

An empirical application of Markowitz mean-variance theory in evaluating portfolio performance: Evidence from Vietnam's VN30 equity constituents

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CHRONICLE

ABSTRACT

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This study explores the application of Markowitz's Modern Portfolio Theory (Mean-Variance Theory) under a no short-selling constraint in Vietnam's stock market, focusing on the VN30 index from 2012 to 2024. It compares three investment strategies: (A) naive 1/N allocation, (B) Markowitz optimization targeting the same standard deviation as strategy A, and (C) Markowitz optimization maximizing the Sharpe ratio. The findings reveal that strategy C significantly outperforms cumulative returns and risk-adjusted performance, demonstrating that when properly adjusted, the Markowitz model can still generate alpha in emerging markets like Vietnam. However, the study emphasizes prudent risk management and warns against overreliance on theoretical Optimization without considering market realities.

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1. Introduction

The Modern Portfolio Theory (MPT), pioneered by Harry Markowitz (Markowitz, 1952), laid the foundational framework for optimizing a securities portfolio by balancing expected return against risk - typically measured by variance or standard deviation (And et al., 2006). Under this framework, investors can construct an efficient frontier of optimal portfolios that maximize expected return for a given acceptable risk level. Over the decades, Markowitz's theory has become a cornerstone in asset pricing and portfolio management. However, the practical implementation of the mean-variance model has often faced empirical challenges (Avramov & Wermers, 2006). Numerous studies in developed markets have shown that Markowitz-optimized portfolios may underperform when deployed in real-world settings and, in some cases, yield inferior results compared to naive allocation strategies such as the equally weighted 1/N rule. A primary explanation lies in estimation risk - errors in estimating expected returns and the covariance matrix can lead to unstable and unreliable optimization outcomes (Michaud & Ma, 2001; DeMiguel, Garlappi, & Uppal, 2009). Nevertheless, Markowitz's theoretical contributions remain influential and have been extended and refined by subsequent research. Enhanced models, such as the Black-Litterman framework (Black & Litterman, 1992), integrate investors' subjective views into the optimization process, resulting in more coherent asset allocations. Likewise, improved estimation techniques, including covariance shrinkage methods by Ledoit and Wolf (2003), have helped mitigate the noise inherent in historical data. Furthermore, Michaud and Ma (2001) introduced the resampled efficient frontier, which smooths the traditional frontier by incorporating estimation uncertainty. These advancements collectively aim to preserve the core insights of the Markowitz framework while enhancing its robustness and practical relevance. In the Vietnamese context - particularly within a transitional market evolving from frontier to emerging status - the applicability and efficacy of the Markowitz model remain underexplored. Vietnam's stock market over the past decade (2012-2024) has experienced periods of rapid growth and sharp corrections, creating a compelling setting for testing

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the performance of various investment strategies. Whether a Markowitz-based optimization strategy can outperform a naive allocation strategy within the VN30 universe (comprising the 30 largest-cap stocks) is a critical and timely question for domestic asset managers and investment funds. Addressing this question contributes to local portfolio practice and the broader literature on market efficiency in emerging economies (Carhart, 1997; Fama & French, 1993, 2015).

This study applies the Markowitz model to optimize a VN30 stock portfolio over the 2012-2024 period (from its initial publication to the present) and compares its performance with an equally weighted (1/N) strategy. We construct and backtest three portfolio strategies: (A) the naive 1/N portfolio, where each stock receives equal weight; (B) a Markowitz portfolio constrained to match the standard deviation of portfolio A in each month ($\sigma_{\text{target}} = \sigma_A$); and (C) a Markowitz portfolio that maximizes the Sharpe ratio (i.e., the most efficient portfolio), subject to a no short-selling constraint. All portfolios are rebalanced monthly, using a rolling window of the most recent six months of historical data for parameter estimation.

2. Theoretical Framework and Literature Review

2.1 Markowitz's Portfolio Theory and Its Extensions

Harry Markowitz (1952) was the first to introduce a portfolio selection model based on two parameters: expected return (mean) and risk (variance). According to his framework, a portfolio is considered efficient if no other portfolio exists with the same level of risk but a higher expected return, or vice versa, the same return with lower risk. The set of all such efficient portfolios forms the efficient frontier in the return-risk space. Any portfolio below this frontier is deemed inefficient, as it could be improved without increasing risk. Markowitz's solution recommends allocating assets to optimize an objective function that reflects the investor's risk-return preferences - equivalent to maximizing expected return under a risk constraint or minimizing risk subject to a target return.

The mean-variance model assumes that investors only care about asset return distributions through the first two moments: mean and variance. This implies that returns follow a normal distribution or that the investor's utility function is quadratic. Although these assumptions simplify reality, they make Optimization tractable and have laid the groundwork for subsequent theories, such as the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory.

Despite its strengths, the Markowitz model also has notable limitations in practical implementation. Its main challenge is accurately estimating key inputs - namely, expected returns and the covariance matrix of asset returns. These estimations are often error-prone, particularly in volatile markets or with limited historical data. Minor estimation errors can lead to significant shifts in the optimal portfolio structure (Michaud & Ma, 2001). Michaud and Ma (2001) famously dubbed this the "Markowitz optimization enigma," noting that theoretically optimal portfolios are often suboptimal. The resulting weights can be erratic, excessively concentrated in a few assets (sometimes those with spuriously high returns), and may involve significant short positions. Such extreme solutions are typically not robust and may be unsuitable for most investors (Bertsimas et al., 2011).

In response, researchers have proposed a variety of extensions to address these issues. One significant advancement is the Black-Litterman model (Black & Litterman, 1992), which combines the mean-variance framework with market equilibrium (CAPM) and incorporates investor views. Rather than relying solely on historical data, the model includes subjective expectations about relative asset returns, which are then blended with market information to form more stable expected returns. The resulting portfolios are less extreme and more aligned with investor beliefs, improving practical applicability.

Another line of research has focused on improving parameter estimation. Ledoit and Wolf (2004) proposed a shrinkage estimator for the covariance matrix, which blends the sample covariance matrix with a structured target (e.g., identity matrix). This technique acknowledges that the actual covariance matrix often resembles a more straightforward structure more closely than noisy empirical estimates - especially when the number of observations is limited. Shrinking the sample matrix toward a structured form helps reduce estimation noise. Empirical results show that shrinkage-based Optimization outperforms standard mean-variance optimization in out-of-sample tests.

Further improvements involve imposing additional constraints on the optimization problem. Jagannathan and Ma (2003) demonstrated that constraints such as long-only (no short-selling) and position limits can serve as implicit parameter regularization. In some cases, they even cause the naive 1/N portfolio to emerge as the optimal solution. These constraints help prevent extreme allocations and reduce the impact of estimation errors. Fund managers commonly use them to ensure a minimum level of diversification.

Beyond refinements to the Markowitz model, a new class of strategies - known as risk-based portfolios - has emerged. These approaches, including risk parity and minimum-variance portfolios, aim to avoid estimating expected returns, which is the most error-prone component. Instead, they construct portfolios based solely on the risk structure (De Carvalho, Lu, & Moulin, 2012). By removing the need to forecast expected returns, these strategies can offer more stable and robust performance. DeMiguel, Garlappi, and Uppal (2009) conducted a comprehensive study comparing over a dozen portfolio optimization methods and found that none consistently outperformed the naive 1/N strategy. This striking conclusion - "simple yet effective" - posed a significant challenge to the optimization literature, underscoring the need for models that are not only theoretically sound but also practically resilient (Kan & Zhou, 2007; Brodie et al., 2009; Hou et al., 2021).

In this study, we adopt a traditional Markowitz mean-variance approach with an added no-short-selling constraint and compare its effectiveness against the naive 1/N strategy using data from the Vietnamese stock market.

2.2 Empirical Literature Review

Numerous empirical studies have assessed the performance of Markowitz's portfolio theory and optimization-based investment strategies under various market conditions. In developed markets such as the U.S. and Europe, evidence often points to the difficulty of outperforming simple allocation strategies unless improved estimation techniques are applied. For example, Chopra and Ziemba (1993) found that minor errors in return estimates can significantly deteriorate portfolio performance, while Frost and Savarino (1988) showed that imposing constraints on portfolio weights can enhance out-of-sample results. Using actual market data, DeMiguel et al. (2009) demonstrated that the naive 1/N portfolio often outperforms standard Markowitz portfolios, highlighting the crucial role of mitigating estimation risk. Optimization-based strategies may have more room to generate alpha in emerging markets, where market efficiency may be lower. For instance, Mishra, Pisipati, and Vyas (2011) compared the Black-Litterman and traditional Markowitz models in the Indian equity market. Their results indicated that the Black-Litterman approach, incorporating expert views, generated superior risk-adjusted returns. However, even the conventional Markowitz model somewhat outperformed the market-weighted benchmark. These findings suggest that subjective information can enhance optimization effectiveness in emerging markets and that even standard Markowitz optimization may yield added value over passive strategies. In China, Xie (2021) conducted an empirical study applying Markowitz's framework to the Shanghai Stock Exchange. Using weekly data on A-share stocks and a two-tier filtering process to select five stocks, the author then applied a constrained mean-variance optimization (no short-selling). The resulting portfolio outperformed the Shanghai Composite Index in most periods, indicating the practical relevance of the Markowitz model in China. The study also emphasized the importance of effective stock selection in indirectly reducing noise in input estimation.

Several empirical studies have investigated portfolio optimization techniques in Brazil - the largest stock market in Latin America. Santos and Tessari (2012) tested various quantitative optimization methods, including Markowitz-style models, on Brazilian market data and found that some methods, especially those with estimation adjustments, improved portfolio returns relative to the market index. Similarly, Zanini and Figueiredo (2021) studied the application of Markowitz and Sharpe ratio optimization on Brazilian stocks from 1995 to 2000 and found that Sharpe-maximizing portfolios outperformed the market. These results suggest that optimization models can uncover mispricing opportunities in less efficient markets like Brazil when carefully implemented (Merton, 1972).

Overall, the literature reveals a heterogeneous landscape regarding the efficacy of portfolio optimization. Traditional Markowitz strategies face limitations in highly efficient markets but have more significant potential in less efficient emerging markets. The key to success lies in improved estimation and careful portfolio design, such as imposing sensible constraints and incorporating auxiliary information.

3. Data and Methodology

3.1 Research Data

This study utilizes data from the constituent stocks of the VN30 Index, published by the Ho Chi Minh Stock Exchange (HOSE). The VN30 comprises the 30 most extensive market capitalization stocks in Vietnam, serving as a representative proxy for the broader equity market. The analysis period spans from February 2012 to December 2024. The choice of VN30 is motivated by its high liquidity and capitalization, which reflect a substantial share of total market value, as well as the availability and reliability of historical price data relative to smaller-cap stocks.

Two datasets are employed in this study:

- (i) Monthly return data for all VN30 stocks, and
- (ii) Monthly composition data of the VN30 Index.

Table 1

Monthly Composition of the VN30 Index

Time	Stock_1	Stock_2	Stock_3	Stock_4	...	Stock_28	Stock_29	Stock_30
2012-02	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-03	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-04	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-05	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-06	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-07	STB	VIC	SSI	MSN	...	DIG	HVG	GMD
2012-08	STB	VIC	SSI	MSN	...	DIG	MBB	GMD
...
2024-07	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2024-08	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2024-09	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2024-10	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2024-11	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2024-12	STB	VIC	SSI	MSN	...	HDB	MBB	POW
2025-01	STB	VIC	SSI	MSN	...	HDB	MBB	POW

Source: Compiled by the authors

Dataset (i) is structured in tabular form, as shown in Table 1, with each stock represented by a time series of simple monthly returns. These returns are computed from adjusted prices - prices adjusted for dividends, stock splits, and other corporate actions where applicable.

Dataset (ii) provides the list of stock tickers included in the VN30 for each month (Table 2). Since the VN30 composition is reviewed and potentially adjusted semiannually, it is necessary to merge both datasets to determine which stocks were included in the VN30 in any given month.

All data were compiled by the research team from publicly available sources, primarily the official website of the Ho Chi Minh Stock Exchange (HOSE).

Table 2
Monthly Returns of VN30 Stocks (Unit: %)

Time	ACB	BMP	CII	DHG	DRC	FPT	GMD	
Feb-07	25.62	-4.39	13.36	...	2.88	62.83	-2.01	4.41
Mar-07	3.66	-7.53	-12.26	...	-6.05	-9.92	-14.33	-6.88
Apr-07	-7.87	-24.10	-16.02	...	-10.67	-9.65	-21.45	-13.63
May-07	-11.50	26.29	17.45	...	32.19	0.50	23.19	7.23
Jun-07	-21.66	-7.56	-9.47	...	50.33	-2.99	-12.02	-9.82
...
Jul-24	3.15	-3.20	-1.59	...	0.44	2.51	-1.46	-1.02
Aug-24	1.02	7.44	0.00	...	-2.19	-2.56	4.82	0.62
Sep-24	3.83	14.03	0.60	...	-0.65	-7.06	-0.22	-3.11
Oct-24	-1.36	13.41	-1.60	...	-2.33	-10.46	1.04	-1.70
Nov-24	-0.79	-1.69	-7.54	...	-3.44	0.90	6.18	-0.92
Dec-24	2.38	3.64	2.42	...	6.72	1.53	6.42	0.77

Source: Calculated by the authors¹

We process the dataset and align the time series as follows: First, we compute monthly returns for all stocks that have ever been included in the VN30 Index during the 2012-2024 period. Then, for each month in the sample, we identify the set of stocks that were constituents of the VN30 at that specific time based on the monthly index composition data. Stocks that newly enter the VN30 begin to be included in the portfolio from the month of entry onward; conversely, stocks removed from the VN30 are excluded from the portfolio starting the month of exclusion. This approach replicates the behaviour of an investment fund that strictly tracks the current VN30 constituents without holding stocks that have been removed from the index. Most stocks entering the VN30 had already been listed for a sufficient period beforehand, so data availability is not a concern. The monthly return of the portfolio is computed as a simple weighted return: assuming the portfolio holds weights w_i for each stock i at the beginning of month t , and that each stock earns a return r_i during month t , the portfolio return in month t is given by:

$$R_t = \sum_{i=1}^N w_i r_i$$

The cumulative return of the portfolio over the entire period is calculated as the compound product of one plus the monthly returns:

$$\prod_{t=Feb\ 2012}^{Dec\ 2024} (1 + R_t) - 1$$

3.2 Research Methodology

Portfolio Strategies and Backtesting Framework

We design three portfolio strategies and conduct backtesting - i.e., simulated historical performance testing - over the period from 2012 to 2024:

i) Strategy A - Naive 1/N Allocation

This strategy equally allocates capital across all stocks in the VN30 Index. Specifically, at each rebalancing point, if there are NN stocks in the current VN30 portfolio, each stock is assigned a weight of $w_i = 1/N$. This simple "naive diversification" strategy does not require any parameter estimation and serves as the baseline for comparison. The portfolio is rebalanced monthly, meaning that at the beginning of each month, all weights are reset to equal allocation to prevent distortion due to price fluctuations over time.

ii) Strategy B - Markowitz with Target Volatility (σ_{target})

This strategy employs Markowitz mean-variance optimization, targeting a portfolio standard deviation equal to Strategy A each month. The implementation is as follows: At the beginning of each month, we use a six-month rolling window of historical data to estimate the expected returns and the covariance matrix:

¹ The monthly return at time t is calculated as the adjusted price in month t divided by the adjusted price in month $t-1$ minus one.

This portfolio applies Markowitz optimization to match the portfolio's monthly standard deviation to that of Strategy A. Implementation proceeds as follows: At the beginning of each month, we use a six-month rolling window of historical data to estimate the expected returns and the covariance matrix for the current set of VN30 stocks.

$$\bar{R} = \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \dots \\ \bar{r}_N \end{bmatrix}; \quad \Delta = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \dots & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_{2,2} & \dots & \sigma_{2,N} \\ \dots & \dots & \dots & \dots \\ \sigma_{1,N} & \sigma_{2,N} & \dots & \sigma_{N,N} \end{bmatrix}$$

We then solve a constrained optimization problem: maximize expected return subject to a fixed portfolio standard deviation (equal to that of Strategy A in the same month), with the additional constraints that portfolio weights must sum to one and be non-negative (i.e., no short-selling). Specifically:

Constraint 1: The portfolio weights must sum to one and be non-negative

$$\sum_{i=1}^N w_i = [1 \ 1 \ \dots \ 1] \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix} = I^T W = 1; \text{ with } I = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}; w_i \geq 0 \ \forall i$$

Constraint 2: The portfolio's standard deviation must equal the target volatility, denoted as σ_{target}

$$\sigma_P^2 = \sum_{i=1}^N \sum_{j=1}^N \sigma_{i,j} w_i w_j = [w_1 \ w_2 \ \dots \ w_N] \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \dots & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_{2,2} & \dots & \sigma_{2,N} \\ \dots & \dots & \dots & \dots \\ \sigma_{1,N} & \sigma_{2,N} & \dots & \sigma_{N,N} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix} = W^T \Delta W$$

Constraint 3: The expected return of the portfolio must be maximized

$$\bar{r}_P = w_1 \bar{r}_1 + w_2 \bar{r}_2 + \dots + w_N \bar{r}_N = \sum_{i=1}^N w_i \bar{r}_i = [\bar{r}_1 \ \bar{r}_2 \ \dots \ \bar{r}_N] \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix} = \bar{R}^T W.$$

Based on these three constraints, we apply the method of Lagrange multipliers (Lagrange, 1797) to solve the constrained optimization problem that maximizes the expected portfolio return. The solution yields the optimal weight vector $W = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix}$,

corresponding to the asset allocation under Strategy B.

In summary, this optimization problem identifies a portfolio on the efficient frontier that exhibits the same level of risk as the naive 1/N portfolio but achieves the highest expected return. The optimal solution yields the asset weights for Portfolio B in that month. Portfolio B is rebalanced monthly using the newly computed optimal weights (Rockafellar & Uryasev, 2000).

iii) Strategy C - Markowitz Max Sharpe

This portfolio is optimized to maximize the Sharpe ratio. Similar to Strategy B, at the beginning of each month, we use a six-month rolling window of historical data ending at the end of the previous month. The optimization problem is then solved to maximize the Sharpe ratio under the standard constraints that the portfolio weights sum to one and are non-negative (i.e., no short-selling).

Constraint 1: Portfolio weights must sum to one and be non-negative

$$\sum_{i=1}^N w_i = [1 \ 1 \ \dots \ 1] \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix} = I^T W = 1; w_i \geq 0 \ \forall i$$

Constraint 2: The Sharpe ratio of the portfolio is maximized

$$S_P = \frac{\bar{r}_P - r_f}{\sigma_P}$$

where r_f is the monthly risk-free rate. This study uses the one-year government bond yield issued by the State Treasury of Vietnam, converted into an equivalent monthly risk-free rate. Given an annual risk-free rate of 2%, the corresponding monthly rate is computed using the following formula:

$$r_f = (1 + 2\%)^{1/12} - 1 = 0.17\%$$

With these two constraints, we again apply the method of Lagrange multipliers (Lagrange, 1797) to solve the Sharpe ratio maximization problem. The solution yields the weight vector: $W = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix}$, corresponding to the asset allocation under Strategy C.

This optimization problem identifies the tangency portfolio on the efficient frontier - i.e., the portfolio with the highest reward-to-risk ratio (Sharpe ratio). Based on recent six-month data, the optimal weight vector tends to allocate more capital to stocks with higher return-to-risk ratios. Given the non-negativity constraint on portfolio weights, Strategy C is also a long-only strategy. Portfolio C is rebalanced monthly using the newly optimized weights.

All three strategies are designed to satisfy long-only (no short-selling) and fully invested (sum of weights equals 100%) constraints. The key difference lies in their allocation mechanisms: Strategy A follows equal weighting, while Strategies B and C rely on Markowitz optimization with distinct objectives. The six-month rolling window reflects the assumption that more recent data better captures current market conditions while providing sufficient length for reasonably stable parameter estimation. This window length (~126 trading days if using daily data) is commonly used in studies on momentum and volatility. We also conducted preliminary tests using a 12-month window and observed similar rankings among the strategies, though with some variation in performance differentials.

2. Handling the Covariance Matrix via Pseudo-Inverse

The covariance matrix is central to Markowitz portfolio optimization, as it determines how assets interact regarding risk. However, when the length of historical data (e.g., six months) is shorter than or close to the number of assets (VN30), the sample covariance matrix tends to be ill-conditioned or nearly singular. This can result in unstable and unreliable optimization outcomes due to overfitting and estimation noise.

To address this issue, we apply the Moore-Penrose pseudo-inverse instead of the conventional matrix inverse. This technique allows the optimization problem to be solvable when the matrix is non-invertible and mitigates overfitting by smoothing out the influence of small eigenvalues—components often responsible for large distortions in optimization when data is limited.

According to Ledoit and Wolf (2003), improving covariance matrix estimation through pseudo-inverse or shrinkage techniques enhances optimized portfolios' stability and out-of-sample robustness. The pseudo-inverse retains the essential structure of the covariance matrix while filtering out noise from minor variance components, thus improving practical reliability.

As such, applying the pseudo-inverse in this study is a technical necessity for solving the Markowitz problem under limited data and a methodological enhancement that strengthens the robustness of the portfolio strategy across repeated rebalancing periods.

Backtesting Procedure

The backtest begins in February 2012 - the month when the VN30 Index was officially launched in Vietnam - and follows these steps:

i) Initialization (February 2012):

We begin by identifying the constituent stocks of the VN30 Index as of February 2012. Using historical data from August 2011 to January 2012 (a six-month estimation window), we calculate \bar{R} (the average return of each stock) and Δ (the covariance matrix of stock returns). These parameters serve as inputs for solving the optimization problems corresponding to strategies B and C, thereby determining the portfolio weights at the beginning of February 2012. For Strategy A, the weights are equally allocated at $1/N$ per stock. These weights are applied throughout February 2012, and the portfolio return for each strategy is computed at the end of the month.

ii) Monthly Updating:

We then proceed to the next month (March 2012). If there are updates to the VN30 composition - typically announced semiannually in January and July and taking effect in February and August - the stock list is revised accordingly. A rolling window of the most recent six months (e.g., from September 2011 to February 2012 for March) is used to re-estimate \bar{R} and Δ , followed by re-optimization of portfolio weights for strategies B and C. Strategy A continues to maintain equal weighting (with $N = 30$, resulting in fixed weights of $1/30$ per stock). The portfolio returns for March are then calculated based on these weights and the realized stock returns.

iii) Iteration Until End of Sample (December 2024):

This monthly process is repeated through December 2024 - the final period of the study. Each month, the portfolios are rebalanced based on the most up-to-date information.

Following this backtesting procedure, we obtain a monthly return series for the three strategies (A, B, and C) over the entire sample period from 2012 to 2024. Based on these return series, we compute key performance metrics, including cumulative

return (from the start of 2012 through December 2024) and the Sharpe ratio (the mean excess return over the risk-free rate, divided by the standard deviation of returns every month).

To assess the statistical significance of performance differentials across strategies, we conduct paired t-tests on the return differentials. The comparison between Portfolio C and Portfolio A is of particular interest - that is, the Sharpe-optimal portfolio versus the naive 1/N portfolio. The hypotheses tested are as follows:

H_0 : "The average monthly return of Portfolio C is not higher than that of Portfolio A" (i.e., $\mu_C \leq \mu_A$)

H_1 : "The average monthly return of Portfolio C is higher than that of Portfolio A" (i.e., $\mu_C > \mu_A$)

A one-sided t-test is conducted at a 10% significance level. If the p-value is below this threshold, we reject the null hypothesis and conclude that Portfolio C delivers statistically superior returns compared to Portfolio A. Similar tests can be performed between C and B or B and A if needed. However, given the consistent outperformance of C over A in our empirical results, we focus primarily on the C-A comparison to support the core argument.

The entire backtesting framework is automated and implemented in Python, ensuring reproducibility and computational accuracy. Transaction costs are assumed to be zero (i.e., an idealized frictionless market), meaning the reported performance reflects theoretical maximum efficiency. Nonetheless, the relative performance rankings remain informative and robust since all strategies are evaluated under the same assumptions.

4. Empirical Results

4.1 Cumulative Performance of the Strategies

The empirical findings reveal substantial disparities in long-term investment performance across the three portfolio strategies: A, B, and C. Fig. 1 illustrates the cumulative profit and loss (P&L) trajectories of Portfolio A (1/N), Portfolio B (Markowitz with target volatility σ_{target}), and Portfolio C (Markowitz with maximum Sharpe ratio) over the period from early 2012 to the end of 2024. Portfolio C, represented by the green line, consistently outperforms the other two portfolios - A (blue) and B (orange) - regarding cumulative returns. All portfolios are initialized with a starting cumulative return of 0% as of early February 2012. During the initial years (2012 - 2013), the performance of all three strategies remained closely aligned. In the subsequent phase (2013 - 2015), while the market experienced modest fluctuations, all portfolios demonstrated steady growth with negligible divergence.

Starting in 2016, however, Portfolio C began to outperform more distinctly, suggesting that Sharpe ratio optimization gradually conferred a structural advantage. The performance divergence became particularly pronounced during the 2020 - 2021 period amid the post-COVID-19 rally in the Vietnamese equity market. During this time, Portfolio C benefited significantly from its optimization framework, which concentrated capital into the most strongly performing stocks, thereby generating exceptional returns. At its peak in late 2021, Portfolio C achieved a cumulative return close to 1000% - representing an approximate eleven-fold increase over a 10-year horizon - while Portfolios A and B only reached cumulative returns in the 300-400% range.

Following this surge, the market underwent a substantial correction in 2022, during which Portfolio C experienced a notable drawdown (from ~1000% to approximately 400%). Nevertheless, this outcome should not be interpreted as a structural weakness of the strategy. Instead, it reflects the Sharpe-optimal portfolio's inherent capacity to dynamically adapt to market volatility and capitalize effectively on valuation trends. Despite its sensitivity to short-term reversals, the strategy's ability to capture substantial upside in bullish phases underpins its superior long-term performance. By early 2023, Portfolio C had recovered strongly and maintained a sizable lead over its counterparts. As of December 2024, Portfolio C stabilized around a cumulative return of +525%, compared to +250-350% for Portfolios A and B - a significant and persistent outperformance.

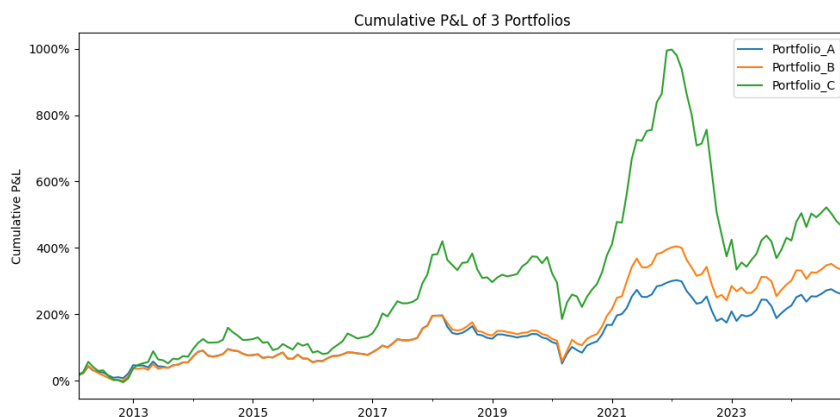


Fig. 1. Cumulative returns of Portfolios A, B, and C over the period 02/2012-12/2024

Source: Author's visualization using Python programming language on Visual Studio Code

Strategy C (Markowitz with maximum Sharpe ratio) delivered the highest cumulative return over the study period. Strategy B (Markowitz with target volatility) ranked second, slightly outperforming Strategy A (equal-weighted 1/N) by the end of the sample. This result implies that even when imposing a volatility constraint equivalent to the 1/N benchmark, the Markowitz framework can still allocate capital more efficiently - favouring assets with higher expected returns - thereby generating marginally superior performance. In contrast, when risk constraints are relaxed, as in Strategy C, the Markowitz model can fully leverage expectations to amplify returns, albeit at the cost of greater portfolio volatility.

4.2 Portfolio Performance Statistics

To provide a more quantitative assessment of the risk-return profile of each strategy, **Table 3** summarizes the key performance statistics of Portfolios A, B, and C, computed from the empirical series of monthly returns.

Table 3
Performance and Risk Metrics of Portfolio Strategies (02/2012-12/2024)

Strategy	Cumulative Gross Return	Average Monthly Return	Average Monthly Standard Deviation	Average Annual Return	Sharpe Ratio
A - Naive 1/N	267%	1.05%	6.48%	13.38%	0.136
B - Markowitz σ target	341%	1.18%	6.56%	15.07%	0.154
C - Markowitz max Sharpe	525%	1.53%	8.32%	20.03%	0.164

Source: Author's calculations

The figures in Table 3 further reinforce the observations drawn from Figure 1. Portfolio A achieved an average monthly return of approximately 1.05%, corresponding to an annualized return of 13.38%, with a standard deviation of around 6.48%, resulting in a Sharpe ratio 0.136. Portfolio B, which incorporated slightly higher expected returns while maintaining a target volatility comparable to Portfolio A, generated a monthly return of approximately 1.18% - equivalent to 15.07% annually - with a similar standard deviation of 6.56%, yielding a Sharpe ratio of 0.154. Portfolio C distinguished itself with the highest monthly return of around 1.53%, translating into an annualized return of 20.03%, accompanied by higher monthly volatility of 8.32%, leading to a Sharpe ratio of 0.164 - the highest among the three. Regarding return-to-risk efficiency, Portfolio C emerged as the most effective, followed by B and A.

Interestingly, even after adjusting for risk, the Sharpe-optimized strategy (C) outperformed the equally weighted 1/N strategy. This result contrasts several studies conducted in developed markets, where 1/N often performs comparably or even better in the Sharpe ratio, mainly due to its estimation-free nature. This divergence suggests that the estimations employed by the Markowitz model in this specific context proved sufficiently reliable to enhance returns and compensate for the associated increase in risk.

Regarding absolute risk, Portfolio C exhibits a monthly standard deviation of approximately 8.32%, which is moderately higher than the roughly 6.5% observed in Portfolios A and B. This difference reflects that Portfolio C concentrates on a smaller number of high-growth stocks, resulting in lower diversification and, consequently, greater overall volatility. However, the disparity is not excessively large, as the VN30 index constituents generally exhibit high pairwise correlations. Moreover, optimizing the Sharpe ratio in Portfolio C prevents excessive allocation to highly volatile stocks unless returns adequately compensate for such volatility. Otherwise, it would drag down the Sharpe ratio. Portfolio B was explicitly constructed to match the monthly risk profile of Portfolio A, and the empirical results confirm that its annualized volatility is approximately equal to that of Portfolio A, hovering around 6.5%.

4.3 Comparison of Returns and Statistical Testing

To examine whether the observed differences in investment performance across strategies are statistically significant or merely due to randomness, we conduct a paired t-test between Portfolios C and A. The test yields a t-statistic of approximately 1.51 and a one-tailed p-value of around 0.066 (testing the null hypothesis $H_0: \mu_C \leq \mu_A$). The result is not statistically significant at the conventional 5% significance level, as the p-value (0.066) exceeds the threshold (0.05); hence, we fail to reject H_0 . However, under more relaxed significance levels - such as 7% or 10% - the null hypothesis can be rejected, suggesting that Portfolio C's average monthly return is statistically higher than that of Portfolio A at confidence levels of approximately 90-93%. In other words, statistical evidence (with about 93% confidence) shows that the Sharpe-optimal Markowitz strategy outperforms the naive 1/N strategy in terms of mean return.

These results are consistent with visual intuition based on the return time series: Portfolio C demonstrates superior performance across many periods. However, due to its higher volatility, the statistical certainty does not reach the conventional 95% confidence level. Nonetheless, the overall trend of $C > A$ is evident, and a p-value of approximately 0.066 suggests that statistical confirmation would likely follow with a more extended sample or a 10% significance threshold.

We also observe that Portfolio B yields a higher average return than Portfolio A, albeit with a smaller margin. Indeed, Portfolio B's higher Sharpe ratio indicates that it delivers more return per unit of risk than Portfolio A. A direct comparison reveals that "Portfolio B has a higher Sharpe ratio than Portfolio A" and "Portfolio C has a higher Sharpe ratio than Portfolio B." While we did not formally conduct t-tests for the B-A and C-B pairs, the consistent ranking of Sharpe ratios suggests an explicit ordering: $C > B > A$.

In summary, the empirical results demonstrate that the Markowitz strategy, applied to the VN30 market from 2012 to 2024, exhibits superior return-generating potential. Notably, the Sharpe-optimal portfolio (C) delivers significantly higher returns and offers a materially improved risk-adjusted performance compared to the equal-weighted strategy. Portfolio B, which was constructed to match the risk level of Portfolio A, also modestly outperforms it, providing evidence that investment efficiency can be enhanced through more intelligent asset allocation - without the need to assume additional overall risk.

5. Discussion of Findings

The empirical findings provide several important insights into the efficiency of Vietnam's stock market and the practical value of the Markowitz model within the context of an emerging market environment.

First, the clear outperformance of the Sharpe-optimal Markowitz strategy (Portfolio C) relative to the naive 1/N allocation strategy (Portfolio A) suggests that the Vietnamese stock market during the 2012-2024 period may still exhibit information asymmetries or inefficiencies that quantitative models can effectively exploit. In a truly efficient market, no pre-specified investment strategy - particularly those relying on historical data - should be able to consistently outperform the equally weighted strategy, which is inherently passive and maximally diversified once risk is taken into account. However, the empirical evidence shows that Portfolio C achieved a significantly higher Sharpe ratio than Portfolio A, implying the presence of systematic return-generating characteristics in the most recent six-month data, which the Markowitz model was able to capitalize on effectively. One plausible explanation lies in a momentum effect in the Vietnamese market: stocks that have performed well over the past 3 to 6 months tend to continue performing well in the short term. Since the Sharpe-optimal strategy maximizes expected returns using trailing 6-month data, it inherently assigns more weight to stocks exhibiting positive momentum (i.e., higher average 6-month returns) while de-emphasizing weaker performers. Portfolio C naturally benefits if momentum persists - a pattern clearly observed during 2020-2021, when many VN30 constituents experienced explosive growth, and Portfolio C had already shifted its exposure toward them. Conversely, the 1/N strategy - due to its uniform allocation across all assets - tends to "buy the good and the bad alike," which dilutes the momentum impact and leads to underperformance during strongly trending markets. The momentum effect has been extensively documented in global markets (e.g., Jegadeesh & Titman, 1993), and Vietnam appears to be no exception. These findings are consistent with prior evidence in emerging markets; notably, Rouwenhorst (1999) found significant momentum effects across 20 emerging equity markets. The implications are clear: institutional investors in Vietnam should consider integrating momentum-based signals or implementing short-term return-optimization frameworks to enhance fund performance.

Second, the success of Strategy C should be considered alongside its higher risk level. The P&L trajectory of Portfolio C exhibits more pronounced fluctuations, with sharp drawdowns and rallies - such as the substantial gains during 2020-2021, followed by a steep decline in 2022. This reflects a common empirical phenomenon: portfolios with higher expected returns typically come with elevated levels of risk. Although Strategy C is designed to maximize the Sharpe ratio, the ratio itself only captures the return-to-risk trade-off. When the model identifies opportunities to significantly enhance returns at the cost of only marginally higher risk, it will exploit them, which can still result in an overall increase in portfolio volatility. Indeed, Portfolio C exhibited a more significant maximum drawdown than Portfolio A. In practical fund management, not all investors are willing to tolerate such levels of volatility - even if, in the long run, they may be compensated accordingly. Therefore, Strategy C may be more suitable for active investors comfortable taking on higher risks in pursuit of superior returns. In contrast, Strategy B offers a more conservative alternative: it maintains the same risk profile as the 1/N strategy while delivering moderately higher returns, making it a good fit for investors seeking to "enhance performance within the same risk budget." Although the Sharpe improvement of approximately 0.018 over Portfolio A may appear modest, in professional asset management, any incremental gain in the Sharpe ratio is valuable - mainly when achieved without increasing risk exposure.

Third, from the perspective of traditional methodology, these results may appear somewhat surprising compared to findings in the classical literature. For example, DeMiguel et al. (2009) famously concluded that it is difficult for any mean-variance optimization method to outperform the naive 1/N strategy consistently. However, in the Vietnamese context, a relatively basic implementation of the Markowitz framework - absent any advanced enhancements such as shrinkage estimators or the Black-Litterman model and using a short rolling window along with simple no-short-selling constraints - produced clear and substantial outperformance. This raises a critical question: What makes the Vietnamese market different? One possible explanation lies in the high degree of cross-sectional dispersion in stock returns within the VN30 universe during the examined period. In environments where return differentials are large - where a few stocks experience explosive gains while others stagnate or decline - a mean-variance optimizer can more easily "pick winners" and allocate capital away from underperformers, thus outperforming a naive, equally weighted strategy. Indeed, during the 2020-2021 period, several bank and brokerage stocks in the VN30 delivered multi-bagger returns, while others showed only marginal gains or moved sideways. The 1/N strategy, by averaging across all constituents, necessarily dilutes exposure to the top performers, making it inferior to a strategy that concentrates bets on the market leaders. This phenomenon is not uncommon in markets that are less efficient or exhibit high levels of speculation, where a few "superstar" stocks often drive index performance. In such cases, optimization-based strategies can significantly outperform if they successfully identify the winners.

On the other hand, it is important to acknowledge that the superior performance of Portfolio C may also reflect a degree of favourable alignment with historical market conditions. Since Strategy C relies on past data to forecast near-term returns, it is backward-looking and potentially momentum-driven. In a prolonged bull market, this backwards-looking behaviour can be

highly effective - rising stocks tend to keep rising, reinforcing positive momentum. However, if the market were to experience a sharp reversal, such a strategy could suffer disproportionately.

Fourth, from a technical perspective, this study demonstrates that the no-short-selling constraint does not hinder the ability to achieve strong portfolio performance; on the contrary, it may actually contribute to the stability of the optimized portfolio. While allowing unrestricted short-selling could lead to even higher Sharpe ratios - enabling leverage and short exposure to poorly performing stocks - it also introduces substantial risk and high transaction costs, particularly in Vietnam's capital markets. Therefore, imposing a non-negativity constraint on portfolio weights is not only practically justified but may also function as a natural shrinkage mechanism, akin to the approach discussed by Jagannathan and Ma (2003), helping to reduce the extremity of optimal weights. Empirically, the optimal Sharpe portfolio (Portfolio C) allocates capital to a concentrated set of 5-10 high-potential stocks rather than excessively loading on a single name. This outcome stems from the implicit diversification effect of the long-only constraint. If the optimizer were to concentrate fully on a single asset, it would forgo diversification benefits and ultimately suffer a decline in the Sharpe ratio. Thus, the long-only constraint contributes to identifying a more balanced and robust allocation.

Finally, the study also carries implications for market behaviour. The results suggest that Vietnamese investors may not yet fully exploit arbitrage opportunities made possible through quantitative Optimization. If more prominent market participants systematically applied Markowitz-style strategies, outperforming stocks might be bid up earlier, eroding the performance differential. However, many individual investors rely heavily on intuition or discretionary judgment, while domestic institutional funds largely follow benchmark-constrained strategies. This leaves room for even a relatively simple quantitative model to generate alpha (i.e., excess returns). In this sense, the findings serve both as a testament to the enduring practical value of Markowitz's classical framework - even more than half a century after its inception - and as a reminder that emerging markets often operate under their own behavioural and structural dynamics, which offer opportunities that can be systematically captured.

6. Conclusion and Implications

This study conducted an in-depth examination of the applicability of Markowitz's portfolio theory in the context of the Vietnamese stock market, focusing specifically on the VN30 index over the period from February 2012 to December 2024. We constructed and compared three investment strategies: (A) an equally weighted 1/N portfolio, (B) a Markowitz portfolio with a target volatility equal to that of the 1/N portfolio, and (C) a Sharpe-optimal Markowitz portfolio. All portfolios were rebalanced monthly using a trailing six-month data window. The empirical results show that Strategy C - the Sharpe-maximizing portfolio - outperformed the naive 1/N strategy in terms of both cumulative returns and risk-adjusted performance (Sharpe ratio), generating approximately +525% cumulative return compared to +250% for the 1/N portfolio. Strategy B also outperformed A by a modest margin, highlighting the potential benefits of Optimization even under tight risk controls. Statistical tests indicate that the performance advantage of C over A is significant at approximately the 90-95% confidence level, reinforcing the interpretation that the outperformance is unlikely to be due to chance alone.

From an academic perspective, the study contributes new empirical evidence demonstrating that the Markowitz framework retains substantial practical relevance in the context of emerging markets. While previous research has raised doubts about the efficacy of portfolio optimization in real-world settings (e.g., Michaud & Ma, 2001; DeMiguel et al., 2009), our findings from Vietnam suggest that when implemented with reasonable constraints (e.g., short horizons, no short-selling), the Markowitz model can indeed deliver substantial alpha. This supports the view that traditional methodologies - when contextually adapted - can remain highly effective. The study also adds to the growing literature on emerging markets such as India, China, and Brazil, where heterogeneous market efficiency levels can lead to divergent outcomes in portfolio optimization. Vietnam remains in a phase where active, quantitatively driven strategies can still yield excess returns.

Practical Implications

The study offers several actionable insights for investors and fund managers in Vietnam:

(i) *For Portfolio Managers:* Optimizing models such as Markowitz can enhance investment outcomes rather than relying solely on intuition or closely tracking passive benchmarks. In particular, the Sharpe-optimal Strategy C demonstrates the potential for superior returns, albeit with higher volatility. Active funds may consider integrating a suitably adjusted version of the Markowitz model into their asset allocation process. That said, careful attention must be paid to parameter selection - such as lookback window length and rebalancing frequency - and implementing additional risk management mechanisms to mitigate losses during abrupt market reversals.

(ii) *For Individual Investors:* The findings suggest that simple heuristics like equal weighting may miss out on significant return opportunities. Individual investors can improve their portfolio outcomes by adopting principles from Markowitz's theory - for instance, by overweighting stocks with strong fundamentals or upward momentum and underweighting weaker performers rather than maintaining static allocations. However, it is crucial to acknowledge the trade-off: higher expected returns are typically associated with greater risk, and each investor must calibrate their level of portfolio aggressiveness according to their risk tolerance.

(iii) *For Market Participants and Regulators:* As more investors adopt optimization-based strategies, the market may gradually become more efficient, and arbitrage opportunities will likely be arbitrated away more quickly. This would diminish

the relative advantage of strategies like Sharpe-maximizing Markowitz, as has already been observed in developed markets. Consequently, the current phase may represent a window of opportunity for quantitatively sophisticated funds before the market transitions toward a higher-efficiency regime. Regulators should also monitor the growing participation of institutional and quantitative investors to better understand the evolving dynamics of market behaviour.

Limitations and Future Directions

Despite its encouraging results, this study acknowledges several limitations. First, the data scope is confined to the VN30 index, which includes large-cap stocks and may not reflect the broader behaviour of mid- and small-cap stocks. Extending the analysis to other market segments - such as VNMidcap or VNSmallcap - could provide a more comprehensive view. Second, while the study covers more than a decade, the 2012-2024 window may not fully span multiple macroeconomic cycles, and results might differ if more extended periods, including prolonged crises or bubbles, were considered. Third, the model assumes zero transaction costs and perfect liquidity, which is unrealistic. Monthly rebalancing with significant weight shifts - especially in Strategy C - can incur meaningful trading costs, potentially offsetting the performance edge. Fourth, using short-term historical data makes the model susceptible to transitory shocks. The study does not explore stability-enhancing techniques like Ledoit-Wolf shrinkage or Bayesian approaches like the Black-Litterman model in the Vietnamese context. Finally, this paper only tests a limited set of basic strategies; other quantitative approaches - such as those incorporating value, size, long-term momentum, or machine learning - remain unexplored. These limitations suggest fruitful directions for future research to deepen further our understanding of portfolio optimization in Vietnam's evolving financial markets.

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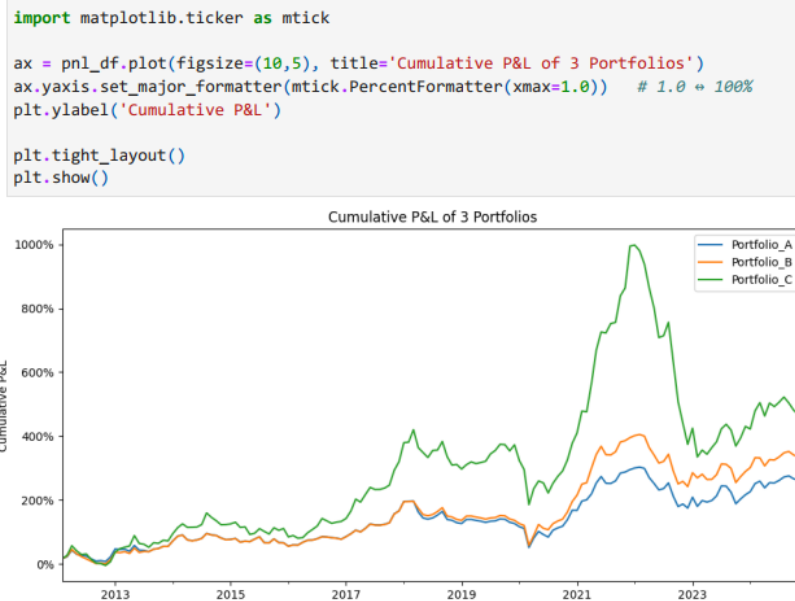
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List of Appendices

1. Appendix 1: Cumulative returns of Portfolios A, B, and C over the period 02/2012-12/2024



2. Appendix 2: Statistical Test Results

```
from scipy.stats import ttest_rel

print("**** Paired t-test results for Portfolios C and A ****")
# 1) Hypothesis: ser_C > ser_A
t_stat, p_value = ttest_rel(ser_C, ser_A, alternative='greater')

# 2) Print results
print(f"Test H0: ser_C <= ser_A vs H1: ser_C > ser_A")
print(f"T-statistic = {t_stat:.4f}")
print(f"P-value = {p_value:.4f}")

# 3) Interpretation at 7% significance level
alpha = 0.07
if p_value < alpha:
    print("✔ There is statistical evidence to conclude: ser_C > ser_A at 7% confidence level")
else:
    print("✘ Not enough evidence to conclude ser_C outperforms ser_A at 7% confidence level")

print("**** Paired t-test results for Portfolios C and A ****")
Test H0: ser_C <= ser_A vs H1: ser_C > ser_A
T-statistic = -1.5130
P-value = 0.0322
✔ There is statistical evidence to conclude: ser_C > ser_A at 7% confidence level
```

3. Appendix 3: Risk Level of Each Portfolio

```
# Standard deviation of the 3 portfolios
print("**** RESULTS ****")
print("Standard deviation of Portfolio A: ", ser_A.std() * 100, "%")
print("Standard deviation of Portfolio B: ", ser_B.std() * 100, "%")
print("Standard deviation of Portfolio C: ", ser_C.std() * 100, "%")

[108] Python

... **** RESULTS ****
Standard deviation of Portfolio A: 6.481877458732767 %
Standard deviation of Portfolio B: 6.55526751698281 %
Standard deviation of Portfolio C: 8.3254443641117 %
```

4. Appendix 4: Average Monthly and Annual Returns of Each Portfolio

```
print("**** RESULTS ****")
print("Average monthly return of Portfolio A: ", ser_A.mean()*100, "%")
print("Average monthly return of Portfolio B: ", ser_B.mean()*100, "%")
print("Average monthly return of Portfolio C: ", ser_C.mean()*100, "%")
print("Annualized average return of Portfolio A: ", ((1+ser_A.mean())**12-1)*100, "%")
print("Annualized average return of Portfolio B: ", ((1+ser_B.mean())**12-1)*100, "%")
print("Annualized average return of Portfolio C: ", ((1+ser_C.mean())**12-1)*100, "%")

[109] Python

... **** RESULTS ****
Average monthly return of Portfolio A: 1.8521041603066161 %
Average monthly return of Portfolio B: 1.1763252234250632 %
Average monthly return of Portfolio C: 1.5328702220460488 %
Annualized average return of Portfolio A: 13.382857227611277 %
Annualized average return of Portfolio B: 15.865947692132497 %
Annualized average return of Portfolio C: 20.027278937864374 %
```

5. Appendix 5: Cumulative Returns Achieved by Each Portfolio

```
# Cumulative returns of 3 portfolios
print("**** RESULTS ****")
print("Cumulative return of Portfolio A on 31/12/2024:")
print(f"{cum_A.tail(1).values[0]*100:.2f}%")
print("Cumulative return of Portfolio B on 31/12/2024:")
print(f"{cum_B.tail(1).values[0]*100:.2f}%")
print("Cumulative return of Portfolio C on 31/12/2024:")
print(f"{cum_C.tail(1).values[0]*100:.2f}%")

[102] Python

... **** RESULTS ***
Cumulative return of Portfolio A on 31/12/2024:
266.82%
Cumulative return of Portfolio B on 31/12/2024:
341.48%
Cumulative return of Portfolio C on 31/12/2024:
525.25%
```

6. Appendix 6: Sharpe Ratio of Each Portfolio

```
# Sharpe ratio of 3 portfolios
print("**** RESULTS ****")
print("Sharpe ratio of Portfolio A: ", (ser_A.mean() - 0.17/100) / ser_A.std())
print("Sharpe ratio of Portfolio B: ", (ser_B.mean() - 0.17/100) / ser_B.std())
print("Sharpe ratio of Portfolio C: ", (ser_C.mean() - 0.17/100) / ser_C.std())

[104] Python

... **** RESULTS ***
Sharpe ratio of Portfolio A: 0.13680775635185658
Sharpe ratio of Portfolio B: 0.15351397788592354
Sharpe ratio of Portfolio C: 0.16369939556446433
```



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