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Dynamic pricing and advertising strategy for a perishable product with an expiration date under a price ceiling constraint

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ABSTRACT

This study develops a dynamic pricing and advertising model for perishable products under an admissible price ceiling, where prices may vary over time but cannot exceed a market-accepted upper bound, while dynamic discounts are allowed. The demand function jointly depends on the selling price, advertising-induced goodwill, and the remaining time until product expiration, a combination rarely addressed in prior research. The problem is formulated as a finite-horizon optimal control model, explicitly incorporating perishability and the price ceiling constraint. By applying Pontryagin's maximum principle, time-dependent optimal trajectories for price and advertising are derived. Numerical experiments and sensitivity analyses illustrate how changes in price sensitivity, advertising effectiveness, and the remaining shelf life effect on demand influence optimal strategies and profitability. The results reveal critical trade-offs between pricing flexibility, advertising intensity, and perishability effects, offering practical guidance for decision-makers in food, pharmaceutical, and fast-moving consumer goods markets with fixed price tags compared to more flexible pricing environments.

1. Introduction

Pricing perishable products is challenging, as unsold inventory quickly becomes waste, leading to lost revenue. As products approach their expiration date, both physical quality and consumer willingness to pay decline, making demand highly time-dependent. Effective pricing strategies must therefore consider remaining shelf life and changing market conditions to maximize profit and minimize waste. In many real-world markets, such as packaged snacks, dairy products, and pharmaceuticals, products have a hard expiration date. They retain full market value up to this point but become unsellable or require deep discounts afterward. Capturing this hard perishability is essential for developing realistic pricing and inventory models. Another important factor is the admissible price ceiling, which sets the maximum selling price during the sales period. This ceiling may arise from regulations, printed packaging labels (common in countries like Iran), or consumer expectations. While downward price adjustments are allowed, prices cannot exceed this ceiling, creating asymmetric pricing flexibility. In markets without such restrictions, sellers may strategically set the ceiling. When combined with perishability, this constraint can fundamentally change optimal pricing patterns. Given these pricing constraints, advertising also plays a key role in managing perishable goods, especially for low-loyalty or impulse products. Advertising builds goodwill, raises awareness, and stimulates demand, but its effectiveness gradually declines as expiration approaches and consumers become more price-sensitive. This creates a dynamic interaction between pricing, advertising, and remaining shelf life.

Despite their practical relevance, most existing studies:

1. Ignore hard expiration dates in modeling demand,
2. Assume unconstrained or symmetric pricing flexibility, or

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3. Treat advertising as exogenous or fixed over time.

To address these gaps, this study develops a dynamic pricing and advertising model for perishable products under a fixed admissible price ceiling. Prices can vary over time but must not exceed a market-accepted maximum. The demand function simultaneously incorporates selling price, advertising-induced goodwill, and remaining shelf life. The problem is formulated as a finite-horizon optimal control model, explicitly accounting for perishability and the price ceiling constraint. Pontryagin's maximum principle is applied to derive the optimal time-dependent trajectories for both price and advertising.

The key contributions of this work are as follows:

- **Integration of perishability and price ceiling constraints:** Incorporating an admissible price ceiling into a dynamic pricing model for perishable goods, allowing discounts but prohibiting upward adjustments beyond the ceiling.
- **Joint modeling of price, goodwill, and expiration effects:** Developing a composite demand function that captures the combined influence of these factors.
- **Optimal control solution:** Providing analytical and numerical results for optimal pricing and advertising strategies in the presence of perishability and ceiling restrictions.
- **Practical insights for managers:** Offering sensitivity-based guidance on how factors such as expiration time, price ceiling levels, and advertising budgets influence profitability in sectors like food, pharmaceuticals, and fast-moving consumer goods (FMCG).

By unifying these behavioral and operational elements, the proposed model delivers a robust and practical decision-making framework for firms managing short-shelf-life products in competitive and constraint-driven markets. Beyond its theoretical contributions, the study also offers direct managerial implications: it helps managers coordinate price discounts and advertising budgets in the presence of regulatory or market-imposed price ceilings. For instance, in markets with printed price tags (e.g., Iran), advertising is the main tool to sustain demand when prices cannot rise. In more flexible markets, dynamic pricing plays the dominant role. These insights allow decision-makers in food, pharmaceutical, and FMCG sectors to align pricing and marketing policies with perishability constraints, thereby improving profitability while reducing waste.

2. Literature Review

This section reviews prior research in three key areas relevant to our study: (1) dynamic pricing and inventory control for perishable products, (2) integration of pricing and advertising strategies, and (3) empirical insights on price ceilings. The review highlights existing contributions, limitations, and the research gap our model addresses.

2.1 Dynamic Pricing and Inventory Control for Perishable Products

Dynamic pricing for perishable goods has been extensively studied, often within the framework of optimal control theory using Pontryagin's maximum principle (Herbon & Khmelnitsky, 2017; Sethi & Thompson, 2000). These models typically consider how demand evolves over time in response to price, perishability, and sometimes visible inventory levels. For instance, Zhang et al. (2015) developed an optimal replenishment and dynamic pricing model for non-instantaneous deteriorating items, while Tashakkor et al. (2018) incorporated a variable deterioration rate to better reflect real-world perishability. Lin Feng (2019) developed an optimal replenishment model for perishable products that simultaneously accounts for quality and quantity deterioration, integrating dynamic pricing with quality investment strategies. Liu et al. (2015) developed an inventory model for perishable food items where demand depends on both price and quality. They proved the existence of an optimal solution using the Filippov–Cesari theorem and derived the necessary conditions with Pontryagin's maximum principle. Lu et al. (2018) analyzed dynamic pricing for perishable products with price adjustment costs, reformulating the problem as a standard optimal control framework and obtaining the optimal solution through Pontryagin's maximum principle.

Recent studies have added further realism to pricing models by incorporating freshness-preservation strategies (Shi & You, 2023) and stochastic demand dynamics (Luo & Chu, 2023). Machine learning–driven approaches have also been applied for multi-period dynamic pricing under demand uncertainty (Hua et al., 2021). More recent works extend these models by combining freshness, sustainability, and preservation investments. Katariya and Shukla (2025) developed a model for non-instantaneously deteriorating products, where demand depends on price, freshness, and greening efforts, optimizing pricing and cycle time under discounting during deterioration. Modak et al. (2025) studied dynamic pricing for perishable items under demand dependent on price, stock level, and freshness, incorporating investments in preservation and lead-time reduction. Both studies used Pontryagin's Maximum Principle to characterize optimal pricing trajectories.

However, most studies still do not address hard expiration dates or binding price ceilings, which are common in real-world markets such as packaged food, pharmaceuticals, and other FMCG products (fast-moving consumer goods). Our work extends this literature by incorporating both a hard expiration constraint and an admissible price ceiling, allowing for dynamic discounting but prohibiting upward price adjustments beyond the ceiling.

2.2 Integration of Pricing and Advertising Strategies

The joint optimization of pricing and advertising has been explored in both competitive and monopolistic contexts. Chutani and Sethi (2012) investigated optimal advertising and pricing strategies within a dynamic duopoly setting, whereas Giri et al. (2015) examined the coordination of price and promotional efforts in a two-echelon supply chain with demand dependent on both factors. Feng et al. (2015) proposed a dynamic pricing and advertising model for perishable goods, incorporating goodwill accumulation and price-dependent demand. Wang et al. (2020) extended this by including production and shortage effects in a goodwill-based framework.

More recently, Dye (2020) examined the joint optimization of pricing, advertising, and inventory control for perishable products, introducing the concept of the "psychic stock effect." This effect establishes a link between inventory levels and consumer perception. Schlosser (2016) developed a stochastic dynamic model integrating both pricing and advertising under time-varying demand. Additionally, Dye and Hsieh (2024) extended this line of research by embedding green-innovation investment into a sustainable pricing–advertising framework. In parallel, digital marketing strategies, including demand learning via dynamic advertising schemes, have gained traction (Agrawal et al., 2023). However, existing models typically assume unconstrained pricing. When a price ceiling binds, advertising may serve as the principal lever to drive demand, a strategic interaction that remains underexplored. Our model addresses this gap by explicitly modeling how advertising intensity adjusts in response to price ceilings and product perishability.

2.3 Empirical Insights on Price Ceilings

Empirical research on price ceilings, particularly in perishable product markets, is relatively limited. Studies such as Aparicio and Cavallo (2021) have shown that regulatory price controls on supermarket products can temporarily affect inflation and alter product variety within narrow categories, as firms introduce higher-priced variants in response to controls. Fan and Zhang (2022) demonstrate, in the context of Chinese cell phones, that tying subsidies to price ceilings can mitigate unintended price increases, enhancing consumer and total surplus. Theoretical insights by Bennett and Chioveanu (2019) further suggest that under regulatory uncertainty, optimal price ceilings may be set relatively high and adapt to competitive pressure. Experimental evidence from Engel and Heine (2017) indicates potential welfare losses from cap regulation. In markets like Iran, price ceilings are commonly enforced via printed packaging labels, effectively capping the maximum price across the sales horizon. Elsewhere, firms may self-impose ceilings to preserve brand positioning. Such ceilings produce asymmetric flexibility: prices can be lowered over time but not raised above the ceiling. This asymmetry reshapes the optimal pricing trajectory and elevates the strategic importance of alternative demand levers such as advertising.

2.4 Summary and Research Gap

The existing literature provides strong foundations in dynamic pricing, perishability modeling, and goodwill-based advertising, but falls short of capturing the combined effects of:

- A hard expiration date for perishable goods;
- A binding price ceiling allowing only downward adjustments; and
- Dynamic advertising as an endogenous goodwill-building mechanism.

By integrating these features into a finite-horizon optimal control framework, our study addresses a practical and underexplored problem in perishable product management. The model provides analytical and numerical solutions for optimal pricing and advertising strategies under realistic market constraints, offering actionable insights for sectors such as food, pharmaceuticals, and FMCG.

After defining the problem, notation, and assumptions, we derive the optimality conditions for the joint dynamic pricing and advertising model of perishable products using Pontryagin's Maximum Principle. Since no closed-form solution is available, the problem is solved numerically. The approach is demonstrated through a numerical example and further examined via sensitivity analysis of key parameters. The findings show that the solution is robust under most conditions, with deviations arising only when the optimal price trajectory surpasses the admissible ceiling.

3. Problem Definition

In this section, the notation is first defined, followed by the introduction of the proposed problem, the related optimal control model, and a numerical example. The notations adopted in this paper are as follows:

$I(t)$: Inventory level at time t

I_0: Initial inventory level

$p(t)$: Unit selling price at time t

p_{max}: Maximum acceptable price

$a(t)$: Advertising level at time t

A: Maximum advertising budget capacity

$G(t)$: Goodwill level at time t

$D(p, G, t)$: Demand rate as a function of price, goodwill, and remaining shelf life

E: Expiration date of the product

T: Length of the replenishment period ($T \leq E$)

k: Cost coefficient related to advertising investment

h: Holding cost per unit inventory per unit time

α: Market potential for demand

β: Price elasticity of demand

γ: Intensity of brand equity effect on demand

μ: Coefficient of remaining shelf life effect on demand

ρ: Rate of goodwill decay

J: Total profit over the planning horizon
$J_{\bar{p}}$: Total profit over the planning horizon, in cases where $p(t) = p_{max}$

$J_c(t)$: Cumulative profit at time t

3.1 Problem Description

In this problem, a monopolistic firm sells a single perishable product with a fixed shelf life and an initial inventory over a finite sales horizon. The firm's objective is to determine the optimal dynamic pricing strategy and advertising investment level that maximize total profit, while considering a market-imposed price ceiling.

The model is based on the following assumptions. The planning horizon is finite, and the system involves a single perishable product with a fixed shelf life and initial inventory level. The product has an admissible price ceiling, above which demand drops to zero, and stockouts are not permitted. Any remaining inventory at the end of the period has zero value. The demand rate is deterministic and, similar to the model of Nerlove and Arrow (1962), depends on the selling price and brand goodwill. In addition, our model further extends this by including the remaining time until product expiration, as consumers' willingness to purchase decreases near the expiration date.

The demand function is defined as:

$$D(t) = \alpha - \beta p(t) + \gamma G(t) + \mu(1 - \frac{t}{T}) \tag{1}$$

As can be seen, the demand decreases as price increases, while higher brand goodwill raises demand. Additionally, demand declines as the product approaches its expiration date, reflecting reduced customer willingness to purchase near the end of the shelf life.

It is worth noting that in the proposed model, the demand function may become negative under certain conditions. However, by selecting appropriate parameter values in the numerical example, demand remains positive throughout the replenishment cycle. Therefore, this condition is considered implicitly in the numerical solution rather than explicitly in the mathematical formulation.

The changes in inventory level in this problem are given by Eq. (2):

$$\dot{I}(t) = -\alpha + \beta p(t) - \gamma G(t) - \mu\left(1 - \frac{t}{T}\right) \quad I(0) = I_0 \tag{2}$$

Considering the above explanations, the components of the total profit in a period are as follows:

1. Sales revenue, $\int_0^T p(t)(\alpha - \beta p(t) + \gamma G(t) + \mu(1 - \frac{t}{T}))\, dt$
2. Holding cost, $\int_0^T hI(t)dt$

3. Advertising cost, $\int_0^T \frac{1}{2}ka(t)^2dt$

Furthermore, the optimization problem is formulated as model (1) as follows:

$$\max_{u(.),a(.)} J = \left\{\int_0^T (p(t)(\alpha - \beta p(t) + \gamma G(t) + \mu(1 - \frac{t}{T})) - hI(t) - \frac{1}{2}ka(t)^2)dt\right\} \tag{3}$$

$$\dot{I}(t) = -\alpha + \beta p(t) - \gamma G(t) - \mu\left(1 - \frac{t}{T}\right) \quad I(0) = I_0\,, \qquad I(T) = 0 \tag{4}$$

$$\dot{G}(t) = a(t) - \rho G(t) \quad 0 \le a(t) \le A\,,\ G(0) = G_0 \tag{5}$$

$$0 \le p(t) \le p_{max} \tag{6}$$

Eq. (3) represents the objective function of the problem, which aims to maximize the total profit over the replenishment cycle. The first term of the function corresponds to the revenue from product sales, while the subsequent terms represent, respectively, the holding and advertising costs. Eq. (4) describes the change in inventory level over time. Since the inventory level remains positive throughout the cycle and no shortages occur, the ending inventory is assumed to be zero in order to maximize total profit during the period. Eq. (5) defines the differential equation for brand goodwill, which depends on the level of investment in advertising. Eq. (6) specifies that the product price cannot exceed the acceptable price ceiling.

3.2 Determining the Optimality Conditions for the Second Problem

Given that the problem is formulated as an optimal control problem, Pontryagin's Maximum Principle can be directly applied to obtain the optimal dynamic pricing and advertising policy. The Hamiltonian function for this problem is formulated as follows:

$$H = p(t)(\alpha - \beta p(t) + \gamma G(t) + \mu(1 - \frac{t}{T})) - hI(t) - \frac{1}{2}ka(t)^2 + \lambda_1(t)(-\alpha + \beta p(t) \\ -\gamma G(t) - \mu(1 - \frac{t}{T})) + \lambda_2(t)(a(t) - \rho G(t)) \tag{7}$$

The differential equations related to the marginal profits are given by equations (8) and (9) as follows:

$$\dot{\lambda}_1(t) = -\frac{\partial H}{\partial I} = h \tag{8}$$

$$\dot{\lambda}_2(t) = -\frac{\partial H}{\partial G} = -\gamma p(t) + \gamma\lambda_1(t) + \rho\lambda_2(t) \tag{9}$$

Moreover, the Hamiltonian maximization condition is as follows:

$$H(p^*, a^*, I^*, G^*, \lambda_1, \lambda_2, t) = \max_{p(t),a(t)} H(p, a, I^*, G^*, \lambda_1, \lambda_2, t) \tag{10}$$

In this problem, due to the free terminal value of $G(t)$ at the end of the replenishment period, the transversality condition (11) holds:

$$\lambda_2(T) = 0 \tag{11}$$

To solve the proposed optimal control problem, Pontryagin's Maximum Principle is applied. According to this principle, in order to determine the optimal trajectory of the control variables (price, $p(t)$ and advertising, $a(t)$), the derivatives of the Hamiltonian function (7) with respect to these variables must be calculated and set equal to zero. This condition defines the critical points of the Hamiltonian function. However, to ensure that these critical points are indeed optimal (maxima), it is necessary to examine the curvature structure of the Hamiltonian function.

In the present model, the Hamiltonian is a function of the control variables price, $p(t)$ and advertising $a(t)$. Given the linear decreasing structure of the demand function with respect to price (which guarantees the negativity of its first derivative) and the assumed quadratic form of the advertising cost as $\frac{1}{2}ka(t)^2$, the second derivative of the Hamiltonian function with respect to both control variables is negative ($\frac{\partial^2 H}{\partial a^2} < 0, \frac{\partial^2 H}{\partial p^2} < 0$). These results indicate that the Hamiltonian function is concave with respect to $p(t)$ and $a(t)$. Therefore, the first-order condition is not only a necessary condition but also a sufficient condition for optimality. In other words, the calculated critical points are not only local maxima but indeed global optima over the considered decision horizon. This feature enhances the mathematical validity and stability of the model results and theoretically ensures the correctness of the obtained solution.

Accordingly, based on Eq. (10), to determine the optimal values of the control variables $p(t)$ and $a(t)$, the following conditions must be satisfied:

$$a^*(t) = \begin{cases} 0 & \lambda_2(t) < 0 \\ \frac{1}{k}\lambda_2(t) & 0 \le \lambda_2(t) < kA \\ A & \lambda_2(t) \ge kA \end{cases} \tag{12}$$

$$p^*(t) = \begin{cases} 0 & \gamma G(t) + \mu(1 - \frac{t}{T}) + \beta\lambda_1(t) < -\alpha \\ \dfrac{\alpha + \gamma G(t) + \mu\left(1 - \frac{t}{T}\right) + \beta\lambda_1(t)}{2\beta} & -\alpha \le \gamma G(t) + \mu(1 - \frac{t}{T}) + \beta\lambda_1(t) < \alpha \\ \dfrac{\alpha + \gamma G(t) + \mu\left(1 - \frac{t}{T}\right)}{\beta} & Otherwise \end{cases} \tag{13}$$

Since boundary solutions yield no profit, the analysis focuses exclusively on interior solutions. In this problem, the Hamiltonian function is given by relation (7), leading to the following system of four equations with four unknowns $\lambda_1(t)$, $\lambda_2(t)$, $G(t)$ and $I(t)$:

$$\dot{\lambda}_1(t) = h \qquad \lambda_1(0) = \lambda_{10} \tag{14}$$

$$\dot{\lambda}_2(t) = -\gamma\left(\frac{\alpha + \gamma G(t) + \mu\left(1 - \frac{t}{T}\right) + \beta\lambda_1(t)}{2\beta}\right) + \gamma\lambda_1(t) + \rho\lambda_2(t) \qquad \lambda_2(T) = 0 \tag{15}$$

$$\dot{G}(t) = a(t) - \rho G(t) \qquad 0 \le a(t) \le A, \qquad G(0) = G_0 \tag{16}$$

$$\dot{I}(t) = -\alpha + \beta p(t) - \gamma G(t) - \mu\left(1 - \frac{t}{T}\right) \qquad I(0) = I_0 \tag{17}$$

By solving the above equations, $\lambda_1(t)$, $\lambda_2(t)$, $G(t)$ and $I(t)$ are obtained, and then, by applying the boundary condition $I(T) = 0$, the constant λ_{10} is determined. Due to the complexity and length of these equations, their explicit form is omitted from the text. To clarify the analysis process and examine the system's behavior under different conditions, the results obtained from these equations are analyzed through numerical examples. The details of solving the equations numerically are presented, along with graphs and tables, in the chapter dedicated to numerical analysis and results.

3.3 Numerical Example for the Second Problem

In this section, a numerical example is presented where parameter values are inspired by previous studies and adjusted to reflect real market conditions. While some coefficients are taken directly from the literature, others are chosen hypothetically in the absence of precise data, yet consistent with economic logic and consumer behavior. To reflect practical constraints such as fixed printed prices on packaged goods and feasible advertising budgets, the parameters are set so that the optimal trajectories of price and advertising naturally remain within realistic ranges without the need for imposing explicit bounds.

Example: The parameters used in this case are presented as follows:

$\alpha = 25, \beta = 3, \gamma = 2, k = 1, h = 1, I_0 = 20, T = 3, E = 3, G_0 = 15, \mu = 1.5, \rho = 0.2,$

$A = 15, p_{max} = 20$

By solving Model (1), the total profit over the horizon is obtained as $J = 307.1851$, and the control variables $p(t)$ and $a(t)$ are derived in the forms of relations (18) and (19), respectively. Their corresponding graphs are presented in Fig. 1 and 2.

$$\begin{gathered} p^*(t) = 2t - 5.1\cos(0.43t + 1.3) - 2.4\sin(0.43t + 1.3) + 7.5\cos(0.43t) \\ +5.1\cos(0.43t - 1.3) - 6.7\sin(0.43t) - 2.4\sin(0.43t - 1.3) + 9.7 \end{gathered} \tag{18}$$

$$\begin{gathered} a^*(t) = 9.8t - 6.1\cos(0.43t + 1.3) + 5.1\sin(0.43t + 1.3) - 4.1\cos(0.43t) - 14\sin(0.43t) \\ -8\sin(0.43t - 1.3) + 3.4 \end{gathered} \tag{19}$$

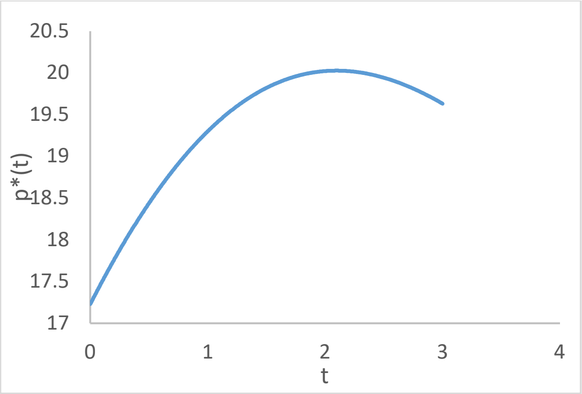

Fig. 1. Optimal product price over the replenishment cycle

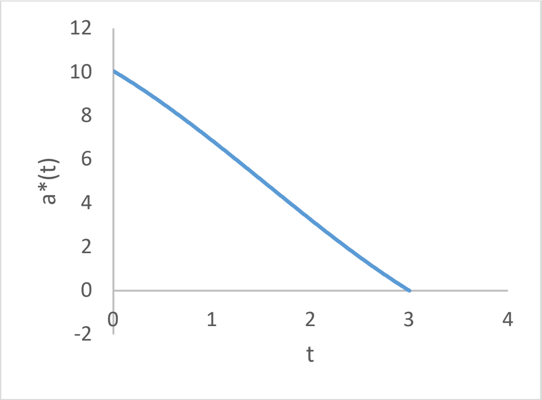

Fig. 2. Optimal advertising over the replenishment cycle

Fig. 1 illustrates the trajectory of the optimal price throughout the selling horizon. As shown, the price initially exhibits an upward trend, reflecting the seller's attempt to maximize profit while sufficient time remains before expiration and the perceived value of the product is still high. Over time, as the end of the horizon approaches, the optimal price decreases. This decline results from the urgency to sell products before expiration and the gradual reduction in consumer willingness to pay. This behavior is consistent with the model assumption that demand decreases with shorter time to expiration.

Fig. 2 depicts the optimal advertising effort over time. Advertising is initially set at its highest level and then gradually declines during the horizon. This pattern aligns with the model's goodwill dynamics, where accumulated advertising gradually builds brand awareness, reducing the marginal effectiveness of new advertising. Consequently, intensive advertising is optimal at the beginning of the sales horizon to stimulate demand and establish product positioning, while in later stages, as goodwill accumulates, less advertising is required.

Additional results regarding demand $D(t)$, inventory level $I(t)$, brand goodwill $G(t)$, and cumulative profit $J_c(t)$ are presented in Figs. 3-6.

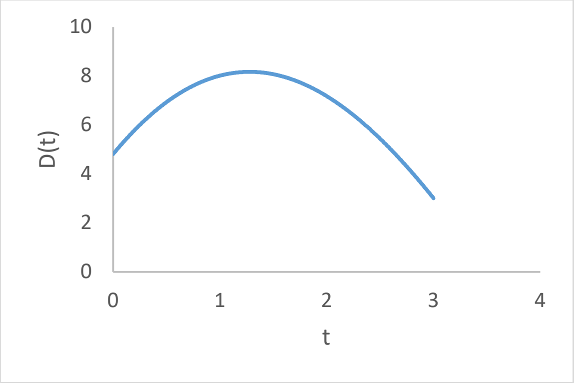

Fig. 3. Product demand over the replenishment cycle

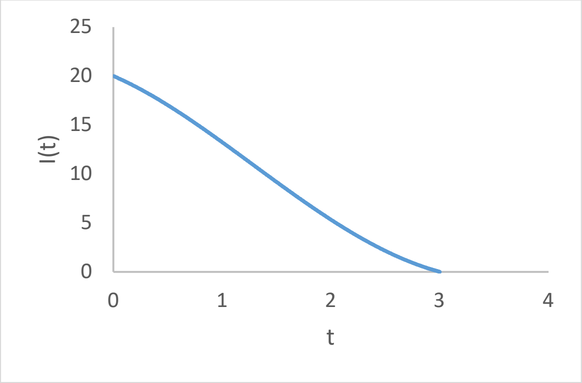

Fig. 4. Inventory level over the replenishment cycle

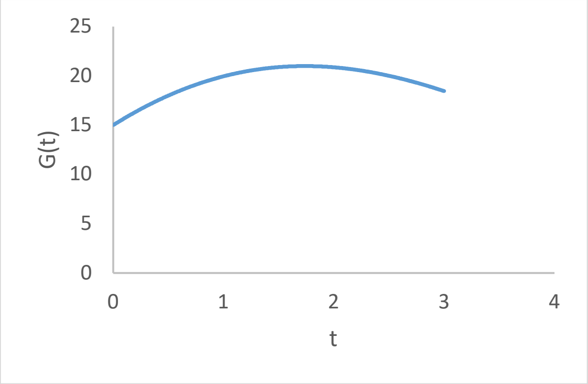

Fig. 5. Brand goodwill over the replenishment cycle

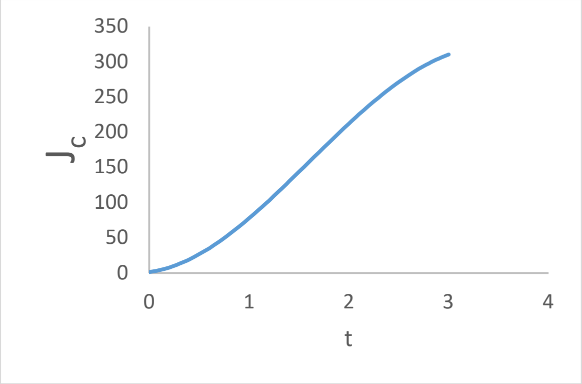

Fig. 6. Cumulative profit over the replenishment cycle

Fig. 3 shows that demand initially increases, reaches a peak, and then declines. This pattern reflects the combined effects of price, advertising, and time-to-expiration. In the early phase, with strong advertising and sufficient remaining shelf life, customers exhibit higher willingness to purchase. Over time, as advertising intensity decreases and the product approaches expiration, perceived value declines and demand falls. This trend is typical of perishable products such as chips and snacks.

Fig. 4 illustrates inventory depletion across the sales horizon. Inventory decreases steadily and approaches zero at the end of the period, indicating well-coordinated supply and demand over time. The complete depletion of inventory by the horizon's end reflects an effective sales strategy for perishable goods, minimizing waste and maximizing total profit.

Fig. 5 presents the trajectory of brand goodwill. Initially, goodwill rises due to intensive advertising, reflecting the cumulative impact of marketing efforts on consumer perception and brand awareness. As advertising diminishes in the latter half of the horizon, the modeled exponential decay of goodwill becomes dominant, leading to a gradual decline. This behavior mirrors real-world markets, where sustained advertising is often necessary to maintain brand recognition.

Finally, Fig. 6 shows cumulative profit over the sales horizon. Profit grows consistently, demonstrating continuous profitability throughout the selling period. The slope of the curve is steeper in the early phase, reflecting higher profitability due to strong advertising and higher prices. As expiration nears, the slope flattens, reflecting declining prices and lower demand. This dynamic is consistent with the model's structure, where optimal pricing and advertising strategies yield positive but diminishing incremental profits as the horizon progresses.

3.4 Sensitivity Analysis

In the sensitivity analysis of this study, the main model parameters, including price elasticity β, advertising effect γ, and the rate of demand decline due to the remaining time to expiration μ were systematically varied to examine their influence on the behavior of control variables (price, $p(t)$ and advertising, $a(t)$), and on the total profit. Because the model is solved numerically using Pontryagin's Maximum Principle, closed-form solutions for the optimal trajectories are not available. Therefore, the sensitivity analysis must be performed numerically by re-solving the system for different parameter values. This numerical approach is standard in optimal control studies of dynamic pricing for perishable goods. For each parameter, three different values were considered, and in each case, the time paths of the optimal price and advertising level were derived. These Figures allow for comparing trends and identifying both the direct and indirect effects of each parameter on optimal decisions.

In the base model, the price ceiling was set to the maximum optimal value obtained in the main scenario (20 units), as exceeding this level is practically infeasible given market constraints and prevailing pricing strategies. By contrast, the advertising ceiling was set slightly above its optimal level in the base case (15 units), since the advertising budget is fully under the firm's control and can be adjusted upward if necessary. This design ensures that, during the sensitivity analysis, advertising levels remain unconstrained and do not require clipping even when parameters vary.

In some sensitivity analysis scenarios, the optimal price trajectory exceeded the admissible ceiling. Instead of enforcing the ceiling point by point, which would complicate the solution procedure, an alternative scenario was introduced in which the price was fixed at the ceiling level throughout the planning horizon. In such cases, the optimal advertising trajectory and the corresponding total profit under the fixed ceiling price were also computed for comparison purposes.

The subsequent subsections analyze the three key parameters of the model: price elasticity of demand (β), the influence of brand goodwill on demand(γ), and the effect of remaining shelf life on demand decay (μ).

3.4.1 Sensitivity Analysis of Price Elasticity of Demand (β)

To investigate the effect of price elasticity on system behavior, the parameter β was tested at three different levels. First, changes in total profit were analyzed for each value of β. Then, to better understand system behavior, the optimal time paths of price and advertising were presented graphically.

Table 1

J and $J_{\bar{p}}$ under different values of β

β	2.8	3	3.2
J	336.0518	307.1851	281.9541
$J_{\bar{p}}$	329.2485	-	-

Table 1 shows that as the price elasticity of demand β, increases, the overall profit J, decreases. This trend indicates that in markets where customers are more sensitive to price changes, price increase policies lead to a sharper decline in demand, resulting in lower final profits. In the case of relatively lower elasticity $(\beta = 2.8)$, the optimal price curve exceeds the acceptable price ceiling, as shown in Fig. 7; therefore, to avoid computational complexity, the price is set at the ceiling level throughout the entire time horizon, and the total profit is calculated accordingly, as indicated by $J_{\bar{p}}$ in the table.

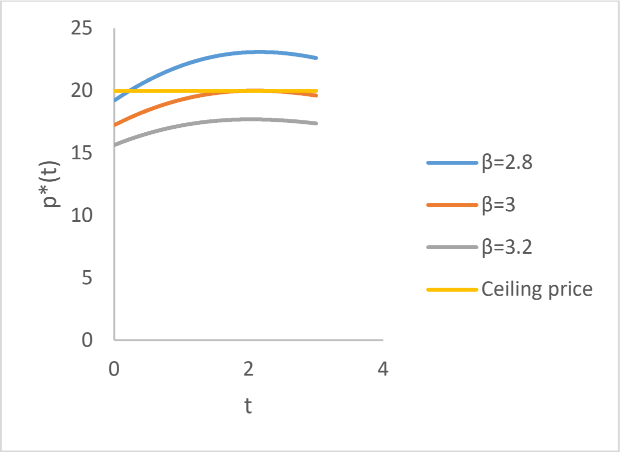

Fig. 7. Price paths for different values of β

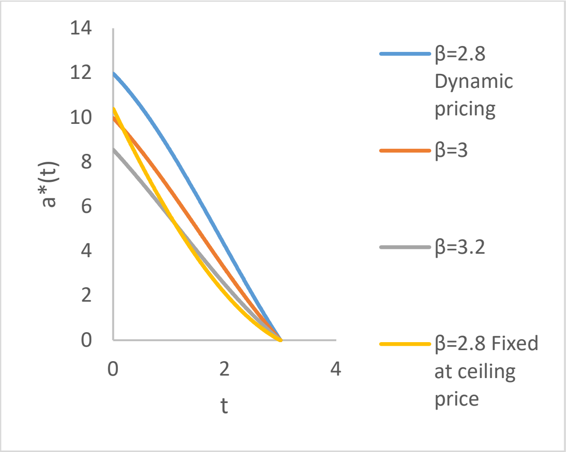

Fig. 8. Advertising levels for different values of β

Fig. 7 illustrates the optimal price trajectories for three different elasticity values. As observed, higher elasticity increases the sensitivity of demand to price, leading the optimal pricing strategy to gradually reduce prices over time in order to avoid sharp demand losses. In all cases, the price path initially rises and then slightly declines, reflecting a strategy of exploiting early demand at higher prices before applying discounts later to stimulate additional sales. In the lower elasticity scenario($\beta = 2.8$), the unconstrained optimal price exceeded the allowable ceiling (e.g., the printed package price) during certain intervals. To ensure realism, a horizontal ceiling line was added, and profits were recalculated under this price-cap constraint, with the price fixed at the ceiling throughout the horizon. As expected, this adjustment reduced the firm's total profit, since it was unable to fully capitalize on the market's initial willingness to pay higher prices.

Fig. 8 illustrates the optimal advertising trajectories under different values of price elasticity (β). As elasticity increases (i.e., when consumers become more sensitive to price changes), the optimal level of advertising decreases. This is because when customers react strongly to price changes, advertising has less effect in offsetting the negative impact of higher prices. In the lower β scenario($\beta = 2.8$), the optimal price exceeds the ceiling. To represent this situation, the advertising trajectory is plotted under two scenarios: one with dynamic pricing below the ceiling and the other with fixed pricing at the ceiling. These constraints not only reduce profit compared to the unconstrained case but also alter advertising behavior. Under fixed pricing at the ceiling, advertising is less flexible and declines more rapidly, whereas under dynamic pricing, advertising remains more responsive over time and generates relatively higher profit.

In all cases, advertising expenditure decreases over time. This reflects a front-loading strategy: firms invest more at the beginning of the product life cycle to build awareness and stimulate initial demand, then gradually reduce spending as the product becomes established and the marginal return on advertising diminishes.

3.4.2 *Sensitivity Analysis of Brand Goodwill Effect* (γ)

To study the effect of brand goodwill on demand, parameter γ was tested at three levels. First, changes in total profit were compared across the scenarios. Then, optimal trajectories of price and advertising were examined to understand how increased responsiveness of demand to goodwill influences managerial decisions.

Table 2
Total profit under different values of γ

γ	1.8	2	2.2
J	280.5120	307.1851	335.4939
$J_{\bar{p}}$	-	-	327.3043

Table 2 shows that as the parameter γ increases, total profit J exhibits a rising trend. This indicates that in markets with higher γ, optimal pricing policies have a stronger effect, leading to higher total profit. In the case of high γ ($\gamma = 2.2$), the optimal price path exceeds the allowable price ceiling, as shown in Fig. 9. To avoid computational complexity, the price is set at the ceiling level throughout the entire time horizon, and the corresponding total profit is calculated accordingly. As observed, the price ceiling reduces total profit over the planning horizon.

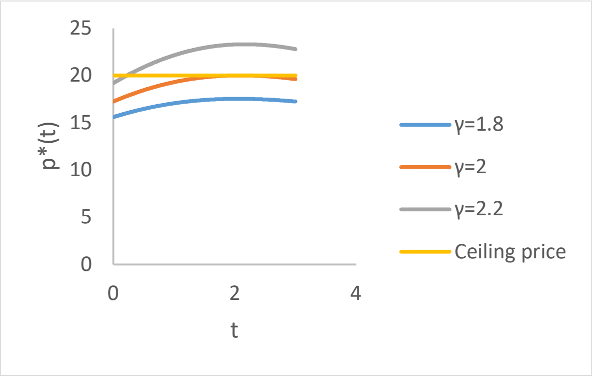

Fig. 9. Price paths for different values of γ

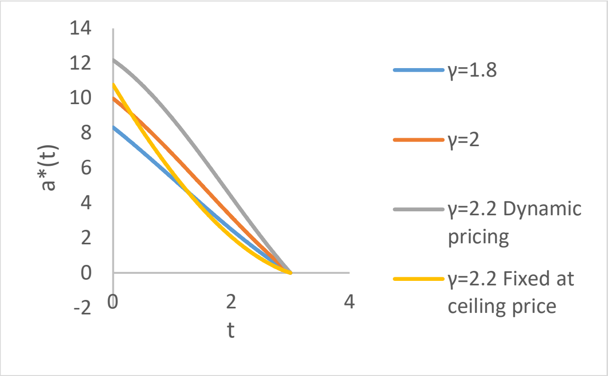

Fig. 10. Advertising levels for different values of γ

Fig. 9 shows that as γ increases, the overall optimal price path also rises. This occurs because more effective advertising generates stronger demand, allowing producers to charge higher prices. However, in the high scenario $(\gamma = 2.2)$, the unconstrained optimal price exceeded the market ceiling. To account for this, a horizontal ceiling line was added, and the price was capped at this level throughout the relevant period. As a result, total profit decreased, since the model could not fully exploit the higher willingness of the market to pay. Fig. 10 shows that as γ increases, the optimal advertising trajectory also rises. Since each unit of advertising becomes more effective in sustaining demand, the model allocates more resources to advertising, supporting both stronger demand and higher prices. In the high γ case $(\gamma = 2.2)$, the optimal price exceeded the ceiling. To capture this, the advertising trajectory was illustrated under two scenarios: one with dynamic pricing under the ceiling and another with fixed pricing at the ceiling. While both caps reduce profit compared to the unconstrained case, they also alter advertising behavior. Under fixed pricing at the ceiling, advertising is less adaptive and flattens earlier, whereas under dynamic pricing with the ceiling constraint, advertising remains more responsive over time, leading to relatively higher profit.

3.4.3 Sensitivity Analysis of the Remaining Shelf Life Effect on Demand (μ)

To study the effect of the remaining shelf life on demand, parameter μ was varied at three levels. In the first step, total profits across the scenarios were calculated and compared. Subsequently, the trajectories of optimal price and advertising were analyzed to provide further insight into how changes in shelf life influence managerial decisions.

Table 3
Total profit under different values of μ

μ	1	1.5	2
J	305.3972	307.1851	308.9782
$J_{\bar{p}}$	–	––	302.6925

Table 3 shows that as the parameter μ, representing the rate of demand decline near the expiration date, increases, the system's total profit rises.

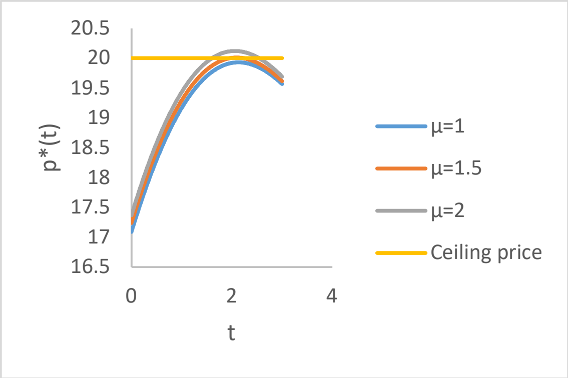

Fig. 11. Price paths for different values of μ

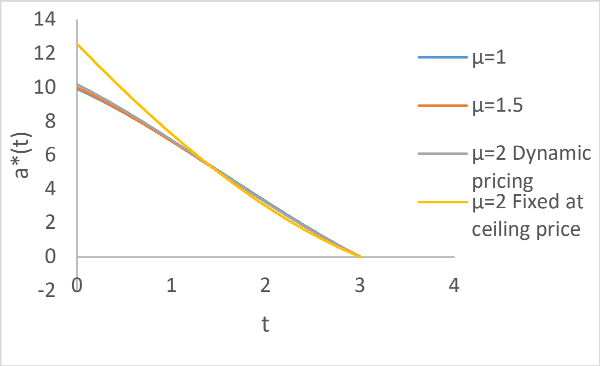

Fig. 12. Advertising levels for different values of μ

In the case of high $\mu (\mu = 2)$, the optimal price exceeded the allowable price ceiling, as shown in Fig. 11; therefore, to avoid computational complexity, the price was set at the ceiling level throughout the entire time horizon, and total profit $J_{\bar{p}}$ was calculated accordingly. As observed, the price ceiling reduces total profit over the time horizon. Fig. 11 shows that higher values of μ lead to higher optimal prices over the planning horizon, as the system aims to extract greater value before demand declines. In the high scenario $(\mu = 2)$, the unconstrained optimal price surpassed the ceiling. To reflect market constraints, a horizontal ceiling line was added, and the price was capped at this level during the corresponding interval. This adjustment lowered total profit, as the firm could not fully capitalize on the elevated willingness to pay. Fig. 12 shows that higher μ leads to greater advertising effort in the early periods. This reflects a strategy of boosting awareness and stimulating demand before expiration reduces sales potential. In the high μ case $(\mu = 2)$, the optimal price again exceeded the ceiling, so the advertising trajectory was drawn under two scenarios: one with dynamic pricing under the ceiling and another with fixed pricing at the ceiling. Beyond profit differences, advertising behavior also diverges: with fixed pricing, advertising intensity declines more sharply after the initial peak, while dynamic pricing allows a smoother and more gradual adjustment, maintaining demand more effectively.

3.4.4 Managerial Insights from Sensitivity Analysis

The sensitivity analysis shows that both brand goodwill (γ) and remaining shelf life effect (μ) significantly influence optimal pricing and advertising strategies. Higher γ values increase the effectiveness of advertising, leading to higher prices and profits; however, when the unconstrained optimal price exceeds the ceiling, total profit is limited, and advertising strategies must adjust accordingly. Similarly, higher μ values prompt firms to set higher prices and allocate more advertising effort early in the sales horizon to capture demand before expiration reduces sales potential. These results suggest that managers should actively monitor brand strength and product perishability: in markets with strict price ceilings, advertising becomes a crucial lever to sustain demand and maximize revenue, whereas in more flexible pricing environments, dynamic pricing can be more effectively exploited. By aligning pricing and promotional efforts with these key parameters, firms can enhance profitability while minimizing waste.

4. Conclusion

This study developed a dynamic pricing and advertising model for perishable products under a binding price ceiling. The model explicitly incorporates the joint effects of a hard expiration date, goodwill-based demand, and a price ceiling that restricts upward adjustments beyond an admissible level. By applying Pontryagin's Maximum Principle, we derived the time-dependent optimal trajectories for both pricing and advertising. Numerical experiments and sensitivity analyses further revealed how price sensitivity, advertising effectiveness, and remaining shelf life shape optimal decisions.

The results show that firms benefit from setting relatively higher prices and allocating greater advertising efforts in the early stages of the selling horizon, when expiration effects are less pronounced. As the expiration date approaches, both the perceived value of the product and the marginal effectiveness of advertising decline, leading to lower optimal prices and reduced advertising intensity. Importantly, when the optimal unconstrained price exceeds the admissible ceiling, total profit decreases, and advertising becomes a more critical tool for sustaining demand.

From a managerial perspective, these findings highlight the importance of front-loading advertising investments and carefully coordinating discounting policies as expiration approaches. In markets with strict price ceilings (e.g., printed price tags in emerging economies), advertising intensity plays a compensatory role by offsetting the loss of pricing flexibility. In more flexible environments, dynamic pricing dominates, but perishability still necessitates early promotional efforts. Overall, the study provides actionable guidance for managers in food, pharmaceutical, and FMCG sectors on how to align pricing and advertising strategies with perishability and regulatory constraints to enhance profitability and reduce waste.

While the proposed model offers a comprehensive framework for joint pricing and advertising decisions under perishability and price ceilings, several extensions could enrich future research. First, incorporating competition among multiple firms would provide deeper insights into strategic interactions in markets with strict regulatory constraints. Second, relaxing the assumption of deterministic demand and considering stochastic variations could make the model more robust to real-world uncertainty. Finally, integrating inventory and supply chain coordination aspects with dynamic pricing and advertising decisions may help managers optimize across multiple operational levers simultaneously.

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