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## A literature survey of HUB location problems and methods with emphasis on the marine transportations

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### ABSTRACT

This article presents a broad review of Hub location problem (HLP), and its applications. The goal of this article is to provide a review on the newest and most recent publications on Hub location and its application in real world practices since 2013. While, there are some articles which reviewed hub location problem literature but some models were not covered in those articles. In this paper, we try to include capacitated models. We survey advances in analysis and modeling of the hub location problem including its variants and solution algorithm of HLPs. We emphasize the most applicable areas for Marine Transpiration. We show that first, majority of the articles consider network domain, while there are some articles which execute HLP in discrete and continuous domain. Also, the capacitated case attracts more attention in recent years. Finally, the contribution of any type of HLP in the literature is shown schematically.

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## 1. Introduction

Facility location problem (FLP) is an interesting area in location problem. Theoretically speaking, FLP seeks to find an optimal location for new facility, and tries to minimize some objective function of total travel time, physical distance, costs, and covering distance, etc. In the FLP, usually we assume that each facility has sufficient capacity to meet the demands of any customers. The application of FLP is not restricted to locating facilities. Some other real-world applications can also be suggested in this area of knowledge including (i) selecting sites for emergency service facilities, such as hospitals and fire stations, (ii) choosing sites for warehouses and distribution centers, (iii) selecting subcontractors and assigning appropriate work to each of them, and (iv) selecting the best sites for an obnoxious facility. There are also some pure applications in transportation, which involve finding the most appropriate location of vehicle charging site (He et al., 2016), location of terminals and other urban facilities (Lin & Lin, 2016).

Marine transportation is one of the most important transportation means, with great contributions in cargo handling, which accounts for 90% of the total trade volume (UNCTAD, 2009). Marine transportation has

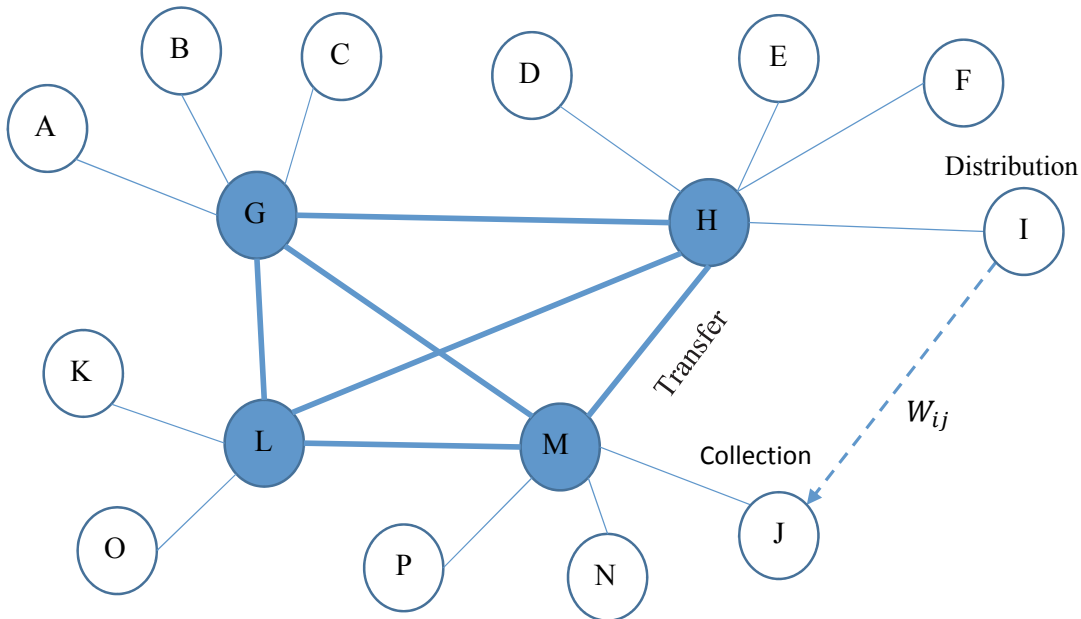
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several advantages over other modes of transportation, most notably for the transportation of large quantity of goods. This advantage can be classified in three groups: higher safety, lower costs and environmentally-friendly issues. In practice, to transfer large quantity of goods by rail and air, we face considerable challenges, such as tight schedules and unforeseen problems, which could lead to further complications for the entire distribution network (Gelareh, et al., 2013). From the economic perspective, it is not possible to transfer cargo directly between each and every node in a network. This increases transportation costs, and requires that a direct route between each two nodes be established. Hub location problem (HLP) is one of the popular topics in FLP, known as hub and spoke problem, which develops a network of nodes involving a smaller set of connections between Origins – Destination ( $O - D$ ). In the real world, some liner shipping companies benefit from HLP. They use ships with huge capacities to serve the majority of nodes, and ships with smaller capacities are used to transfer condensed cargo from some feeder ports to hub ports. For example, Maersk Company benefits from 10 very large ships with capacities above 18,000 Twenty Foot Equivalent Unit (TEU) for trade lines between Europe and Asia.

Consider a complete graph with  $N$  nodes that each nodes can be either an origin or destination node. If each node has a defined demand from another node of the graph and  $i - j$  pair isn't different from  $j - i$  pair, there are  $N(N - 1)/2$  ( $O - D$ ) pairs of nodes. For example a graph with 16 nodes has 120 ( $O - D$ ) pairs. In practice, building a communication channel like road between each ( $O - D$ ) nodes is not affordable and require high unnecessary investment cost. Assume that we select  $P$  nodes in this graph as Hub and the remaining nodes must transfer their demand and supply through one of the hub nodes. Using this idea in spite of using a complete graph with same number nodes, we decrease the number of arcs and the costs accordingly. Generally speaking a hub node can simultaneously have three different functionalities, namely: (Alumur & Kara, 2008)

- i. Consolidation or concentration function of flows that receives, in order to have a larger flow and letting economy of scale to be exploited;
- ii. Switch or transfer function which allow the flows to be re-directed at the node;
- iii. Distribution or decomposition functions of large flows into smaller ones.



**Fig. 1.** An example of hub location with 4 hub nodes

An example of a graph with 16 nodes is shown in Fig. 1. In this figure we choose four nodes including  $G, H, L, M$  as hub nodes and other remaining nodes are non-hub i.e. spoke nodes. Consider two non-hub nodes  $i$  and  $j$  in the graph which are assigned to the hub that located at nodes  $H$  and  $M$ , respectively. The flow between two non-hub nodes  $i$  and  $j$  ( $W_{ij}$ ), first go through node  $i$  to hub  $H$ , then transfer between

to hub nodes  $H$ ,  $M$  and finally from hub  $M$  to node  $j$ . In other word, demand of node  $j$  from node  $i$  follow path  $i \rightarrow H \rightarrow M \rightarrow j$ . In this path the two hub nodes  $H$  and  $M$  play the role of aggregation and distribution function. In this structure, volume of goods, or number of people, etc. which is transferred between two hub nodes is greater than the respective amount of goods or people carried from non-hub nodes to their connected hub. This higher volume allow us to use giant vehicle with higher capacity such as huge vessels, trains, airplanes etc. for carry commodity and pay less cost for transferring operation which is known as the concept of economy of scale that is defined as the coefficient of cost of carrying goods between two hubs (Farahani & Hekmatfar, 2009). Beside of this advantage, there are some other benefits such as economy of scope, economy of density and multiplier effect in use of HLP in network. Some disadvantage of such structure can be (i) Longer travel times and higher costs of some routes (ii) Capacity overload (iii) Higher risk of accident (congestion phenomena) and (iv) missing connecting facilities due to the unforeseen delay (interrupt) at some parts of the network.

In 1965, he proofed that the optimal location of switching center in a graph of communication network is located at the  $p$ -median of the corresponding weighted graph (Hakimi, 1964, 1965). HLPs originally were initiated from the idea by Goldman (1969). He showed that Hakimi's results holds true even in more general cases. He suggested two different formulations as below:

First he assumed that goods are transferred from origins to destinations and pass through processing and collect-and-dispatch centers. The unit transportation cost of a source-center can be different from the unit transportation cost of the processed goods. Second, he assumed that the flows can be processed in more than one center and transportation costs of origin-center, center-center and center-destination can be different. Therefore, the problem is to find  $P$  centers and then assign the flows to them, in order to minimize the transportation costs which is the  $p$ -hub median problem though he used the term center (Campbell et al., 2002). O'Kelly (1986a) for the first time shed light on the way for the future study of HLP. The first work in the area is authored by O'Kelly (1987), where he formulated the first mathematical model for HLP. He suggested the formulation for Single Allocation  $p$ -Hub Median Problem (SApHMP) which is also known as Uncapacitated Single Allocation  $p$ -Hub Median Problem (USApHMP). Since, HLP provide some advantages on network that reduce the costs, many companies in the world have applied HLP in practice. Transportation (Yaman et al., 2007), airline (Saber & Mahmassani, 2013), telecommunication (Carello et al., 2004), Civil Aviation Studies Association (O'Kelly, 1986b), postal and emergencies service (Drezner & Hamacher, 2001) and maritime industry (Zheng et al., 2015) can be considered as several examples of HLP application in real world situations. Altogether, two of the most important and well-known application areas of HLPs are telecommunications and transportations. In telecommunication industries, flows to be sent are digital data which are supposed to be delivered from origins to destinations. Such applications include video conferences, telephone networks, distributed processing and so on. Hub nodes are multiplexors, gates, switches etc. The hub edges are considered as different types of physical media, like optical fiber, coaxial cable etc. There are some study in this area such as Carello et al. (2004), Klincewicz (1998) and Lee et al. (1996). In transportation applications, like public transportation systems, airlines, air freight, express shipment - overnight deliver-, large trucking system, postal delivery and rapid transit, the demands are physical flows in form of passengers or goods to be transferred between any of the origins and destinations. Also, different transportation vehicles can be considered for hub level facilities like, buses, trucks, trains, taxis, planes, fast-lanes etc. Some works which have been done in this area of knowledge are Nickel et al. (2001), Gelareh and Nickel (2011), Campbell (2009), Gelareh and Nickel (2008), Yaman et al. (2007), Lumsden et al. (1999). The hub facilities in postal delivery applications are post offices where the items for different destinations are collected, sorted into several groups and then distributed among destinations. The hub edges can be seen as the planes or trains transferring shipments between districts corresponding to hub nodes. In public transport, the hubs are regional airports, central railway stations or bus terminals. People from corresponding regions (geographical zones containing the nodes assigned to a given hub node) are transferred there to use the inter-hub facilities working on the hub edges. The hub edges are wide body aircrafts flying longer distances or take more people or special type of trains

and/or buses with special functionalities. In freight transportation, hubs are the break-bulk terminals where the smaller trucks unload (load) their shipments onto (from) larger vehicles or any other transportation facilities operating on the hub edges. The models discussed in Ebery et al. (2000), Ernst and Krishnamoorthy (1996) and Krishnamoorthy et al. (1994).

## 2. Model and classifications of HLP

In this section, we present different popular classifications and modeling formulations of HLP.

### 2.1 Classifications

Since several models have been presented in the literature, classification of these models can help us to have clear understanding of the problem from different perspectives. There are several criteria for classification of HLPs, among which only some of them are explained below. Solution domain of HLPs can be divided into three parts: Network, discrete, and continuous. In the network part, the domain of candidate hub nodes is the set of all of the network nodes; and network can be a complete or incomplete graph. The domain of candidate hub nodes in the discrete domain is a series of particular nodes, and in the continuous domain, the domain of hub nodes is a plane or a sphere.

While each problem tries to optimize one or more objective function(s), the criterion of HLPs may be Mini-Max, Mini-Sum, or multi-objective. In the Mini-Max problems, the maximum transportation cost from origin nodes to destination nodes is minimized; whereas in Mini-Sum problem, the total cost incurred by locating hub nodes and allocation of non-hub nodes to hub nodes is minimized. There are some problems that have more than one objective and try to minimize or maximize objectives, simultaneously. The number of hub nodes to be located, can be exogenous or endogenous. In the exogenous problems, the number of hub nodes to be located, is predetermined and it is considered as one of the inputs of the model; while in the endogenous problem, the optimal number of hub nodes is considered as one of the outputs of the model. Meanwhile, the number of hub nodes in the problem can be one or more. Capacity is another criterion for classification of HLPs. Capacity can be constrained as the number of allocations of non-hub nodes, the amount of transferring goods through hub nodes and so on. Hub nodes can have limited or unlimited capacity which in turn will categorize the HLP problems into capacitated and incapacitated problems respectively.

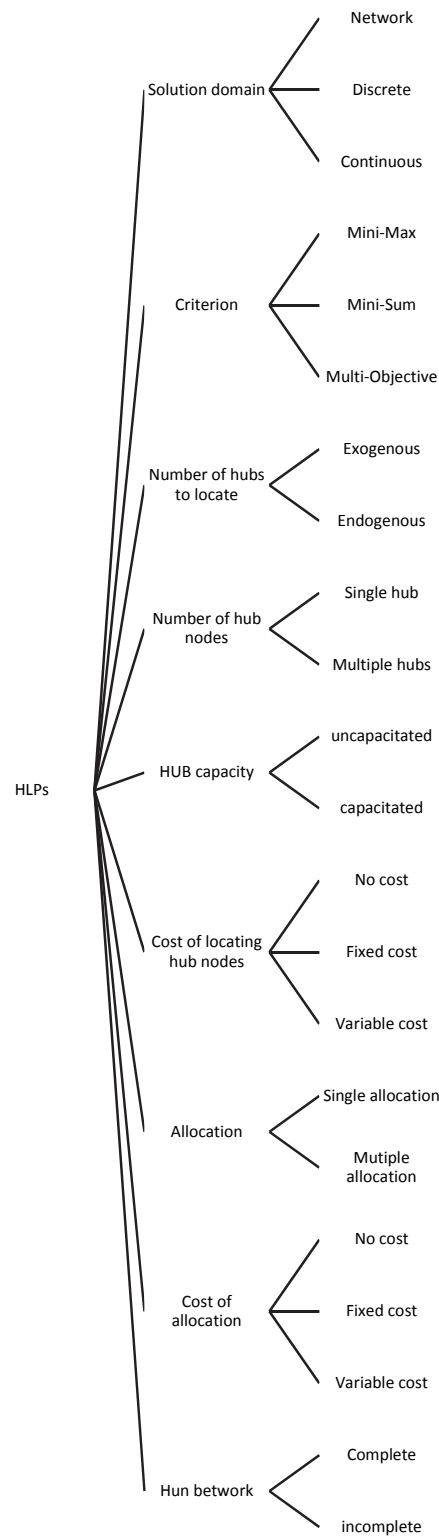
Non-hub nodes can be allocated to hubs in three ways. If non-hub node satisfies its demand from one hub, the problem is called single-allocation. Otherwise, if demand of non-hub nodes can be satisfied from more than one hub, the problem is called multiple-allocation. Generally speaking, the allocation of non-hub nodes to hubs has no cost, neither fixed nor variable costs. One of the popular assumptions in HLP modeling is that the graph which is obtained with hub nodes is considered to be a complete graph. In recent years, no comprehensive research has addressed this assumption. Tree HLP and ring hub problems are two examples of these models. In this paper we provide a broad classification based on the most important and widely used criteria. Our classification is illustrated schematically in Fig. 2.

### 2.2 Basic Model

In this section, we present the basic models in the literature that are widely applied in the researched.

#### 2.2.1 Single HLP

Single HLP model was introduced by O'Kelly (1987) for the first time. In this formulation, the solution domain is a network; while the objective function is Mini-Sum; the number of hub nodes to be located are predetermined and equal to one (single hub). Capacity is limited and cost of establishing the hub and cost of allocation is ignored. Also, each non-hub nodes allocates to single hub node (single allocation).



**Fig. 2.** Classification of HLP

Assume  $h_{ij}$  is the amount of flow between node  $i$  and node  $j$ ;  $C_{ij}$  is the cost of transportation of one unit from non-hub node  $i$  to hub node  $j$ ; and  $Y_{ij}$  is equal to 1, if node  $i$  is allocated to a hub located at node  $j$  (otherwise, 0). In this model,  $h_{ij}$  and  $C_{ij}$  are input parameters of the model, and  $Y_{ij}$  is output of the model

(decision variable). If  $Y_{jj}$  is equal to one, it means that node  $j$  is allocated to itself and in fact it is a hub node.

Mathematical formulation of this model is as follow:

$$\min \sum_i \sum_j \sum_k h_{ik}(C_{ij} + C_{jk})Y_{ij}Y_{kj} \quad (1)$$

subject to

$$\sum_j Y_{jj} = 1 \quad (2)$$

$$Y_{ij} \leq Y_{jj} \quad \forall i, j \quad (3)$$

$$Y_{ij} \in \{0,1\} \quad \forall i, j \quad (4)$$

Eq.(1) minimizes the total cost of transferring goods through hub nodes. Eq.(2) ensures that there is only one hub node in the network. Non-hub node  $i$  cannot be allocated to hub  $j$  before node  $j$  is selected as hub. This important issue is checked in Eq.(3). Finally, Eq. (4) represents type of decision variable.

### 2.2.2 $P$ -hub location problem ( $P$ -HLP)

This section defines a  $p$ -HLP and proposes its mathematical formulation. This problem is also proposed by O'Kelly (1987). In  $p$ -HLP, unlike Single HLP, there are more than one hub nodes. Solution domain is network. Similar to single HLP, each non-hub node must be assigned to one hub node. Criterion of this problem is Mini-Sum, and the number of hub nodes is exogenous and equals to  $p$ . Also, there is no cost for both establishing hub node and allocating non-hub nodes to hub nodes. This is important to say that the graph obtained from hub nodes is complete and at least one or at most two hub nodes have to be traversed for traveling between two non-hub nodes.

The input and output of this model are similar to single HLP with this exception that, in  $p$ -HLP,  $\alpha$  is defined as discount factor denoting economic scale for transferring goods between two hub nodes ( $0 \leq \alpha < 1$ ); while in single HLP, transferring cost includes two parts, in  $p$ -HLP, this cost includes three parts: cost of transferring goods from the origin node to this hub, transferring cost between two hub nodes, and transferring cost from hub to the destination node. Therefore, the cost of transferring between two hubs is multiplied by  $\alpha$ . In other words, the path to transfer between two non-hub nodes  $i$  and  $j$  is  $i \rightarrow k \rightarrow m \rightarrow j$ ; while  $k$  and  $m$  are hub nodes that nodes  $i$  and  $j$  are allocated to this node, respectively. According to this explanation, mathematical formulation is presented as following:

$$\text{Min} \sum_i \sum_k Y_{ik}C_{ik} \left( \sum_j h_{ij} \right) + \sum_j \sum_m Y_{jm}C_{jm} \left( \sum_i h_{ij} \right) \quad (5)$$

$$+ \alpha \sum_i \sum_j \sum_k \sum_m h_{ij}C_{km}Y_{ik}Y_{jm}$$

subject to

$$\sum_j Y_{ij} = 1 \quad \forall i \quad (6)$$

$$\sum_j Y_{jj} = p \quad (7)$$

$$Y_{ij} \leq Y_{jj} \quad \forall i, j \quad (8)$$

$$Y_{ij} \in \{0,1\} \quad \forall i, j \quad (9)$$

Eq. (5) minimizes total transferring cost between network nodes. The first term of this equation represents cost of transferring outgoing flow from non-hub  $i$  to hub  $k$ . The second term presents the cost of transferring ingoing flow from hub  $m$  to non-hub  $j$ . While the third term shows transferring cost between

two hub nodes  $k$  and  $m$ . Moreover, Eq. (6) ensures that each non-hub node is assigned to exactly one hub node. Eq.(7) guarantees that the number of hub nodes are equal to  $p$ . Eq.(8) reflects that node  $i$  can't be assigned to hub  $j$ , before node  $j$  is selected as hub. Finally, Eq. (9) shows type of decision variable.

### 2.2.3 P-Hub median location problem (multiple allocation p-HLP)

In the objective function of p-HLP, two binary variables ( $Y_{ik}, Y_{jm}$ ) multiplied by together and this makes the model quadratic. To simplify the model, Campbell, J. F. (1991) proposed a linear mathematical formulation. Because of similarity of this formulation to p-median problem, this formulation is called p-hub median location problem. In this model, each non-hub node can be assigned to more than one hub node (multiple allocation). Like p-HLP, criterion is Mini-Sum; the number of hub nodes is exogenous and equals to  $p$ ; no cost is considered for both establishing hub node and allocating non-hub nodes to hub nodes. In this model, two variables are considered.  $C_{ij}^{km}$  denotes unit transportation cost between the origin node  $i$ , the destination node  $j$  and hub nodes  $k$  and  $m$ . Also,  $Z_{ij}^{km}$  denotes flow from the origin node  $i$  to the destination node  $j$  via hub facilities located at nodes  $k$  and  $m$ . Finally, mathematical formulation of the p-hub median location problem is defined as below:

$$\min \sum_i \sum_j \sum_k \sum_m C_{ij}^{km} h_{ij} Z_{ij}^{km} \quad (10)$$

subject to

$$\sum_k X_k = p \quad (11)$$

$$\sum_k \sum_m Z_{ij}^{km} = 1 \quad \forall i, j \quad (12)$$

$$Z_{ij}^{km} \leq X_m \quad \forall i, j, k, m \quad (13)$$

$$Z_{ij}^{km} \leq X_k \quad \forall i, j, k, m \quad (14)$$

$$Z_{ij}^{km} \geq 0 \quad \forall i, j, k, m \quad (15)$$

$$X_k \in \{0,1\} \quad \forall k \quad (16)$$

where  $C_{ij}^{km}$  is calculated as Eq.(17), and  $X_k$  is 1 when a hub facility is located at node  $j$  (otherwise, 0).

$$C_{ij}^{km} = C_{ik} + \alpha C_{km} + C_{mj} \quad (17)$$

In Eq.(10), total transportation cost is minimized. The number of hub nodes is checked in Eq.(11). While  $Z_{ij}^{km}$  indicates allocation variable, Eq.(12) ensures that each origin–destination pair ( $i, j$ ) is allocated to one pair of hub nodes ( $k, m$ ). If origin-destination pair ( $i, j$ ) allocated to single hub, then indices  $k$  and  $m$  must be equal. Eq.(13) and Eq.(14) stipulate that firstly node  $k$  and node  $m$  must be selected as hub, then flow from node  $i$  to node  $j$  can transfer. Eq.(15) and Eq.(16) define type of decision variables. Also single allocation p-HLP is proposed by Daskin (1995).

### 2.2.4 P-hub center location problem

One of the most important variations of facility location problems is the p-center location problem that at first was introduced by Hakimi (1965).

This problem is frequently used to find the location of emergency facilities such as fire station or hospital (Suzuki & Drezner, 1996). In the center problem, demand nodes demonstrate origin-destination pairs in

HLP. Campbell, J. F. (1994) was the first one who formulated and discussed about the p-hub center problem. He introduced three different types of p-hub center location problems:

- ❖ The maximum cost for any origin–destination pair is minimized.
- ❖ The maximum cost for movement on any single link (origin-to-hub, hub-to-hub and hub to-destination) is minimized.
- ❖ The maximum cost of movement between a hub and an origin/destination is minimized (vertex center).

According to Campbell (1994), the first type is important for perishable or time sensitive items. An example of the second type of p-hub center problem is items that require some preserving/processing such as heating or cooling which are available at hub locations. For the third type, similar examples to the second type can be given considering that hub-to-hub links may have some special attributes. Campbell (1994) presented formulations for both single and multiple allocation versions for all three types of p-hub center problem. The aim of p-hub center location problem is minimizing the maximum cost of origin-destination pairs. So the criterion of this problem is defined as Mini-Max. This problem is modeled as p-hub median location problem, except that its objective function is defined as below:

$$\min \max_{i,j,k,m} \{C_{ij}^{km} h_{ij} Z_{ij}^{km}\} \quad (18)$$

While, the criterion seeks to minimize the maximum transportation cost from the origin nodes to the destination ones, constraints are like those of the p-hub median location problem. In 2007, Campbell, A. M. et al. (2007) Proposed a novel improvement of p-hub center location problems. Ernst et al. (2009) introduced a new mathematical formulation for single allocation p-hub center location problems. Yaman and Elloumi (2012) studied application of hub center location problem in the star-star network. Afterward in one paper, Liang (2013) at first showed that the star p-hub center problem does not admit a  $(1.25 - \epsilon)$ -approximation algorithm for any  $\epsilon > 0$ , unless  $P = NP$ , and then presented complementing the in-approximation results, a purely combinatorial 3.5-approximation algorithm for the star p-hub center problem. Recently, Brimberg et al. (2015) have proposed a general variable neighborhood heuristic search for solving the incapacitated single allocation p-hub center problem.

### 2.2.5 P-hub covering location problem

According to the history, for the first time Hakimi (1965) introduced covering problems. The first mathematical model in covering problems was developed by Toregas et al. (1971). They considered modeling the location of emergency service facilities. Two extensions of classical covering location problem are set covering location problem and maximal covering location problem. Initially, Campbell (1994) developed covering location problem into HLPs, which are named as hub set covering location and hub maximal covering location problems. Subsequently, Kara and Tansel (2003) and Wagner (2008) proposed new mathematical formulation for the single allocation p-hub covering location problem and the p-hub covering location problem with bounded path lengths, respectively. In both extensions of p-hub covering location problems, models seek to find the best location of hub facilities in a way that each origin-destination pair of non-hub nodes is covered by a pair or single hub nodes. In fact, if origin-destination pair node located at pre-specified distances (a maximum acceptable service distance) from hub nodes, they are covered. Also, for covering non-hub nodes, the cost of transportation from origin  $i$  to destination  $j$  through the selected hub nodes must not be greater than or equal to a predetermined value.

$$C_{ij}^{km} \leq \varphi_{ij} \quad (19)$$

As in the p-hub center problem, Campbell (1994b) defined three coverage criteria for hubs. The origin-destination pair  $(i, j)$  is covered by hubs  $k$  and  $m$  if:

- ❖ The cost from  $i$  to  $j$  via  $k$  and  $m$  does not exceed a specified value,
- ❖ The cost for each link in the path from  $i$  to  $j$  via  $k$  and  $m$  does not exceed a specified value,
- ❖ Each of the origin-hub and hub-destination links meets separate specified values.



### 2.2.5.1 Hub set covering location problem

One of the particular extensions of hub covering problem is hub set covering location problem. The hub set-covering problem tries to minimize the number of hubs to ensure that the length of between each pair of nodes is less than or equal to a predefined cover radius  $\beta$ . In order to formulate this problem, all assumptions of p-hub median location problem are valid, except that the number of hub nodes are unknown i.e. endogenous. Therefore, fixed cost of establishing hub facilities should be taken into account. Let  $F_k$  be the fixed cost of opening hub facilities in node  $k$ . Also,  $\varphi_{ij}$  represents maximum cost for covering links connecting demand nodes  $i$  to  $j$ , and  $V_{ij}^{km}$  is equal to one, if hubs located at nodes  $k$  and  $m$  could cover demand pair  $(i, j)$ , otherwise it is zero. Like p-hub median location problem,  $C_{ij}^{km}$  transfers cost from the origin node  $i$  to the destination node  $j$  via hub nodes located at nodes  $k$  and  $m$ . By considering this notation, the objective function is formulated as Eq. (20) and is consistent with Eq.(12) - Eq.(16) and Eq.(21).

$$\min \sum_k F_k X_k \quad (20)$$

subject to

$$\sum_k \sum_m V_{ij}^{km} Z_{ij}^{km} \geq 1 \quad \forall i, j \quad (21)$$

(12) -(16)

In Eq. (20), total cost of opening new hub facilities is minimized, and Eq.(21) ensures that each demand pair is covered at least one time by a hub pair.

### 2.2.5.2 P-hub maximal covering location problem

Another particular extension of hub covering problem is hub maximal covering. Like hub set covering, this problem is also formulated as p-hub median location problem, except that in this problem the number of hub nodes are determined previously –exogenous- and input of the model is a parameter. Since the constraint of the model is like that of p-hub median location problem, its objective function is defined as follows:

$$\max \sum_i \sum_j \sum_k \sum_m h_{ij} V_{ij}^{km} Z_{ij}^{km} \quad (22)$$

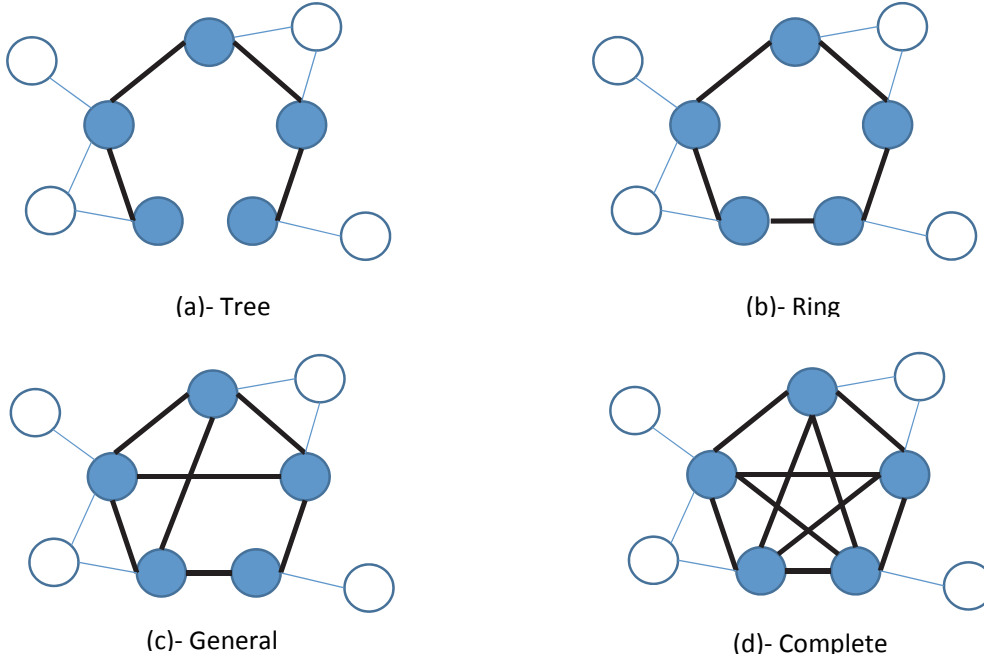
While, the amount of transportation demand is covered, it is maximized in Eq.(22).

### 2.2.6 Incomplete hub location

As mentioned above, fully connected hub networks are studied in this area. However, in the hub location literature, there are some articles that did not suppose completed hub network topology. We classify incomplete hub network into four categories that are reviewed in the following.

*Tree hub network.* In this topology as shown in Fig. 3(a), hub networks are connected without any cycles. In fact, in order to go from one hub to the other hub, there is only one path. This network structure was originally introduced by Kim and Tcha (1992) in HLPs. In their two-level network, single allocation strategy was studied at first level (i.e., Spoke links), and the hub network structure was a tree as the hub level.

*Ring hub network.* In the ring topology, the number of hub links is the same as the number of hubs and there is just one cycle in the hub network (Fig. 3(b)). In addition, each hub node is connected exactly to two hubs. There are few studies in this area of hub networks (Campbell & O'Kelly, 2012; Klineciewicz, 1998).



**Fig. 3.** Examples of the hub networks

*Special form hub network.* In the special topology, hub networks are restricted by decision makers, and their topology is different from that of tree and ring structures. For example, Wasner and Zäpfel (2004) introduced a model that all of its hubs just have been linked with one hub as a central hub. Their model is a mixed integer nonlinear programming, and multiple allocation strategy is employed in it.

*General hub network.* In the general topology, the shape of hub network is determined by the model, and each structure such as tree, ring, completed can be created. Actually, there is no restriction on the hub network design, so we call it the general structure. Yoon et al. (2000) suggested a general form of hub networks. The proposed model did not consider fixed cost of hubs. Then, they proposed a model with many variables and constraints in this area. This model takes into account costs of opening hubs, hub links, spoke links, and flows shipping. Yoon and Current (2008) afterwards presented another model, which did not consider shipping cost of flows. Like HLP, several models can be extended by considering several assumptions such as allocation strategy and capacity strategy in incomplete network. For example, single allocation incomplete p-hub median location problem is modeled as below:

$$\min \sum_{i \in N} \sum_{k \in H} C_{ik} O_i Y_{ij} + \sum_{i \in H} \sum_{j \in H} \sum_{k \in N} \alpha C_{ij} f_{ij}^k + \sum_{i \in N} \sum_{k \in H} C_{ki} D_i Y_{ik} \quad (23)$$

subject to: (6)-(9)

$$b_{km} \leq Y_{kk} \quad \forall k, m \in H \quad (24)$$

$$b_{km} \leq Y_{mm} \quad \forall k, m \in H \quad (25)$$

$$\sum_k \sum_m b_{km} = q \quad \forall k, m \in H \quad (26)$$

$$\sum_{j \in H} f_{ij}^k + O_i Y_{ik} = \sum_{j \in H} f_{ij}^k + \sum_{l \in N} h_{il} Y_{lk} \quad \forall i \in N, k \in H \quad (27)$$

$$f_{ij}^k + f_{ji}^k \leq O_k b_{ij} \quad \forall i, j \in H, k \in N \quad (28)$$

$$f_{ij}^k \geq 0 \quad (29)$$

where  $b_{ij}$  is a binary variable which is one, if a hub link is established between hubs  $i \in H$  and  $j \in H$  (otherwise it is zero); and  $H$  and  $N$  denote hub node and node set, respectively. Also,  $f_{ij}^k$  represents the

total amount of flow originating from node  $k \in N$  to be routed on hub link  $\{i, j\}$  in the direction from  $i \in H$  to  $j \in H$ .

Constraints (24) and (25) enforce that a hub link can only be established if both of the end nodes are hubs. Note that since we are designing an undirected hub network, we defined  $b_{ij}$  variables only for  $i < j$ . Constraint (26) ensures that exactly  $q$  hub links are to be established. Constraint (27) explains flow conservation constraint. In fact, for each node  $k$  and hub node  $i$ , the total flow originating from the node  $k$  entering into the hub node  $i$  must be equal to the outgoing flow. The first terms on the left and right-hand sides of Constraint (27) calculate the flow within the hub network ( $f$  variables); whereas the second terms correspond to the flows via the allocations. Constraint (28) restricts the  $f$  variables to be positive only on the established hub links. Eq (29) represents non-negativity requirements.

2.2.7 Hub arc location problem (HALP)

This model initially was introduced by Campbell et al. (2005a) in two parts. In part I, introduction of HALP was presented. The formulation and optimal algorithm were proposed in part II, subsequently. The optimal solution to the hub arc location problem may include three types of arcs:

- ❖ Hub arcs joining two hubs (with reduced unit flow cost)
- ❖ Access arcs joining a non-hub origin/destination and a hub, and
- ❖ Bridge arcs joining two hubs, but without the reduced unit flow costs.

The hub arc location problem tries to locate a given number of hub arcs in such a way that the total flow cost is minimized. For more details, refer to Campbell, J. F. et al. (2005b). Let  $q$  be the number of hub arcs and  $k, l \in H$ . Also, consider  $d_{ij}$  denotes distance between two nodes  $i$  and  $j$ . While HALP have three types of arcs, the parameters  $\chi$ ,  $\delta$ , and  $\beta$  represent the unit transportation costs for collection (origin-hub), distribution (hub-destination), and transfer along a bridge arc between two hubs, respectively. A summary of the notations is presented in Table 1.

Finally, HALP can be formulated as MILP as follows.

**Table 1**  
Notation for HALP

Variable	Description
$\tilde{Y}_{kl}^i$	transfer flow on a bridge arc from hub $k$ to hub $l$ that originates at $i$
$\hat{Y}_{kl}^i$	transfer flow on a hub arc from hub $k$ to hub $l$ that originates at $i$
$X_{lj}^i$	distribution flow from hub $l$ to destination $j$ that originates at $i$
$H_k$	1 if node $k$ is a hub, and 0 otherwise

$$\min \sum_i \left[ \sum_k \chi d_{ik} h_{ik} + \sum_k \sum_l d_{kl} (\beta \tilde{Y}_{kl}^i + \alpha \hat{Y}_{kl}^i) + \sum_l \sum_j \delta d_{lj} X_{lj}^i \right] \tag{30}$$

subject to

$$\sum_k h_{ik} = O_i \quad \forall i \tag{31}$$

$$\sum_l X_{lj}^i = h_{ij} \quad \forall i, j \tag{32}$$

$$h_{ik} + \sum_l (\tilde{Y}_{lk}^i + \hat{Y}_{lk}^i) = \sum_l (\tilde{Y}_{kl}^i + \hat{Y}_{kl}^i) + \sum_j X_{kj}^i \quad \forall i, k \tag{33}$$

$$\sum_l \sum_{k < l} b_{kl} = q \tag{34}$$

$$\hat{Y}_{kl}^i + \hat{Y}_{lk}^i \leq \tilde{M}_{ilk}^y b_{lk} \quad \forall i, l, k, l < k \tag{35}$$

$$\tilde{Y}_{kl}^i \leq \tilde{M}_{ikl}^y H_k \quad \forall i, l, k \tag{36}$$

$$\tilde{Y}_{kl}^i \leq \tilde{M}_{ikl}^y H_l \quad \forall i, l, k \quad (37)$$

$$X_{kj}^i \leq h_{ij} H_k \quad \forall i, l, k \quad (38)$$

$$h_{ik} \leq O_i H_k \quad \forall i, k \quad (39)$$

$$H_k \leq \sum_{l < k} b_{lk} + \sum_{l > k} b_{kl} \quad \forall k \quad (40)$$

$$h_{ik}, \hat{Y}_{kl}^i, \tilde{Y}_{kl}^i, X_{kj}^i \geq 0 \quad \forall i, j, k, l \quad (41)$$

$$b_{lk}, H_k \in \{0,1\} \quad \forall k, l, \quad l < k \quad (42)$$

in which M-values are some large constants. In Eq.(30), the total sum of the costs for collection, transfer, and distribution, is minimized. Eq. (31) ensures that all flows from each origin leave the origin. Eq. (32) ensures that all flows for each origin-destination pair arrives at the proper destination. The constraint (33) is the flow conservation equation at the hubs. Eq. (34) ensures that the appropriate number of hub arcs is selected. Eq. (35) establishes a hub arc for every hub arc flow. The constraints (36)-(37) ensure that the end points of bridge arcs will be hubs. Eqs. (38)-(39) ensure that hub nodes are established for every distribution and collection movement, respectively. Eq. (40) ensures that hub nodes are established only at the end points of hub arcs.

### 3. Solution algorithm of HLP

The optimization methods can be classified into four categories: exact algorithms, heuristic algorithms, meta-heuristic algorithms, and combinatorial algorithms. These algorithms have witnessed a spectacular growth in recent years and different types of HLPs are being solved via a large variety of these algorithms. In this section, the relevant research is reviewed and some of the represented algorithms and solution approaches are introduced. Also, we use some notations to introduce various kinds of HLPs properly.

**Table 2**

Notations for different type of HLPs

# of hub	Type of HLPs	Allocation strategy	Hub capacity
Fixed to P ( <i>P</i> )	Median ( <i>M</i> )	Single Allocations ( <i>SA</i> )	Capacitated ( <i>C</i> )
Not fixed ( <i>N</i> )	Center ( <i>T</i> )	Multiple Allocation ( <i>MA</i> )	Uncapacitated ( <i>U</i> )
	Covering ( <i>V</i> )	r-allocation ( <i>r</i> )	Capacity on route ( <i>CR</i> )
	Set Covering ( <i>SC</i> )		
	Maximal Covering ( <i>MC</i> )		
	Multi-objective ( <i>MO</i> )		

These notations are listed in Table 2 as mentioned above, a lot of HLPs cannot be solved in a reasonable time. To address this issue, heuristic and meta-heuristic algorithms are applied in many studies. However, some studies use mimetic algorithms which are combination of Evolutionary Algorithms (EA) with Local Search operators that work within the EA loop and solve the problem in a reasonable time. Literature of application of these algorithms is presented in Table 4. It is necessary to mention that some articles consider capacity constraint on route which connect the hub nodes together. In order to show this point, we use ‘CR’ as it is used in Table 2. Also, some data sets are shown as abbreviations. These data sets are: Civil Aeronautics Board (CAB), Australian Post (AP), Iranian Aviation Dataset (IAD), Turkish Network (TR) and Random Generation (RG).

#### 3.1 Application of exact algorithm in HLPs

Two independent heterogeneous streams, coming from very different scientific communities, had significant success in solving HLPs:

- Integer Programming (IP) as an exact approach, is coming from the operations research community and is based on concepts of linear programming (Dantzig, 1963).
- Local search with various extensions and independently developed variants, is called meta-heuristics, and is considered as a heuristic approach.

Therefore, even though integer programming optimization approaches are applied to solve small hub problems, larger instances of HLPs need to be solved by heuristic procedures or meta-heuristic procedures. Studies that use exact algorithm for solving HLPs are summarized in Table .

According to Table 3, most of the articles in this field considered uncapacitated cases. However, there are some articles that consider capacitated cases. The percentages of articles that consider capacitated cases have been increased from 21% in the period before 2013, to 33% afterwards. Also, a lot of attention has been paid to single allocation strategy (57%), median problem (65%) and fixed number of hub nodes (51%) in recent years.

### *3.2 Application of heuristic and meta-heuristic algorithm in HLP*

As mentioned above, a lot of HLP models cannot be solved in a reasonable time. To address this issue, a lot of heuristic and meta-heuristic algorithms are applied in many studies. However, some research used mimetic algorithms which are combination of Evolutionary Algorithms (EA) with Local Search operators that work within the EA loop and solve the problem in a reasonable time. The literature survey of applying these algorithms is presented in Table 4. It is necessary to mention that some articles considered capacity constraint on route which connects the hub nodes together. In order to show this point, we use 'CR' as it is used in Table 2. Also, some data sets are shown as abbreviations. These data sets include: Civil Aeronautics Board (CAB), Australian Post (AP), Iranian Aviation Dataset (IAD), Turkish Network (TR) and Random Generation (RG).

As indicated in Table 4, heuristic and meta-heuristic algorithms are often used for single allocation (85%) with respect to allocation strategy. Also, 24% of the published articles considered capacitated cases. Another observation is that, like exact algorithms, the median problem has attracted more attention than other types of HLPs (58%). One attractive research area presented in Table 4 is the continuous HLP, which is not considered in the published articles. While HLP belongs to a difficult class of problems (NP-hard) (Abdinnour-Helm & Venkataramanan, 1998), majority of the studies have applied heuristics and meta-heuristics algorithms.

### *3.3 Application of hybrid algorithm in HLP*

Since, each of the exact heuristic, and meta-heuristic algorithms have some problems such as speed, intensification, diversification, some works used a methodology which includes any pair of these algorithms to overcome above-mentioned problems. In HLP literature, there are few studies that used this methodology to solve HLPs. These studies are listed in Table 5.

As shown in Table 5, all of these studies considered median problems. Also, majority of them considered single allocation strategy, incapacitated cases. In fact, most of these studies have been published in last two years.

**Table 3**  
Application of exact algorithm in HLPs

Authors	Capacity	Assignment strategy	Model type	# of hub	Problem size	Data	Solving procedure
(Puerto et al., 2013)	U	SA	M	P	S	AP	Branch-and-bound-and-cut (B&B & cut)
(de Camargo et al., 2013)	U	SA	M	N	L	AP	Benders decomposition algorithm
(de Sá et al., 2013)	U	SA	M	N	S&M&L	AP	Benders decomposition algorithm
(Ghodratnama et al., 2013)	C	SA	V	N	S&M&L	RG	Branch-and-bound
(Campbell, J. F., 2013)	U	MA	M	N	S	CAB	Benders decomposition algorithm
(de Sá et al., 2013)	U	MA	M	P	S&M&L	CAB & AP	Benders branch-and-cut scheme
(Wang et al., 2013)	U	SA	M	N	L	Global liner shipping company	ILP
(Zarandi et al., 2013)	U	MA	V	N	M & L	CAB & AP & TR	MIP
(Zheng et al., 2014)	U	SA	M	N	L	Asia – Europe– Oceania shipping services	Lagrangian relaxation based solution method
(Karimi & Setak, 2014)	U	MA	M	N	S	CAB & IAD	Lagrangian relaxation approach and the valid inequalities
(Yang et al., 2014)	U	SA	T	P	S	-	Mixed-integer programming problem with generalized credibility constraints
(Rodríguez-Martin et al., 2014)	U	SA	M	P	S & M	CAB & AP	Branch-and-cut algorithm
(Hult et al., 2014)	U	SA	T	P	S & M	CAB & AP	Cutting plane method
(Correia et al., 2014)	C	SA	M	N	S & M	AP	Tighter linear relaxation bounds
(Boukani et al., 2014)	C	SA & MA	M	N	M	Iranian aviation dataset	MILP solver
(Yang, K. et al., 2014)	U	SA	T	P	S	RG	MIP solver
(Plum et al., 2014)	CR&U	SA	M	N	S	Liner-lib-2012	SFM mixed integer program
(Shahabi & Ummkrishnan, 2014)	U	SA & MA	M	N	S	CAB	MIP
(O’Kelly et al., 2015)	U	MA	M	P	S&M&L	CAB	Improved benders decomposition algorithm
(Xavier et al., 2015)	U	MA	M	P	S&M&L	German towns & AP & TSPLIB	The hyperbolic smoothing approach
(Ghaffari-Nasab et al., 2015b)	C	SA & MA	M	N	L	AP	Standard optimization package
(Ahmadi et al., 2015)	U	SA	M	P	S	An automobile part distribution system	Three main phases, namely location modeling, risk modeling, and decision making

**Table 3**  
Application of exact algorithm in HLPs (Continued)

Authors	Capacity	Assignment strategy	Model type	# of hub	Problem size	Data	Solving procedure
(Chaffari-Nasab et al., 2015a)	U	SA	M	N	S&M	Cheong et al. (2007)	Standard optimization package
(Yildiz & Karaşan, 2015)	U	SA	V	N	S&M&L	Global mobile data traffic forecast update, 2011–2016	Branch and cut algorithms
(Peker & Kara, 2015)	U	SA & MA	MC	P	S&M&L	CAB & TPS	MIP solver
(Rastani et al., 2015)	CR & C	SA	M	N	S&M	AP	CPLEX solver
(Yang, T.-H. & Huang, 2015)	U	SA	M	N	S	Air freight data in Taiwan and china	Wait-and-see model (WS model) & expected value (EV)
(Yang, T.-H. & Chiu, 2015)	C	SA	M	N	S	Air passenger market in Taiwan and mainland china	The expected value of perfect information (EVP), and the value of the stochastic solution (VSS)
(Puerto et al., 2015)	C	SA	M	N	S	AP	B&B
(An et al., 2015)	U	SA	M	P	M&L	CAB	Branch and bound & Lagrangian relaxation
(Alumur, S. A. et al., 2015)	C	SA & MA	M	N	S	CAB	Branch-and-bound tree and LP relaxation
(Chodratnama et al., 2015)	C	SA	MO (M & V & T & fixed cost)	N	S	Uniform	Fuzzy multi-objective goal programming (FMOGP) - Torabi and Hassini's (TH)
(Adibi & Razmi, 2015)	U	MA	M	P	S	Iranian aviation in 2010	MIP solver
(O'Kelly, 2015)	U	MA	M	P	S	CAB	Standard cuts
(Bray et al., 2015)	-	-	PE <sup>1</sup>	-	-	Asia and Europe	DEA
(Sarıççek & Akkuş, 2015)	U	SA	M	P	M	Turkish airport	ELECTERE iii & IP
(Çiftçi & Şevkli, 2015)	CR & U	SA	M	P	M	Turkey's biggest air carriers	MIP
(Mahmutogullari & Kara, 2016)	U	MA	M & T	P	L & M & S	CAB & TPS	Linear programming models
(Fang et al., 2016)	U	MA	M	N	S	CAB	ILP
(Corberán et al., 2016)	C	SA	M	N	S & M & L	-	Strategic oscillation

<sup>1</sup> Performance Evaluation

**Table 3**  
Application of exact algorithm in HLPs (Continued)

Authors	Capacity	Assignment strategy	Model type	# of hub	Problem size	Data	Solving procedure
(Puerto et al., 2013)	U	SA	M	P	S	AP	Branch-and-bound-and-cut (B&B & cut)
(de Camargo et al., 2013)	U	SA	M	N	L	AP	Benders decomposition algorithm
(de Sá et al., 2013)	U	SA	M	N	S&M&L	AP	Benders decomposition algorithm
(Ghodratnama et al., 2013)	C	SA	V	N	S&M&L	RG	Branch-and-bound
(Campbell., 2013)	U	MA	M	N	S	CAB	Benders decomposition algorithm
(de Sá et al., 2013)	U	MA	M	P	S&M&L	CAB & AP	Benders branch-and-cut scheme
(Wang et al., 2013)	U	SA	M	N	L	Global liner shipping company	ILP
(Zarandi et al., 2013)	U	MA	V	N	M & L	CAB & AP & TR	MIP
(Zheng et al., 2014)	U	SA	M	N	L	Asia – Europe–Oceania shipping services	Lagrangian relaxation based solution method
(Karimi & Setak, 2014)	U	MA	M	N	S	CAB & IAD	Lagrangian relaxation approach and the valid inequalities
(Yang, K. et al., 2014)	U	SA	T	P	S	-	Mixed-integer programming problem with generalized credibility constraints
(Rodríguez-Martín et al., 2014)	U	SA	M	P	S & M	CAB & AP	Branch-and-cut algorithm
(Hult et al., 2014)	U	SA	T	P	S & M	CAB & AP	Cutting plane method
(Correia et al., 2014)	C	SA	M	N	S & M	AP	Tighter linear relaxation bounds
(Boukani et al., 2014)	C	SA & MA	M	N	M	Iranian aviation dataset	MILP solver
(Yang, K. et al., 2014)	U	SA	T	P	S	RG	MIP solver
(Plum et al., 2014)	CR&U	SA	M	N	S	Liner-lib-2012	SFM mixed integer program
(Shahabi & Unnikrishnan, 2014)	U	SA & MA	M	N	S	CAB	MIP
(O’Kelly et al., 2015)	U	MA	M	P	S&M&L	CAB	Improved benders decomposition algorithm
(Xavier et al., 2015)	U	MA	M	P	S&M&L	German towns & AP & TSPLIB	The hyperbolic smoothing approach
(Ghaffari-Nasab et al., 2015b)	C	SA & MA	M	N	L	AP	Standard optimization package
(Ahmadi et al., 2015)	U	SA	M	P	S	An automobile part distribution system	Three main phases, namely location modeling, risk modeling, and decision making
(Ghaffari-Nasab et al., 2015a)	U	SA	M	N	S&M	Cheong et al. (2007)	Standard optimization package
(Yildiz & Karaşan, 2015)	U	SA	V	N	S&M&L	Global mobile data traffic forecast update, 2011–2016	Branch and cut algorithms
(Peker & Kara, 2015)	U	SA & MA	MC	P	S&M&L	CAB & TPS	MIP solver
(Rastani et al., 2015)	CR & C	SA	M	N	S&M	AP	CPLEX solver



**Table 4**  
**Application of heuristic and meta-heuristic algorithm in HLPs**

Author	Capacity	Assignment Strategy	Model Type	# of Hub	Problem Size	Data	Solving Procedure
(Bashiri et al., 2013)	C	SA	T	P	S&M	AP	GA
(Mohammadi, Mehrdad et al., 2013)	C	SA	V	P	S&M&L	Uniform	MOICA-NSGA II-PAES
(Luter-Villagra & Marjanov, 2013)	U	SA	M	N	S	CAB	GA
(Yang et al., 2013a)	U	SA	T	P	S	Uniform	GALS
(Lin et al., 2013)	C	SA	M	N	-	-	Based on Greedy Heuristic
(Yang et al., 2013b)	U	SA	T	P	S	Simulated Data	PSOGALS
(Aros-Vera et al., 2013)	U	SA	M	P	S	Queens borough, NY, to commuters traveling to Manhattan	Heuristic Concentration Integer (HCI) procedure
(Davari et al., 2013)	U	SA	V	N	S	CAB	simulation-embedded Variable Neighborhood Search (VNS)
(Hosapujari & Verma, 2013)	CR	SA	M	P	M	Mandal's Swiss network	GA
(Zade et al., 2014)	U	SA	MO (MC)	P	S&M	TR	NSGA II
(Ting & Wang, 2014)	U	SA	M	N	S&M&L	CAB & AP	threshold accepting (TA) algorithm
(Rieck et al., 2014)	U	SA	M	N	S&M&L	Geo data containing positions of sawmills and wood-manufacturers	GA with multi-start procedure
(Mohammadi et al., 2014)	C	SA	M	P	S&M&L	Randomly	simulated annealing (SA) - imperialist competitive algorithm (ICA)
(Nikokalam-Mozafar et al., 2014)	C	SA	MO (V)	N	-	S&M&L	multi-objective invasive weed optimization-NSGA
(Parvareh et al., 2014)	U	MA	M	P	S&M&L	CAB & TR	MOSA-MOTS
(Mokhtari & Abbasi, 2014)	U	SA	M	N	L	RG	VNPSO
(Qin & Gao, 2014)	U	SA	M	P	L	RG	GA
(Eghbali et al., 2014)	U	SA	MO (V)	N	S	TR	NSGA II
(Peiró et al., 2014)	U	r	M	P	S&M&L	CAB & AP & USA423	Heuristic based on the GRASP methodology
(Sasaki et al., 2014)	U	MA	HALP	N	S	CAB	H
(Sadeghi et al., 2015)	CR	SA	V	P	S&M&L	RG	DE & GA
(Zheng et al., 2015)	CR	SA	M	N	M	Asia-Europe-Oceania shipping network	Genetic algorithm embedded with a multi-stage decomposition approach
(Estrada-Romeu & Robusci, 2015)	CR	SA	M	N	S&M	the line-haul network design of the largest Spanish freight carrier	H
(Rahmani et al., 2015)	CR	SA	M	N	S&M&L	RG	Non-trivial extensions of the nearest neighbor and insertion approaches.
(Todorosjević et al., 2015)	U	-	M	P	L	AP & USA423 data set	general variable neighborhood search heuristic
(He et al., 2015)	C	SA	M	N	L	RG	H
(Ebrahimi-zade et al., 2015)	U	SA	SC	N	S	cargo delivery firm which serves in 11 provinces of Iran	GA & ICA
(Rahimi et al., 2015)	C	SA	M	P	M	RG	DE
(Pasandideh et al., 2015)	U	SA	MC	P	S & M	RG	SA-GA
(Shahbaga et al., 2015)	C	SA	M	P	-	Uniform	H
(Meier & Clausen, 2015)	U	SA	M	N	S & M	data from a large European road freight company	H
(Martí et al., 2015)	U	MA	M	P	S&M&L	CAB & AP	Scatter search
(Rabbani & Kazemi, 2015)	U	MA	T	P	M	CAB & AP	SA-GA by use Dijkstra's algorithm in fitness function
(Rahimi et al., 2015)	U	SA	MO (T)	P	S&M&L	RG	self-adaptive differential evolution (SADE)
(Yang, K. & Liu, 2015)	U	SA	T	P	S	RG	Hybrid PSO
(Azizi et al., 2016)	U	SA	M	P	S&M&L	CAB & TR	GA-based algorithm

**Table 5**  
Application of hybrid algorithm for HLPs

Author	Capacity	Assignment Strategy	Model Type	Number of Hub nodes	Problem Size	Data	Solving Procedure		
							Exact	Meta-heuristic	Heuristic
(Marić et al., 2013)	U	SA	M	N	S&M&L	CAB & AP	Local Search	EA	-
(Stanojević et al., 2015)	C	SA	M	N	L	AP	-	EA	Branch and Bound
(Zarandi et al., 2015)	U	SA	M	P	-	CAB	-	Simulated Annealing (SA)	Iterated Local Search (ILS)
(Gelareh et al., 2015)	U	MA	M	N	L	CAB & AP	-	Meta-heuristic method	Benders decomposition approach
(Lopes et al., 2015)	C	SA	M	P	M	-	local search heuristic	-	branch-and-cut approach
(Martins de Sá et al., 2015)	U	SA	M	N	M&L	CAB & AP	VND <sup>2</sup> - GRASP <sup>3</sup> - ALNS <sup>4</sup>	-	Benders decomposition
(Ghadiri & Rahmani, 2015)	U	SA	M	P	M	AP	novel heuristic based on the discrete PSO algorithm	First stage solves by VNS <sup>5</sup> and Second stage solve by TS <sup>6</sup>	-
(de Sá et al., 2015)	U	SA	M	P	S&M&L	CAB & AP	Benders – branch-and-cut algorithm	GRASP - ALNS	H

### 3.4 Application of HLP in marine transportation

In previous sections, we reviewed applications of solving procedures in HLPs. As mentioned above, HLPs was applied in liner shipping companies to serve the destination nodes in their trading lines. Hub networks can be applied in transportation systems and they provide more flexibility through focuses on the cargo flows. Moreover they benefit from lower costs for shipping. HLP improves the cost structure in ports through changes in transportation distance and uses the economy of scale in the ocean routes. Therefore, in this section application of HLP in marine transportation is reviewed. Table 6 presents the works that implemented HLP in marine transportation.

According to the researches summarized in Table 6 except MCDM model, all of the articles used median problem in order to formulate HLP and there is no research which apply any kind of other major HLPs such as: hub center problem, hub covering problem. Also, all of the articles assume discrete domain, except Zhao et al. (2016) which adopted continuous domain for HLP.

<sup>2</sup> Variable neighborhood descent

<sup>3</sup> Greedy randomized adaptive search procedure

<sup>4</sup> Adaptive large neighborhood search

<sup>5</sup> variable neighborhood search

<sup>6</sup> Tabu search

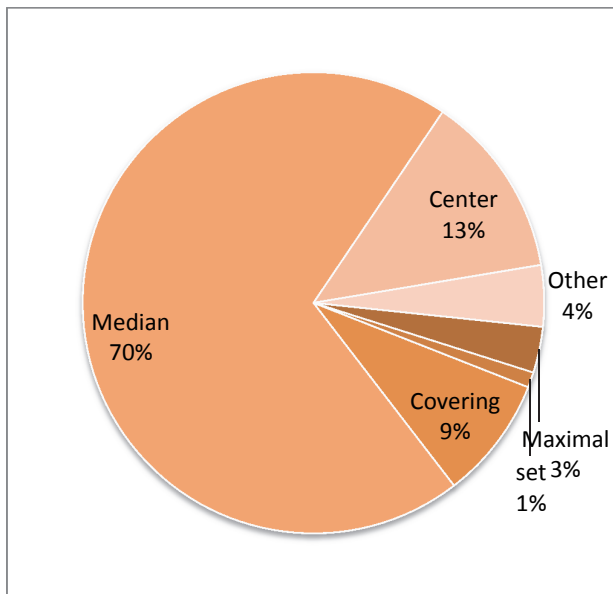
**Table 6**  
Application of HLP in marine transportation

Authors	Capa city	Assignment strategy	Model type	Number of hub nodes	Problem size	Data	Solving procedure
(Gelareh, Shahin et al., 2013)	U	MA	M	-	M	Europe–North American ports	H
(Gelareh, S et al., 2013)	CR & U	SA	M	P	S	Asia–Europe trade route	Lagrangian decomposition approach which uses a heuristic procedure
(Meng & Wang, 2011)	CR	SA	M	N	M	Southeast Asian countries	Hybrid GA & solution quality
(Gelareh & Nickel, 2011)	CR	MA	M	N	L	North America and Europe	Decomposition algorithm
(Jiang et al., 2012)	CR & C	MA	M	N	M	RG	-
(Gelareh et al., 2010)	U	MA	M	N	S & M	OR library	Lagrangian algorithm
(Asgari et al., 2013)	U	SA	M	N	S	Asian hub ports: Singapore and Hong Kong	Interval branch and bound
(Zabih et al., 2016)	-	-	MCD M	-	-	South ports of Iran	AHP & TOPSIS
(Yang, Y.-C. & Chen, 2016)	-	-	MCD M	-	-	Ports in northeast Asia	AHP & GRA
(Merakli & Yaman, 2016)	U	SA	M	P	L	CAB & TR	Benders decomposition
(Zhao et al., 2016)	U	SA	M	N	L	RG	Lagrange relaxation
(Yang, Y.-C. & Chen, 2016)	-	-	MCD M	-	-	Taiwan, Korea, and Japan ports	AHP & GRA
(Su et al., 2016)	-	-	MCD M	-	-	Hong Kong and Xiamen ports, Kaohsiung Port	AHP & Entropy & GRA
(Sun & Zheng, 2016)	CR	SA	M	N	M	Arctic waterways	Branch-and bound method

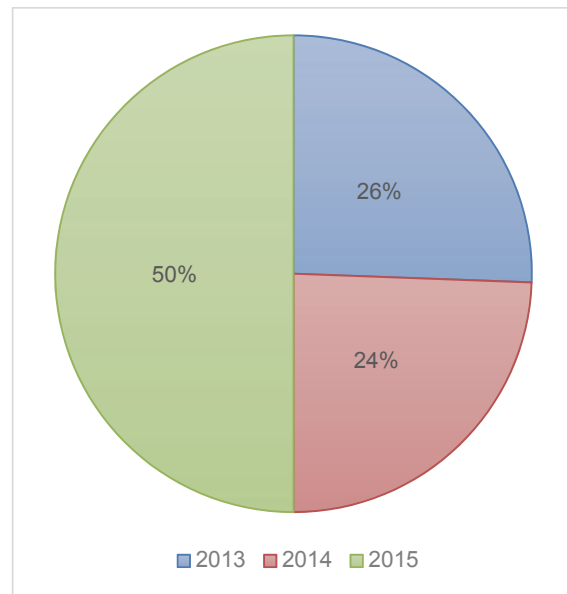
#### 4. Conclusion

In this paper we reviewed the related literature of HLP problem during time period 2013 to 2016. We updated the classifications, and presented the basic models for different variations of HLP. In addition, we reviewed some new models that are not considered in previous review papers. Also, a categorization based on the solution approach is presented, which is divided into three categories, including: exact algorithm, heuristic and meta-heuristic algorithms and hybrid algorithm.

Fig. 4 shows the total number of the publications based on applied models and Fig. 5 shows the distribution of publications in recent years.



**Fig. 4.** The total number of publications based on underlying models



**Fig. 5.** The number of publications of HLP in recent years

The overall publication trend of HLP is as follows: before the year 2000, focus of studies are on defining and formulating new problems. After that, gradually, some extensions of HLP are presented like HLP with capacity constraint are considered. Also, researchers focus on finding different solution methodologies for these problems. However, HLP is a NP-HARD problem, but in recent years, the number of articles that used exact algorithm for solving HLP has increased. From model type viewpoint, hub median problem has attracted more attention than hub covering location problems and hub center location problems in HLP literature. This can be observed clearly in Fig. 4.

Based on Fig. 5, we can observe that the number of articles in HLP area has increased substantially in recent years. It is predicted that this growth will continue in the future. Since in real world economics, the companies are trying to get more and more market share and this requires selling their products with lower prices, which can be achieved by reducing the transportation costs that in turn can be considered as an economic issue in HLP.

There are few articles in the literature that consider more than one objective. Usually in HLPs, objective function is minimizing the total cost of transportation. However, in real world, there are a lot of other important criteria which must be considered in problem formulation. For example, some of these criteria are including but not limited to: time, distance, profit, utilization, covering, opening, reopening, activating costs of vehicles that are located in the hubs and etc. Therefore, incorporating multi-objective formulation in the HLP modeling and developing new solution procedures for such models can be a possible future research direction.

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