

## Uncertain Supply Chain Management

homepage: [www.GrowingScience.com/uscm](http://www.GrowingScience.com/uscm)**Stochastic transfer point location problem: A probabilistic rule-based approach****Amir Yousefli<sup>a\*</sup>, Hamed Kalantari<sup>b</sup> and Mehdi Ghazanfari<sup>b</sup>**<sup>a</sup>*Department of Industrial Management, Imam Khomeini International University, Qazvin, Iran*<sup>b</sup>*Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran***CHRONICLE***Article history:*

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In this paper, a weighted transfer point location problem is developed in which demand points have probabilistic coordinates. The proposed model is formulated as a probabilistic unconstrained nonlinear programming and the optimum values of decision variables are obtained in the form of probability distribution functions. Because of the complexity in solving the developed model by using the traditional solution approaches, a new stochastic inference method called Probabilistic Role Base (PRB) is developed based on the derived optimum probability distribution functions of the decision variables. This method is used to infer the optimum or near optimum values of all decision variables without solving nonlinear programming model, directly. Finally, to demonstrate the efficiency of the developed algorithm, a numerical example is presented and the results are compared with the optimum solution.

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**1. Introduction**

Transfer point location problem (TPLP) is one of the latest related models to hub location problem. According to O'Kelly (1986a), Hub refers to a central facility which connects a set of interacting points. Although the network hub location problem was first addressed by Goldman (1969), the investigation on hub location started with pioneering researches of O'Kelly (1986a, 1986b, 1987). For more studies, one can refer to Contreras et al. (2009) and Ta et al. (2012). The TPLP was first studied by the pioneering works of Berman et al. (2007). According to Berman et al. (2007), the transfer point is a new established central facility which is able to serve  $n$  demand points. In their model, the factors  $\alpha \in (0,1)$  and  $w_i \in (0,1)$  introduce the traveling cost unit from such transfer point to the facility and the demand point  $i$  to the transfer point, respectively. Berman et al. (2005) formulated the problem of locating one facility and  $p$  transfer points to serve a set of  $n$  demand points, in the both plane and on a network with two minisum and minimax objectives. They called their model as Facility and Transfer Points Location Problem (FTPLP). They investigated the minisum version of the problem on a network and used some heuristic algorithms such as descent approach, simulated annealing and tabu search to solve it. Berman et al. (2007) analyzed the TPLP and established properties and solution approaches to

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find the best location of the transfer point for both minimax and minisum objectives, both on the plane and network. Berman et al. (2008) investigated the Multiple Transfer Points Location Problem (MTPLP) as an extension of the TPLP with both the minisum and minimax objectives, on the plane and network. In this model, they formulated the problem of locating multiple transfer points to serve a set of the demand points with a given facility location. Sasaki et al. (2008) considered the multiple location of transfer points (MLTP) and facility and transfer points location problem (FTPLP) on a network. They formulated the minisum and minimax MLTP as p-median and p-center problems, respectively. They also proposed a new flow-based formulation for both minisum and minimax FTPLP. Fiker et al. (2016) presented a decision support system to simulate disasters and plan shipments of relief good via transfer points demand points in the affected area. In other research, Kalantari et al. (2014) considered transfer point location problem in fuzzy environment and developed a rule-based decision support system using fuzzy controllers to infer optimum or near optimum location of transfer point directly. Mohamadi et al. (2015) developed a multi-objective non-linear in fuzzy environment which considers two vital needs in disaster time including medical services and relief commodities through location of hospitals, transfer points, and location routing of relief depots.

In addition to the mentioned papers, some researchers investigated the stochastic forms of the TPLP. Hosseinjou and Bashiri (2011) presented a stochastic model of TPLP in which demand points' locations have uniform probability distributions. They used expected value method to solve their proposed model. Also, some works were done in related areas to TPLP such as hub and spoke location, traveling salesman (TSP), one center and round trip problems. Yang (2009) investigated the air freight hub location and flight route planning in stochastic environment. He assumed that demand has discrete probability distribution with finite number of possible realization called "scenario". Contreras et al. (2011) studied uncapacitated stochastic hub location problem in which demands and transportation costs were stochastic in nature. They proved equivalence of this stochastic model to a deterministic expected value problem. Sim et al. (2009) presented the uncertain p-hub center problem with stochastic hub nodes and used a chance-constrained formulation to model the minimum service level requirement. Ghiani et al. (2008) studied waiting strategies for the traveling salesman problem with respect to the probabilistic characterization of customer demands. Weyland et al. (2012) studied how to set the vehicle capacity for traveling salesman problems where some of the customer demands are stochastic. Zhang et al. (2010) investigated a container transportation problem with truck deployment in a local area with shippers, receivers, multiple depots, and multiple terminals. Zhang and Wu (2010) considered uncertainty Euclidean facility location problems and obtained robust optimal solution of the Euclidean facility location problem with unknown-but-bounded uncertainty using the existing robust optimization methodology. Also, for more studies about TSP, one can refer to Braekers et al (2012). In the aforesaid stochastic TPLP and related models, parameters and coefficients of decision variables have probability distribution functions whereas decision variables are considered to be crisp. This means that in an uncertain environment a crisp decision is made to meet some decision criteria.

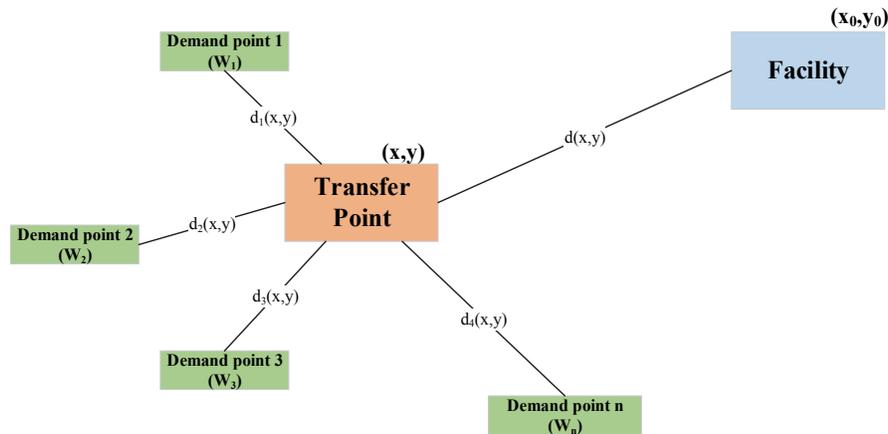
In this paper, a new TPLP model is developed in the stochastic environment in which the locations of the demand points have probability distribution functions. This model is named Stochastic Transfer Point Location Problem (STPLP). Obviously, when the locations of the demand points are estimated using probability distributions, optimum location of transfer point should not be deterministic. In other words, demand points might be realized in the various locations, based on which the optimum location of the transfer point can be calculated. So, the transfer point might be located optimally in the several points. Therefore, deriving a probability distribution function for optimum location of the transfer point seems to be rational. For this end, we present an algorithm to obtain the optimum probability distribution functions of the transfer point location coordinates. Obtained optimum probability distribution functions gathers all possible optimum values of the transfer point location coordinates regarding the future realization of the uncertain parameters. Hence this helps decision maker to have a wide vision of the possible situations and solutions that improves his/her knowledge over the problem. Since the proposed model is very complex probabilistic nonlinear programming problem and might be

a bit hard to comprehend by practitioners, a new probabilistic inference system called Probabilistic Rule Base (PRB) is developed to infer the optimum or near optimum values of the transfer point location without solving transfer point location problem directly.

The organization of this paper is as follows: In the section 2, the stochastic transfer point location problem is presented. Section 3 consists of the developed probabilistic rule base and its designing approach. Performance of the PRB to infer the optimum or near optimum solution is illustrated via a numerical example in the section 4. Finally, in section 5, conclusions and future researches are remarked.

## 2. Stochastic Transfer Point Location Problem

In this section, the Stochastic Transfer Point Location Problem is formulated in which the location of each demand point has a probability distribution function. Fig. 1 depicts the general geometric representation of the developed model.



**Fig. 1.** Geometric representation of the STPLP

Different probability distribution functions could be considered for demand points coordinates,  $\bar{x}_i$  and  $\bar{y}_i$ . Also, each demand point has its own weight. The minisum version of STPLP model is developed under the following notations and assumptions:

### Notations

$n$	Number of demand points
$\alpha$	Weight associated with the facility
$(x_0, y_0)$	Location of the facility
$w_i$	Weight associated with demand point $i$
$(\bar{x}, \bar{y})$	probabilistic location of the transfer point (decision variables)
$(\bar{x}_i, \bar{y}_i)$	probabilistic coordinate of the demand point $i$ ( $\bar{C}_i$ )
$d(x, y)$	distance between the transfer point and the facility
$d_i(x, y)$	distance between the demand point $i$ and the transfer point

### Assumptions

- I: Coordinates of the demand points could have any type of the probability distribution function.
- II: Distance between the demand point  $i$  and the transfer point and also between the transfer point and the facility are measured by  $d_i(x, y) = (\bar{x}_i - \bar{x})^2 + (\bar{y}_i - \bar{y})^2$  and  $d(x, y) = (\bar{x} - x_0)^2 + (\bar{y} - y_0)^2$ , respectively.

The developed STPLP is formulated as model (1).

$$\begin{aligned}
F &= \min_{x,y} \left\{ \sum_{i=1}^n w_i d_i(x,y) + \alpha d(x,y) \right\} \\
&= \min_{x,y} \left\{ \sum_{i=1}^n w_i [(\bar{x}_i - \bar{x})^2 + (\bar{y}_i - \bar{y})^2] + \alpha [(\bar{x} - x_0)^2 + (\bar{y} - y_0)^2] \right\} \quad (1)
\end{aligned}$$

The weights of the facility ( $\alpha$ ) and the demand point  $i$  ( $w_i$ ) must be defined in order to develop a theoretical explanation for the applicability of the model (1).

The facility weight ( $\alpha$ ) is defined as a function of the following three factors:

- The unit cost of transportation between the transfer point and the facility,  $TC_f$ : which is a function of the vehicle and path properties.
- Travel speed between the transfer point and the facility,  $TS_f$ : which is the function of the vehicle speed used in moving between the transfer point and the facility. Thus, the higher speed vehicle is used in the path from the transfer point to the facility, the lower value of the  $\alpha$  will be assigned.
- Service level risk,  $SL_f$ : which is affected by the emergency service frequency, presented in the transfer point, and the state of the vehicle, used in traveling between the transfer point and the facility. It means that, the service cannot be assigned to the other emergency needs, when the service vehicle is busy.

Now, we have  $\alpha = f(TC_f, TS_f, SL_f)$ . The  $\alpha$  value has a direct relation with  $TC_f$  and  $SL_f$ , i.e., if each of the  $TC_f$  and  $SL_f$  increase,  $\alpha$  will be increased to allow the transfer point to be located as near as the facility to reduce the vehicle travel risk and cost. Also, there is a reverse relation between  $TS_f$  and the  $\alpha$  value. It means that, if  $TS_f$  increases, then  $\alpha$  will be decreased in order to locate the transfer point farther from the facility.

Let define  $w_i$  as a function of three factors as follows:

- Importance degree of the demand point  $i$ ,  $I_i$ .
- Unit cost of transportation between the demand point  $i$  and the transfer point,  $TC_i$ .
- Travel speed between the demand point  $i$  and the transfer point,  $S_i$ .

The demand point  $i$  population number and emergency needs will affect  $I_i$ . On the other hand,  $TC_i$  and  $S_i$  may be functions of the transportation vehicle, and path features. It must be noted that  $I_i$  and  $TC_i$  have direct relationship with  $w_i$ , while there exists a reverse relationship between  $S_i$  and  $w_i$ .

The optimal location of the transfer point  $(x, y)$  is placed somewhere among the demand points and the facility. By increasing the  $\alpha$  value, the transfer point location moves toward the facility.

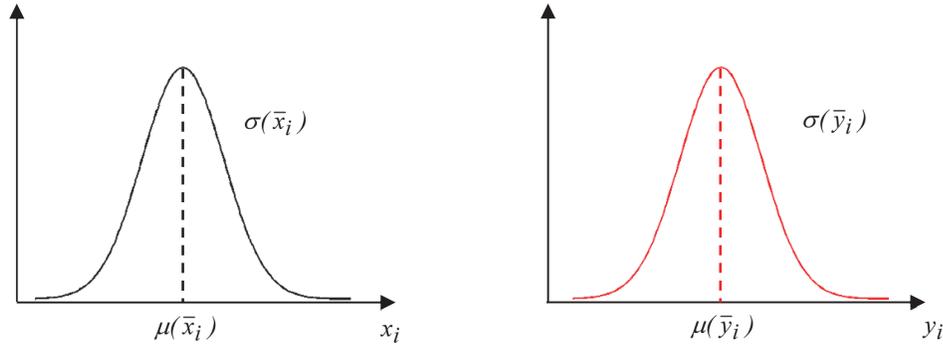
Model (1) is a probabilistic unconstrained nonlinear programming problem which can be solved using conventional stochastic programming approaches such as expected value or chance constrained programming method. These methods convert the stochastic model into a crisp equivalent and solve the crisp version instead of the original uncertain problem and present crisp values for decision variables. As mentioned before, according to this fact in model (1), parameters are stochastic in nature, thus optimum values of decision variables must be uncertain too. In the next section, a new method is presented to obtain optimum probability distribution functions of the decision variables of model (1).

### 3. Probabilistic Rule Base (PRB)

Rule bases have been used in many practical studies so far. Most of these works have used stochastic rule bases as inference engines in the developed decision support systems in which each crisp rule has

a probability degree of accuracy. In this paper, we develop a new rule-based inference system in which the antecedents and consequents have probability distribution functions. This inference system is called “Probabilistic Rule Base (PRB)”. The developed PRB is used to infer the optimum or near optimum values of the transfer point location coordinates  $\bar{x}$  and  $\bar{y}$  and the objective function  $\bar{F}$ . The design stages of the PRB are as follows:

- (1) Determine probability distribution functions of the stochastic input parameters: In the STPLP, define the probability distribution functions of the coordinates  $\bar{x}_i$  and  $\bar{y}_i$  for each demand point  $i$  as Fig. 2.



**Fig. 2.** Probability distribution functions of one set of stochastic input parameters  $\bar{x}_i, \bar{y}_i$

- (2) Consequents derivation: The consequents are derived using a new method based on the optimum knowledge as follows:

Randomly generate  $a_i$  and  $b_i$ , respectively, with respect to the probability distribution functions of the antecedents  $\bar{x}_i$  and  $\bar{y}_i$  and convert stochastic model (1) in to following crisp equivalent:

$$F = \min_{x,y} \left\{ \sum_{i=1}^n w_i [(a_i - x)^2 + (b_i - y)^2] + \alpha [(x - x_0)^2 + (y - y_0)^2] \right\} \quad (2)$$

It is easy to verify that the model (2) is a convex unconstrained nonlinear programming problem (Ta et al., 2012). Thus, the optimum values of the decision variables and objective function can be derived using following equations:

$$x^* = \frac{\alpha x_0 + \sum_{i=1}^n w_i a_i}{\alpha + \sum_{i=1}^n w_i} \quad (3)$$

$$y^* = \frac{\alpha y_0 + \sum_{i=1}^n w_i b_i}{\alpha + \sum_{i=1}^n w_i} \quad (4)$$

$$F^* = \sum_{i=1}^n w_i [(a_i - x^*)^2 + (b_i - y^*)^2] + \alpha [(x^* - x_0)^2 + (y^* - y_0)^2] \quad (5)$$

Now, compute the decision variables  $(x^*, y^*)$  and the objective function  $(F^*)$  by Eqs. (3-5) for  $N$  times in which  $N$  is a sufficient large number. Then, fit a suitable probability distribution function to each derived consequents and name them as  $\bar{x}, \bar{y}$  and  $\bar{F}$ .

- (3) Rule base construction: The Probabilistic Rule Base (PRB) must be designed using stochastic parameters as the antecedents and the stochastic decision variables as the consequents.
- (4) Implication method. Here, a new implication method is developed named Correlation Coefficient based Implication (CCI). The steps of the proposed method are as follows:

**Step 1:** Calculate  $\rho_x^i$  ( $i = 1, \dots, n$ ) as the correlation coefficient between the antecedent  $x_i$  ( $i = 1, \dots, n$ ) and the consequent  $\bar{x}$ , respectively, with respect to the probability distribution functions of them. Also, calculate  $\rho_y^i$  ( $i = 1, \dots, n$ ) as the correlation coefficient between each antecedent  $y_i$  ( $i = 1, \dots, n$ ) and the consequent  $\bar{y}$ , respectively, as same as  $\rho_x^i$ .

**Step 2:** Put  $\rho_x^* = \max_i |\rho_x^i|$  and  $\rho_y^* = \max_i |\rho_y^i|$  as the most correlated parameters with  $\bar{x}$ ,  $\bar{y}$  and name related parameters as  $\bar{X}$  and  $\bar{Y}$  respectively.

**Step 3:** If  $a_0$  and  $b_0$  be the inputs of the candidate antecedents  $\bar{X}$  and  $\bar{Y}$ , then calculate  $\gamma$  and  $\beta$  as follows:

$$\gamma = F_{\bar{X}}(a_0) = P(\bar{X} \leq a_0) = \int_{-\infty}^{a_0} f(\bar{X}) d\bar{X} \quad (6)$$

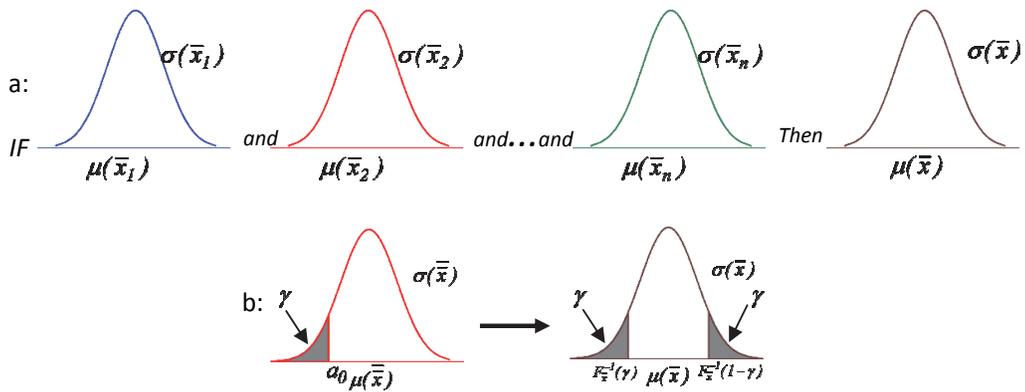
$$\beta = F_{\bar{Y}}(b_0) = P(\bar{Y} \leq b_0) = \int_{-\infty}^{b_0} f(\bar{Y}) d\bar{Y} \quad (7)$$

In the above equations,  $f(\cdot)$  is the probability distribution of the related antecedent.

**Step 4:** Depending on the correlation sign (negative or positive) between the antecedent  $\bar{X}$  and consequent  $\bar{x}$ , one of the case 1 or 2 will be fired to derive  $x$ . Also  $y$  will be obtained through case 3 or 4 subject to the correlation sign between the antecedent  $\bar{Y}$  and consequent  $\bar{y}$ .

- Case1:
  - If  $\text{Correlation}(\bar{X}, \bar{x}) > 0$ , then  $x = F_{\bar{x}}^{-1}(\gamma)$ .
- Case2:
  - If  $\text{Correlation}(\bar{X}, \bar{x}) < 0$ , then  $x = F_{\bar{x}}^{-1}(1 - \gamma)$ .
- Case3:
  - If  $\text{Correlation}(\bar{Y}, \bar{y}) > 0$ , then  $y = F_{\bar{y}}^{-1}(\beta)$ .
- Case4:
  - If  $\text{Correlation}(\bar{Y}, \bar{y}) < 0$ , then  $y = F_{\bar{y}}^{-1}(1 - \beta)$ .

Developed CCI algorithm is shown in Fig. 3.



**Fig. 3.** a: Representation of a rule from the Probabilistic Rule Base (PRB); b: CCI method

The implication of the developed PRB method is demonstrated using a numerical example in the next section.

#### 4. Numerical Example

In this section five demand points are considered and the STPLP model is formulated as follows:

$$F = \min_{x,y} \left\{ \sum_{i=1}^5 w_i [(\bar{x}_i - \bar{x})^2 + (\bar{y}_i - \bar{y})^2] + \alpha [(\bar{x} - x_0)^2 + (\bar{y} - y_0)^2] \right\} \tag{8}$$

Consider the location of the facility as  $(x_0, y_0) = (35, 20)$  and the facility weight as  $\alpha = 0.45$ . Although probabilistic parameters of model (8) could have any type of probability distribution functions, but for simplicity, Normal distribution function is considered for all demand points' coordinates with parameters shown in Table 1.

**Table 1**  
The Normal distribution functions' parameters and the weights of the demand points

$\bar{C}_i$	$\bar{x}_i$		$\bar{y}_i$		$w_i$
	$\mu$	$\sigma$	$\mu$	$\sigma$	
$\bar{C}_1$	7	1	17	1	0.6
$\bar{C}_2$	6	0.67	10	1	0.75
$\bar{C}_3$	11.5	0.83	6.5	0.5	0.45
$\bar{C}_4$	17.5	0.83	17	0.67	0.72
$\bar{C}_5$	18.5	1.33	8.5	1.17	0.35

The model (8) with the mentioned parameters is solved in order to derive the optimum probability distribution functions of the transfer point coordinates. Using the second stage of the proposed algorithm for the PRB in the previous section, the coordinates of the transfer point are obtained as  $(\bar{x}, \bar{y}) \sim (N(14.68, 0.35), N(13.48, 0.35))$  and the objective function is calculated as  $\bar{F} \sim N(371.6, 16.49)$ .

Based on the probabilistic parameters and the decision variables  $\bar{x}$  and  $\bar{y}$ , following If-Then rule bases could be structured. The first one is the PRB to infer coordinate  $x$  and the second one is used for reasoning coordinate  $y$  of the transfer point:

*If  $\bar{x}_1 \sim N(7, 1)$  and  $\bar{x}_2 \sim N(6, 0.67)$  and  $\bar{x}_3 \sim N(11.5, 0.83)$  and  $\bar{x}_4 \sim N(17.5, 0.83)$  and  $\bar{x}_5 \sim N(18.5, 1.33)$  Then  $\bar{x} \sim N(14.68, 0.35)$ .*

*If  $\bar{y}_1 \sim N(17, 1)$  and  $\bar{y}_2 \sim N(10, 1)$  and  $\bar{y}_3 \sim N(6.5, 0.5)$  and  $\bar{y}_4 \sim N(17, 0.67)$  and  $\bar{y}_5 \sim N(8.5, 1.17)$  Then  $\bar{y} \sim N(13.48, 0.35)$ .*

**Table 2**  
Some examples of 100,000 samples

E.g.	$(\bar{x}_1, \bar{y}_1)$	$(\bar{x}_2, \bar{y}_2)$	$(\bar{x}_3, \bar{y}_3)$	$(\bar{x}_4, \bar{y}_4)$	$(\bar{x}_5, \bar{y}_5)$	$F^*(x, y)$	$F(x^*, y^*)$	Error (%)
1	(7.5,17.09)	(7.37,11.52)	(11.63,5.67)	(19.19,17.58)	(20.81,19.22)	373.79	373.89	0.03
2	(6.54,17.25)	(5.76,9.57)	(10.33,6.02)	(18.95,17.23)	(17.35,10.03)	403.66	405.42	0.44
3	(7.17,18.96)	(7.61,9.9)	(12.16,5.88)	(18.16,16.62)	(18.96,7.94)	345.79	346.23	0.13
4	(7.84,20.4)	(5.69,9.2)	(13.31,6.58)	(16.61,17.65)	(20.40,7.9)	352.09	352.59	0.14
5	(8.23,18.81)	(6.23,8.93)	(11.92,6.56)	(16.98,16.8)	(18.81,9.5)	342.92	343.05	0.04
6	(6.3,19.5)	(6.24,10.76)	(11.23,6.5)	(16.31,17.92)	(19.5,8.48)	405.73	406.08	0.09
7	(6.96,15.66)	(5.78,10.22)	(11.37,6.53)	(17,19.17)	(15.66,8.77)	378.45	379.57	0.30
8	(6.46,16.97)	(6.2,10.39)	(10.68,6.22)	(17.65,16.12)	(16.97,7.03)	367.62	369.83	0.60
9	(7.53,19.09)	(5.48,10.04)	(12.18,6.47)	(17.19,16.64)	(19.09, 8.12)	354.71	355.40	0.19
10	(8.04,15.73)	(4.99,12.1)	(12.37,5.99)	(17,16.8)	(15.73,8.42)	377.97	378.04	0.02
11	(7.08,17.91)	(5.77,9.61)	(9.72,6.16)	(16.78,17.27)	(17.91,9.2)	394.09	395.14	0.27
12	(6.31,21.58)	(5.49,8.82)	(10.92,6.3)	(18.84,17.38)	(21.58,9.87)	394.30	395.27	0.25
13	(9.11,16.48)	(4.69,9.51)	(10.91,7.01)	(18.64,17.56)	(16.48,8.68)	361.43	361.87	0.12
14	(7.5,17.09)	(7.37,11.52)	(11.63,5.67)	(19.19,17.58)	(20.81,19.22)	373.79	373.89	0.03
15	(6.54,17.25)	(5.76,9.57)	(10.33,6.02)	(18.95,17.23)	(17.35,10.03)	403.66	405.42	0.44

Designed probabilistic rule bases can efficiently solve problems in the form of model (8) with deterministic parameters. 100,000 samples are generated randomly and solved by the developed PRBs and also outputs are compared with the optimum solutions. Some of these examples are presented in Table 2. In this table,  $F(x^*, y^*)$  is the objective function calculated using above PRBs' outputs and  $F^*(x, y)$  is the optimum solution.

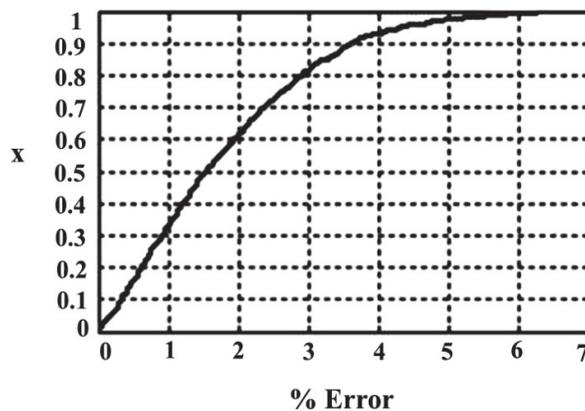
Table 3 shows the results of comparison between the objective function values obtained from the developed PRBs and the optimum values for 100,000 randomly generated examples, when  $\alpha = 0.45$ .

**Table 3**

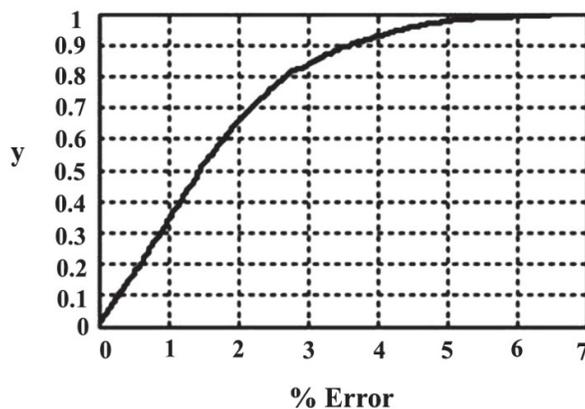
Results of comparison between the developed PRB output and the optimum solution

Deviation from optimum solution (%)	$x$	$y$	$F(x, y)$
Minimum deviation	0	0	0
Maximum deviation	8.03	7.52	1.34
Average deviation	1.84	1.68	0.17

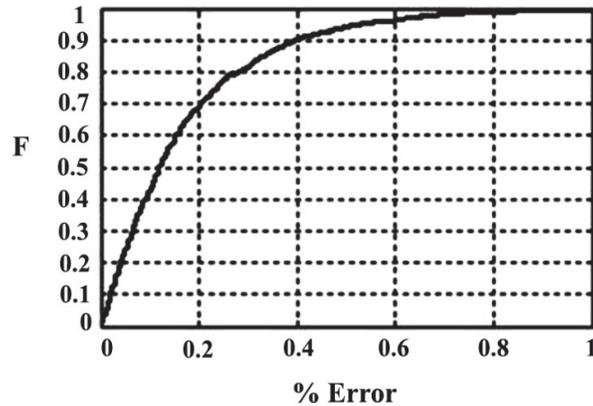
As shown in Table 3, the average deviations of  $x$ ,  $y$  and  $F(x, y)$  are 1.84%, 1.68% and 0.17%, respectively. To demonstrate how much the value of the average deviations for 100,000 different samples are robust and also to show the pattern of the deviations dispersion, the cumulative proportion curve of the  $x$ ,  $y$  and  $F(x, y)$  deviations for 100,000 samples are provided in Figs. (4-6), respectively.



**Fig. 4.** Cumulative proportion curve of the  $x$  deviation for 100,000 samples



**Fig. 5.** Cumulative proportion curve of the  $y$  deviation for 100,000 samples



**Fig. 6.** Cumulative proportion curve of the  $F(x, y)$  deviations for 100,000 samples

Fig. 4 and Fig. 5 depict that about 97% of the errors are less than 5% for coordinate  $x$  and for coordinate  $y$ , about 98% of errors are less than 5%. Also figure 6 shows the great efficiency of the developed approach to calculate the objective function, so that over than 98.5% of deviations between PRB solution and optimum solution are less than 0.8%.

## 5. Conclusion

In this paper, a new transfer point location problem in which the demand points are weighted and have stochastic coordinates was formulated as a probabilistic unconstrained nonlinear programming model. Due to the complexity of the proposed model for the practitioners such as managers, a probabilistic rule base was developed to reach the optimum or near optimum values of the decision variables without solving the nonlinear programming problem directly.

To demonstrate the efficiency of the developed algorithm, a numerical example was presented and the validation tests were provided. The results show that the developed PRB model is highly reliable and can be used as a powerful Decision Support System for decision makers. More developments in this problem and solution method are possible. For example, accuracy of the developed inference system could be improved in case of the probability density functions of decision variables being estimated using more accurate approaches such as Bayesian inference system or Kernel estimator. Moreover, the probabilistic version of the multiple transfer point locations problem can be considered as an extension of the presented model. Also, researchers can use the presented inference system to analyse the facility and transfer point location problem with probabilistic demand and coordinates of demand points.

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