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A two-warehouse production inventory model with trade credit under reliability consideration

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A B S T R A C T

A two warehouse production inventory model is developed for deteriorating items under reliability consideration. The effect of trade credit is considered under inflation. Since, formulating a suitable inventory model is one of the major concerns for an industry, the main objective of this paper is to optimize the total related cost for reliable production process. The model is illustrated through numerical example. The sensitivity analyses of the cost function are performed due to different measures and some managerial inferences are presented.

1. Introduction

In classical economic production quantity model, it is assumed that the production set-up cost is fixed and all the items produced are of perfect quality. But, in reality all the items produced are not always perfect but directly affected by the reliability of the production process. The quality of the product can be improved by investment in the reliability of the production process. During the past few years, researchers have started focusing on this concept. Lin and Hou (2005) presented a model with improvement in process reliability, quality and reduction in set up time. Cheng (1989) developed economic production quantity model with flexibility and reliability consideration. Pal et al. (2007) focused on policy with variable demand under reliability consideration. Leung (2007) generalized geometric programming solution to economic production quantity model with flexibility and reliability consideration. Yadav et al. (2010) focused on production model under limited storage with flexibility and reliability consideration. Singhal (2013) developed volume flexible inventory system in Fuzzy environment. Panda and Maiti (2009) developed a multi-item inventory model with price dependent demand with flexibility and reliability consideration under fuzzy environment using geometric programming approach. In today’s competitive environment, it is more common to see that retailer is

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allowed a fixed time period before they settle their account to the supplier. This period is also known as trade credit period. Before the end of the end of trade credit period the retailer can sell the goods and accumulate revenue and earn interest. A high interest is charged if the payment is not settled by the end of this period. Shah (2006) developed an inventory model for items deteriorating with time under permissible delay in period. Soni et al. (2006) presented an EOQ model under DCF approach for progressive payment scheme. Yang (2006) developed a two warehouse inventory model under conditionally delay in payment. He focused on warehouse problem considering partial backlogging. Singh and Saxena (2014) presented two-warehouse inventory model under trade credit in fuzzy environment.

The effect of inflation is observed in many countries; first, it has been introduced by Buzacot (1975) where economic order quantity model under inflation is presented. Several researchers extended the work of in different ways. Dey et al. (2008) developed two storage inventory models under inflation and time value of money. Singh et al. (2009) highlighted an integrated model with multivariable demand with credit period. Hadidi et al. (2011) focused on integrated cost model under perfect maintenance. Soni et al. (2006) illustrated optimal policies involving various service level constraints. Park (1983) presented integrated production model for deteriorating item. Yan and Cheng (1988) developed economic order quantity model under assumption of production stopping and restarting times. Maity and Maiti (2009) presented optimal inventory policies under different substitute item with deterioration. Singh et al. (2013) illustrated an EOQ model considering trapezoidal demand and trade credit. They presented economic order quantity model with variable demand rate under inflation. Tayal et al. (2014) developed a production inventory model under space restriction. Singh and Prasher (2014) presented production inventory model considering machine breakdown and stochastic repair time.

In this paper we have presented two warehouse production inventory models for deteriorating items with permissible delay in payment under inflation and reliability consideration. Demand rate is considered as an exponential function of time. Finally, some numerical examples for illustration are provided and sensitivity analysis is performed.

2. Assumptions and Notations

2.1. Assumptions

- Production rate is greater than demand rate. Also, it is linear combination of on-hand inventory and demand rate i.e. $P(t)=\left[I(t)+bD(t)\right]\left(1-e^{-dt}\right)$.
- Demand rate is exponentially an increasing function of time .i.e. $D(t)=\mu e^{\lambda t}, 0\le\lambda\le 1$.
- Deterioration is taken as time dependent for owned warehouse, while, Wei-bull distribution for rented warehouse.
- Planning Horizon is finite.
- Model is considered for imperfect items and inflation is also taken in this model.
- Shortages are not allowed.
- Lead time is zero, and no replenishment or repair of deteriorated items is made during a given cycle.
- A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.
- The owned warehouse (O.W) has a fixed capacity of W units, and the rented warehouse (R.W) has unlimited capacity.
- The supplier provides the retailer a permissible delay of payments. During the trade credit period the account is not settled, the revenue is deposited in an interest bearing account. At the end of the permissible delay, the retailer pays off the items ordered, and starts to pay the interest charged on the items in stock.

- Total cost of interest and depreciation per production cycle is inversely related to the set up cost and directly related to process reliability i.e. $f(C_s, v) = c\, C_s^{-e} v^f$ where c, e, f > 0 are constant real numbers chosen to provide the best fit of the estimated cost function. The process reliability level v means of all the items produced in a production run only v% are acceptable quality that can be used to meet demand.

2.2 Notations

d	Imperfect production rate
W	Owned warehouse capacity
C_S	Set up cost per production run. (a decision variable)
C_{RW}	Holding cost rented warehouse per unit time
C_{OW}	Holding cost owned warehouse per unit time
C_D	Deterioration cost
$I_{O_1}(t)$	Inventory level in owned warehouse $0 \le t \le t_1$
$I_{R_2}(t)$	Inventory level in rented warehouse $t_1 \le t \le t_2$
$I_{R_3}(t)$	Inventory level in rented warehouse $t_2 \le t \le t_3$
$I_{O_4}(t)$	Inventory level in owned warehouse $t_3 \le t \le T$
$I_{O_5}(t)$	Inventory level in owned warehouse $t_1 \le t \le t_3$
t_1, t_2	Production period for owned warehouse and rented warehouse
t_3, T	Non-Production period
T	Total cycle time
M	Retailer's trade credit period offered by supplier in years
s	Unit selling price
C	Unit purchase cost
I_e	Interest which can be earned per \$ per year
I_C	Interest charges per \$ in stocks per year by the supplier
IDC	Cost of interest and depreciation per production cycle
$TC_1(C_S, v, T)$	Present worth of total cost per unit time, when $M \le t_3 < T$
$TC_2(C_S, v, T)$	Present worth of total cost per unit time, when $t_3 < M \le T$
$TC_3(C_S, v, T)$	Present worth of total cost per unit time, when $M > T$

3. Mathematical model

The production starts at time $t = 0$, and items accumulate from 0 up to W units in owned warehouse. After time t_1 any production quantity exceeding W will be stored in rented warehouse. After this production stopped and the inventory level in rented warehouse begins to decrease at t_2 and will reach 0 units at t_3 due to combined effect of demand and deterioration. The inventory level in owned warehouse comes to decrease at t_1 and then falls below W at t_2 & t_3 due to deterioration. But, during $[t_3, T]$, the inventory is depleted due to both demand and deterioration. By the time to T, both warehouses are empty Fig. 1 shows the behavior of inventory system.

These are the differential equations showing the inventory with the variation in time.

$$\frac{dI_{O_1}(t)}{dt} + \theta\, I_{O_1}(t) = P(t) - D(t) \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_{R_2}(t)}{dt} + \theta\, I_{R_2}(t) = P(t) - D(t) \qquad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI_{R_3}(t)}{dt}+\theta\, I_{R_3}(t)=-D(t) \qquad t_2\le t\le t_3 \tag{3}$$

$$\frac{dI_{O_4}(t)}{dt}+\theta\, I_{O_4}(t)=-D(t) \qquad t_3\le t\le T \tag{4}$$

$$\frac{dI_{O_5}(t)}{dt}+\theta\, I_{O_5}(t)=0 \qquad t_1\le t\le t_3 \tag{5}$$

With boundary conditions:

$$I_{O_1}(0)=0,\ I_{R_2}(t_1)=0,\ I_{R_3}(t_3)=0,\ I_{O_4}(T)=0,\ I_{O_5}(t_1)=W \tag{6}$$

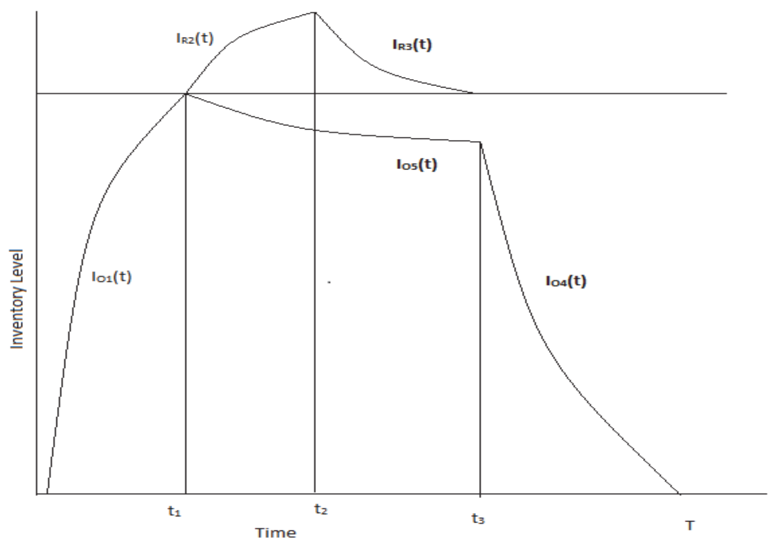

Fig. 1. Graphical representation of the two-warehouse inventory system

The solution of these above mentioned equations are given as follow:

$$I_{O_1}(t)=(b-1)\mu\left(\frac{dt^2}{2}+\frac{\lambda dt^3}{3}-\frac{(\theta-d)dt^4}{4}\right)-\mu\left(t+\frac{\lambda t^2}{2}-(\theta-d)\frac{t^3}{2}\right) \qquad 0\le t\le t_1 \tag{7}$$

$$I_{R_2}(t)=(b-1)\,\mu\left(\frac{d\left(t^2-t_1^2\right)}{2}-\frac{\alpha d t^{\beta}\left(t^2-t_1^2\right)}{2}+\frac{\left(t^3-t_1^3\right)\lambda d}{3}\right)-\mu\left(\left(t-t_1\right)-\alpha t^{\beta}\left(t-t_1\right)+\frac{\lambda\left(t^2-t_1^2\right)}{2}-\left(t^2-t_1^2\right)\frac{\lambda\alpha t^{\beta}}{2}\right), \qquad t_1\le t\le t_2 \tag{8}$$

$$I_{R_3}(t)=\mu\left(\left(t_3-t\right)-\alpha t^{\beta}\left(t_3-t\right)+\frac{\lambda}{2}\left(t_3^{\,2}-t^2\right)-\frac{\lambda\alpha t^{\beta}}{2}\left(t_3^{\,2}-t^2\right)\right) \qquad t_2\le t\le t_3 \tag{9}$$

$$I_{O_4}(t)=\mu\left(\left(T-t\right)-\frac{\theta t^2}{2}\left(T-t\right)+\frac{\lambda}{2}\left(T^2-t^2\right)-\frac{\lambda\theta t^2}{4}\left(T^2-t^2\right)\right) \qquad t_3\le t\le T \tag{10}$$

$$I_{O_5}(t)=W\,e^{-\left(\frac{\theta}{2}\right)\left(t_1^{\,2}-t^2\right)} \qquad t_1\le t\le t_3 \tag{11}$$

Based on the assumptions and description of the model, the total annual costs, TC, include the following elements:

The present worth ordering cost is given by C_S (12)

The present worth inventory holding cost in rented warehouse and owned warehouse are

Holding Cost rented warehouse $= c_{RW}\left(\int_{t_1}^{t_2} I_{R_2}(t)\ e^{-rt}dt + \int_{t_2}^{t_3} I_{R_3}(t)\ e^{-rt}dt\right)$ (13)

Holding Cost owned warehouse $= c_{OW}\left(\int_{0}^{t_1} I_{O_1}(t)\ e^{-rt}dt + \int_{t_1}^{t_3} I_{O_5}(t)\ e^{-rt}dt + \int_{t_3}^{T} I_{O_4}(t)\ e^{-rt}dt\right)$ (14)

The present worth deterioration cost=

$$I_D = c_D\left(\int_0^{t_1}\theta t I_{O_1}(t)e^{-rt}dt + \int_{t_1}^{t_3}\theta t I_{O_5}(t)e^{-rt}dt + \int_{t_3}^{T}\theta t I_{O_4}(t)e^{-rt}dt + \int_{t_1}^{t_2}\alpha\beta t^{\beta-1} I_{R_2}(t)e^{-rt}dt + \int_{t_2}^{t_3}\alpha\beta t^{\beta-1} I_{R_3}(t)e^{-rt}dt\right) \quad (15)$$

Now,we will calculate interest paid and earned by the retailer, for this there are three cases:

Case 1: ($M \le t_3 < T$)

In this case, the permissible delay period *M* expires before the total inventory depletion period *T*. Therefore, retailer will have to pay interest charged on unsold items during (*M*, *T*).

In this case, $M \le T$ the present worth of interest earned is

$$IE_1 = sI_e\int_0^M Dt\,e^{-rt}dt = \mu\, sI_e\left(\frac{M^2}{2} - (r-\lambda)\frac{M^3}{3} + \frac{\lambda r M^4}{4}\right) \quad (16)$$

Hence, present worth of interest payable by retailer is given by

$$IC_1 = cI_C\left(\int_M^{t_3} I_{R_3}(t)e^{-rt}dt + \int_M^{t_3} I_{O_5}(t)e^{-rt}dt + \int_{t_3}^{T} I_{O_4}(t)e^{-rt}dt\right) \quad (17)$$

Case 2: ($t_3 < M \le T$) In this case, present worth interest payable is

$$IC_2 = cI_C\int_M^T I_{O_4}(t)e^{-rt}dt \quad (18)$$

Case 3: ($M > T$)‚ In this case, no interest charges are paid for the items

$$IC_3 = 0$$

Hence, the present worth interest earned is given by

$$IE_2 = sI_e\left(\int_0^T Dt\,e^{-rt}dt + DT(M-T)\right) \quad (19)$$

Therefore, the present worth annual total relevant costs for the retailer can be expressed as

TC(t_3,C_S,v,T) = Ordering Cost + Holding cost in Rented Warehouse + Holding cost in Owned Warehouse + Deteriorating cost + Depreciation Cost +Interest payable cost –Interest earned.

Therefore, the total cost per unit time of the given inventory model as a function of t_1, t_2, t_3, v, C_S and T say $TC\ (t_1, t_2, t_3, v, C_S, T)$ is given by

$$\text{TC }(t_3, \text{CS}, v, T) = \begin{cases} TC_1, \text{ if } M \le t_3 < T \\ TC_{2,} \text{ if } t_3 < M \le T \\ TC_{3,}\text{if } M > T \end{cases} \tag{20}$$

Eq. (20) denotes the cost function of the system in terms of t_1, t_2, t_3, v, C_S and T. To find out the optimal solution of this system we have to find out the optimal values of t_1, t_2, t_3, C_S, v and T. We have relation between these variables.

$$0 \le t_1 \le t_2 \le t_3 \le T \tag{21}$$

4. Numerical Analysis

The above discussed inventory model is illustrated through the numerical example for which the input values are considered in proper units as follows:

$\theta = 0.01, r = 1, \lambda = 2, \mu = 20, C_D = 2, I_C = 0.15, M = 0.25, c = 0.2, f = 1, e = 1$

$\alpha = 0.06, \beta = 0.01, d = 0.1, b = 5, C_{RW} = 0.6, W = 500, C_{OW} = 1.2, C = 0.2$

There are three cases according to the permissible delay period as follows:

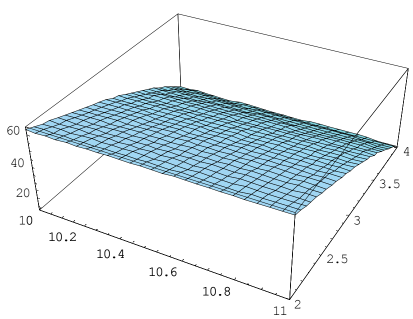

Fig.2. (Case: M≤ t3<T) Convexity of TC 1 w.r.t. v and Cs

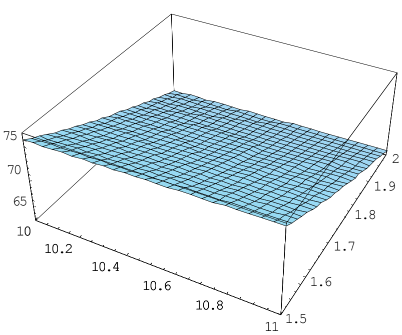

Fig. 3. (Case: t3 < M < T) Convexity of TC 2 w.r.t. v and Cs

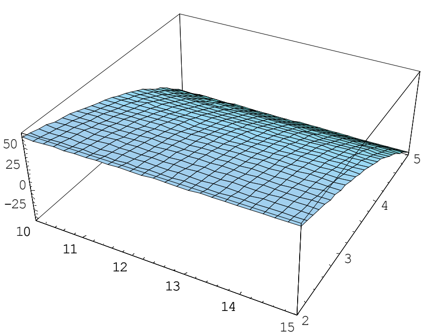

Fig.4. (Case: M > T) Convexity of TC 3 w.r.t. v and Cs

Case 1: ($M \le t_3 < T$), Using mathematical software Mathematica 7 the output results are as follows: T=1.56547, v=0.209467, t_3 =0.800002, C_S= 10.32, TC_1 = 96.5648.

Case 2: ($t_3 < M \le T$), Using mathematical software Mathematica 7 the output results are as follows: T=1.52193, v=0.233164, t_3 =0.800001, C_S= 10.30, TC_1 = 90.8913.

Case 3: (M > T), Using mathematical software Mathematica 7 the output results are as follows: T=2.64105, v=0.629393, t_3 =0.800475, C_S= 10.13, TC_1 = 50.9644.

5. Sensitivity Analysis

Corresponding to different associated parameters, a sensitivity analysis is carried out to check the stability of the model. The analysis has been done with the parameters M, α, β, μ, W taking one parameter at a time and other variables unchanged and is shown in Table 1, Table 2 and Table 3.

Table 1
Case: $M \le t_3 < T$

% variation in M	M	v	C_S	T	TC_1
-20%	0.20	0.212076	10.39	1.57032	97.0438
-15%	0.2125	0.211429	10.37	1.56911	96.9242
-10%	0.225	0.210778	10.35	1.5679	96.8045
-5%	0.2375	0.210124	10.34	1.56668	96.6847
0%	0.25	0.21	10.32	1.565	96.5648
5%	0.2625	0.208806	10.31	1.56425	96.4448
10%	0.275	0.208141	10.30	1.56304	96.3247
15%	0.2875	0.207473	10.29	1.56182	96.2045
20%	0.3	0.206802	10.27	1.5606	96.0843
% variation in α	**α**	**v**	**C_S**	**T**	**TC_1**
-20%	0.048	0.209361	10.37	1.56799	96.813
-15%	0.051	0.209385	10.36	1.56741	96.7562
-10%	0.054	0.209411	10.34	1.5668	96.6958
-5%	0.057	0.209438	10.33	1.56615	96.632
0%	0.06	0.21	10.32	1.565	96.5648
5%	0.063	0.209497	10.30	1.56475	96.494
10%	0.066	0.209528	10.28	1.564	96.4198
15%	0.069	0.209561	10.27	1.56321	96.3421
20%	0.072	0.209595	10.26	1.56238	96.2608
% variation in β	**β**	**v**	**C_S**	**T**	**TC_1**
-20%	0.0080	0.209466	10.25	1056545	96.563
-15%	0.0085	0.209467	10.28	1.56546	96.5634
-5%	0.0095	0.209467	10.30	1056547	96.5643
0%	0.01	0.21	10.32	10565	96.5648
5%	0.0105	0.209467	10.34	1.56547	96.5652
15%	0.0115	0.209467	10.36	1.56548	96.5661
20%	0.012	0.209467	10.39	1.56549	96.5665
% variation in μ	**μ**	**v**	**C_S**	**T**	**TC_1**
-20%	16	0.288898	10.26	1.68971	87.4423
-15%	17	0.270418	10.28	1.65509	89.8131
-5%	19	0.230654	10.30	1.59324	94.3693
0%	20	0.21	10.32	1.565	96.5648
5%	21	0.18731	10.34	1.53951	98.7107
15%	23	0.224061	10.36	1.49219	102.876
20%	24	0.197361	10.39	1.47069	104.894
% variation in W	**W**	**v**	**C_S**	**T**	**TC_1**
-20%	325	0.464319	10.24	1.38236	79.4442
-15%	425	0.162829	10.28	1.49239	89.4711
-5%	475	0.175646	10.30	1.54182	94.236
0%	500	0.21	10.32	1.565	96.5648
5%	525	0.239235	10.40	1.58846	98.8582
15%	575	0.268156	10.45	1.63291	103.345
20%	600	0.210269	10.48	1.65537	105.537

Table 2

Case: $t_3 < M < T$

% variation in M	M	v	C_S	T	TC_2
-20%	0.20	0.232929	10.40	1.52145	90.9673
-15%	0.2125	0.232987	10.36	1.52157	90.9481
-10%	0.225	0.233045	10.34	1.52169	90.929
-5%	0.2375	0.233104	10.32	1.52181	90.9101
0%	0.25	0.233164	10.30	1.52193	90.8913
5%	0.2625	0.233225	10.28	1.52206	90.8727
10%	0.275	0.233287	10.26	1.52218	90.8543
15%	0.2875	0.233349	10.24	1.52231	90.836
20%	0.3	0.233412	10.20	1.52244	90.8179
% variation in α	α	v	C_S	T	TC_2
-20%	0.048	0.233004	10.34	1.52466	91.1466
-15%	0.051	0.233041	10.33	1.52403	91.0881
-10%	0.054	0.23308	10.32	1.52337	91.0261
-5%	0.057	0.233121	10.31	1.52267	90.9605
0%	0.06	0.233164	10.30	1.52193	90.8913
5%	0.063	0.233209	10.29	1.52115	90.8186
10%	0.066	0.233257	10.28	1.52034	90.7422
15%	0.069	0.233306	10.27	1.51948	90.6623
20%	0.072	0.233357	10.26	1.51859	90.5787
% variation in β	β	v	C_S	T	TC_2
-20%	0.0080	0.233164	10.31	1.52191	90.8895
-15%	0.0085	0.233164	10.32	1.52192	90.89
-5%	0.0095	0.233164	10.32	1.52193	90.8909
0%	0.01	0.233164	10.30	1.52193	90.8913
5%	0.0105	0.233164	10.33	1.52194	90.8918
15%	0.0115	0.233164	10.34	1.52195	90.8927
20%	0.012	0.233164	10.35	1.52195	90.8931
% variation in μ	μ	v	C_S	T	TC_2
-20%	16	0.243621	10.27	1.64455	82.1459
-15%	17	0.281719	10.29	1.60968	84.4194
-5%	19	0.254041	10.30	1.54904	88.7858
0%	20	0.233164	10.30	1.52193	90.8913
5%	21	0.210584	10.36	1.49662	92.9506
15%	23	0.161105	10.38	1.45061	96.9425
20%	24	0.133974	10.40	1.42963	98.8804
% variation in W	W	v	C_S	T	TC_2
-20%	325	0.265854	10.26	1.34846	75.2939
-15%	425	0.105896	10.28	1.45194	84.4201
-5%	475	0.196621	10.30	1.49925	88.7693
0%	500	0.233164	10.30	1.52193	90.8913
5%	525	0.255453	10.35	1.54409	92.9808
15%	575	0.237146	10.38	1.58701	97.0676
20%	600	0.237146	10.40	1.60777	99.0641

Table 3
Case: M > T

% variation in M	M	v	C_S	T	TC_3
-20%	0.20	0.631124	10.01	2.68024	48.7894
-15%	0.2125	0.630693	10.08	2.67038	49.3402
-10%	0.225	0.630261	10.10	2.66056	49.8863
-5%	0.2375	0.629828	10.12	2.65078	50.4276
0%	0.25	0.629393	10.13	2.64105	50.9644
5%	0.2625	0.628956	10.14	2.63137	51.4965
10%	0.275	0.628578	10.15	2.62173	52.0241
15%	0.2875	0.628079	10.16	2.61214	52.5473
20%	0.3	0.627638	10.17	2.6026	53.066
% variation in α	α	v	C_S	T	TC_3
-20%	0.048	0.629083	10.22	2.64152	51.1025
-15%	0.051	0.629155	10.20	2.6414	51.0712
-10%	0.054	0.629231	10.17	2.64128	51.0378
-5%	0.057	0.62931	10.15	2.64116	51.0022
0%	0.06	0.629393	10.13	2.64105	50.9644
5%	0.063	0.629478	10.11	2.64095	50.9243
10%	0.066	0.629566	10.09	2.64087	50.882
15%	0.069	0.629657	10.07	2.6408	50.8372
20%	0.072	0.62975	10.05	2.64075	50.79
% variation in β	β	v	C_S	T	TC_3
-20%	0.0080	0.629395	10.22	2.64105	50.9634
-15%	0.0085	0.629328	10.20	2.64114	50.9994
-5%	0.0095	0.62932	10.18	2.64115	51.0037
0%	0.01	0.629393	10.13	2.64105	50.9644
5%	0.0105	0.629392	10.15	2.64105	50.9646
15%	0.0115	0.629391	10.17	2.64105	50.9651
20%	0.012	0.629391	10.19	2.64105	50.9654
% variation in μ	μ	v	C_S	T	TC_3
-20%	16	0.617807	10.20	2.2989	60.0347
-15%	17	0.604202	10.18	2.41641	58.412
-5%	19	0.627078	10.16	2.50576	55.1593
0%	20	0.629393	10.13	2.64105	50.9644
5%	21	0.807184	10.14	1.66519	70.7492
15%	23	0.811206	10.15	1.70868	69.4602
20%	24	0.814062	10.16	1.73548	68.3626
% variation in W	W	v	C_S	T	TC_3
-20%	325	0.790605	10.21	1.67133	43.0109
-15%	425	0.678981	10.18	2.78981	44.4998
-5%	475	0.621668	10.15	2.6738	49.1142
0%	500	0.629393	10.13	2.64105	50.9644
5%	525	0.619647	10.10	2.76927	50.526
15%	575	0.671908	10.09	2.62214	50.0103
20%	600	0.591017	10.01	2.74682	50

5.1 Observations

Case 1: ($M \leq t_3 < T$)

1. An increase in M and α reduces v, T, C_S and TC_1.
2. An increase in β results in decrement of v but increases in C_S, T and TC_1.
3. An increase in μ results in decrement of v, T but increment in C_S and TC_1.
4. An increase in W result in increment in v, C_S, T and TC_1.

Case 2: ($t_3 < M \leq T$)

1. An increase in β yields an increment in T, TC_2 and C_S but v remains unchanged.
2. An increase in μ results in decrement of v, T but increment in C_S and TC_2.
3. An increase in W results in increment in v, C_S, T and TC_2.
4. An increase in α results in increment in v but decrement in C_S, T and TC_2.
5. An increase in M result in increment of v and T but decrement in C_S and TC_2.

Case 3: ($M > T$)

1. An increase in M results in increment of C_S and TC_3 but reduces v and T.
2. An increase in α results in increment in v but reduces C_S ,T and TC_3.
3. An increase in β results in decrement of v, C_S but increment in TC_3 and T remains unchanged.
4. An increase in μ results in decrement of T but increment in v, C_S and TC_3.
5. An increase in W results in decrement in v, C_S and TC_3 and increment in T.

6 Conclusion

In this paper we have presented an integrated production inventory model under reliability consideration. It is impossible to consider that every production system is perfect. The demand rate was taken as increasing function of time which shows a very realistic phenomenon. The effect of permissible delay in payment and inflation has also been considered. A numerical example has been shown to illustrate the model. The model is optimized and the convexity of the model is shown. A sensitivity analysis is also performed to check the stability of the model. For future scope the model can be extended for stochastic demand rate and with learning and forgetting effects for production and manufacturing.

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