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Pricing and inventory decisions in a vendor managed inventory system with revenue sharing contract

Alireza Haji[a*], Maryam Afzalabadi[a] and Rasoul Haji[a]

[a]*Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran*

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A B S T R A C T

In this paper, we design a revenue sharing contract to coordinate pricing and inventory control decisions in a serial supply chain consisting of one supplier, one manufacturer and one retailer. We assume that the retailer faces Poisson demand and his unsatisfied demands will be lost. The retailer applies one-for-one period policy in which he constantly places an order for one unit of product to the manufacturer in a predetermined time interval which results in a deterministic demand for the manufacturer. Solution procedures are developed to find the equilibrium in the Vendor Managed Inventory (VMI) program with a revenue sharing contract, in which no party is willing to deviate from for the sake of maximizing his own profit. Furthermore, we formulate the total profit of the system and obtain the optimal retail price and order cycle for the retailer and the inventory policy of manufacturer which maximizes the supply chain's total profit and at the end we will highlight the revenue sharing contract performance.

1. Introduction

Interaction between supply chain members has been extensively discussed in the past. There has been a vast body of work on centralized and decentralized inventory systems. Bernstein and Federgruen (2005) studied the equilibrium behaviour of a decentralized supply chain with competing retailers under demand uncertainty. Cachon and Zipkin (1999) showed that efficiency is reduced when there is a competition in a two-stage serial supply chain. They also explained that the value of cooperation was more context specific. Guan and Zhao (2010) formulated models to optimize sales volumes in a multi-retailer system operated on an infinite time horizon with stochastic demands. Considering cournot competition, they simultaneously optimized the expected sales volumes and (r, Q) policies for all retailers.

In the traditional supply chain, it is the retailer who controls his own inventory and places orders to the supplier. In Vendor Managed Inventory (VMI) systems, it is the supplier who controls the retailers' inventories and makes decisions on when and how to replenish. In recent years, contract designs for VMI programs are recognized to be an important issue. Yu et al. (2009) studied a VMI system of a

* Corresponding author.
E-mail address: ahaji@sharif.edu (A. Haji)

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single manufacturer and several retailers and they provided a solution for maximizing profits of both the manufacturer and retailers. Yu et al. (2013) extended their work to find the best strategy to select retailers by providing a nonlinear, mixed-integer, game-theoretic and analytically intractable Stackelberg game model to help the manufacturer to optimally select his retailers.

Capacity-constrained manufacturer in a VMI system was the focus of study in Almehdawe and Mantin (2010). They compared dominance of retailers and the manufacturer and showed that the retailer dominance, in general, results in higher supply chain efficiency and the highest overall efficiency is when the leader has the lowest market scale among other retailers.

Mateen et al. (2015) provided formulations for minimizing the expected total cost for the VMI when overstocking is penalized. They also studied four different VMI models in Palsule-Desai (2013) and formulated the costs for different aspects of inventory synchronization such as replenishment policy and batch size in product delivery and showed that the all VMI models have less cost than retailer-controlled system. Within VMI models, the one which uses sub-batch size deliveries is proven to have less cost. While, Hoque (2011) follows the same approach in his research, Hariga et al. (2014) allow unequal shipments to retailers. Hariga et al. (2013) showed that taking equal replenishment intervals as the base solution, an algorithm could be devised to find a near-optimal solution for unequal intervals in an iterative approach.

Lee et al. (2014) studied a VMI contract with (r, Q) policy and showed that although VMI may result in significant cost savings for both parties, the retailer may not always benefit from VMI, especially when the ratio of the supplier's fixed cost to that of the retailer's is small and/or when the physical storage cost is relatively large compared with the cost of capital.

In this paper, we focus on the contract design for VMI programs. In contrast, many papers study the design of contracts for non-VMI modes, e.g., retailer managed inventory, or RMI for short. For instance, Krichen et al. (2011) discussed a retailer-managed inventory system in which supplier proposes discounts and delay in payments based on the quantity of orders. They proposed a decision rule that generates preferred coalitions for each retailer to reduce the number of explored coalition structures. In some papers, the supply chain efficiency has been studied under different inventory management structures. Govindan (2015) compared VMI and non-VMI systems using pharmaceutical industry data and showed that the VMI system performs better than non-VMI system.

In order to satisfy customers' heightened expectations, the enterprises increasingly find that they must rely on effective supply chains. A non-efficient supply chain may carry a high cost. Consequently, supply chain performance largely depends on the coordination of materials, information, and financial flows of several separated firms. The supply chain members are autonomous and primarily concerned with optimizing their own objectives. This self-interest may result in poor performance. In order to align each member's objective with the supply chain's objective, contracting is a way to coordinate supply chain members such that optimal performance is achievable. Contract characterizes the information and financial flow and specifies the duties and rights of each member which coordinate contractual members by set of transfer payments. In the research literature on supply chain management, coordination issues have attracted an extensive research. Some researchers have studied coordination problem by contract.

Since video cassette rental issues were raised by BlockBuster, Inc., revenue sharing contracts have come to spotlight. Providing a contract for sharing profits between retailers and manufacturers changes the inventory parameters drastically. Although it may seem that video cassette rental business is not the case anymore, revenue sharing contract has been used as the basis of many industries such as mobile application, aftermarket sales of automotive parts, online retailers, electronic home appliances and health and medicine products. Cachon et al. (2005) comprehensively investigate the use of revenue-sharing for coordination of the distribution channel. They show that revenue-sharing induces the retailer to choose optimal actions for the supply chain (quantity and price) and distributes the channel profits between the supplier and the retailer. Kunter (2012) showed that channel coordination requires cost and revenue sharing via a sharing rate and marketing effort participation on both manufacturer and retailer level and efficiency requires that a retailer's participation of at least 50% in the manufacturer's cost of marketing effort. Wang et al. (2004) demonstrated that under a revenue sharing contract between a supplier and a manufacturer where supplier decides on the retail price and delivery quantity for this product, both channel performance and the performance of individual firm, rely critically on demand price elasticity and on the retailer's share of channel cost. Yao et al. (2008) explained that in case of competitive retailers, the provision of revenue-sharing in the contract could provide better performance than a price-only contract. Liu et al. (2013) presented a nonlinear programming framework to provide a fair revenue-sharing solutions between logistic service integrator and functional service provider when they face stochastic demand condition. Palsule-Desai (2013) compared revenue-dependant revenue-sharing contracts with revenue-independent revenue-sharing contracts and explained that there were some situations in which revenue-dependant contracts outperform revenue-independent contracts. El Ouardighi (2014) extended the revenue-sharing idea to collaboration of manufacturer and supplier to improve the quality of the product. Chakraborty et al. (2015) studied comparing revenue-sharing contracts with wholesale price in common retailer channel and concluded that when common retailer has the lead role, it is beneficial for retailer to offer salvage revenue-sharing contract. When manufacturers are leaders, then it is beneficial for the manufacturers to provide wholesale price contract although revenue-sharing contract contributes to the channel performance. Zhang et al. (2015) concentrated on how to improve the efficiency of a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. Guan et al. (2011) showed that in a VMI system with (r, Q) policy, when there is a franchising contract for retailer with ownership, the system achieves the same performance as in centralized control.

Besides the supply chain structure and the kind of interactions and coordination between supply chain members, the replenishment policy has significant effect on supply chain efficiency. In many cases, by choosing suitable combination of replenishment policy and coordination mechanism we may increase supply chain efficiency tremendously and even in some cases it may lead to perfect coordination. Supply chain environmental conditions such as demand function, shortage type and the cost functions are some substantial parameters for each member to choose the suitable inventory policy.

Finding replenishment policy for the supply chain in which the unsatisfied demand lost is studied by a few papers. Bijvank and Vis (2011) reviewed and classified review policies for lost-sales inventory system and reported that for continuous reviews, comparing (s, Q) policies, (S-1, S) policies and (s, S) policies, most of the work has been done so far was on (s, Q); although still not much is known about an optimal replenishment policy when excess demand is lost. Andersson and Melchiors (2001) proposed a heuristic for cost-effective base-stock policy to prevent stock outs when both warehouse and all retailers use (S-1, S) review policy. They showed that the cost of the policies is just 0.4% above optimal policy. Seifbarghy and Jokar (2006) developed an approximate cost function to find the optimal re-order points for given batch sizes in all installations when retailers are working under continuous review inventory policy (r, Q).

While it is well known that in case of stochastic demand, there is no inventory control policy in which both the order size and the order interval are constant with an optimal solution, Haji and Haji (2007) and later Haji et al. (2009) proposed a new One-for-One period policy, in which one unit of product is ordered in each ordering period and so eliminating uncertainty in demand for suppliers. They derived the long-run average total inventory cost, consisting of holding and shortage costs in terms of the average inventory and showed that their cost function has a unique solution and they also formulated the optimal value of the time interval between two consecutive orders. For the one for one period policy, discussed in Haji and Haji (2007) an order for one unit of item is placed in a pre-determined time interval. Hence, both the order size and the order interval are constant. As such, this policy prevents expanding the demand uncertainty for supplier. That is, the demand for the supplier is deterministic, one unit every T units of time.

In our study, we consider a supply chain consisting of a single manufacturer and a single retailer facing stochastic demand. Clark and Scarf (1960), Federgruen and Zipkin (1984), Chen and Zheng (1994) and Cachon and Zipkin (1999) demonstrate that an echelon base stock policy is the optimal policy in this system. Furthermore, Haji and Haji (2007) compared the performance of base stock policy and one for one period (1,T) policy and established advantages for the new policy as follows:

1- The safety stock in supplier is eliminated. (cost reduction)
2- Shortage cost in supplier due to uncertainty in demand is eliminated.
3- Information exchange cost for supplier due to the elimination of certainty of its demand is eliminated.
4- Inventory control and production planning in supplier are simplified.
5- This policy is very easy to apply.

Utilizing these advantages, we use one-for-one period ordering (1,T) policy for the retailer. We design a revenue sharing contract for the VMI program to improve the supply chain efficiency and maximize its total profit. We formulate the centralised control and the VMI program of a serial supply chain consisting of a supplier, a manufacturer and a retailer. We show that the revenue sharing contract will lead to perfect coordination of the supply chain.

2. Problem statement

We study a revenue sharing contract and its further improvement for a serial supply chain consisting of a supplier, a manufacturer and a retailer as shown in Fig. 1.

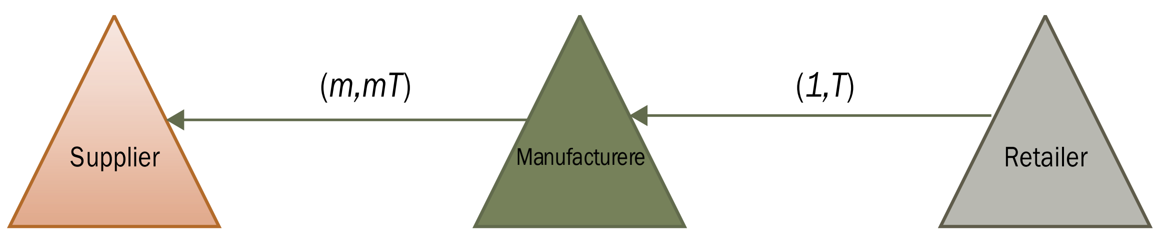

Fig. 1. A serial supply chain consisting of a supplier, a manufacturer and a retailer

The manufacturers complete the order by delivering the goods to retailer facing stochastic demand. The demand rate is a function of retailer's price. The retailer uses a new ordering policy called one-for-one period ordering $(1, T)$ policy in which an order of size one is placed at every fixed cycle time T which results in a deterministic demand for the manufacturer. The manufacturer must obtain the optimum ordering times of the retailer and his own to satisfy the orders of retailer. That is, at every Tunits of time he delivers 1 unit of the product to the retailer and at every mT units of time he places an order (of size m) to the outside supplier. Furthermore, since the lead time to the manufacturer from the outside supplier is constant, to further reduce his holding cost due to satisfying the retailer's orders, he places his orders to outside supplier in such a way that the arrival times of the orders coincide with the arrival times of the retailer's orders.

Since the decision makers are independent, the manufacturers and the retailer will decide to maximize their own profit rate. Moreover, the retailer determines the optimal retail price with the objective of maximizing his profit rate. Since the retail price affects the demand and consequently affects the manufacturers' operational profit, the manufacturers should consider the retailer's reaction in their decision. Besides, the decision of manufacturers on his wholesale price affects the retailer's costs and consequently its profit. So, both the manufacturer and the retailer should consider each other's decision in their objective.

To facilitate comparisons, we analyze the system performance in centralized control, which plays the role of benchmark. Then we will investigate coordination of the supply chain through a revenue sharing contract in a VMI program in which the retailer agrees to give a percentage of his revenue to the manufacturer. Because the manufacturer is responsible for supply chain inventory levels and incurs the related costs from the fluctuations of these levels, the manufacturer decides about the inventory policy of the supply chain.

2.1 Assumptions

The research problem can be defined by the following assumptions:

1) The fixed ordering cost is zero or negligible at the retailor.
2) The demand process is Poisson and unsatisfied demands will be lost at the retailer.
3) The demand rate is a function of retail price.
4) The replenishment lead time is constant both at the retailer and the manufacturer.
5) The transportation time of each order placed by the retailer is assumed to be constant.
6) Shortage is not allowed for the manufacturer.
7) The transportation time from the supplier to the manufacturer is constant and the supplier has enough stock that would never face shortages.

To satisfy the retailer's demand, the manufacturer should determine times and quantities of his orders to supplier, such that the retailer's demand is satisfied on time and his total inventory cost is minimized.

2.2 Notation

We introduce the following notations:

P: The product sales price to customer

$\bar{P}$: The maximum product sales price to customer

$\mu(P)$: Demand rate of retailer as the function of her retail price P, $\mu(p) = KP^{-e_p}$

K: A constant in the demand function of retailer, which represents her market scale

e_p: Price sensitivity factor in the retailer's demand function

π: Unit cost of a lost sale at the retailer.

h_r: Holding cost of a product unit during one period at the retailer

T: Ordering cycle of the retailer

I: Average inventory level at the retailer

NP_R: Expected net profit for the retailer

P_m: The wholesale price of the manufacturer

C_m: Manufacturing cost of the product

A: Ordering cost for the manufacturer

h_m: Holding cost of a product unit during one period in manufacturer's warehouse

m: Integer value which determines the ordering cycle of the manufacturer to the supplier (mT)

φ: The percentage of the revenue the retailer keeps. The percentage $(1-\varphi)$ is delivered to the manufacturer. φ is determined through bargaining between the two parties, $\varphi \epsilon [0,1]$

NP_m: Expected net profit of the manufacturer

NP_{sc}: Total net profit of the supply chain,$NP_{sc} = NP_R + NP_m$.

3. Model formulation

3.1 Profit function of the retailer

The retailer uses a new ordering policy called one-for-one period ordering $(1, T)$ policy Haji and Haji (2007) in which an order of size one is placed at every fixed cycle time T.

One-for-one period ordering $(1, T)$ policy is an alternative algorithm for base stock policy for solving inventory control problems in which the ordering cost is negligible. Haji and Haji (2007) demonstrated that $(1, T)$ is outperforming $(S - 1, S)$, specially in cases where lead time increases. Moreover, $(1, T)$ policy has many advantages for upstream members of the supply chain, which has been already mentioned in section 1.

To use a One-for-one period ordering $(1, T)$ policy, the inventory problem can be interpreted as a $D/M/1$, a single channel queuing system in which the inter-arrival, T, times are constant and the service times have exponential distribution with mean value of $1/\mu$. Thus, the arrival rate of units to the system is $\lambda = 1/T$ and the service rate is μ. The net profit of the retailer equals his revenue minus the total costs. The revenue is the payment from customers and the cost includes the wholesale price which he pays to the manufacturer to buy products, the inventory cost and the shortage cost as follows:

$$NP_R = (P - P_m)\mu\rho - h_r I - \pi\mu(1 - \rho), \tag{1}$$

where ρ is the proportion of the demand satisfied by the retailer and $(P - P_m)\mu\rho$ is the revenue obtained by the retailer from selling the product minus the price he should pay to the manufacturer to buy the product. Also, $(1 - \rho)$ is the proportion of time that the retailer is out of stock, as such, $\pi\mu(1 - \rho)$ is his lost sales cost per unit time, since

$$\rho = \frac{\lambda}{\mu} = \frac{1}{T\mu} \tag{2}$$

Haji and Haji (2007) also show that the average inventory is

$$I = \frac{\rho}{1 - \beta} \tag{3}$$

where

$$\beta = e^{-\frac{1-\beta}{\rho}} = e^{-\frac{1}{I}} \tag{4}$$

Consequently,

$$\rho = I(1 - \beta) = I\left(1 - e^{-\frac{1}{I}}\right) \tag{5}$$

So, we can represent average net profit of the retailer as follows

$$NP_R = (P - P_m)\mu I\left(1 - e^{-\frac{1}{I}}\right) - h_r I - \pi\mu\left(1 - I\left(1 - e^{-\frac{1}{I}}\right)\right) \tag{6}$$

We find the optimal value of I which maximizes the retailer's profit. Consequently, we can find T from Eq. (2) and Eq. (5) as

$$T = \frac{1}{\mu I\left(1 - e^{-\frac{1}{I}}\right)} \tag{7}$$

3.2 Profit function of the manufacturer

The manufacturer receives orders from the retailer based on the optimal replenishment cycles. In this scenario, the retailer orders one unit of one type of product at specific intervals (T) to the manufacturer. In other words, order quantities are discrete and periodic. So, the manufacturer should decide on his ordering pattern (the number and time of his orders along with the quantity of his orders) to the supplier. As shortage is not allowed and the manufacturer avoids keeping excessive inventory, the manufacturer's ordering quantities should be obtained at the ordering times of the retailer. Regarding the retailer's ordering times and quantities, the manufacturer decides about the frequency and the quantity of orders to the supplier to minimize the holding and ordering costs besides fulfilling retailer's demand without any shortage. So the manufacturer uses the integer-ratio policy (see also Guan & Zhao, 2010; Yu et al., 2009; Mateen et al., 2015; Hariga et al., 2013), in which the replenishment cycle of the manufacturer equals mT. In this inventory policy, the replenishment cycle of the manufacturer is an integer multiple (m) of the common replenishment cycle (T).

To satisfy the retailer's orders, the manufacturer should order m units of the product in every mT units of time to his supplier. So, the manufacturer should obtain the optimum value of m to minimize the holding and ordering in the long term. The inventory level of the manufacturer by this policy is depicted in Fig. 2.

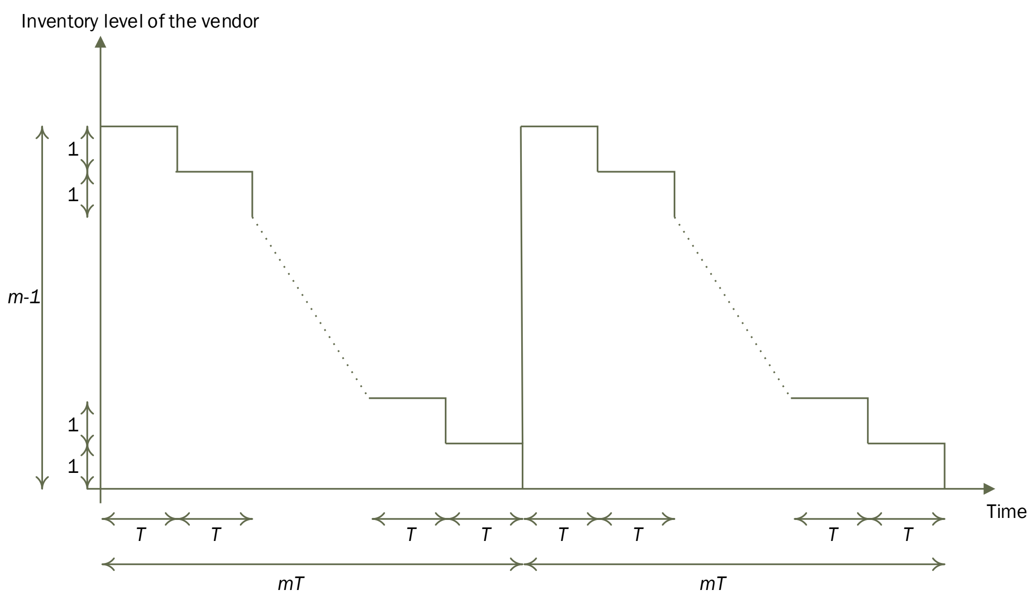

Fig. 2. The inventory level of the product at the manufacturer

Then the profit function of the manufacturer based on his replenishment cycle mT is

$$NP_m = (P_m - C_m)\mu\rho - \frac{A}{mT} - \frac{h_m(m-1)}{2} \tag{8}$$

By substituting Eq. (7) for T we have

$$NP_m = \mu(P_m - C_m)I\left(1 - e^{-\frac{1}{I}}\right) - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} - \frac{(m-1)h_m}{2} \tag{9}$$

3.3 Centralized control

To facilitate the comparison, we analyze the system performance for the case of centralized control, which plays the role of a benchmark. From Eq. (6) and Eq. (9) we can formulate the total profit of the supply chain as follows

$$NP_{SC} = \mu(P - C_m)I\left(1 - e^{-\frac{1}{I}}\right) - \pi\mu\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right] - h_R I - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} - \frac{(m-1)h_m}{2} \quad (10)$$

We will use Theorems 1, 2 and 3 to investigate the properties of $I$and m for the optimal inventory policy which minimizes the supply chain cost. To achieve this purpose, we propose a convergence algorithm to find the optimal inventory policy for the supply chain in a centralized control environment.

Theorem 1: For any fixed I and m the optimal P can be uniquely determined by satisfying the following condition:

$$P^* = \frac{e_p}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} + \frac{A}{m}\right) \quad (11)$$

Proof: see proof in Appendix A.

□

Theorem 2: For any fixed I the optimal m can be uniquely determined by satisfying the following condition:

$$m^*(m^* - 1) \leq \frac{2A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{h_m} \leq m^*(m^* + 1) \quad (12)$$

Proof: see proof in Appendix B.

□

Theorem 3: The optimal value of I (denoted as I^*) to minimize Eq. (10) is to satisfy

$$\left.\frac{\partial NP_{SC}}{\partial I}\right|_{I=I^*} = 0 \quad (13)$$

Proof: see proof in Appendix C.

□

By solving and simplifying Eq. (13) we have

$$\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) = \frac{h_R}{\mu\left(P - C_m + \pi - \frac{A}{m}\right)} \quad (14)$$

The right-hand side of Eq. (14) is always positive for all values of I. So, from Eq. (14) we know that $P - C_m + \pi - \frac{A}{m}$ should be bigger than zero. Thus we can find the minimum value of m as follow:

$$m_{min} = \left\lceil \frac{A}{P - C_m + \pi} \right\rceil \quad (15)$$

In which $\lceil x \rceil$ represents the least integer greater than or equal to x. Now that we have found a lower bound for the value of m using Eq. (15), by Theorem (4) we can show that the optimum value of I is bounded.

Theorem 4: The optimal value of I for Eq (14) is bounded.

Proof: See proof in Appendix D.

□

Using Theorems 1, 2, 3 and 4 we have developed an enumerative algorithm to find the optimal inventory policy for the supply chain participating in centralized control policy. Then, we show that the algorithm is capable of finding the optimal solution in Theorem 4.

3.3.1 The algorithm for the case of centralized control

Now, the unique solution can be obtained by following Algorithm:

Algorithm 1: Finding Centralized control optimal solution

Step 1. (Initiation) Set i =1. Find m_{min} from Eq. (15). Set $P = C_m$ and calculate candidate solutions I^* for $m = m_{min}, m_{min}+1, m_{min}+2, \ldots$ separately by Eq. (14) and for each m check whether Eq. (12) is satisfied. Once Eq. (12) is satisfied, m_1 and I_1 are obtained. Calculate optimal P_1 from Eq. (11).

Step 2. (Iteration step) set $i = i + 1$. calculate m_i and I_i. Considering these new amounts of m_i and I_i, calculate updated P_i using Eq. (14) and Eq. (11).

Step 3. (Stopping criteria) repeat step (2) until the difference of P in two successive iteration be equal or less than an specific amount ε.

Theorem 5: The algorithm 1 finds the unique optimal policy in the case of centralized control.

Proof: see proof in Appendix E. □

3.4 Revenue sharing contract under VMI control

In this model we assume that the manufacturer controls the replenishment process of both parties and determines the optimal replenishment cycles of himself and the retailer. Hence, the manufacturer bears the expenses of his own setup cost A per replenishment, his expected holding h_m per time unit and expected holding cost of the retailer h_R per time unit. The retailer receives an expected revenue per time unit, and incurs an expected penalty cost for lost sales per time unit.
The retailer returns a percentage of sales revenue, $1-\varphi$, to the manufacturer, which is determined through bargaining between the two parties. When the manufacturer manages the inventory according to one-for-one period policy the resultant profit at the retailer is given by

$$NP_R = \mu\varphi PI\left(1-e^{-\frac{1}{I}}\right) - \pi\mu\left[1 - I\left(1-e^{-\frac{1}{I}}\right)\right], \tag{16}$$

where $I\left(1-e^{-\frac{1}{I}}\right)$ is the proportion of the demand satisfied by the retailer and $\mu\varphi PI\left(1-e^{-\frac{1}{I}}\right)$ is the proportion of revenue obtained by the retailer from selling the product under revenue sharing contract. Also, $\left(1 - I\left(1-e^{-\frac{1}{I}}\right)\right)$ is the proportion of time that the retailer is out of stock, as such, $\pi\mu\left[1 - I\left(1-e^{-\frac{1}{I}}\right)\right]$ is his lost sales cost per unit time.

And the manufacturer's profit function is:

$$NP_m = \mu(1-\varphi)PI\left(1-e^{-\frac{1}{I}}\right) - \mu C_m I\left(1-e^{-\frac{1}{I}}\right) - \frac{A\mu I\left(1-e^{-\frac{1}{I}}\right)}{m} - \frac{(m-1)h_m}{2} - h_R I \tag{17}$$

From Eq. (16) and Eq. (17) the system profit function is then

$$NP_{SC} = \mu(P - C_m)I\left(1 - e^{-\frac{1}{I}}\right) - \pi\mu\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right] - h_R I - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} - \frac{(m-1)h_m}{2}, \tag{18}$$

which is the same as Eq. (10). Hence, it is seen from Eq. (18) that the proportion φ does not influence the system profit.
The manufacturer hopes for the replenishment policy to maximize Eq. (17), which includes the setup cost and the holding cost, whereas the retailer prefers the replenishment policy that generates lower penalty cost in Eq. (16).

Now, considering the interaction of manufacturers as the leader and the retailer as follower in the Stackelberg game, we will use an iterative algorithm as follows to obtain the final optimal solution. The optimal value of P will be obtained considering the manufacturer's inventory policy. Also by assuming that P is constant, we can calculate the equilibrium solution $I^*$and m^*.

The same as the centralized supply chain in section 3-3, there are some considerations for the optimal solution in VMI program which have been presented in Theorems 6 and 7. We have used these Theorems to develop a convergent algorithm to find the optimal pricing and inventory policy in VMI program with revenue sharing contract.

Theorem 6: For any fixed I and m the optimal P can be uniquely determined by satisfying the following condition:

$$P^* = \frac{e_p}{\varphi(e_p - 1)} \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} \tag{19}$$

Proof: see proof in Appendix F. □

Theorem 7: The optimal value of I (denoted as $\tilde{I}$) to minimize (17) is to satisfy

$$\left.\frac{\partial NP_m}{\partial I}\right|_{I=\tilde{I}} = 0 \tag{20}$$

Proof: see proof in Appendix G. □

By solving and simplifying Eq. (19) we have

$$\left(1 - e^{-\frac{1}{\tilde{I}}} - \frac{1}{\tilde{I}} e^{-\frac{1}{\tilde{I}}}\right) = \frac{h_R}{\mu\left((1-\varphi)P - C_m - \frac{A}{m}\right)} \tag{21}$$

The left-hand side of Eq. (21) is always positive for all values of I. So, from Eq. (21) we know that $(1-\varphi)P - C_m - \frac{A}{m}$ should be bigger than zero. Thus, we can find the minimum value of m as follows:

$$m_{min} = \left\lceil \frac{A}{(1-\varphi)P - C_m} \right\rceil \tag{22}$$

Now we have found a lower bound for value of m using Eq. (22), by Theorem (8) we can show that the optimum value of I is bounded as follow.

Theorem 8: The optimal value of I for Eq. (20) is bounded.

Proof: See proof in Appendix H. □

Although the manufacturer possesses the priority to determine the optimal policy, he needs the retailer to cooperate to determine the policy. So, the optimal value of P, the proportion φ and the value of expected inventory level I are determined by both parties through the following algorithm:

Algorithm 2: Revenue Sharing Contract

Step 1: The negotiation starts φ_{max}, which is the maximum revenue sharing coefficient accepted by the two parties to enter the VMI program. In other words, $(1-\varphi_{max})$ is the minimum revenue share the retailer will give to the manufacturer to manage his inventory. The manufacturer and the retailer determine the optimal values of I, m and P which minimize Eq. (16) and Eq. (17) denoted by $(\tilde{I}, \widetilde{m}, \tilde{P})$.

Step 2: The retailer desires to increase the inventory level to gain more profit, so in negotiations he tries to increase I to decrease his shortage cost. Provided $NP_{SC} = NP_R + NP_m$ is increased, both parties can benefit as a win–win result by appropriately modifying φ. The procedure repeats until NP_{SC} becomes increasing, wherein an equilibrium is reached. The resultant policy at the equilibrium is denoted by $(\varphi^*, I^{**}, m^{**}, P^{**})$.

Step 3: Find the solution and stop. Sign a cooperative contract to share the added profit between the manufacturer and the retailer.

For step 1 of Algorithm 2, determining φ_{max} is the main problem for the parties to enter the VMI program. Although, the value of φ_{max} depends on the bargaining power of each party, its feasible interval, which makes the VMI program beneficial for both parties, has been presented in proposition 1. Another important issue for the manufacturer is how to determine values of I and m for any proposed φ of the retailer, which can be obtained by algorithm 3. Finally, using Theorem (9) we have presented the specifications of accepted values of φ, I and m for the revenue sharing contract.

Proposition 1: The maximum revenue sharing coefficient, φ_{max}, should satisfy Eq. (23)

$$\frac{\pi\left[1-I\left(1-e^{-\frac{1}{I}}\right)\right]}{PI\left(1-e^{-\frac{1}{I}}\right)} < \varphi_{max} < 1-\frac{1}{\mu PI\left(1-e^{-\frac{1}{I}}\right)}\left(\frac{A\mu I\left(1-e^{-\frac{1}{I}}\right)}{m}+\frac{(m-1)h_m}{2}+h_R I\right) \tag{23}$$

Proof: see proof in Appendix I. □

Algorithm 3: Finding $(\tilde{I}, \widetilde{m}, \tilde{P})$ for algorithm 2

Step 1. (Initiation) Find m_{min} from Eq. (15). Set $P = \bar{P}$ and calculate candidate solutions I^* for $m = m_{min}, m_{min}+1, m_{min}+2, \ldots$ separately by Eq. (21) and for each m check whether Eq. (12) is satisfied. Once Eq. (12) is satisfied, $\widetilde{m}_1$ and $\tilde{I}_1$ are obtained. Calculate optimal $\tilde{P}$ from Eq. (19).

Step 2. (Iteration step) consider the $\tilde{P}$ in the iterative algorithm (3) and calculate $\widetilde{m_1}$ and $\tilde{I}_1$. Considering these new amounts of $\widetilde{m_1}$ and $\tilde{I}_1$, calculate updated $\tilde{P}$ using equations (21) and (12).

Step 3. (Stopping criteria) repeat step (2) until the difference of $\tilde{P}$ in two successive iteration be equal or less than an specific amount ε.

Using Algorithm (3). We can find the optimal values of P I and m for any given value φ. We have shown that this Algorithm finds the optimal pricing and inventory policy which maximizes the net profit function of the manufacturer and the retailer by Theorem 9.

Theorem (9):Algorithm 3 finds the unique optimal pricing and inventory policy in the VMI program which maximizes the manufacturer and the retailer net profit**.**

Proof: see proof in Appendix J. □

The bargaining process will continue till the value of the profit function of one or both of the parties does not increase by changing the price and the inventory level. In Theorem 10, we have shown that the equilibrium of the revenue sharing contract for VMI program, which maximizes the net profit of both parties, is the optimal pricing and inventory policy of the centralized supply chain regardless of the value of φ.

Theorem (10): The VMI program is coordinated with the revenue sharing contract described by algorithm 2 if the manufacturer and the retailer agree on any contract $(\varphi^*, P^{**}, I^{**}, m^{**})$ that satisfies the conditions

$$(P^{**}, I^{**}, m^{**}) = (P^*, I^*, m^*)$$

And

$$\left(\frac{\pi}{P^*} + \varphi_{max}\frac{\widetilde{P}}{P^*}\right) S_I - \frac{\pi}{P^*} < \varphi^* < 1 - K_I - (1 - \varphi_{max})\frac{\widetilde{P}}{P^*} S_I \tag{24}$$

In which (I^*, m^*) is the optimal policy i-n centralized control and

$$S_I = \frac{\tilde{I}\left(1 - e^{-\frac{1}{\tilde{I}}}\right)}{I^*\left(1 - e^{-\frac{1}{I^*}}\right)} \tag{25}$$

$$K_I = \frac{1}{P^*\mu I^*\left(1 - e^{-\frac{1}{I^*}}\right)}\left(h_r(I^* - \tilde{I}) + \frac{h_v(m^* - \widetilde{m})}{2} + A\mu\left(\frac{I^*\left(1 - e^{-\frac{1}{I^*}}\right)}{m^*} - \frac{\tilde{I}\left(1 - e^{-\frac{1}{\tilde{I}}}\right)}{\widetilde{m}}\right)\right) \tag{26}$$

Proof: see proof in Appendix K. □

The amounts of both φ_{max} and φ depend on the bargaining power of the manufacturer and the retailer. The more powerful the manufacturer, the smaller the values of φ_{max} and φ^*.

3. Numerical example

In this section, a simple example is used to illustrate and test the algorithm. The data for the system are given in Table 1 below.

Table 1
The parameters' values for numerical example

Decision Variables		Retailer's Parameters		Manufacturer's Parameters		
e_p	K	π	h_R	A	C_m	h_m
2	10000	20	14	30	20	12

We have presented the optimal values of decision variables to maximize the total supply chain profit in Table 2.

Table 2
The optimal solution in centralized control

$\boldsymbol{\mu}$	$\boldsymbol{P^*}$	$\boldsymbol{I^*}$	$\boldsymbol{m^*}$	$\boldsymbol{T^*}$	$\boldsymbol{\rho^*}$	$\boldsymbol{NP_{SC}}^*$
1.88	72.93	1.7	3	0.7	0.76	16.04

Now we want to obtain the optimal solution through revenue sharing contract. To start the algorithm, two parties should agree on a revenue sharing coefficient to start the revenue sharing bargaining process which satisfies Eq. (23). The acceptable interval of φ_{max} for this example is $[0.04, 0.54]$. The value of selected φ_{max} depends on the bargaining power of the parties. For this example, we assume that both parties have agreed on $\varphi_{max} = 0.5$.

To obtain the optimal solution for VMI control under revenue sharing contract we have presented one possible bargaining process through algorithm 2 that can be occurred between the two parties step by step in Table 3 as follows

Table 3
Bargaining process through Algorithm 2

Step	P	μ	I	m	ρ	φ	$NP_M{}^*$	$NP_R{}^*$	$NP_{SC}{}^*$	ΔNP_{SC}
1	55.00	3.31	0.80	3.00	0.57	0.500	-16.72	12.39	-4.33	0.00
2	62.23	2.58	0.95	3.00	0.62	0.495	-9.72	16.19	6.46	10.80
3	68.50	2.13	1.12	3.00	0.66	0.485	-4.56	16.71	12.15	5.69
4	71.23	1.97	1.24	3.00	0.69	0.482	-2.67	16.73	14.06	1.91
5	72.02	1.93	1.30	3.00	0.70	0.481	-2.08	16.73	14.66	0.60
6	72.93	1.88	1.70	3.00	0.76	0.480	-0.74	16.78	16.04	1.39

As can be seen in Table 3, the optimal solution which is beneficial to both parties occurs at the optimal solution of the centralized control and so the revenue sharing contract can coordinate the supply chain. From Eq.(24) we know that the optimal revenue sharing coefficient should be in the interval $[0.469,0.478]$ at the final step of this contract. By increasing the inventory level, above 1.70, $\Delta \boldsymbol{NP_{SC}}$ will be a negative value and it will decrease the total supply chain inventory profit. Although, the value of φ_{max} affects the revenue share and consequently net revenue of each party, it does not have any influence on reaching the optimal policy through the bargaining process. To show the impact of φ_{max} in bargaining process and finding the equilibrium we have presented the possible bargaining process for values $\varphi_{max} = 0.2$ and $\varphi_{max} = 0.7$ in Tables 4**.**

Table 4
Final optimal replenishment policy for different values of φ_{max}

φ_{max}	Step	P	μ	I	m	ρ	φ	$NP_M{}^*$	$NP_R{}^*$	$NP_{SC}{}^*$	ΔNP_{SC}
0.2	1	63.40	2.49	1.40	3.00	0.71	0.200	5.24	8.35	13.58	0.00
	2	70.40	2.02	1.51	3.00	0.73	0.198	5.90	9.73	15.63	2.05
	3	71.70	1.95	1.54	3.00	0.74	0.196	6.00	9.82	15.81	0.18
	4	72.93	1.88	1.70	3.00	0.76	0.185	6.04	10.00	16.04	0.23
0.7	1	58.59	2.91	0.70	3.00	0.53	0.600	-54.55	27.25	-27.30	0.00
	2	60.20	2.76	0.76	3.00	0.56	0.595	-50.89	30.47	-20.42	6.88
	3	63.56	2.48	0.89	3.00	0.60	0.590	-47.65	35.99	-11.66	8.76
	4	65.22	2.35	0.99	3.00	0.63	0.585	-42.34	39.04	-3.30	8.36
	5	67.23	2.21	1.11	3.00	0.66	0.580	-40.66	41.78	1.12	4.42
	6	69.34	2.08	1.23	3.00	0.68	0.575	-38.19	43.63	5.44	4.33
	7	70.20	2.03	1.35	3.00	0.71	0.570	-37.45	45.44	7.99	2.54
	8	70.90	1.99	1.44	3.00	0.72	0.565	-35.66	46.35	10.69	2.70
	9	71.10	1.98	1.58	3.00	0.74	0.560	-32.24	48.11	15.87	5.18
	10	72.93	1.88	1.70	3.00	0.76	0.590	-28.22	44.26	16.04	0.17

Since the value of φ_{max} depends on the bargaining power of the parties, a smaller value of φ_{max} presents a more powerful manufacturer. Since for smaller values of φ_{max}, a higher revenue share will be assigned to the manufacturer, the optimal replenishment policy of the manufacturer at the first step of the Algorithm 2 has the value nearest to the centralized control solution value and the bargaining process to achieve the agreement will be easier to handle too. Totally, as have been explained before, the value of φ_{max} just affects the optimal policy of the manufacturer at the first step of the Algorithm 2 and the revenue shares of the parties from the total revenue of the supply chain but does not have any influence on the final agreement of the parties.

4. Conclusions

In this paper we formulated a manufacturer managed inventory, VMI, supply chain that consists of a manufacturer and a retailer. We formulated the supply chain under centralized control and developed an algorithm to design the optimal revenue sharing contract under VMI. Under revenue sharing contract, the system achieves the same performance as in centralized control, so the contract is a perfect contract.

We know that the best pricing and inventory policy for the supply chain will be obtained when all the parties make their pricing and inventory decisions under centralized control, which is not achievable easily in real world and the companies should utilize their bargaining power instead. In this paper we have shown that the optimal pricing and inventory policy of the supply chain can be obtained under a revenue sharing contract by which the equilibrium is the same as the centralized control.

Our research could be extended in several possible directions. We can further find other contract types to coordinate the supply chain for VMI and non-VMI supply chain structures. Moreover in many supply chain structures, pricing decisions and quality properties will affect the demand of consumers which can be considered in further research endeavors.

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Appendix A: Proof of Theorem 1

For any given I and m, the first derivative of Eq. (10) with respect to P can be obtained from

$$\frac{\partial NP_{SC}}{\partial P} = -e_p K P^{-e_p-1}\left(P - C_m - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} - \frac{A}{m}\right) I\left(1 - e^{-\frac{1}{I}}\right) + K P^{-e_p} I\left(1 - e^{-\frac{1}{I}}\right) \quad (A.1)$$

Since $e_p K P^{-e_p-1} I\left(1 - e^{-\frac{1}{I}}\right)$ is positive, if $\left(P - C_m - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} - \frac{A}{m}\right) < 0$ then $\frac{\partial NP_{SC}}{\partial P} > o$ which is not possible. So

$$P - C_m - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} - \frac{A}{m} > 0 \tag{A.2}$$

Second derivative of Eq. (10) with respect to P can be obtained from (B.3)

$$\frac{\partial^2 NP_{SC}}{\partial P^2} = -I\left(1 - e^{-\frac{1}{I}}\right) K e_p P^{-e_p-2}\left(C_s + \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} + \frac{A}{m}\right) < 0 \tag{A.3}$$

Since, NP_{SC} is a concave function of P for any given I and m, we can find the optimal value of P by setting the first derivative of Eq. (10) to zero as follows:

$$-e_p K P^{-e_p-1}\left(P - C_m - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} - \frac{A}{m}\right) I\left(1 - e^{-\frac{1}{I}}\right) + K P^{-e_p} I\left(1 - e^{-\frac{1}{I}}\right) = 0 \tag{A.4}$$

So,

$$P^*(I, m) = \frac{e_p}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)} + \frac{A}{m}\right) \tag{A.5}$$

Appendix B: Proof of Theorem 2

For any given I we have

$$\begin{aligned} NP_m(m+1) - NP_m(m) &= \frac{h_m m}{2} + \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m+1} - \frac{h_m(m-1)}{2} - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} \\ &= \frac{h_m}{2} - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m(m+1)} \end{aligned} \tag{B.1}$$

and

$$\begin{aligned} NP_m(m-1) - NP_m(m) &= \frac{h_m(m-2)}{2} + \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m-1} - \frac{h_m(m-1)}{2} - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} \\ &= -\frac{h_m}{2} + \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m(m-1)} \end{aligned} \tag{B.2}$$

Since

$$[NP_m(m+1) - NP_m(m)] - [NP_m(m-1) - NP_m(m)] = 2\frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m(m-1)(m+1)} > 0 \tag{B.3}$$

We can find the optimal value of m when $NP_m(m+1) - NP_m(m) \geq 0$ and $NP_m(m-1) - NP_m(m) \geq 0$. From $NP_m(m+1) - NP_m(m) \geq 0$ we have

$$\frac{2A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{h_m} \leq m(m+1) \tag{B.4}$$

And from $NP_m(m-1) - NP_m(m) \geq 0$ we have

$$\frac{2A\mu I\left(1-e^{-\frac{1}{I}}\right)}{h_m} \geq m(m-1) \tag{B.5}$$

So the optimal m for any given I should satisfy Eq. (12).

Appendix C: Proof of Theorem 3

For any given P and m, the first derivative of Eq. (10) with respect to I can be obtained from

$$\frac{\partial NP_{SC}}{\partial I} = \mu\left(P - C_m + \pi - \frac{A}{m}\right)\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) - h_R \tag{C.1}$$

Since $\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right)$ is positive, if $\left(P - C_m + \pi - \frac{A}{m}\right) < 0$ then $\frac{\partial NP_{SC}}{\partial I} < o$ which is not possible. So

$$P - C_s + \pi - \frac{A}{m} > 0 \tag{C.2}$$

Second derivative of Eq. (10) with respect to I can be obtained from (C.3)

$$\frac{\partial^2 NP_{SC}}{\partial I^2} = -\mu\left(P - C_m + \pi - \frac{A}{m}\right)\frac{e^{-\frac{1}{I}}}{I^3} < 0 \tag{C.3}$$

Since, NP_{SC} is a concave function of I for any given P and m, we can find the optimal value of I by setting the first derivative of Eq. (10) to zero.

Appendix D: Proof of Theorem 4

From Eq.(14) we can obtain the derivative of optimal I with respect to m as follows:

$$\frac{-h_r \frac{A}{m^2}}{\mu\left(P - C_m + \pi - \frac{A}{m}\right)^2} = -\frac{e^{-\frac{1}{I}}}{I^3}\left(\frac{\partial I}{\partial m}\right) \tag{D.1}$$

So $\frac{\partial I}{\partial m} > 0$ and we can see that the optimal value of I is increasing with m. So, we can find an upper bound for I by setting $m = \infty$. Assuming $m = \infty$, Eq. (14) becomes

$$\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) = \frac{h_R}{\mu(P - C_m + \pi)} \tag{D.2}$$

Also, when $m = 1$, the value of I is in its biggest magnitude. For $m = 1$, Eq. (14) becomes

$$\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) = \frac{h_R}{\mu(P - C_m + \pi - A)} \tag{D.3}$$

The expression $\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right)$ is increasing in I, so we can find an amount of I which satisfies Eq. (D.2) and Eq. (D.3).

Appendix E: Proof of Theorem 5

We show that the iterative procedure always converges to a unique solution. Let $P^{(k)}$ be the solution given by the procedure at the *kth* iteration. Using *induction* we'll show that $P^{(k)}$ is increasing in *k*. Since $P \leq \bar{P}$, so $P^{(k)}$ is Convergent.

According the algorithm, $P^{(0)} = C_m$. It is obvious that $P^{(k)} > C_m$ for all k. In particular, $P^{(1)} > P^{(0)}$. Assume that $P^{(k)} \geq P^{(k-1)}$ for all $k<n$. Consider $k=n$. from Eq. (12) we can show that P^* is decreasing in I and m.

$$\frac{\partial P^*}{\partial I} = -\frac{e_p}{e_p - 1}\frac{\pi\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right)}{I^2\left(1 - e^{-\frac{1}{I}}\right)^2} < 0 \tag{E.1}$$

Then $I^{(n-1)} \leq I^{(n-2)}$ and $m^{(n-1)} \leq m^{(n-2)}$. So we have:

$$\begin{aligned} P^{(n)} &= \frac{e_p}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I^{(n-1)}\left(1 - e^{-\frac{1}{I^{(n-1)}}}\right)\right]}{I^{(n-1)}\left(1 - e^{-\frac{1}{I^{(n-1)}}}\right)} + \frac{A}{m^{(n-1)}}\right) \\ &\geq \frac{e_p}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I^{(n-2)}\left(1 - e^{-\frac{1}{I^{(n-2)}}}\right)\right]}{I^{(n-2)}\left(1 - e^{-\frac{1}{I^{(n-2)}}}\right)} + \frac{A}{m^{(n-2)}}\right) = P^{(n-1)} \end{aligned} \tag{E.2}$$

which completes our induction.

Now we show that the solution obtaining with this algorithm is unique. Suppose that there exists two different solutions, denoted by P_1 and P_2 in which $\mathrm{P}_1 > \mathrm{P}_2$. Since I^* and m^* obtaining from Eq. (12) and Eq. (14) are decreasing in P , we have,

$$(I_1^*, m_1^*) < (I_2^*, m_2^*) \tag{E.3}$$

From Eq. (10), we have

$$\begin{aligned} NP_{SC}{}^2 - NP_{SC}{}^1 = &\frac{\mu_2 I_2\left(1 - e^{-\frac{1}{I_2}}\right)}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I_2\left(1 - e^{-\frac{1}{I_2}}\right)\right]}{I_2\left(1 - e^{-\frac{1}{I_2}}\right)} + \frac{A}{m_2}\right) - h_R(I_2 - I_1) - \frac{h_m}{2}(m_2 - m_1) \\ &- \frac{\mu_1 I_1\left(1 - e^{-\frac{1}{I_1}}\right)}{e_p - 1}\left(C_m + \frac{\pi\left[1 - I_1\left(1 - e^{-\frac{1}{I_1}}\right)\right]}{I_1\left(1 - e^{-\frac{1}{I_1}}\right)} + \frac{A}{m_1}\right) < 0 \end{aligned} \tag{E.4}$$

This contradicts our assumption, so the equilibrium solution must be unique.

Appendix F: Proof of Theorem 6

For any given I and m, the first derivative of Eq. (16) with respect to P can be obtained from

$$\frac{\partial NP_R}{\partial P} = -e_p K P^{-(e_p+1)}\left(\varphi P - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)}\right) I\left(1 - e^{-\frac{1}{I}}\right) + K P^{-e_p}\varphi I\left(1 - e^{-\frac{1}{I}}\right) \tag{F.1}$$

Second derivative of Eq. (16) with respect to P can be obtained from (F.2)

$$\frac{\partial^2 NP_R}{\partial P^2} = e_p{}^2 K P^{-(e_p+2)}\left(\varphi P - \frac{\pi\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right]}{I\left(1 - e^{-\frac{1}{I}}\right)}\right) I\left(1 - e^{-\frac{1}{I}}\right) - e_p K\varphi P^{-(e_p+1)} I\left(1 - e^{-\frac{1}{I}}\right) \tag{F.2}$$

Since, $e_p \frac{\pi\left[1-I\left(1-e^{-\frac{1}{I}}\right)\right]}{I\left(1-e^{-\frac{1}{I}}\right)} > \frac{\pi\left[1-I\left(1-e^{-\frac{1}{I}}\right)\right]}{I\left(1-e^{-\frac{1}{I}}\right)}$ and consequently $e_p \frac{\pi\left[1-I\left(1-e^{-\frac{1}{I}}\right)\right]}{I\left(1-e^{-\frac{1}{I}}\right)} > \frac{P\varphi(e_p-1)}{e_p}$, $\frac{\partial^2 NP_R}{\partial P^2} < 0$ and NP_R is a concave function of P for any given I and m, we can find the optimal value of P by setting the first derivative of Eq. (16) to zero.

Appendix G: Proof of Theorem 7

For any given P and m, the first derivative of Eq. (17) with respect to I can be obtained from

$$\frac{\partial NP_m}{\partial I} = \mu\left((1-\varphi)P - C_m - \frac{A}{m}\right)\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) - h_R \tag{G.1}$$

Since $\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right)$ is positive, if $\left((1-\varphi)P - C_m - \frac{A}{m}\right) < 0$ then $\frac{\partial NP_m}{\partial I} < o$ which is not possible.

So

$$(1-\varphi)P - C_m - \frac{A}{m} > 0 \tag{G.2}$$

Second derivative of Eq. (10) with respect to I can be obtained from (G.3)

$$\frac{\partial^2 NP_m}{\partial I^2} = -\mu\left((1-\varphi)P - C_m - \frac{A}{m}\right)\frac{e^{-\frac{1}{I}}}{I^3} < 0 \tag{G.3}$$

Since, NP_m is a concave function of I for any given P and m, we can find the optimal value of I by setting the first derivative of Eq. (17) to zero.

Appendix H: Proof of Theorem 8

From Eq. (21) we can obtain the derivative of optimal I with respect to m as follows:

$$\frac{-h_r \frac{A}{m^2}}{\mu\left((1-\varphi)P - C_m - \frac{A}{m}\right)^2} = -\frac{e^{-\frac{1}{I}}}{I^3}\left(\frac{\partial I}{\partial m}\right) \tag{H.1}$$

So $\frac{\partial I}{\partial m} > 0$ and we can see that the optimal value of I is increasing with m. So, we can find an upper bound for I by setting $m = \infty$. Assuming $m = \infty$, Eq. (14) becomes

$$\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) = \frac{h_R}{\mu\big((1-\varphi)P - C_m\big)} \tag{H.2}$$

Also, when $m = 1$, the value of I is in its biggest magnitude. For $m = 1$, Eq. (14) becomes

$$\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right) = \frac{h_R}{\mu\big((1-\varphi)P - C_m - A\big)} \tag{H.3}$$

The expression $\left(1 - e^{-\frac{1}{I}} - \frac{1}{I}e^{-\frac{1}{I}}\right)$ is increasing in I, so we can find an amount of I which satisfies Eq. (H.2) and Eq. (H.3).

Appendix I: Proof of Proposition 1

At the VMI program, the maximum revenue sharing coefficient chosen should be such that for both parties, the total net profit is positive. Otherwise, the VMI program will not be feasible. So we should have

$$NP_R = \mu\varphi PI\left(1 - e^{-\frac{1}{I}}\right) - \pi\mu\left[1 - I\left(1 - e^{-\frac{1}{I}}\right)\right] > 0 \tag{I.1}$$

$$NP_m = \mu(1-\varphi)PI\left(1 - e^{-\frac{1}{I}}\right) - \frac{A\mu I\left(1 - e^{-\frac{1}{I}}\right)}{m} - \frac{(m-1)h_m}{2} - h_R I > 0 \tag{I.2}$$

From Eq. (G.1) and Eq. (G.2) we have Eq (23) in Proposition 1.

Appendix J: Proof of Theorem 9

We show that the iterative procedure always converges to a unique solution. Let $P^{(k)}$ be the solution given by the procedure at the *kth* iteration. Using *induction* we'll show that $P^{(k)}$ is decreasing in *k*. Since $P \geq C_m$, so $P^{(k)}$ is Convergent.
According the algorithm, $P^{(0)} = \bar{P}$. It is obvious that $P^{(k)} < \bar{P}$ for all *k*. In particular, $P^{(1)} < P^{(0)}$. Assume that $P^{(k)} \leq P^{(k-1)}$ for all *k*<*n*. Consider *k*=*n*. from Eq. (19) we can show that P^* is decreasing in I and m

$$\frac{\partial P^*}{\partial I} = -\frac{e_p}{\varphi(e_p - 1)} \frac{\pi\left(1 - e^{-\frac{1}{I}} - \frac{1}{I} e^{-\frac{1}{I}}\right)}{I^2\left(1 - e^{-\frac{1}{I}}\right)^2} < 0 \tag{J.1}$$

Then $I^{(n-1)} \geq I^{(n-2)}$. So we have:

$$P^{(n)} = \frac{e_p}{\varphi(e_p - 1)}\left(\frac{\pi\left[1 - I^{(n-1)}\left(1 - e^{-\frac{1}{I^{(n-1)}}}\right)\right]}{I^{(n-1)}\left(1 - e^{-\frac{1}{I^{(n-1)}}}\right)}\right) \leq \frac{e_p}{\varphi(e_p - 1)}\left(\frac{\pi\left[1 - I^{(n-2)}\left(1 - e^{-\frac{1}{I^{(n-2)}}}\right)\right]}{I^{(n-2)}\left(1 - e^{-\frac{1}{I^{(n-2)}}}\right)}\right) = P^{(n-1)} \tag{J.2}$$

This completes our induction.
Now we show that the solution obtaining with this algorithm is unique. Suppose that there exists two different solutions, denoted by P_1 and P_2 in which $\mathrm{P}_1 > \mathrm{P}_2$. Since I^* and m^* obtaining from equations (12) and (20) are decreasing in P , we have,

$$(I_1^*, m_1^*) < (I_2^*, m_2^*) \tag{J.3}$$

From Eq. (18), we have

$$\begin{aligned} NP_{SC}{}^2 - NP_{SC}{}^1 = {} & \mu_2 I_2\left(1 - e^{-\frac{1}{I_2}}\right)\left(\left(\frac{e_p}{\varphi(e_p - 1)} - 1\right)\frac{\pi\left[1 - I_2\left(1 - e^{-\frac{1}{I_2}}\right)\right]}{I_2\left(1 - e^{-\frac{1}{I_2}}\right)} - C_m - \frac{A}{m_2}\right) - h_R(I_2 - I_1) \\ & - \frac{h_m}{2}(m_2 - m_1) \\ & - \mu_1 I_1\left(1 - e^{-\frac{1}{I_1}}\right)\left(\left(\frac{e_p}{\varphi(e_p - 1)} - 1\right)\frac{\pi\left[1 - I_1\left(1 - e^{-\frac{1}{I_1}}\right)\right]}{I_1\left(1 - e^{-\frac{1}{I_1}}\right)} - C_m - \frac{A}{m_1}\right) < 0 \end{aligned} \tag{J.4}$$

This contradicts our assumption, so the equilibrium solution must be unique.

Appendix k: Proof of Theorem 10

The proof is divided into two steps. In step 1, we show that the contract $(\varphi^*, P^*, I^*, m^*)$ maximizes NP_{Sc} and therefore is efficient. In step 2, we show that feasible ranges for φ^* is restricted to the values mentioned in Eq. (24).

Step 1: At the first step of the algorithm, we find the optimal values of $(\tilde{P}, \tilde{I}, \widetilde{m})$ which maximizes the net profit of the manufacturer and the retailer distinctively. Since we do not consider the shortage cost in the cost function of the manufacturer, the manufacturer intends to keep a smaller inventory to lower his holding cost. So the optimum inventory level for Eq. (21) will be smaller than the optimal inventory level of Eq. (14). The retailer will reduce revenue sharing coefficient to increase the manufacturer's revenue share, to encourage him to increase the inventory level. So the shortage cost of the retailer would decrease. The bargaining process at each level will be accepted if and only if their total net profit increases. In other words, in each bargaining level the following conditions are established

$$\begin{aligned} &\Delta I > 0 \\ &\Delta P > 0 \\ &\Delta\varphi < 0 \\ &\Delta NP_R > 0 \\ &\Delta NP_m > 0 \\ &\Delta NP_{sc} = \Delta NP_R + \Delta NP_m > 0 \end{aligned} \tag{K.1}$$

The bargaining process will be continued until the conditions of Eq.(k.1) are being satisfied. Assume that at a bargaining level $\tilde{I} < I < I^*$, we still can reduce the value of NP_{sc} by determining appropriate values of (φ, P, I, m). So, we will continue the bargaining process as long as $I < I^*$. When the accepted inventory level reaches value I^*, since $I^*$is the optimal solution which minimizes NP_{sc}, we cannot increase the value of NP_{sc} by increasing the inventory level of the manufacturer any more. In other words, for $I > I^*$ we have $\Delta NP_{sc} < 0$, so $\Delta NP_R + \Delta NP_m < 0$. This implies that the net profit of one of the parties or both of them decrease regardless of the value of φ. So, the bargaining process will stop at this level and we have found the optimal contract parameters, which make parties not willing to deviate from.

Step 2: The contract parameters as well as the revenue sharing coefficient in particular, should be chosen such that for both parties, the net profit is not lower than the net profit before the bargaining process. Total net profit of the manufacturer before the bargaining process and after the bargaining process is presented in Eq. (k.2) and Eq. (k.3) respectively:

$$\widetilde{NP_m} = \mu(1-\varphi_1)\tilde{P}\tilde{I}\left(1-e^{-\frac{1}{\tilde{I}}}\right) - \frac{A\mu\tilde{I}\left(1-e^{-\frac{1}{\tilde{I}}}\right)}{m} - \frac{(\widetilde{m}-1)h_m}{2} - h_R\tilde{I} \tag{K.2}$$

$$NP_m{}^* = \mu(1-\varphi_2)P^*I^*\left(1-e^{-\frac{1}{I^*}}\right) - \frac{A\mu I^*\left(1-e^{-\frac{1}{I^*}}\right)}{m} - \frac{(m^*-1)h_m}{2} - h_R I^* \tag{K.3}$$

The contract is acceptable to the manufacturer if and only if

$$NP_m{}^* - \widetilde{NP_m} > 0 \tag{K.4}$$

This is also true for the retailer. Net profit of the retailer before and after bargaining process is presented in Eq. (I.5) and Eq. (I.6) respectively:

$$\widetilde{NP}_R = \mu\varphi_1\tilde{P}\tilde{I}\left(1-e^{-\frac{1}{\tilde{I}}}\right) - \pi\mu\left[1-\tilde{I}\left(1-e^{-\frac{1}{\tilde{I}}}\right)\right] \tag{K.5}$$

$$NP_R{}^* = \mu\varphi_2P^*I^*\left(1-e^{-\frac{1}{I^*}}\right) - \pi\mu\left[1-I^*\left(1-e^{-\frac{1}{I^*}}\right)\right] \tag{K.6}$$

So, the retailer propose the revenue sharing coefficient in a manner that

$$NP_R{}^* - \widetilde{NP}_R > 0 \tag{K.7}$$

From (k.4) and (k.7) we have Eq. (24) of Theorem 10.