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Pricing model for instantaneous deteriorating items with partial backlogging and different demand rates

Hetal Patel*

U. V. Patel College of Engineering, Ganpat University, India

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A B S T R A C T

In this study, a single product is considered which starts to deteriorate with constant rate of replenishment and demand rate is time and price dependent exponential function. Shortage is allowed with partial back logging and the relationship between backorder rate and waiting time is considered to be exponential. The aim is to decide pricing strategy and maximize total average profit function. Total profit function is optimized analytically and proved to be concave function of price. Finally, numerical example is given to illustrate the implementation of the algorithm followed by the sensitivity analysis.

1. Introduction

Product deterioration is very critical issue in various systems using inventory (Bakker et al., 2012). Deterioration is considered as damage, vaporization, dryness, spoilage, etc. Blood bank, volatile liquids, medicine, food stuff are deteriorating inventory goods, which deteriorate during their storage period (Dye et al. 2007; Goyal & Giri, 2001). Loss due to deterioration cannot be negligible. Ghare and Schrader (1963) initiated the journey of studying deteriorating inventory product by developing a model for deteriorating inventory item with no shortage and constant deterioration rate. However, against the assumption of constant deterioration rate, Covert and Philip (1973) relaxed this assumption and developed a model by considering two-parameter Weibull distribution deterioration rate (Ouyang et al., 2006). The literature is further extended by Philip (1974) by taking two-parameter Weibull deterioration rate. Further, Aliyu and Boukas (1998) presented discrete-time inventory control problem with deterministic or stochastic demand for deteriorating items having variable deterioration rate. However, Chang and Dye (2001) described EOQ model taking varying deterioration rate of time and allowing permissible delay in payments. Apart, Maity and Maiti (2009) explained multi-item inventory model with real time examples having substitute and complimentary deteriorating items.

* Corresponding author
E-mail address: hrp07@ganpatuniversity.ac.in (H. Patel)

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Distinctively, Mishra and Shah (2008) modeled salvage value taking demand constant and two variable Weibull distribution function of time for varying deterioration rate, having no shortage. Ouyang et al. (2009) formulated EOQ policy assuming demand rate as constant and non-instantaneous deterioration rate as constant with no shortages. Allowing shortages reduces carrying costs and increases the cycle time. If shortage cost is less than carrying cost then lowering the average inventory level by permitting shortage, makes sense. This model allows shortages with partial backlogging. Li et al. (2007) formulated model by considering demand rate as constant and also the deterioration rate as constant having shortage with complete backlogging with postponement strategy. Taleizadeh and Nematollahi (2014) developed a model by allowing delay in payment, complete back logging with constant deterioration rate, and demand rate.

As constant demand is not possible in real and pricing decision is very critical for maximizing the profit, many researchers have adopted pricing strategy with different assumption and conditions. In this context, Abad (2001) developed an inventory model by taking demand as general function of price with time dependent deterioration and shortages are partially backordered. The backlogging rate sometimes behaves exponentially. Abad (2003) developed integrated pricing model allowing backlogging without calculating backorder cost and the lost sale cost. Teng et al. (2007) extended Abad’s (2003) model by calculating backlogging cost and lost sale cost in profit function. Shah et al. (2012) formulated integrated ordering and pricing policy with quadratic demand function of time and power function of price without allowing shortages and deterioration. Mukhopadhyay et al. (2004) computed demand rate as general function of price and deterioration rate as time dependent linear function without provision of shortages. Maihami and Abadi (2012) formulated pricing model by assuming demand as linear function of price and power function of time allowing partial backlogging for non-instantaneous deteriorating product.

Chang et al. (2006) gave pricing policy with constant deterioration rate for finite planning horizon allowing partially backlogging. Widyadanaa et al. (2011) considered finite planning horizon for instantaneous deterioration with planned backlogging. Furthermore, Chang et al. (2006) further examined the EOQ model by taking backorder rate in general form and importantly taken demand as stock dependent. The condition of partial backlogging was relaxed in a study by Dye et al. (2007) to develop pricing strategy by considering full backlogging. In fact, seasonality aspect was considered while developing EOQ model in a multi-echelon system with constant deterioration and partial backlogging. Still, studies performed have overlooked the situations when demand is stock dependent. Guchhait et al. (2013) formulated Lot sizing model with constant deterioration.

Distinctively, Panda et al. (2009) approached a model using selling price discounts along with demand as stock dependent. Wang and Huang (2014) constructed pricing model considering ramp-type dependent demand. Inventory dependent demand with constant rate of deterioration was considered in Tripathi and Mishra (2014) study. Farughi et al. (2014) modeled the inventory system for non-instantaneous deteriorating items where demand is linear function of price and exponential function of time with constant deterioration rate. They also allowed shortages partially with back order rate in fraction form. Kumar and Kumar (2016) studied the salvage worth and learning by considering partial shortages, Tripathi and Kaur (2017) considered time-shortages, which is non-increasing and interestingly since they assumed deterioration as time dependent, which is non-decreasing. Apart, Saha and Sen (2017) studied deterioration as probabilistic with backlogging and demand as negative exponential. Differently, Shah (2017) formulated model taking fixed lifetime with conditional trade credit, however Pandey et al. (2017) offered quantity discounts while, Rastogi et al. (2017) offered credit limits with case discount. Recently, Mashud et al. (2018) used products with different deterioration rates allowing shortages and demand as stock and price dependent.

Among all above literature, very few studies are offering pricing discount. In current study demand rate is different in various time interval where demand depends on price and time exponentially and discounts offering on price during shortages. Shortages are partially backlogged where back order rate is exponential function of waiting time. We consider price discounts and study the effect of weighting coefficient of price on total profit. Notations and assumptions are outlined in the next section. Then, total profit function is optimized theoretically and proved to be a concave function of price and time. Finally, procedure for solving a model is demonstrated through numerical analysis to illustrate algorithm and sensitivity analysis is presented.

2. Notations and assumptions

The assumptions with some notations are listed as follow:

2.1 Notations

p	selling price / unit (decision variable)
w	weighting coefficient $(0 < w \le 1)$
$D(p,t)$	demand function at time t for given p
c_p	purchasing cost /unit $(0 < c_p < p)$
t_1	point of time where inventory is zero (decision variable)
t_2	time duration of shortages (decision variable)
h	cost of holding / unit /unit time
K	cost of ordering / order
c_s	backorder cost / unit /unit time
o	cost of lost sales / unit
I_M	Level of maximum inventory at each cycle
Q	ordering quantity / cycle
S	maximum shortage
$I_1(t)$	inventory at time $t(0 \le t \le t_1)$ where deterioration exists
$I_2(t)$	inventory at time $t(0 \le t \le t_2)$ is negative

2.2 Assumptions

1. Single item instantaneous deterioration with constant rate θ, is considered.
2. Infinite replenishment rate is considered with finite order size.
3. $D(p,t)$ is a "demand function of selling price and time", and is computed by $D(p,t) = \begin{cases} d(p)f(t), & \text{if } 0 \le t \le t_1 \\ d(p_1), & \text{if } 0 \le t \le t_2 \end{cases}$ where $d(p) = ap^{-b}, f(t) = e^{-\varepsilon t}, p_1 = pw(0 < w \le 1)$
4. There is no provision for replacing or repairing of deteriorated units.
5. Backlogging rate is $\beta(x) = e^{-\delta x}$ as shortages are allowed, where x is the waiting time up to the next arrival.

3. Model Formulation

Let I_M units of items arrive at the inventory system at the beginning of replenishment cycle. The inventory level declines during time 0 to t_1, only due to demand rate and deterioration rate to be zero and shortages start during time 0 to t_2 which are backlogged partially. The process is repeated as mentioned above. The model is followed as per following Fig. 1.

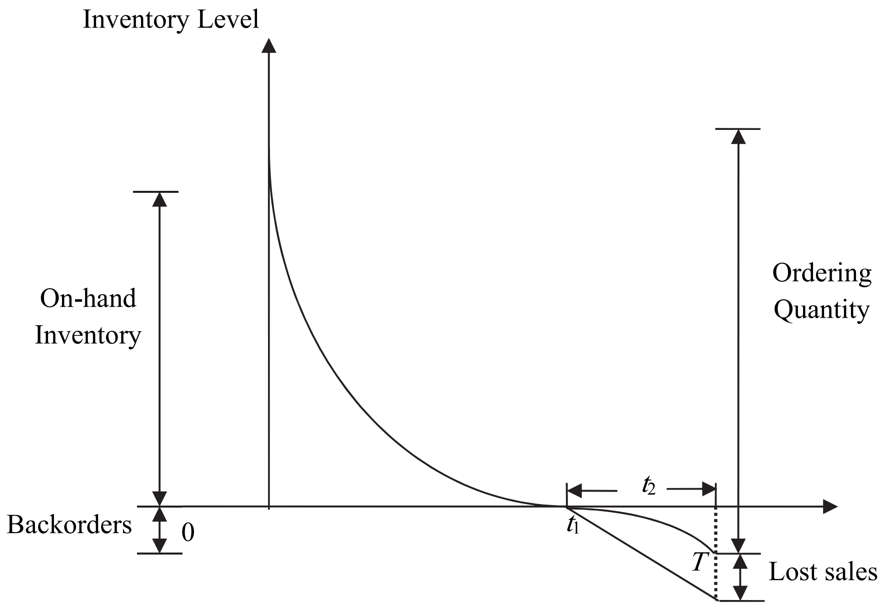

Fig. 1. The inventory system

As the nature of deteriorating inventory item, inventory model is characterized by following differential equation:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -ap^{-b}e^{-\varepsilon t}, \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -a(pw)^{-b}\,\mathrm{e}^{-\delta(t_2 - t)}, \quad 0 \le t \le t_2 \tag{2}$$

With terminal condition,

$$I_1(0) = I_M \text{ and } I_1(t_1) = 0 = I_2(t_1) \tag{3}$$

By solving equations (1) and (2), we get

$$I_1(t) = I_M + \frac{ap^{-b}\left(e^{-\theta t} - e^{-\varepsilon t}\right)}{(\theta - \varepsilon)}, \quad 0 \le t \le t_1 \tag{4}$$

$$I_2(t) = -\frac{a(pw)^{-b}}{\delta}\mathrm{e}^{-\delta t_2}\left(\mathrm{e}^{\delta t} - 1\right), \quad 0 \le t \le t_2 \tag{5}$$

Since $I_1(t_1) = I_2(t_1) = 0$, it follows from Eq. (3) and Eq. (4) that, (6) Here, maximum inventory level is $I_M = \frac{ap^{-b}\left(e^{-\varepsilon t_1} - e^{-\theta t_1}\right)}{(\theta-\varepsilon)}$. Put this value in Eq. (3), we get

$$I_1(t) = \frac{ap^{-b}\left(e^{-\varepsilon t_1} - e^{-\theta t_1}\right)}{(\theta-\varepsilon)} + \frac{ap^{-b}\left(e^{-\theta t} - e^{-\varepsilon t}\right)}{(\theta-\varepsilon)}, \quad 0 \le t \le t_1 \tag{7}$$

The maximum shortages is

$$S = -I_2(t_2) = \frac{a(pw)^{-b}}{\delta}\left(1 - \mathrm{e}^{-\delta t_2}\right) \tag{8}$$

Thus, the order quantity per order is

$$Q = I_M + S = \frac{ap^{-b}\left(e^{-\varepsilon t_1} - e^{-\theta t_1}\right)}{(\theta-\varepsilon)} + \frac{a(pw)^{-b}}{\delta}\left(1 - \mathrm{e}^{-\delta t_2}\right) \tag{9}$$

To compose profit function, following elements are needed:

- The ordering cost is $OC = K$
- The purchase cost is

$$PC = c_p Q = c_p\left[\frac{ap^{-b}\left(e^{-\varepsilon t_1} - e^{-\theta t_1}\right)}{(\theta-\varepsilon)} + \frac{a(pw)^{-b}}{\delta}\left(1 - e^{-\delta t_2}\right)\right]$$

- The holding cost is

$$HC = h\left[\int_0^{t_1} I_1(t)\,dt\right]$$
$$= -\frac{hap^{-b}e^{-(\theta+\varepsilon)t_1}}{(\theta-\varepsilon)\theta\varepsilon}\left[\theta t_1\varepsilon\left(e^{\varepsilon t_1} - e^{\theta t_1}\right) + e^{(\theta+\varepsilon)t_1}(\theta-\varepsilon) - \theta e^{\theta t_1} + \varepsilon e^{\varepsilon t_1}\right]$$

- Considering backlog, the cost of shortage is

$$SC = c_s\int_0^{t_2}\left[-I_2(t)\right]dt = \frac{c_s a(pw)^{-b} e^{-\delta t_2}\left(e^{\delta t_2} - \delta t_2 - 1\right)}{\delta^2}$$

- Realizing lost sales, the opportunity cost is computed as

$$LC = od(p_1)\int_0^{t_2}\left(1 - \beta(t_2 - t)\right)dt$$
$$= \frac{oa(pw)^{-b}}{\delta}\left(e^{-\delta t_2} + \delta t_2 - 1\right)$$

- The sales revenue is

$$SR = p\left[\int_0^{t_1} D(p,t)\,dt + S\right]$$
$$= p\left[-ap^{-b}\left(\frac{e^{-\varepsilon t_1} - 1}{\varepsilon}\right) - a(pw)^{-b}\left(\frac{e^{-\delta t_2} - 1}{\delta}\right)\right]$$

Gathering above element, the total average profit (denoted by $\Pi_A(p,t_1,t_2)$) is computed as,

$$\Pi_A(p,t_1,t_2) = \frac{\mu(p,t_1,t_2)}{(t_1+t_2)}, \tag{10}$$

where, $\mu(p,t_1,t_2)=SR-OC-PC-HC-SC-LC$

$$\mu(p,t_1,t_2)=p\left[-ap^{-b}\left(\frac{e^{-\varepsilon t_1}-1}{\varepsilon}\right)-a(pw)^{-b}\left(\frac{e^{-\delta t_2}-1}{\delta}\right)\right]-K-c_p\left[\frac{ap^{-b}\left(e^{-\varepsilon t_1}-e^{-\theta t_1}\right)}{(\theta-\varepsilon)}+\frac{a(pw)^{-b}}{\delta}\left(1-\mathrm{e}^{-\delta t_2}\right)\right]$$
$$+\frac{hap^{-b}e^{-(\theta+\varepsilon)t_1}}{(\theta-\varepsilon)\theta\varepsilon}\left[\theta t_1\varepsilon\left(e^{\varepsilon t_1}-e^{\theta t_1}\right)+e^{(\theta+\varepsilon)t_1}(\theta-\varepsilon)-\theta e^{\theta t_1}+\varepsilon e^{\varepsilon t_1}\right]-\frac{c_s a(pw)^{-b}e^{-\delta t_2}\left(e^{\delta t_2}-\delta t_2-1\right)}{\delta^2}$$
$$-\frac{oa(pw)^{-b}}{\delta}\left(e^{-\delta t_2}+\delta t_2-1\right) \tag{11}$$

Our optimization problem is to maximize total average profit function by optimizing decision variables p,t_1 and t_2. To prove concavity of total profit function, we follow methodology adopted by Sana (2010). To solve the problem we first find optimal value $\left(t_1^*,t_2^*\right)$ by keeping p fix and then we find optimal value p^*. Now to find optimal value $\left(t_1^*,t_2^*\right)$ we first proceed as below.

By keeping p fixed, taking first and second ordered partial derivatives of equation (8) on both sides with respect to t_1 & t_2 and using necessary condition of optimization $\partial\Pi_A(p,t_1,t_2)/\partial t_1=0$ and $\partial\Pi_A(p,t_1,t_2)/\partial t_2=0$, we have

$$\frac{\partial\mu(p,t_1,t_2)}{\partial t_1}=\frac{\partial\mu(p,t_1,t_2)}{\partial t_2} \tag{12}$$

Next, differentiating $\Pi(p,t_1,t_2)$ from equation (9) partially with respect to t_1 and t_2, one has

$$\frac{\partial\mu(p,t_1,t_2)}{\partial t_1}=\frac{ap^{-b}}{(\theta-\varepsilon)}\left[\left(ht_1+c_p\right)\left(\varepsilon e^{-\varepsilon t_1}-\theta e^{-\theta t_1}\right)+pe^{-\varepsilon t_1}(\theta-\varepsilon)\right] \tag{13}$$

$$\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_1^2}=\frac{-ap^{-b}}{(\theta-\varepsilon)}\left[\left(p(\theta-\varepsilon)\varepsilon e^{-\varepsilon t_1}\right)+\left(\left(ht_1+c_p\right)\varepsilon-h\right)\varepsilon e^{-\varepsilon t_1}-\left(\left(ht_1+c_p\right)\theta-h\right)\theta e^{-\theta t_1}\right] \tag{14}$$

$$\frac{\partial\mu(p,t_1,t_2)}{\partial t_2}=-a(pw)^{-b}\left(e^{-\delta t_2}\left(c_s t_2+c_p-p-o\right)+o\right) \tag{15}$$

$$\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_2^2}=-a(pw)^{-b}e^{-\delta t_2}\left[-\delta c_s t_2+c_s-c_p+o+p\right]$$
$$=-a(pw)^{-b}e^{-\delta t_2}\left[(1-\delta t_2)c_s+p-c_p+o\right] \tag{16}$$

From Eqs. (11-13),

$$\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_1\partial t_2}=0=\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_2\partial t_1} \tag{17}$$

From Eq. (10), we have

$$\frac{ap^{-b}}{(\theta-\varepsilon)}\left[\left(ht_1+c_p\right)\left(\varepsilon e^{-\varepsilon t_1}-\theta e^{-\theta t_1}\right)+pe^{-\varepsilon t_1}\left(\theta-\varepsilon\right)\right]=-a(pw)^{-b}\left(e^{-\delta t_2}\left(c_s t_2+c_p-p-o\right)+o\right) \quad (18)$$

Clearly $\phi(t_1)$=L.H.S. of Eq. (16) and $\varphi(t_2)$=R.H.S. of Eq. (16) is function of t_1 and t_2 respectively. Since $\varphi(t_2)=-a(pw)^{-b}\left(e^{-\delta t_2}\left(c_s t_2+c_p-p-o\right)+o\right)$, differentiating with respect to t_2,

$$\frac{d\varphi(t_2)}{dt_2}=\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_2^2}=-a(pw)^{-b}e^{-\delta t_2}\left[(1-\delta t_2)c_s+p-c_p+o\right]\leq 0$$

Since $\left(1-\delta t_2\right)c_s+p-c_p+o\geq 0\Rightarrow t_2\leq\dfrac{p-c_p+o+c_s}{\delta s}=\tilde{t}_2$ (say)

Therefore $\varphi(t_2)$ is decreasing function of $t_2\in\left(0,\tilde{t}_2\right)$ and increasing function for $t_2\in\left(\tilde{t}_2,\infty\right)$. Hence $\varphi_{\min}=\varphi\left(\tilde{t}_2\right)$ can be found. Besides, $\phi(t_1)=\dfrac{ap^{-b}}{(\theta-\varepsilon)}\left[\left(ht_1+c_p\right)\left(\varepsilon e^{-\varepsilon t_1}-\theta e^{-\theta t_1}\right)+pe^{-\varepsilon t_1}\left(\theta-\varepsilon\right)\right]$ and

$$\frac{d\phi(t_1)}{dt_1}=\frac{\partial^2\mu(p,t_1,t_2)}{\partial t_1^2}=\frac{-ap^{-b}}{(\theta-\varepsilon)}\left[\left(p(\theta-\varepsilon)\varepsilon e^{-\varepsilon t_1}\right)+\left(\left(ht_1+c_p\right)\varepsilon-h\right)\varepsilon e^{-\varepsilon t_1}-\left(\left(ht_1+c_p\right)\theta-h\right)\theta e^{-\theta t_1}\right]$$

<0 using Taylor series expansion and neglecting higher terms.

$\phi(t_1)$ is decreasing function of t_1. Therefore there exists a unique $\tilde{t}_1$ such that $\phi\left(\tilde{t}_1\right)=\varphi_{\min}$.

Hence for any given $t_2^*\in\left(0,\tilde{t}_2\right)\exists$ a unique $t_1^*\in\left(0,\tilde{t}_1\right)$ such that $\phi\left(t_1^*\right)=\varphi\left(t_2^*\right)$. Consequently t_1 can be uniquely determined as a function of t_2(Vidovic & Kim, 2006). Also from Eq. (12), Eq. (14) and Eq. (15);

$$\left(\left.\frac{\partial^2\Pi_A(t_1,t_2)}{\partial t_1^2}\right|_{(t_1^*,t_2^*)}\right)\times\left(\left.\frac{\partial^2\Pi_A(t_1,t_2)}{\partial t_2^2}\right|_{(t_1^*,t_2^*)}\right)-\left[\left.\frac{\partial^2\Pi_A(t_1,t_2)}{\partial t_1\partial t_2}\right|_{(t_1^*,t_2^*)}\right]^2>0$$

Hence, the Hessian matrix at point $\left(t_1^*,t_2^*\right)$ is negative definite. So obtained solution $\left(t_1^*,t_2^*\right)$ is optimal for given p. Now for solving pricing problem, for any given $\left(t_1^*,t_2^*\right)$, the necessary condition for $\Pi_A\left(p,t_1^*,t_2^*\right)$ to be maximum at point p^* is, let $\dfrac{\partial\mu\left(p,t_1^*,t_2^*\right)}{\partial p}=0$ and solve for p^* and $\dfrac{\partial^2\mu\left(p,t_1^*,t_2^*\right)}{\partial p^2}<0$.

Here

$$\frac{\partial\mu\left(p,t_1^*,t_2^*\right)}{\partial p}=$$

$$\frac{1}{\varepsilon\delta^2p(\theta-\varepsilon)\theta}\left\{\begin{array}{l}ap^{-b}bh\delta^2\left[\theta e^{-\varepsilon t_1}\left(1+\varepsilon t_1\right)-\theta+\varepsilon-\varepsilon e^{-\theta t_1}\left(1+\theta t_1\right)\right]+a(pw)^{-b}\theta\varepsilon(\theta-\varepsilon)\\ \left[bc_s-bc_se^{-\delta t_2}+\delta e^{-\delta t_2}\left(b\left(p-c_p+o-c_st_2\right)-p\right)+\delta^2t_2bo-\delta b\left(p-c_p+o\right)+\delta p\right]\\ +ap^{-b}\theta\delta^2\left[p(b-1)(\theta-\varepsilon)\left(e^{-\varepsilon t_1}-1\right)+\varepsilon c_pb\left(e^{-\varepsilon t_1}-e^{-\theta t_1}\right)\right]\end{array}\right\}$$

Using Taylor series expansion and neglecting higher terms,

$$\frac{\partial\mu\left(p,t_1^*,t_2^*\right)}{\partial p}=$$

$$\frac{1}{\varepsilon\delta^2 p(\theta-\varepsilon)\theta}\left\{\begin{array}{l} ap^{-b}bh\delta^2\left[\theta\left(1-\varepsilon^2t_1^2\right)-\theta+\varepsilon-\varepsilon\left(1-\theta^2t_1^2\right)\right]+a(pw)^{-b}\theta\varepsilon(\theta-\varepsilon) \\ \left[bc_s-bc_s(1-\delta t_2)+\delta(1-\delta t_2)\left(b\left(p-c_p+o-c_st_2\right)-p\right)+\delta^2t_2bo-\delta b\left(p-c_p+o\right)+\delta p\right] \\ +ap^{-b}\theta\delta^2\left[p(b-1)(\theta-\varepsilon)\left((1-\varepsilon t_1)-1\right)+\varepsilon c_pb\left((1-\varepsilon t_1)-(1-\theta t_1)\right)\right] \end{array}\right\}$$

$$=\frac{1}{\varepsilon\delta^2 p(\theta-\varepsilon)\theta}\left\{\begin{array}{l} ap^{-b}bh\delta^2\theta\varepsilon t_1^2(\theta-\varepsilon)+a(pw)^{-b}\theta\varepsilon(\theta-\varepsilon) \\ \left[bc_s\delta t_2-\delta c_st_2-\delta^2t_2\left(b\left(p-c_p-c_st_2\right)-p\right)\right] \\ +ap^{-b}\theta\delta^2\left[-\varepsilon t_1p(b-1)(\theta-\varepsilon)+\varepsilon c_pb(\theta-\varepsilon)t_1\right] \end{array}\right\}$$

$$\frac{\partial\mu\left(p,t_1^*,t_2^*\right)}{\partial p}=\frac{p^{-(b+1)}}{\delta}\left\{\begin{array}{l} aw^{-b}\left[bc_st_2-c_st_2-\delta t_2\left(b\left(p-c_p-c_st_2\right)-p\right)\right] \\ +a\delta\left[t_1p-t_1b\left(p-c_p\right)\right]+abh\delta t_1^2 \end{array}\right\} \tag{19}$$

On extension, resulting second order derivation of $\Pi\left(p,t_1^*,t_2^*\right)$ w.r.t. p is

$$\frac{\partial\mu^2\left(p,t_1^*,t_2^*\right)}{\partial p^2}=-(b+1)p^{-(b+2)}\left\{\begin{array}{l} aw^{-b}\left[\frac{c_s}{\delta}t_2(b-1)-t_2b\left(p-c_p\right)+bc_st_2^2+t_2p\right]+ \\ \left[at_1p-at_1b\left(p-c_p\right)\right]+abht_1^2 \end{array}\right\}$$
$$+ap^{-(b+1)}\left\{-w^{-b}t_2(b-1)+t_1(1-b)\right\}$$

This is less than zero. Hence $\mu\left(p,t_1^*,t_2^*\right)$ behaves concavely w.r.t. p for a given t_1^*,t_2^*. Hence optimal value p^* is obtained from equation (17) equating with zero and it is unique. Consequently $\Pi_A\left(p,t_1,t_2\right)$ is optimized at $\left(p^*,t_1^*,t_2^*\right)$.

4. Algorithm for solution

The optimal solution $\left(p^*,t_1^*,t_2^*\right)$ of the problem is attained by applying following four-step algorithm:

Step 1: Start from $j=0$. Then initialize the value of $p_j=p_1$.

Step 2: Equating Eq. (11) and Eq. (13) with zero to find optimal value $\left(t_1^*,t_2^*\right)$ for a given price p_j.

Step 3: Use the result in step 2 to find p_{j+1} by Eq. (25).

Step 4: If (p_j - p_{j+1}) is significantly less, then optimal solution is $\left(p^*,t_1^*,t_2^*\right)$ and the process ends. If not, then set $j=j+1$ and repeat second step.

Hence using $\left(p^*,t_1^*,t_2^*\right)$, we can get optimal Q^* from Eq. (7).

Analytical proof is completed and is illustrated by following numerical example for better understanding.

5. Numerical example

Here deterioration rate is constant and demand is price and time dependent. Back order rate is exponential function of waiting time. Parameter values are given as below to find decision variables.

$K = \$250$ per order, $a = 400000, b = 3.5, c_p = \$3$ per unit, $\delta = 0.9, \varepsilon = 0.96, h = \$0.4$ per unit per year, $o = \$4$ per unit, $c_s = \$0.1$ per unit per year, $\theta = 0.1$

Different discount w is given on selling price and its effect on decision variable is given as below.

Table 1
Effect of weighting coefficient on decision variable

w	p	t_1	t_2	Q	$\Pi_A(p,t_1,t_2)$
0.98	4.27	0.4005	0.1020	746	2134.08
0.95	4.30	0.3645	0.1174	746	2203.53
0.9	4.37	0.2799	0.1412	746	2377.30
0.85	4.49	0 .1426	0.1620	746	2690.59
0.83	4.58	0.0532	0.1689	746	2911.05

Table 1 shows that when discount decrease, price increase and therefore profit increase.

6. Sensitivity analysis

Sensitivity is exhibited to know the effect of parameters on decision variable making change in one parameter by +40%,+20%,0%, -20% and -40% in original value as given in numerical example where $w = 0.95$ and remaining parameters are constant. The results are shown in Table 2. From the results, we have the following observations,

(1) Optimal selling price p^* will decrease when system parameters increases except for ε, K, h. However, p^* remains constant for change in c_s. Admittedly, p^* is highly positive sensitive to c_p and strongly negative sensitive to b. Rest changes are negligible.

(2) It is noted that cycle length (t_1^*) is positively related to $K, b, \delta, o, c_s, \theta$ and c_p and negatively related to a, ε and h. Moreover, (t_1^*) is positive sensitive to b and c_p.

(3) When the values of parameters K, b, ε, h and c_p increase, the cycle length (t_2^*) increases, and parameters a, δ, o, θ and c_s increase, the cycle length (t_2^*) decreases.

(4) It is also observed that optimal total profit per unit time (Π_A^*) is positively related to a, θ and negatively related to $K, b, \delta, o, \varepsilon, h, c_s$ and c_p. Admittedly, it is noteworthy that Π_A^* is highly sensitive to b and c_p. Therefore decision maker should estimate b and c_p very carefully.

(5) Order quantity remains same with changes in all parameters.

Table 2
Sensitive analysis with respect to model parameters

Parameter	Change (%)	Value	p^*	t_1^*	t_2^*	Q^*	$\Pi_4(p,t_1,t_2)$
K	-40	150	4.29	0.2551	0.0981	746	2443.84
	-20	200	4.30	0.3120	0.1084	746	2314.44
	0	250	4.30	0.3645	0.1174	746	2203.53
	20	300	4.31	0.4141	0.1254	746	2105.56
	40	350	4.31	0.4617	0.1327	746	2017.33
a	-40	240000	4.31	0.5233	0.1415	746	1146.82
	-20	320000	4.31	0.4261	0.1273	746	1666.15
	0	400000	4.30	0.3645	0.1174	746	2203.53
	20	480000	4.30	0.3210	0.1100	746	2753.84
	40	560000	4.30	0.2883	0.1042	746	3314.04
b	-40	2.1	5.88	0.0683	0.0762	746	26083.52
	-20	2.8	4.78	0.1735	0.0954	746	7495.46
	0	3.5	4.30	0.3645	0.1174	746	2203.53
	20	4.2	4.02	0.7822	0.1406	746	589.42
	40	4.9	3.86	1.0000	0.1754	746	92.52
δ	-40	0.54	4.33	0.3099	0.1814	746	2315.46
	-20	0.72	4.32	0.3423	0.1426	746	2247.95
	0	0.9	4.30	0.3645	0.1174	746	2203.53
	20	1.08	4.29	0.3805	0.0996	746	2172.16
	40	1.26	4.28	0.3926	0.0865	746	2148.85
ε	-40	0.576	4.28	0.4798	0.1037	746	2357.37
	-20	0.768	4.29	0.4122	0.1110	746	2272.98
	0	0.96	4.30	0.3645	0.1174	746	2203.53
	20	1.152	4.32	0.3284	0.1230	746	2144.82
	40	1.344	4.33	0.2998	0.1280	746	2094.23
h	-40	0.24	4.26	0.3956	0.1113	746	2247.47
	-20	0.32	4.28	0.3793	0.1144	746	2224.81
	0	0.4	4.30	0.3645	0.1174	746	2203.53
	20	0.48	4.32	0.3511	0.1201	746	2183.47
	40	0.56	4.34	0.3388	0.1227	746	2164.51
c	-40	2.4	4.31	0.3266	0.1622	746	2281.81
	-20	3.2	4.31	0.3481	0.1363	746	2236.72
	0	4	4.30	0.3645	0.1174	746	2203.53
	20	4.8	4.30	0.3773	0.1029	746	2178.13
	40	5.6	4.29	0.3876	0.0916	746	2158.10
c_s	-40	0.06	4.30	0.3637	0.1182	746	2205.09
	-20	0.08	4.30	0.3641	0.1178	746	2204.31
	0	0.1	4.30	0.3645	0.1174	746	2203.53
	20	0.12	4.30	0.3649	0.1169	746	2202.76
	40	0.14	4.30	0.3652	0.1165	746	2202.00
θ	-40	0.06	4.33	0.3392	0.1220	746	2169.47
	-20	0.08	4.32	0.3512	0.1198	746	2186.04
	0	0.1	4.30	0.3645	0.1174	746	2203.53
	20	0.12	4.29	0.3793	0.1148	746	2222.06
	40	0.14	4.27	0.3961	0.1120	746	2241.76
c_p	-40	1.8	3.20	0.1513	0.0860	746	8082.13
	-20	2.4	3.44	0.2480	0.0842	746	4216.52
	0	3	4.30	0.3645	0.1174	746	2203.53
	20	3.6	5.16	0.5019	0.1542	746	1265.11
	40	4.2	6.00	0.6652	0.1941	746	772.45

7. Conclusion and future scope

In this study, an inventory system with a single instantaneous deteriorating product was modeled. Two points had been considered: first, demand depends on price and time, which is power function of price & time and second, deterioration rate is constant function. Importantly, price discounts have been given and allowed partially backlogging which is an exponential function of waiting time. Numerical example shows the effect of weighting coefficient on profit. Study demonstrates that price discounts significantly increasing the profits. Sensitivity analysis was carried out to show critical parameters and offer managerial insights. One can extend the model for non-instantaneous deteriorating item with stock dependent demand.

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