

$|P_t|$ the cardinality of the partition $P_t$;

$k_t = 1,2,\cdots,|P_t|$ is the granularity of the partition

$P_t$; $i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers taken from the set $\{1, 2, \cdots, |P_t|\}$ and

$C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t|-k_t)!}$ is binomial coefficient.

*1.3 The Double weighted Partitioned Hamy Mean (DWPHM) operators*

This section presents novel weighted partitioned Hamy mean operators that incorporate weights for both attributes and coalitions. Various properties and special cases of these new operators examined.

**DEFINITION 9. Double Weighted Arithmetic Partitioned Hamy mean (DWAPHM) operators:** Let $x_i$ (i = 1 to n) be a set of nonnegative real numbers, which are partitioned into *d* distinct coalitions $d = \{P_1, P_2, \cdots, P_d\}$. Let $w_i \in [0,1]$ represent the weight of the $i^{th}$ attribute, with the normalization condition $\sum_{i=1}^{n} w_i = 1$. The DWAPHM operator is defined as follows:

$$DWAPHM^{(k_t)}(x,W) = \sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} \left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}} \right) \tag{9}$$

where:

$|P_t|$ the cardinality of the partition,

$P_t$; $k_t = 1,2,\cdots,|P_t|$ is the granularity of the partition,

$P_t$; $i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers taken from the set $\{1, 2, \cdots, |P_t|\}$,

$C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t|-k_t)!}$ Is binomial coefficient.

Example is provided to illustrate the calculation procedure of the proposed PMSM operator. Let $C = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of attributes, which is partitioned into two *d* distinct coalitions: $P_1 = \{x_1, x_2, x_3\}$ and $P_2 = \{x_4, x_5\}$, i.e., $|P_1| = 3$ and $|P_2| = 2$ , where $|P_1|$and $|P_2|$ denote the number of attributes in the partitions $P_1$ and $P_2$, respectively. Assume the degrees corresponding to the attributes$\{x_1, x_2, x_3, x_4, x_5\}$ are {0.6,0.5,0.3,0.7,0.1} and the weighting vector w = (0.4, 0.2, 0.2, 0.1, 0.1). For simplicity we set the granularity parameter k = 2. The aggregated result of the correlated attributes within group $P_1$ is computed as follows:

$$W_{p_1} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_1} \left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_1}^{k_t}} \right) = (0.4 + 0.2 + 0.2) \times \left[ \frac{(0.6^{0.4} \times 0.5^{0.2})^{\frac{1}{0.6}} + (0.6^{0.4} \times 0.3^{0.2})^{\frac{1}{0.6}} + (0.5^{0.2} \times 0.3^{0.2})^{\frac{1}{0.4}}}{3} \right] = 0.381$$

The integrated result of the correlated attributes within group $P_2$ is computed as follows:

$$W_{p_2}\left(\frac{\sum_{1\le i_1<\ldots<i_{k_t}\le p_2}\left(\prod_{j=1}^{k_t}\left(x_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k_t}w_{i_j}}}}{C_{p_2}^{k_t}}\right)=(0.1+0.1)\times\left[\frac{\left(0.7^{0.1}\times0.1^{0.1}\right)^{\frac{1}{0.2}}}{1}\right]=0.05$$

Therefore, according to Eq. (17), we the DWAPHM operator is computed as follows:

$$DWAPHM^{(k_t)}(x,W)=\sum_{t=1}^{d}w_{p_t}\left(\frac{\sum_{1\le i_1<\ldots<i_{k_t}\le p_t}\left(\prod_{j=1}^{k_t}\left(x_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k_t}w_{i_j}}}}{C_{p_t}^{k_t}}\right)=(0.4+0.2+0.2)\times$$
$$\left[\frac{\left(0.6^{0.4}\times0.5^{0.2}\right)^{\frac{1}{0.6}}+\left(0.6^{0.4}\times0.3^{0.2}\right)^{\frac{1}{0.6}}+\left(0.5^{0.2}\times0.3^{0.2}\right)^{\frac{1}{0.4}}}{3}\right]+(0.1+0.1)\times\left[\frac{\left(0.7^{0.1}\times0.1^{0.1}\right)^{\frac{1}{0.2}}}{1}\right]=0.434$$

Note that DWAPHM operator exhibits several special cases under different partitioning scenarios, as outlined below:

**Case 1**. When all attributes belong to a single partition and each attribute interacts with all others i.e. $d=1$ ; the cardinality of of the partition is $|P_t|=n$ and the $k=1,2,\cdots,n$, the $DWAPHM^{(k_t)}(x,W)$ reduces to $WHM\ (x,W)$ operator, expressed as follows:

$$DWAPHM^{(k_t)}(x,W)=\frac{\sum_{1\le i_1<\ldots<i_{k_t}\le p_t}\left(\prod_{j=1}^{k_t}\left(x_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k_t}w_{i_j}}}}{C_{p_t}^{k_t}} \tag{10}$$

**Case 2.** In practical cases, some arguments might not relate to any other arguments as they do not fit into any of the groups. In this scenario, we can divide all the arguments into two distinct coalitions denoted as $P_1$ and $P_2$. The arguments in $P_1$ are interrelated while arguments in $P_2$ are not related to any attributes. We assume that $P_1$ formed is based on existing relationship structure. The DWAPHM operator is modified as follows:

$$DWAPHM^{(k_t)}(x,W)=W_{p_1}\left(\frac{\sum_{1\le i_1<\ldots<i_{k_t}\le p_t}\left(\prod_{j=1}^{k_t}\left(x_{i_j}\right)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k_t}w_{i_j}}}}{C_{p_1}^{k_t}}\right)+W_{p_2}\left(\frac{\sum_{j=1}^{p_2}x_{ij}}{|P_2|}\right) \tag{11}$$

**Case 3.** When argument sets are not partitioned and the type of the interrelationship among the arguments is the same – i.e. cardinality satisfies $|P_t|=n$ and the parameter $k=1,2,\cdots,n$ - We further investigate specific cases of the DWAPHM operator with parameter $k$ by assigning particular values to the parameter *k*.

Case 3.1: If $k=1$, the DWAPHM operator reduces to arithmetic mean as follows:

$$DWAPHM^{(1)}(x,W)=\frac{\sum_{j=1}^{n}x_{ij}}{n} \tag{12}$$

**Case 3. 2.** If $k=n$, the DWAPHM operator reduces to weighted geometric mean as follows:
The next step of the development of the proposed DWAPHM operator is to establish some of its features.

**1. Idempotency.** If $x_i=x\ \forall\ i\in\{1,2,\cdots,n\}$, Then $DWAPHM^{(k_t)}(x,W)=x$

**Proof.** Since $x_1=x_2=\cdots=x$, the DWAPHM simplifies as:

$$WPWHM^{(k_t)}(x,W)=\sum_{t=1}^{d}w_{p_t}\left(\frac{\sum_{1\le i_1<\ldots<i_{k_t}\le p_t}\left(\prod_{j=1}^{k_t}(x)^{w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{k_t}w_{i_j}}}}{C_{p_t}^{k_t}}\right) \tag{13}$$

$$= \sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} \left( x^{\sum_{j=1}^{k_t} w_{ij}} \right)^{\frac{1}{\sum_{j=1}^{k_t} w_{i_j}}}}{C_{p_t}^{k_t}} \right) = \sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} (x)}{C_{p_t}^{k_t}} \right)$$

$$= \sum_{t=1}^{d} w_{p_t} \left( \frac{C_{p_t}^{k_t} \times (x)}{k_t} \right) = x$$

**2. Monotonicity.** *Let* $\{x_1, x_2, \cdots, x_n\}$ *and* $\{y_1, y_2, \cdots, y_n\}$ *be two sets of real numbers, if* $x_i \le y_i$ for all $i$, *then* $DWAPHM^{(k_t)}(x, W) \le DWAPHM^{(k_t)}(y, W)$

**Proof.** Since $x_i \le y_i$, for all i, in the light of the definition of DWAPHM operator, we can see that $x_{ij}^{w_{ij}} \le y_{ij}^{w_{ij}}$. Then this allows us to derive:

$$\left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right) \le \left( \prod_{j=1}^{k_t} \left( y_{i_j} \right)^{W_{i_j}} \right)$$

Thus, we derive:

$$\left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}} \le \left( \prod_{j=1}^{k_t} \left( y_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}$$

Similarly, we derive:

$$\frac{\sum_{\le i_1 < \ldots < i_{k_t} \le p_t} \left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}} \le \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} \left( \prod_{j=1}^{k_t} \left( y_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}}$$

Therefore, we derive:

$$\sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} \left( \prod_{j=1}^{k_t} \left( x_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}} \right) \le \sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le pt} \left( \prod_{j=1}^{k_t} \left( y_{i_j} \right)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}} \right)$$

Hence

$DWAPHM^{(k_t)}(x, W) \le DWAPHM^{(k_t)}(y, W)$

**3. Boundedness.** Let $\{x_1, x_2, \cdots, x_n\}$ be a set of real number $min_i x_i = x^-$ and $max_i x_i = x^+$, where $i \in N$. We have $x^- \le DWAPHM^{(k_t)}(x, W) \le x^+$

**Proof.** From the idempotency of $WPWHM^{(k_t)}(x, W)$ operator, we can claim

$WPWHM^{(k_t)}(x^-, W) = x^-$

$WPWHM^{(k_t)}(x^+, W) = x^+$

Monotonicity of WPWHM operator:

$x^- \le DWAPHM^{(kt)}(x, W) \le x^+$

**DEFINITION 10. Double Weighted Geometric Partitioned Hamy mean operator**

Let $x_i$ (i = 1 to n) be a set of nonnegative real numbers, which are Partitioned into $d$ distinct coalitions $d = \{P_1, P_2, \cdots, P_d\}$. Let $w_i \in [0,1]$ represent the importance, of the $i^{th}$ attribute, such that $\sum_{i=1}^{n} w_i = 1$. The DWGPHM operator is defined as:

$$DWGPHM^{(k_t)}(x, W) = \prod_{t=1}^{d} \left\{ \left\{ \prod_{1 \le i_1 < \ldots < i_k \le p_t} \left( \frac{\sum_{j=1}^{k_t} W_{i_j} x_{i_j}}{\sum_{j=1}^{k_t} W_{i_j}} \right)^{1/C_{p_t}^{k_t}} \right\}^{w_{p_t}} \right\} \tag{14}$$

where $|P_t|$ the cardinality of the partition $P_t$; $k_t = 1,2,\cdots,|P_t|$ is the granularity of the partition $P_t$; $i_1, i_2, \cdots, i_{k_t}$ are $k_t$ natural numbers from the set $\{1, 2, \cdots, |P_t|\}$ and $C_{|P_t|}^{k_t} = \frac{|P_t|!}{k_t!(|P_t|-k_t)!}$ is binomial coefficient.

Some special cases of the WPWGHM operator are presented below:

**Case1.** If all attributes belong to a single partition, meaning all attributes every attribute interacts with every other attribute i.e. $d = 1$ ; the cardinaly of $|P_t| = n$ and the $k = 1,2,\cdots,n$, then the $DWGPHM^{(k_t)}(x, W)$ reduces to $WGHM^{(k_t)}(x, W)$ operator as follows:

$$DWGPHM^{(k_t)}(x, W) = \left\{ \prod_{1 \le i_1 < \ldots < i_k \le n} \left( \frac{\sum_{j=1}^{k} W_{i_j} x_{i_j}}{\sum_{j=1}^{k} W_{i_j}} \right) \right\}^{\frac{1}{C_n^k}} \tag{15}$$

**Case 2.** When some arguments are not related to any other argument as they do not fit into any the groups. In this scenario, we divide these arguments into two sets denoted by $P_1$ and $P_2$. The arguments in $P_1$ are interrelated while arguments in $P_2$ are not related to any attribute. We assume that $P_1$ formed is based on existing relationship structure.

$$DWGPHM^{(k_t)}(x, W) = \left\{ \left\{ \prod_{1 \le i_1 < \ldots < i_k \le p_1} \left( \frac{\sum_{j=1}^{k_t} W_{i_j} x_{i_j}}{\sum_{j=1}^{k_t} W_{i_j}} \right)^{1/C_{p_t}^{k_t}} \right\}^{w_{p_1}} \right\} \times \left\{ \left\{ \left\{ \prod_{1 \le i_1 < \ldots < i_k \le p_2} (x_{ij}) \right\}^{1/|P_2|} \right\}^{w_{p_2}} \right\} \tag{16}$$

**Case 3.** When argument sets are not partitioned and the types of the interrelationship among arguments are same – i.e the cardinality $|P_t| = n$ and the parameter $k = 1,2,\cdots,n$, We further investigate some specific cases of DWGPHM operator with parameter k taking some particular values.

**Case 3.1.** If $k = 1$,the novel DWGPHM operator reduces to weighted geometric mean as follows:

$$DWGPHM^{(1)}(x, W) = WGM(x) = \left\{ \prod_{j=1}^{n} x_j \right\}^{\frac{1}{n}} \tag{17}$$

**Case 3.2.** If $k = n$, the DWGPHM operator reduces to weighted arithmetic mean as follows:

$$DWGPHM^{(n)}(x, W) = \frac{\sum_{j=1}^{n} W_j x_j}{\sum_{j=1}^{n} W_j} \tag{18}$$

In reality, the criteria are not always be correlated with each other. To improve decision-making accuracy, the criteria should be partitioned into distinct categories.

## 2. PHARSA method

This section presents the developed Partitioned Hamy Aggregators -based Ranking and Selection Approach (PHARSA) method based on DWAPHM and DWGPHM operators.

Let $\{A_i\}_{i=1}^{m}$ be a specified feasible alternative and $\{C_j\}_{j=1}^{n}$ is a set of attributes. Let $\{w_j\}_{j=1}^{n}$ be the weights of attributes $C_j$, which are partitioned into *d* distinct coalitions $P_1, P_2, \cdots, P_d$, and $\{w_j\}_{j=1}^{n}$, denoting the weights of each attribute $C_j$, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The following is a novel MADM approach based on the proposed operators:

Step 1. Normalize the decision matrix. Generally, attributes can be divided into tangible and intangible categories. We can utilize the following formula to normalize the decision matrix.

### The case of cardinal attribute value

For benefit-type cardinal attributes, the normalized attribute values $x_{ij}^{L}$'s are computed as follows:

$$x_{ij}^{L} = \frac{x_{ij} - x_j^{--}}{x_j^{++} - x_j^{--}} \tag{19}$$

whereas, for cost-type cardinal attributes $x_{ij}^{L} = -$are computed as follows:

where

$$x_j^{--} = max\left(0, \left(x_j^- - d_j\right)\right) \tag{20}$$

$$x_j^{++} = x_j^+ + d_j, \text{ where } d_j = \frac{x_j^+ - x_j^-}{2} \tag{21}$$

where $x_j^-$ and $x_j^+$ correspond respectively to the minimum and maximum attribute values with respect to the attribute $C_j$ ($j =$ 1 to $n$).

**The case of ordinal attribute values**

The relative position $RP_{ij}$ of alternative $A_i$ on ordinal scale $S_j$ is determined by its rank when alternatives are ordered by their ordinal attribute values $x_{ij}$'s with the most preferred alternative is ranked first. Letting $N_j$ denotes the number of gradations of the scale, the linear normalized attribute values $x_{ij}^L$'s will be of the form

$x_{ij}^L = \frac{RP_{ij}}{N_j}$, $i =$1 to $m$

Step 2. Computing the arithmetic partitioned Hamy integrated values of alternatives

$$APHIV_i^{(k)} = DWAPHM^{(k)}(x_i^C, W) = \sum_{t=1}^{d} w_{p_t} \left( \frac{\sum_{1 \le i_1 < \ldots < i_{k_t} \le p_t} \left( \prod_{j=1}^{k_t} (x)^{W_{i_j}} \right)^{\frac{1}{\sum_{j=1}^{k_t} W_{i_j}}}}{C_{p_t}^{k_t}} \right) \tag{22}$$

Step 3. Computing the geometric partitioned Hamy integrated values of alternatives

$$GPHIV_i^{(k)} = DWGPHM^{(k)}(x_i^C, W) = \prod_{t=1}^{d} \left\{ \left\{ \prod_{1 \le i_1 < \ldots < i_k \le p_t} \left( \frac{\sum_{j=1}^{k_t} W_{i_j} x_{i_j}}{\sum_{j=1}^{k_t} W_{i_j}} \right)^{1/C_{p_t}^{k_t}} \right\}^{w_{p_t}} \right\} \tag{23}$$

Step 4. Computing the partitioned Hamy ranking value ($PHRV_i^{(k)}$) of alternative $A_i$ is expressed as:

$$PHRV_i^{(k)} = (1 - \mu) \times PHIV_i^{(k)} + \mu \times PGHIV_i^{(k)}, \ 0 \le \mu \le 1 \tag{24}$$

Step 5. Ranking all the alternatives: based on the partitioned Hamy ranking value, the higher $PHRV_i^{(k)}$, the better is the alternative $A_i$.

## 3. Case study: Water pipe material selection problem

Access to potable water remains a major challenge in many developing countries, where efficient distribution systems are essential (Rajasärkkä et al., 2016; Tomboulian et al., 2004). Water pipes basically fall into three types: metallic, concrete, and plastic. Common metallic materials include copper, lead, steel, and cast iron, while plastic pipes are typically made from high-density polyethylene (HDPE), polyvinyl chloride (PVC), polypropylene (PP), and cross-linked polyethylene (PEX) (Janajreh et al., 2015; Ashby & Johnson, 2014). Over the last decades, the materials used for water pipes have evolved, with polymers and ceramics increasingly replacing the traditional materials. The selection of pipe material is influenced by a wide range of factors, including material properties, performance requirements, reliability, safety, physical characteristics, environmental conditions, availability, disposability and recyclability, as well as economic considerations (Arthur et al., 2020). The choice of water pipe material depends on the intended use and the required water quality (Petti et al., 2019). Common materials include concrete, reinforced concrete, and metals (e.g., ductile iron, copper, lead, stainless steel, and galvanized steel). Plastic pipes, including high-density polyethylene (HDPE), polyvinyl chloride (PVC), polypropylene (PP), and cross-linked polyethylene (PEX), are also widely used (Charles et al., 2005). Among these, PVC is one of the most extensively used polymeric materials in water pipe construction. It is the second-largest thermoplastic by volume, after PE, for producing water transportation pipes (Ashby & Johnson, 2014).

PVC pipes are commonly used for conveying drinking water due to their relative affordability, durability, chemical inertness, lightweight nature, and ease of processing (Rimdusit et al., 2014; Bais et al., 2019). PVC pipes are typically installed underground to protect them from damage and UV radiation (Petti et al., 2019). However, environmental changes are challenging this conventional practice. Land erosion, exacerbated by climate change, is increasingly exposing buried PVC pipes,

and combined with UV radiation in tropical regions due to ozone layer depletion, this poses a significant threat (Marron et al., 2019; Jassim et al., 2017). Prolonged UV exposure embrittles PVC materials, leading to degradation and an increased vulnerability to fracture (Shams et al., 2019). Furthermore, while PVC material is extensively utilized in household settings, it is unsuitable for hot water transportation (Petti et al., 2019). Numerous materials with diverse and interacting properties are used in water pipe manufacturing. The selection of the most suitable material remains a complex, time-consuming challenge requiring a systematic and efficient approach. The proposed approach was implemented at Comptoir Sfaxien du Plastique (CSP), a medium-sized plastic product manufacturing company established in 1998 in Sfax, Tunisia. With over 25 years of experience, CSP produces a wide range of pipes, including LDPE sheaths for the protection of electrical and telephone cables, irrigation pipes, as well as BD-HD-Black polyethylene pipes for potable water and agricultural irrigation, PVC pipes for drainage, liquid transport, sanitation, and telecommunication networks, corrugated electrical conduits for indoor and underground electrical installations, and red and blue ringed sheaths for the protection of sanitary and plumbing pipes. Certified ISO-9001, CSP holds a strong reputation in domestic and international markets. CSP needs to select a suitable alternative material for transporting drinking water through pipes that are non-toxic, cost-effective, UV- and radiation resistant, lightweight and easy to install, and suitable for deployment remote and hard-to-reach areas. After a preliminary screening, seven candidate materials were identified by CSP, namely Polyethylene (PE-32), Polyethylene (PE-66), Polyethylene (PE-80), Polyethylene (PE-100), Chlorinated Polyvinyl Chloride (CPVC-K67), Polypropylene (PPR), and Polyvinylidene fluoride (PVDF). The alternatives were evaluated with respect to seven attributes: $(C_1)$ Density (g/cm2), $(C_2)$ flexibility, $(C_3)$ Installation costs, $(C_4)$ Price (dt/kg), $(C_5)$ the Minimum Required Strength (MRS) $(C_6)$ pressure resistance and $(C_7)$ UV resistance. Among these attributes, density $(C_1)$, cost $(C_4)$ and MRS $(C_5)$ are considered cardinal attributes while the remaining criteria are ordinal. Furthermore, based on the knowledge and expertise of CSP manager, the attributes were partitioned into two sub-sets: P1 and P2
Where:

$$\text{P1}= \left\{\text{Density (gcm2)}, \textbf{f}\text{lexibility}, \text{Installation costs}, \text{Price } \left(\frac{\text{TND}}{\text{kg}}\right)\right\}.$$

$$P_2 = \{\text{Minimum Required Strength}, \text{ Pressure resistance}, \text{and UV resistance}\}.$$

In fact, there are interrelationships between the attributes in the set of P1su-bsets, while there are no interrelationships among the attributes in the P2- set. In this problem, the attribute weights are given as $w = (0.1, 0.09, 0.22, 0.16, 0.2, 0.11, 0.12)$.Then, the decision table for this material selection problem is presented in Table 1 below.

**Table 1**
The decision matrix

| Materials | Density ($g/cm^3$) | Flexibility | Installation cost | Price (TND/Kg) | MRS (MPa) | Pressure Resistance | UV Resistance |
|---|---|---|---|---|---|---|---|
| **PE32** | 0.920 | 4 | 3 | 4 | 3.2 | 3 | 3 |
| **PE63** | 0.951 | 3 | 3 | 4.05 | 6.3 | 3 | 3 |
| **PE80** | 0.956 | 3 | 3 | 4.1 | 8 | 3 | 3 |
| **PE100** | 0.960 | 2 | 3 | 4.3 | 10 | 3 | 3 |
| **CPVC K67** | 1.750 | **1** | **2** | **7** | 25 | 5 | 5 |
| **PPR 70%** | 0.905 | **1** | **2** | **5** | 10 | 3 | 2 |
| **PVDF 50%** | 1.800 | **1** | **2** | **7** | 10 | 3 | 5 |

### *3.1 Application of the proposed PHARSA method*

The rest of this section details the application of the proposed method using the above data.
Step 1. Since attributes C1, C3 and C4 are cost-type criteria, while C2, C5, C6 and C7 are benefit-type criteria, a normalized decision matrix must be constructed using Eq. (1) and Eq. (2). The resulting normalized values are presented in Table 2.

**Table 2**
Normalized matrix

| Materials | Density ($g/cm^3$) | Flexibility | Installation cost | Price (TND/Kg) | MRS (MPa) | Pressure Resistance | UV Resistance |
|---|---|---|---|---|---|---|---|
| **PE32** | 0.794 | 0.8 | 0.6 | 0.824 | 0.089 | 0.6 | 0.6 |
| **PE63** | 0.781 | 0.6 | 0.6 | 0.818 | 0.175 | 0.6 | 0.6 |
| **PE80** | 0.778 | 0.6 | 0.6 | 0.812 | 0.223 | 0.6 | 0.6 |
| **PE100** | 0.777 | 0.4 | 0.6 | 0.788 | 0.279 | 0.6 | 0.6 |
| **CPVC K67** | 0.425 | **0.2** | **0.4** | **0.471** | 0.696 | 1 | 1 |
| **PPR 70%** | 0.801 | **0.2** | **0.4** | **0.706** | 0.279 | 0.6 | 0.4 |
| **PVDF 50%** | 0.403 | **0.2** | **0.4** | **0.471** | 0.279 | 0.6 | 1 |

Step 2. The proposed DWAPHM and DWGPHM operators were applied to aggregate the evaluations for each alternative. Given the parameters $d = 2, |P_1| = 4, and\ |P_2| = 3$, along with the granularity parameters $k_1 = 2\ and\ k_2 = 1$ , the computation yields aggregated matrix presented in Table 3.

**Table 3**
The aggregated matrix

| | PE32 | PE63 | PE80 | PE100 | CPVC K67 | PPR 70% | PVDF 50% |
|---|---|---|---|---|---|---|---|
| $DWAPAHM^{(2,1)}$ | 0.599 | 0.589 | 0.595 | 0.574 | 0.608 | 0.462 | 0.486 |
| $DWGPHM^{(2,1)}$ | 0.506 | 0.541 | 0.56 | 0.557 | 0.554 | 0.461 | 0.446 |

Step 3. Water pipe materials were ranked by computing the partitioned Hamy ranking values for $\boldsymbol{\mu} = 0.5$, using Eq. (23), alternative ranking is presented in Table 4.

**Table 4**
The partitioned hamy ranking values

| | PE32 | PE63 | PE80 | PE100 | CPVC K67 | PPR 70% | PVDF 50% |
|---|---|---|---|---|---|---|---|
| ***PHRV*** | 0.533 | 0.565 | 0.577 | 0.57 | **0.581** | 0.421 | 0.466 |
| **Rank** | 5 | 4 | 2 | 3 | 1 | 7 | 6 |

According to PHARSA method, water pipe material are ranked as follows:
CPVC K67> PE80 > PE100 > PE63 > PE32 > PVDF 50% > *PVDF* 70%. Note that CPVC K67 is the highest ranked waste-pipe material.

### *3.2 Sensitivity Analysis*

In the PHARSA approach, the influence of the parameter $\mu$ on the decision-making process is examined to enhance accuracy and efficiency. The ranking orders with the different parameters is illustrated in Fig. 1. As shown, the analysis reveals that the top-ranked alternative remains consistent across various parameter values. This demonstrates that the proposed approach exhibits strong stability with respect to parameter changes.

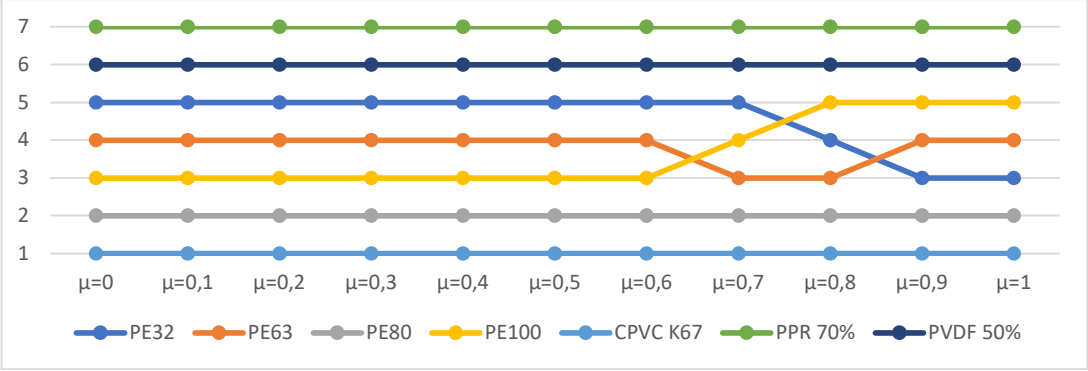


**Fig. 1.** Ranking orders with different parameters

### *3.3 Dynamic decision matrix effects*

To assess how dynamic matrices impact alternative rankings, seven different scenarios were created. In each scenario removed the lowest-ranked alternative from the previous step and then re-evaluate the rankings of the remaining options. This process allows us to observe the changes in ranking results as alternative are sequentially eliminated.

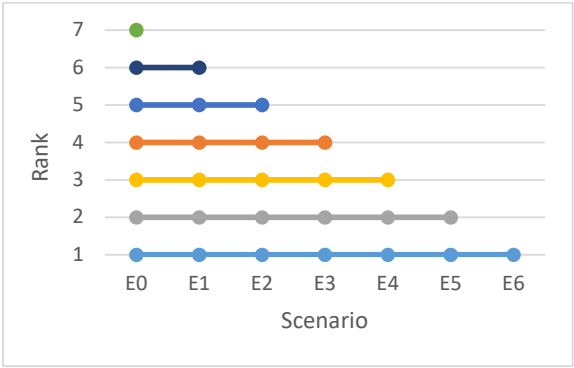


**Fig. 2.** The summary of experimental results

In the initial ranking (scenario S0), PPR 70% was identified as the worst-performing alternative. In the first scenario (S1), PVDF alternative PVDF 50% was dropped from consideration, creating a new decision matrix with five alternatives. The resulting ranking was as follows: CPVC K67 ≻ PE80 ≻ PE100 ≻ PE63 ≻ PE32. Clearly, alternative $M_1$ emerged as the new lowest-ranked alternative. Therefore, in the second scenario (S2), PE32 was removed in order to form a new decision matrix. This process continued until only alternative CPVC K67 remained. Fig. 2 illustrates the experimental results. As we can observe that the relative lines standing for alternatives do not intersect. This indicates that the relative rankings of the alternatives are consistent across all experiments. This means that the proposed method preserves the indication of the 'best' alternative. The absence of any rank reversal in this example confirms the stability and robustness of the proposed method in the dynamic decision-making contexts.

### *3.4 Comparison with other methods*

This subsection presents a comparative analysis of solution results between the proposed method and other MADM techniques for water pipe material selection, including Complex Proportional Assessment (COPRAS) (Zavadskas et al., 2007), Evaluation Based on the Distance from the Average Solution (EDAS) (Keshavarz Ghorabaee et al., 2015), Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) (Stević et al., 2020), MACONT (Wen et al., 2020), WASPAS (Zavadskas et al., 2012), Root Assessment Method (RAM) (Sotoudeh Anvari, 2023), Alternative by Alternative Comparison (ABAC) based on a multi-criteria decision-making method (Biswas et al., 2022), and Compromise Ranking of Alternatives from Distance to Ideal Solutions (CRADIS) (Chakraborty et al., 2023). Fig. 3 presents a visual comparison of all ranking results, demonstrating the relative performance of each method in resolving the material selection problem. Fig. 3 presents the decision results of all MADM methods, excepting MACONT. As illustrated, the water-pipe material CPVC K67 is the highest-ranked alternative. This indicates that the results yielded by the proposed method are consistent with other MADM approaches, supporting its applicability in real-life decision-making.

PE 80 is ranked as the second-best water pipe material by the proposed method, CRADIS, MARCOS, and WASPAS, while ABAC, EDAS, and MACONT rank it third and even fourth in some other methods. Similarly, PE 100 is placed third by the proposed method, COPRAS, RAM, and WASPAS, while the remaining methods assign it different rankings. Notably, EDAS and ABAC both rank PE 100 as the second-best option. Finally, PPR (70%) and PVDF (50%) are ranked as the worst-performing alternatives across all MCDM methods, except COPRAS and RAM.

Although PHARSA's rankings differ significantly from other MCDM methods, these differences may stem from two key factors:

Normalization Framework: PHARSA employ distinct normalization procedures for cardinal and ordinal attribute, unlike conventional methods

Attribute interdependencies: Existing MADM methods fail to model interrelationships and partitioning structure among the attribute. In practice, attributes often exhibit dependencies; ignoring these relationships compromises ranking accuracy. Consequently, PHARSA's ability to both capture attribute interrelationships and analyze partition structures within the attribute set explains its divergent, and more reliable, rankings compared to traditional methods.

**Fig. 3.** The summary of the experimental results

## 4. Conclusion

This paper proposes a double weighting system for partitioned Hamy mean (PHM) operators by weighting both the criteria and the partitions. Specifically, we defined the Double-Weighted Arithmetic partitioned Hamy mean (DWAPHM) and the

Double-Weighted Geometric Partitioned Hamy Mean (DWGPHM) operators. These operators constitute the theoretical foundation of the proposed ranking and selection approach PHARSA.

To demonstrate the effectiveness of the method, a real-world case study involving the selection of water pipe was conducted. The results clearly indicate the proposed method outperforms the alternatives in terms of accuracy and decision quality.

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