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Trade-off in robustness, cost and performance by a multi-objective robust production optimization method

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A B S T R A C T

Designing a production process normally is involved with some important constraints such as uncertainty, trade-off between production costs and quality, customer's expectations and production tolerances. In this paper, a novel multi-objective robust optimization model is introduced to investigate the best levels of design variables. The primary objective is to minimize the production cost while increasing robustness and performance. The response surface methodology is utilized as a common approximation model to fit the relationship between responses and design variables in the worst-case scenario of uncertainties. The target mean ratio α is applied to ensure the quality of the process by providing the robustness for all types of quality characteristics and with a trade-off between variability and deviance from the ideal point. The Lp metric method is used to integrate all objectives in one overall function. In order to estimate target value of the quality loss by considering production tolerances, the process capability ratio (C_{pm}) is applied. At the end, a numerical chemical mixture problem is served to show the applicability of the proposed method.

1. Introduction

Nowadays, most engineering design methods try to assist decision makers for optimizing the processes and achieving the highest quality with minimum costs. The process of finding the accurate design parameters is stated as an optimization. Typically, any optimization technique needs to consider design constraints. It is the engineer's duty to choose the design parameters x according to an (or some) objective function(s) $f(X)$ (Beyer & Sendhoff, 2007). Process optimization is one of the intensive aspects of product development (Lukic et al., 2017). During the optimization process, we need to maximize one or more parameters, while keeping all others within their constraints. The main goal is to reach a desired performance for the process that manufactures some products, by minimizing the cost of operation in a production process, or the variability of a quality characteristics by maximizing the yield of the production process. Furthermore, due to noisy data and/or uncertainty affecting some parameters of the model, achieving robust performance plays an essential role for engineering design problems.

In practice, most processes are affected by external uncontrollable factors which cause that quality characteristics being far from the ideal points with variation in their exact values. Taguchi's Robust

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design aims to reduce the impact of these types of environmental factors on a product or process, and leads to greater customer satisfaction and higher operational performance. The objective of robust design is to minimize the total quality loss in products or processes. Robust design is the most powerful method available for reducing product cost, improving quality, and simultaneously reducing development time. In process robustness studies, it is desirable to minimize the influence of noises and uncertainty in the process and simultaneously determine the levels of input and control factors, by optimizing the overall responses, or in another sense, optimizing product and process, which are less sensitive to various causes of variances. By employing the information of experiments about the relationships between input control factors and output responses, robust design methods can disclose robust solutions that are less sensitive to causes of variations (Nha et al., 2013).

There are different robust optimization models proposed in the literature for design processes in engineering problems. Nevertheless, there is still a gap between theory and practice in optimization, being evident in the fact that optimization methods are still not used for many real-world problems, (Bertsimas et al., 2011; Beyer & Sendhoff, 2007). In order to increase the reliability in optimization results, uncertainty and the tradeoff between three aspects of production cost, robustness, and performance are important circumstances which need to be considered in production problems. The primary aim of this paper is to propose a new mathematical formulation of robust optimization model to find the best levels of design variables in the production process under minimum computational cost when uncertainty and the tradeoff between three aspects of production cost, robustness, and performance are attended in the problem. In addition, physical constraints to satisfy customer's requirements and obligation to satisfy production tolerances are also considered in the model. In robust design approach, both the robustness of the objective functions (optimal results) and the constraints (feasibility) are considered, simultaneously. The proposed model is formulated by considering three different types of quality characteristics such as of Nominal The Best (NTB), Smaller The Better (STB), and Larger The Better (LTB). In order to estimate the target point applied in the expected quality loss function, a new approach is suggested by using C_{pm} for all three types of characteristics. However, since we wish to design the model with the customer's point of view, the terms of customer tolerance (LSL, USL) and process capability index are used in the proposed model. In addition, the trade-off between production cost and performance with insensitivity against environmental factors is attended in designing the model, while most existing methods are just concentrated on seeking the best levels of design variables which maximizes the robustness (Gabrel et al., 2014).

The rest of the paper is organized as follows. The application of integrating robust design optimization and response surface modeling (RSM) in the literature is briefly reviewed in Section 2. In Section 3, the methodology including the required steps for constructing the proposed method is explained. This section also includes two different mathematical formulations based on process's cost and quality loss. A numerical example (mixture problem) is served in Section 4 to illustrate the applicability of the proposed models. Finally, this paper is concluded in Section 5.

2. Literature review

It is commonly accepted that the Taguchi's principles are useful and very appropriate for industrial product design (Simpson et al., 2001). Taguchi also represented the concept of quality loss as an average amount of total loss that compels to society because of deviating from the ideal point and variability in responses. Moreover, this function tries to make a trade-off between the mean and variance of each type of quality characteristics (Park & Antony, 2008). Fig. 1 depicts the graphical concepts of expected loss function based on the classification of quality characteristics into three different types including NTB, LTB, and STB.

Expected quality loss functions based on Taguchi's approach for all three types of quality characteristics are:

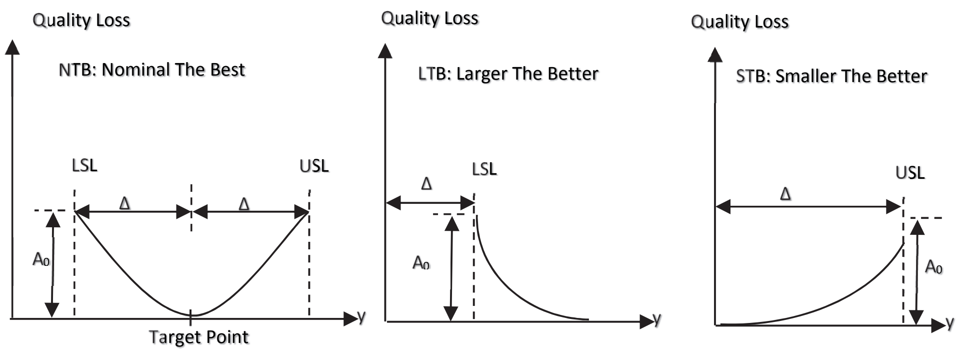

Fig. 1. The expected loss function for three types of quality characteristics

NTB $$L(y) = C_0[(\mu - T)^2 + \sigma^2] \quad (1)$$

STB $$L(y) = C_0[\mu^2 + \sigma^2] \quad (2)$$

LTB $$L(y) = C_0[(\frac{1}{\mu^2})(1 + 3\sigma^2/\mu^2)] \quad (3)$$

where, μ , σ^2, T respectively are mean, variance, and target of response and C_0 is the loss coefficient. The value of C_0 is computed by $\frac{A_0}{\Delta^2}$ for NTB and STB and $A_0\Delta^2$ for LTB. The quality loss coefficient C_0 can be determined on the basis of the necessary information on the losses in monetary terms caused by falling outside the customer tolerance. The coefficient C_0 plays an important role to make the expected loss function in monetary loss scales. In addition, A_0 is introduced as a cost of repair or replacement when the quality characteristic performance has the distance of Δ from target point (Phadke, 1989).

Recently, the robust optimization under uncertainty has been interested where treatments of uncertainty are described in different scenarios. A common approach in robustness studies is associated with minimizing objectives in the worst-case scenario. The min-max robustness (also called strict robustness) has been appropriately elucidated by Ben-Tal et al. (2009). The robust optimization methodology has been adopted in many applications of interest in different sciences, and it is widely used in practice for optimizing, planning, and scheduling of real processes. In Boyaci et al. (2017), a fuzzy mathematical model was developed by RSM technique and fuzzy logic to optimize drilling process optimization with multiple responses. Investigate the literature shows interesting issues in application of robust design optimization in production and manufacturing processes (e.g. Parnianifard et al., 2018).

In practice, the designer often has to deal by conflicting objectives and source of uncertainty. In the process and product optimization, a common problem is to determine optimal operating condition that balances the multiple quality characteristics of a product. There are different methods in literature for Multi-Objective Robust Optimization (MORO). The robust design approach has been combined with different methods in multi-objective optimization such as the weighted sum method (Zadeh, 1963), goal programming (Charnes & Cooper, 1977), physical programming (Messac & Ismail-Yahaya, 2002), compromise programming (Chen et al., 1999), desirability function (Chen et al., 2012; Costa et al., 2011), different metric methods (Hwang & Masud, 2012; Miettinen, 2012), and evolutionary algorithms (Deb, 2011). Computation-intensive in design problems are becoming increasingly common in production industries. Investigating all Pareto optimal solutions is computationally expensive and time-consuming, because in most cases, Pareto optimal solutions are usually exponentially large (Chinchuluun & Pardalos, 2007). In practice, difficulties arise because of different units of measurement, criteria, and levels of importance among the multiple responses or quality measurements. Moreover, some different methods have been presented which try to tackle the problem of optimizing multiple responses simultaneously, (e.g. Marler & Arora, 2004; Miettinen, 2012). Notably, preference of each method than other strongly depends on the role of decision maker and information on hand based on different purposes of the problem, (i.e. none of existing methods in the multi-objective problem can be claimed to be superior to the others in every aspect), (Miettinen, 2001).

The computation burden is often caused by expensive analysis and simulation processes in order for physical testing of data. To address such a challenge, approximation techniques (also known as metamodels or surrogate models) are often used. Approxiamtion methods have been developed in statistics, mathematics, computer science, and various engineering disciplines. These methods have been used to avoid intensive computational and numerical models, which might squander time and resources for estimating model's parameters. If input or design variables (X) and responses or outputs (y) have a relationship as $y = f(X)$, then a model can fit to approximate that relationship is $\hat{y} = g(X)$, so $\hat{y} = y + \varepsilon$ where ε represents an error of approximation (Simpson et al., 2001). Some number of common approximation methods are polynomial regression (also called Response Surface Methodology (RSM)), Kriging, Artificial Neural Network (ANN), Radial Basis Functions (RBF), see (Simpson et al., 2001; Wang & Shan, 2007). The name of RSM might be somewhat misleading since all types of approximation methods constitute a “surface” which enables the user to predict the response at untried points. However, the common use of RSM, which is also adopted here, is to address polynomial regression models. The response surface approach facilitates understanding the system by modeling the response functions for process mean and variance, respectively. RSM is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing process. The overview of the second-order response surface model is shown as:

$$y = f(X) = \hat{\beta}_0 + \sum_{i=1}^{m} \hat{\beta}_i\, x_i + \sum_{i=1}^{m} \hat{\beta}_{ii}\, x_i^2 + \sum \sum_{i<p=2}^{m} \hat{\beta}_{ip}\, x_i x_p + \varepsilon, \tag{4}$$

where $\widehat{\beta}_0$, $\hat{\beta}_i$ and $\hat{\beta}_{ip}$ are unknown regression coefficients and the term ε is the usual random error (noise) component. The accuracy of the approximation model strongly depends on designing appropriate sample points. Some experimental sampling methods are Central Composite Design (CCD), fractional factorial, Box-Behnken, alphabetical optimal, and Plackett-Burman (Myers et al., 2016).

3. Methodology

In the current work, some main assumptions and outstanding points are followed as below:

- In this study, uncertainty is assumed to be fixed in the worst scenario, and under this condition we try to minimize the expected loss for each quality characteristic (response) and minimize constraint variation region. In the worst-case scenario of uncertainties, it is assumed that all variations of system performance may occur simultaneously in the worst possible combinations of design variables. Respect to the min-max approach, we try to minimize the maximum variability in the process performance due to the existence of uncertainties in their worst framework. The highest amount of process’s cost is raised due to facing process in the worst combinations of uncertainties. In addition, the variability due to fluctuating input variables is assumed as a stochastic term in the problem.
- To reduce the computational complexity of the model, first we standardize all design variables into $[-1, 1]$, then resulted in magnitudes use in RSM proceeding for utilizing simpler regression coefficients in the formulation. To normalize in $[-1, 1]$ can be used $x' = 2.\,(\frac{x - x_{min}}{x_{max} - x_{min}}) - 1$.

- The fluctuating of input factors around its specific value is assumed that constructed by the existence of environmental factors (uncontrollable in practice), and it is desirable to responses do not have much variability due to its fluctuation (He et al., 2010).

3.1 Nomenclature

The parameters and symbols which used in the proposed method are revealed in Table 1.

Table 1
The table of nomenclature

Notation	Description
i, t	Indexes for design variables, $\{i, t = 1, 2, \ldots, m\}$
m	Number of design variables
N	Number of quality characteristics with nominal the best type (NTB)
S	Number of quality characteristics with smaller the better type (STB)
L	Number of quality characteristic with larger the better type (LTB)
k, K	Index and Number of all three types of quality characteristics as responses (NTB, STB, and LTB), $k = (1, 2, \cdots, K), So\ that\ K = N + S + L$
y_k	The function which shows relationship (second order model) between k_{th} quality characteristic and design variables set, $y_k = f_k(X)$
$\overline{L(y_k)}$	The target point of expected loss for k_{th} quality characteristic
$E(y_k)$	Expected value of k_{th} expected loss function
$V(y_k)$	Variance of k_{th} expected loss function
$L(y_k)$	The expected loss function of k_{th} quality characteristic
$f_{Cost}(X)$	The relationship between cost of production and design variables set
j, J	Index and number of constraint function
k_j	The penalty factors which associated to j_{th} constraint
g_j	The relationship between j_{th} constraints and design variables set, $g_j = f_j(X)$
$E(g_j)$	Expected value of j_{th} constraint function
$V(g_j)$	Variance of j_{th} constraint function
x_i^U	The upper feasible bound for i_{th} design variable
x_i^L	The lower feasible bound for i_{th} design variable
σ_i^2	The variance of i_{th} design variable
σ_i	The standard deviation of i_{th} design variable
σ_{it}	The covariance between i_{th} and t_{th} design variables
LSL_k	The upper bound of k_{th} quality characteristic
USL_k	The lower bound of k_{th} quality characteristic
C_k	Quality loss coefficient for k_{th} quality characteristic
U	The overall objective of all k objective functions (L_p metric method)
D	Depicts upper limitation for the overall distances of all expected quality losses from relevant target points
B	The whole budget which associated to production

3.2 Robustness in objective functions

Clearly, any product which fails to reach the target value is termed as a loss in robust design, in contrast to the traditional design approach where a product in a tolerance range is accepted as a product of good quality (Khan et al., 2015). For constructing the robustness in all three types of quality characteristics, three different expected losses based on Taguchi's approach have been introduced, see Eqs.(1-3). The loss coefficient (constant) C_0 generally plays an important role in optimal parameter settings to make trade-offs among characteristics in multiple quality characteristic problems. In the Taguchi's expected loss for STB type, the target point was placed to zero, whereas for NTB type, an infinite target was considered. However, in practice for real condition of the process particularly in the production process, this kind of targeting are exaggerating (Sharma & Cudney, 2011). Also for optimizing the process, we need functions of expected quality loss that be comparable to one another in three cases of NTB, LTB, and STB. Sharma et al. (2007) proposed the target mean ratio that has a common formula for all three types and brings similarity among them. Based on their proposed target mean ratio α, the expected quality loss is described as below:

$$L(y_k) = C_k[E(y_k)^2(1-\alpha_k)^2 + V(y_k)], for\ k = 1,2,\ldots,K \tag{5}$$

where the α_k is equal to $T_k/E(y_k)$ and $0 \le \alpha_k \le M$ when M is a large number and T_k is a target point for k^{th} characteristic. The α_k could be defined by the decision maker and based on the type of k^{th} quality characteristic. For different values of α_k, the expected loss represents different magnitudes for each type of NTB, LTB, and STB. This value shows the shifting of $E(y_k)$ to the right or left side of the target point and can be chosen zero for STB type, a larger number more than one for LTB type and also 1 for NTB. But, it is strongly recommended that the target point and specially α_k do not need to be a large number or infinity for LTB cases, but it just needs to be significantly greater than one, for more information see Sharma and Cudney (2011) and Sharma et al. (2007). In order to follow the customer's satisfaction in the production process, let's consider the target point is in the center of production tolerances, so $T_k = (USL_k + LSL_k)/2$.

3.3 Robustness in constraints set

The constraints of the production process which are classified into two groups. First the physical constraints g_j, and second the limiting magnitude of design variables. The preferences of the designer or available resources for choosing the interest levels for design variables are some instances of physical constraints (Messac & Ismail-Yahaya, 2002). In robust design optimization, robustness in both objectives set and constraints set needs to be considered. Moreover, to study the variation of constraints, we employ the worst-case scenario approach. In the worst-case scenario of uncertainties, it is assumed that all variations of system performance may occur simultaneously in the worst possible combination of variability sources. The original constraints are modified by adding the penalty term separately to each of them as below:

$$E(g_j) + k_j.\sqrt{V(g_j)} \le 0, \quad for\ j = 0,1,2,\ldots,J \tag{6}$$

where k_j is penalty factor of j^{th} constraint which can be determined by the decision maker. This penalty factor or confidence coefficient can control the degree of robustness (Sahali et al., 2015). To achieve the feasibility of the constraint under uncertainty, a general probabilistic feasibility formulation can be considered as $p(g_i \le 0) \ge p_j^*$ $, j = 1,2,\ldots,J$ where p_j^* is the desired probability for satisfying j^{th} constriants. If we assume g_i is normally distributed, $k_j = \Phi^{-1}(p_j^*)$ have been suggested by Parkinson et al. (1993) while Φ^{-1} is the inverse function of the cumulative density function in a standard normal distribution.

The bounds of design variables are also modified to ensure feasibility under deviations:

$$x_i^L + \sigma_i \le X_i \le x_i^U - \sigma_i \qquad , for\ i = 1,2,\cdots,m \tag{7}$$

3.4 Estimating of model's parameters

Based on unknown terms in expected quality loss functions and constraints set, the common estimating equations are computed as blow:

$$E(y_k) = f_k(X) + \frac{1}{2}\sum_{i=1}^{m}\sum_{t=1}^{m}\frac{\partial^2 f_k(X)}{\partial x_i\,\partial x_t}\,.\Delta_{it} \quad for\ k = 1+2+\cdots+K \tag{8}$$

$$V(y_k) = \sum_{i=1}^{m}\sum_{t=1}^{m}\frac{\partial f_k(X)}{\partial x_i}\,.\frac{\partial f_k(X)}{\partial x_t}\,.\Delta_{it} \quad for\ k = 1+2+\cdots+K \tag{9}$$

$$E(g_j) = f_j(X) + \frac{1}{2}\sum_{i=1}^{m}\sum_{t=1}^{m}\frac{\partial^2 f_j(X)}{\partial x_i\,\partial t}\,.\Delta_{it} \quad for\ j = 1+2+\cdots+J \tag{10}$$

$$V(g_j) = \sum_{i=1}^{m}\sum_{t=1}^{m}\frac{\partial f_j(X)}{\partial x_i}\,.\frac{\partial f_j(X)}{\partial x_t}\,.\Delta_{it} \quad for\ j = 1+2+\cdots+J \tag{11}$$

where $\Delta_{it} = \sigma_{it}$ if $i \neq t$ and σ_i^2 if $i = t$. The expressions of the mean and variance of the relevant quality characteristic for each objective function and also mean and variance of each physical constraint are respectively estimated by the second-order terms of Taylor's expansion about $E(y_k)$ and $E(g_j)$. Also, the derived equations are valid for any probability density function of y_k and g_j. The fluctuating of the design variables around their specific values are due to the effects of environmental factors in the process.

3.5 Multi-response optimization method

In the current paper, the weighted L_p metric is used to integrate multiple objectives for all types of quality characteristics, due to two main reasons. First, needing less information from decision maker and second compared to other multi-objective method is the ease of application in practice, (See Miettinen, 2001). Also capability ratio C_{pm} is used as a supplement of the L_p metric to estimate the target value of each expected loss. The weighted L_p metric method can define the desired point and try to find an optimal solution that is as close as possible to this point (Chinchuluun & Pardalos, 2007). This method appropriately has been applied in the robust multi-objective to find a Pareto optimal solution, (See Ardakani & Noorossana, 2008).

3.5.1 Overall function

In the current work, the L_p metric is used to measure the distance between the expected loss of each quality characteristic and the relevant target point. Notable that all responses have the same scales due to the existence of coefficient C_0 in expected loss formulation, which make them in scale of monetary. The overall function which is utilized to integrate all responses is:

$$\min \quad U = \left(\sum_{k=1}^{K} w_k\ \left|L(y_k) - \overline{L(y_k)}\right|^p\right)^{\frac{1}{p}}. \tag{12}$$

Here $\overline{L(y_k)}$ is the target point for k^{th} expected loss, the quantity of w_k shows the importance of k^{th} expected loss compared to others and can take a value between zero and one, so that $\sum_{k=1}^{K} w_k = 1$ and assigned by the decision maker. Different weights in this metric can be produced by different deviation of each function from the target point. Generally the cases of $p = 1,2,\ldots,\infty$ is more common to employ in computational models, (See Miettinen, 2012).

3.5.2 Estimating the target point

In current paper, the desired capability of the process is used to estimate the target magnitude of each expected loss function. The process capability C_{pm} was proposed by Chan et al. (1988). In this index, the numerator is the range of the tolerance interval ($USL - LSL$) of the process which illustrates customer's limitations. The denominator is a combined measure of the standard deviation and the deviation of the mean from the target value. This ratio derives the mean square deviation related to Taguchi's loss function. The capability index C_{pm} for NTB is clearer than STB and NTB type. In the production process for quality characteristics with NTB type we do not need to allocate a large number or infinity for upper specification level (Sharma et al., 2007). Also for the same reason for STB types, the value of zero for the lower specification is exaggerative. So we can assume the upper and lower specification level is q times greater than LSL for NTB types and q times smaller than USL for STB types of quality characteristics, while $q > 1$. The twofold more than the target point for q in the case of LTB have been recommended by Sharma and Cudney (2011). So, if the middle value between upper and lower specification assumes the ideal value for the performance of quality characteristics, then $q = 4$ is suggested to be used for upper customer's limitation in LTB types and $q = \frac{1}{4}$ for lower customer's limitation in STB types. Therefore, we can estimate the target point of the expected loss while the goal is to achieve the target of process capability ($\overline{C_{pm}^k}$) which is defined by the decision maker for k^{th} quality characteristic. Moreover, the target points for expected loss based on types of quality characteristics are computed as below:

NTB: $$\overline{L(y_k)} = C_1\left(\frac{(USL_k - LSL_k)}{6\overline{C_{pm}^k}}\right)^2 \quad , for\ k = 1,2,\dots,N \tag{13}$$

LTB: $$\overline{L(y_k)} = C_2\left(\frac{LSL_k(q-1)}{6\overline{C_{pm}^k}}\right)^2 \quad , for\ k = 1,2,\dots,L \tag{14}$$

STB: $$\overline{L(y_k)} = C_3\left(\frac{USL_k(1-q^{-1})}{6\overline{C_{pm}^k}}\right)^2 \quad , for\ k = 1,2,\dots,S \tag{15}$$

3.6 Mathematical formulations

Here, based on the importance of production cost than the overall expected quality loss, two different mathematical formulations are proposed, while choosing an adequate formulation depends on the real process requirements. The function $f_{Cost}(X)$ is associated with the production cost according to values of design variables to satisfy the process tolerances.

3.6.1 Model I: A mathematical model based on the overall expected quality loss

min $$U = \left(\sum_{k=1}^{K} w_k \left|L(y_k) - \overline{L(y_k)}\right|^p\right)^{\frac{1}{p}} \tag{16}$$

subject to: $$f_{Cost}(X) \le B \tag{17}$$

$$E(g_j) + k_j.\sqrt{V(g_j)} \le 0 \quad , for\ j = 0,1,2,\dots,J \tag{18}$$

$$x_i^L + \sigma_i \leq X_i \leq x_i^U - \sigma_i \,, for\ i = 1,2,\cdots,m \tag{19}$$

This model tries to minimize an overall expected loss of all quality characteristics. The value B shows the limitation of the allocated budget for optimizing the process. As mentioned before the physical constraints and the design variables limitation are placed into constraints set.

3.6.2 Model II: A mathematical model based on the process production cost

$$\min \quad f_{Cost}(X) \tag{20}$$

$$\text{subject to:} \quad U = \left(\sum_{k=1}^{K} w_k \, \left| L(y_k) - \overline{L(y_k)} \right|^p \right)^{\frac{1}{p}} \leq D \tag{21}$$

$$E(g_j) + k_j . \sqrt{V(g_j)} \leq 0 \qquad , for\ j = 0,1,2,\dots,J \tag{22}$$

$$x_i^L + \sigma_i \leq X_i \leq x_i^U - \sigma_i \,, for\ i = 1,2,\cdots,m \tag{23}$$

where D depicts upper limit for the overall distances between all expected quality losses from their relevant target points. Notably, the threshold D is selected in such a way that feasible solutions always exist.

4. Numerical example

Here, in order to show the applicability of the model a chemical mixture problem is chosen due to applicability of this model in different aspects of engineering such as chemical, oil, and food production. So, let use the numerical case which was taken from Myers et al. (2016) and has been used by He et al. (2010). For this chemical process, two input variables (time and temperature) and three responses (yield, viscosity, and number average of molecular weight) are assumed. The first step is to construct the required experiments and collect the necessary data through running the designed experiments. Here the central composite design is used for designing experiments, see Table 2.

Table 2
Design of experiments and collected results (two input variables and three responses)

Input Variables (Coded Values)		**Experiments Results**		
x_1 (Time)	x_2 (Temperature)	y_1 (Yield)	y_2 (Viscosity)	y_3 (Molecular Weight)
-1	-1	76.5	62	2940
+1	-1	78	66	3680
-1	+1	77	60	3470
+1	+1	79.5	59	3890
-1.4142	0	75.6	71	3020
+1.4142	0	78.4	68	3360
0	-1.4142	77	57	3150
0	+1.4142	78.5	58	3630
0	0	79.9	72	3480
0	0	80.3	69	3200
0	0	80	68	3410
0	0	79.7	70	3290
0	0	79.8	71	3500

We assume all experiments were executed in the worst combination of uncertainty (environmental factors) in the problem. Note that for simplicity of the formulation, input variables are normalized in [-1, 1]. Here, the objectives are maximizing yield (LTB), minimizing molecular weight (STB), and keeping viscosity in relevant target point (NTB). The RSM is used to approximate the relationship between each response and input variables, over input/output data obtained by CCD design. The experiment results were evaluated in the Design Expert (V.10) software and the outputs. The second-order model of three responses are formulated as below:

$$y_1 = 79.94 + 0.99x_1 + 0.52x_2 + 0.25x_1x_2 - 1.38x_1^2 - 1.00x_2^2 \tag{24}$$

$$y_2 = 70.00 - 0.16x_1 - 0.95x_2 - 1.25x_1x_2 - 0.69x_1^2 - 6.69x_2^2 \tag{25}$$

$$y_3 = 3376.00 + 205.10x_1 + 177.35x_2 - 80.00x_1x_2 - 41.75x_1^2 + 58.25x_2^2 \tag{26}$$

The 3D surface and contour plot of responses are shown in Fig. 2.

Next, we add a physical constraint into a problem with the following inequality:

$$g(x_1, x_2): \quad -1.37x_1 - 3.25x_2 + 8.70x_1x_2 \leq 0 \tag{27}$$

The procedure of the collecting data from the production process is based on designing experiments which has been executed in the worst combinations of uncertainties (environmental variables), so, the maximum variation is imposed to each response. The procedure of robust optimization model (min-max method) has been followed in such a way that minimizes this variation (Ben-Tal et al., 2009).

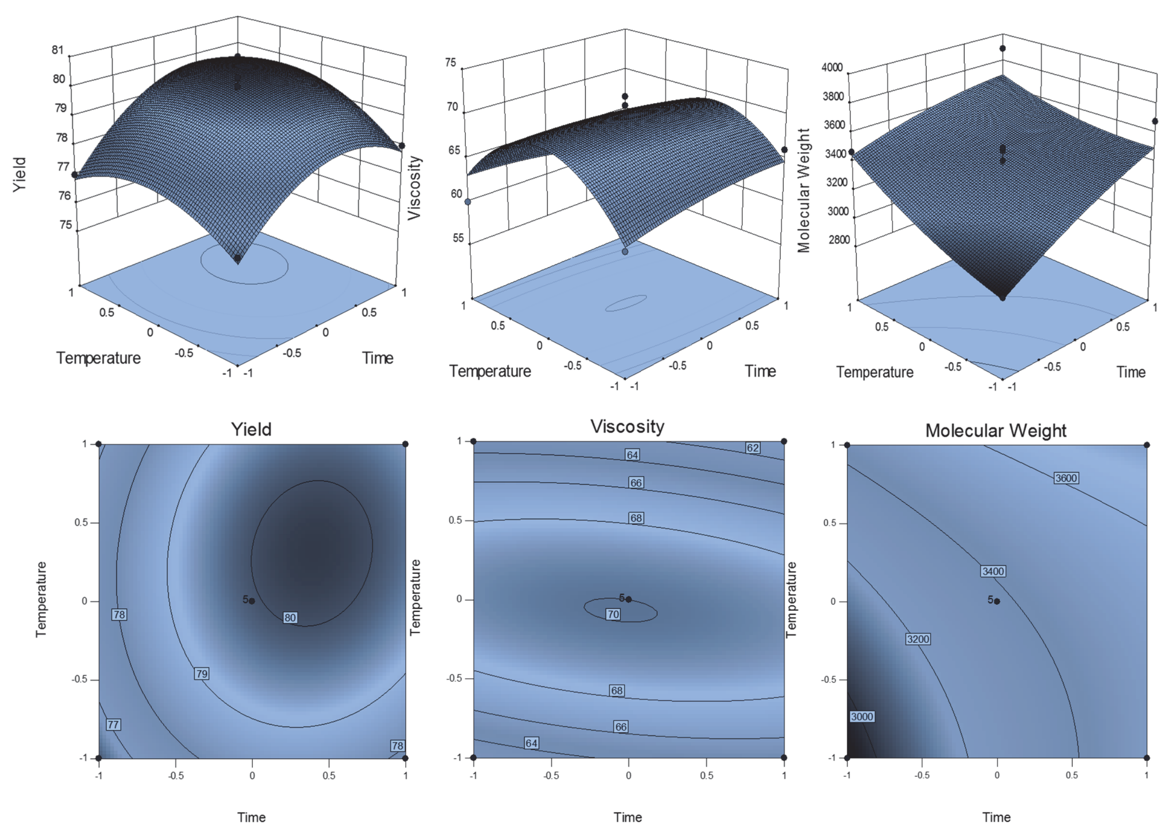

Fig. 2. The 3D surface and contour plot of three responses based on two input variables

We assume, due to the existence of noises in the process, each input variable is fluctuated around its exact value with a variance of 0.02 unit ($\sigma_1^2 = \sigma_2^2 = 0.02$). We assume there is no correlation between time and temperature, so $\sigma_{12} = \sigma_{21} = 0$. Moreover, regarding to Eq.(8) until Eq.(11), the mean and variance of each response are approximated as below:

$$E(y_1) = 79.92 + 0.99x_1 + 0.52x_2 + 0.25x_1x_2 - 1.38x_1^2 - 1.00x_2^2 \tag{28}$$

$$V(y_1) = 0.03 - 0.10x_1 - 0.03x_2 - 0.05x_1x_2 + 0.15x_1^2 + 0.08x_2^2 \tag{29}$$

$$E(y_2) = 69.93 - 0.16x_1 - 0.95x_2 - 1.25x_1x_2 - 0.69x_1^2 - 6.69x_2^2 \tag{30}$$

$$V(y_2) = 0.02 + 0.06x_1 + 0.52x_2 + 0.74x_1x_2 + 0.07x_1^2 + 3.61x_2^2 \tag{31}$$

$$E(y_3) = 3376.17 + 205.10x_1 + 177.35x_2 - 80.00x_1x_2 - 41.75x_1^2 + 58.25x_2^2 \tag{32}$$

$$V(y_3) = 1470.38 - 1252.55x_1 + 170.13x_2 - 105.60x_1x_2 + 267.45x_1^2 + 399.45x_2^2\,. \tag{33}$$

With the same procedure, the mean and variance of the constraint are defined as below:

$$E(g) = -1.37x_1 - 3.25x_2 + 8.70x_1x_2 \tag{34}$$

$$V(g) = 0.25 - 1.13x_1 - 0.48x_2 + 1.51x_1^2 + 1.51x_2^2 \tag{35}$$

We consider the production limitation for responses as $LSL_1 = 76$ for yield, $LSL_2 = 60$, $USL_2 = 70$ for viscosity, and $USL_3 = 3700$ for molecular weight. The upper specification of yield is assumed four times more than LSL_1, so $USL_1 = 304$, and $LSL_3 = 925$ that is four times less than upper specification. However, the target point for all cases is estimated by $T_k = (USL_k + LSL_k)/2\,; for\ k = 1,2,3$. As mentioned before, the quality loss coefficients play an important role for making the monetary scale in expected losses. In the current instance, we assume $C_1 = 6 \times 10^{-3}$, $C_2 = 8$, and $C_3 = 2 \times 10^{-4}$. Furthermore, the expected loss function for each three responses can be approximated by following functions:

$$\begin{aligned} L(y_1) = 6 \times 10^{-3}[&(79.92 + 0.99x_1 + 0.52x_2 + 0.25x_1x_2 - 1.38x_1^2 - 1.00x_2^2 - 190)^2 \\ &+ 0.03 - 0.10x_1 - 0.03x_2 - 0.05x_1x_2 + 0.15x_1^2 + 0.08x_2^2] \end{aligned} \tag{36}$$

$$\begin{aligned} L(y_2) = 8[&(69.93 - 0.16x_1 - 0.95x_2 - 1.25x_1x_2 - 0.69x_1^2 - 6.69x_2^2 - 65)^2 + 0.02 \\ &+ 0.06x_1 + 0.52x_2 + 0.74x_1x_2 + 0.07x_1^2 + 3.61x_2^2] \end{aligned} \tag{37}$$

$$\begin{aligned} L(y_3) = 2 \times 10^{-4}[&(3376.17 + 205.10x_1 + 177.35x_2 - 80.00x_1x_2 - 41.75x_1^2 \\ &+ 58.25x_2^2 - 2312.5)^2 + 1470.38 - 1252.55x_1 + 170.13x_2 \\ &- 105.60x_1x_2 + 267.45x_1^2 + 399.45x_2^2] \end{aligned} \tag{38}$$

If we assume $x_1, x_2 = 0$ for the current condition, so the capabilities are $C_{pm}^1 = 0.345, C_{pm}^2 = 0.338$ and $C_{pm}^3 = 0.435$. The decision maker wish to reach 20 percent improvement in the performance of process for each quality characteristic. Thus, the new goals for performances are $C_{pm}^1 = 0.432$ for yeild, $C_{pm}^2 = 0.422$, and $C_{pm}^3 = 0.543$. Thus, according to Eqs.(13-15), the target point for each response is computed as $\overline{L(y_1)}$ =51, $\overline{L(y_2)}$ =142, and $\overline{L(y_3)}$ =264. So, the overall objective with L_p metric method is formulated as follow:

$$U = [w_1(L(y_1) - 51)^p + w_2(L(y_2) - 142)^p + w_3(L(y_3) - 264)^p]^{\frac{1}{p}} \tag{39}$$

P= 2 is considered for this model to show the emphasizing of the model in the amount of deviation from the target point. The terms of w_1, w_2 and w_3 depict the importance of each response compared to others, and for the current instance we examine different combinations of w_1, w_2 and w_3.

Finally, the mathematical formulations of the problem are constructed based on the importance of cost compared to expected loss in the process. Let's consider in current instance the cost of mixture problem is followed by $f_{cost} = 120 + 50x_1 + 35x_2 - 15x_1x_2$, and total budget allocated to process for production is 150. Also, the penalty factors which associated to the physical constraint is $k = 2$.

Model I: Robust optimization model based on overall expected loss in the process

$$\min \quad U = [w_1(L(y_1) - 51)^2 + w_2(L(y_2) - 142)^2 + w_3(L(y_3) - 264)^2]^{\frac{1}{2}} \tag{40}$$

$$\text{subject to:} \quad 120 + 50x_1 + 35x_2 - 15x_1x_2 \leq 150 \tag{41}$$

$$-1.37x_1 - 3.25x_2 + 8.70x_1x_2 + 2\sqrt{0.25 - 1.13x_1 - 0.48x_2 + 1.51x_1^2 + 1.51x_2^2} \leq 0 \tag{42}$$

$$-0.98 \leq x_1, x_2 \leq 0.98 \tag{43}$$

Model II: robust optimization model based on production cost in the process:

$$\min \quad 120 + 50x_1 + 35x_2 - 15x_1x_2 \tag{44}$$

$$\text{subject to:} \quad U = [w_1(L(y_1) - 51)^2 + w_2(L(y_2) - 142)^2 + w_3(L(y_3) - 264)^2]^{\frac{1}{2}} \leq 20 \tag{45}$$

$$-1.37x_1 - 3.25x_2 + 8.70x_1x_2 + 2\sqrt{0.25 - 1.13x_1 - 0.48x_2 + 1.51x_1^2 + 1.51x_2^2} \leq 0 \tag{46}$$

$$-0.98 \leq x_1, x_2 \leq 0.98 \tag{47}$$

We assume the value of $D = 20$ for the upper bound of overall expected loss. This threshold can be settled based on the importance of the maximum distances between expected loss and the relevant target point for warranty the existence of feasible solutions.

Table 3
The results of model I based on different combination of weights

No	w_1	w_2	w_3	x_1	x_2	U	$L(y_1)$	$L(y_2)$	$L(y_3)$	Cost	Total
1	0.25	0.5	0.25	-0.122	0.98	59.475	73.530	72.343	326.756	149.993	622.623
2	0.25	0.25	0.5	-0.814	0.98	46.998	75.872	51.896	270.997	125.566	524.331
3	0.5	0	0.5	0.291	0.175	15.120	72.383	156.453	264.073	139.911	632.821
4	0.33	0.33	0.33	-0.098	0.957	59.479	73.437	61.354	324.912	150.002	609.704
5	0.75	0.25	0	-0.122	0.98	39.917	73.530	72.343	326.756	149.993	622.623
6	0.75	0	0.25	0.292	0.176	18.518	72.383	156.159	264.230	139.989	632.761
7	1	0	0	0.387	0.308	21.347	72.347	113.997	282.207	148.342	616.893
8	0	0	1	0.142	0.307	0.000	72.456	127.221	264.042	137.191	600.910
9	0	1	0	-0.122	0.98	69.648	73.530	72.343	326.756	149.993	622.623
10	0	0.75	0.25	-0.122	0.98	67.993	73.530	72.343	326.756	149.993	622.623
11	0	0.5	0.5	-0.685	0.98	63.654	75.298	53.885	282.426	130.120	541.729
12	0.5	0.5	0	-0.122	0.98	51.761	73.530	72.343	326.756	149.993	622.623
13	0.25	0.75	0	-0.122	0.98	61.359	73.530	72.343	326.756	149.993	622.623
14	0.25	0	0.75	0.291	0.175	10.692	72.383	156.453	264.073	139.911	632.821
15	**0**	**0.25**	**0.75**	**-0.85**	**0.98**	**45.383**	**76.044**	**51.464**	**267.736**	**124.295**	**519.539**
16	0.5	0.25	0.25	-0.627	0.98	48.148	75.061	55.015	287.426	132.167	549.669

5. Results and discussion

We have used MATLAB® optimization toolbox, "fmincon" function to solve both nonlinear mathematical formulations. The results of the both models for 16 different combinations of w_1, w_2 and w_3

have been compared in Tables 3 and Table 4 while $0 \leq w_1, w_2, w_3 \leq 1$ and $w_1 + w_2 + w_3 = 1$. As can be seen from the results, choosing the best solutions for this problem strongly depends on the appropriate combinations of weighting w_1, w_2 and w_3 which are determined by the decision maker. However, according to first model (see again Table 3), the best result can be achieved when the first expected loss has the zero weight, and second and third expected losses are 0.25 and 0.75, respectively. In this condition, the best result is $x_1 = -0.85, x_2 = 0.98$ and the minimum value is obtained for summation of cost and expected losses by 519.539. By turning to second model (see Table 4), the feasibility of solutions is strongly related to the value of D (upper bound of overall expected loss). It can be seen, a minimum total cost and losses is reached when $x_1 = -0.091$ and $x_2 = -0238$. In this point just the first objective proceeds in overall Lp metric function (i.e. the weight one is allocated to yield's expected loss and two others, viscosity and molecular weight are weighted zero). In general, in terms of lower total expected losses and production cost, the first model shows the better performance than the second model, while in term of robustness (i.e. variability of results due to changing in the weight combinations) the second model gives more robust results, see Fig. 3. Notably, the obtained results significantly depend on allocating magnitudes of D and B (total budget allocated to process) in model. It must be mentioned that input factor levels can determine how big of a change in the response can be gotten. Moreover, for the current instance to ensure the adequate change of each expected loss to be moved as close as possible to the target point, the bounds of changing in levels of input factors must be chosen far enough apart to make the adequate change in responses.

Table 4
The results of model II based on different combination of weights

No	w_1	w_2	w_3	x_1	x_2	Cost	$L(y_1)$	$L(y_2)$	$L(y_3)$	Total
1	0.25	0.5	0.25	-0.965	0.98	120.230	72.706	194.599	226.573	614.108
2	0.25	0.25	0.5	-0.158	0.347	125.075	72.706	194.599	226.573	618.953
3	0.5	0	0.5	-0.98	0.958	118.622	72.706	194.599	226.573	612.500
4	0.33	0.33	0.33	-0.163	0.348	124.863	72.706	194.599	226.573	618.741
5	0.75	0.25	0	-0.98	0.98	119.706	72.706	194.599	226.573	613.584
6	0.75	0	0.25	-0.412	0.649	126.105	72.706	194.599	226.573	619.983
7	**1**	**0**	**0**	**-0.091**	**-0.238**	**106.813**	**72.706**	**194.599**	**226.573**	**600.691**
8	0	0	1	-0.98	0.908	116.143	72.706	194.599	226.573	610.021
9	0	1	0	-0.252	0.361	121.391	72.706	194.599	226.573	615.268
10	0	0.75	0.25	-0.203	0.354	123.303	72.706	194.599	226.573	617.181
11	0	0.5	0.5	-0.18	0.351	124.205	72.706	194.599	226.573	618.083
12	0.5	0.5	0	-0.173	0.349	125.205	72.706	194.599	226.573	619.083
13	0.25	0.75	0	-0.223	0.357	122.542	72.706	194.599	226.573	616.420
14	0.25	0	0.75	-0.98	0.922	116.839	72.706	194.599	226.573	610.717
15	0	0.25	0.75	-0.165	0.348	124.787	72.706	194.599	226.573	618.665
16	0.5	0.25	0.25	-0.14	0.344	125.748	72.706	194.599	226.573	619.626

6. Conclusion

In current paper, a new production optimization model by integrating robust design and approximation method is proposed. This model is able to optimize different types of production processes with considering important circumstances which could be occurred repeatedly in practice. The proposed model handles the tradeoff between three aspects of production cost, robustness, and process performance. This model is able to investigate the best levels of design variables to cover model's requirements with at least computational cost. Robustness in physical constraints to satisfy customer's requirements and obligation to satisfy production tolerances are placed on the model's formulation. Note that, both the robustness of the objective functions and the constraints are considered simultaneously. Specialization and generalization of existing robust optimization models to be ease applied in the practice by attending other main parameters in production processes such as fuzzy conditions, dynamic objectives, and discrete and continues value of design variables can be suggested for feauture research. Also, applying other approxiamtion techniques such as Kriging, RBF, ANN can be interested for future research subjects.

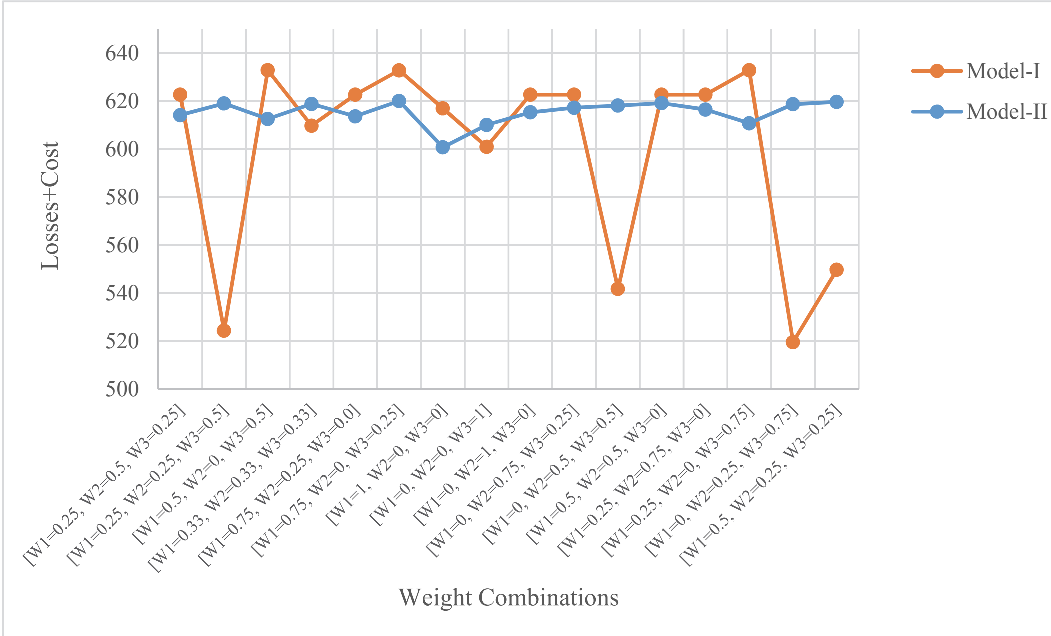

Fig. 3. Comparison of optimization results obtained by Model-I and Model-II according to 16 different combinations of allocated weights to three objectives based on Lp metric overall function method. The y-axis shows the total expected losses (i.e. objectives are designed based on Taguchi expected losses) and production cost that resulted from optimization models in each weight combination.

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