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Supply chain model with multi distributer and multi retailer with partial backlogging

Rekha Rani Chaudhary[a] and Vikas Sharma[b*]

[a]H.O.D Department of Mathematics, Government Engineering College Bharatpur, Rajasthan, India
[b]Ph.D Scholar, Banasthali Vidyapith Rajasthan India

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A B S T R A C T

This paper deals with the co-ordination of a single producer, multi distributors and multi retailers for a supply chain management to get the maximum profit at minimum investment when shortages is permitted at the retailer‘s end and they are the partially backlogged. Most previous studies on supply chain have dealt with a moderately simpler chain with a single producer and a single buyer. The requirement of the producer is directly proportional to demand of the distributor, while the demand of the distributor is dependent on retailers' requirement. This passes on rationally to the whole supply chain. The proposed model of this paper considers deteriorating items where the deterioration rate is considered as constant.

1. Introduction

Supply chain coordination is used to find the maximum profit at the minimum investment. Due to lack of coordination in supply chain management, different stages of the supply chain have various objectives because information moving between stages is delayed and distorted. As a result, each stage tries to maximize its personal profits, resulting in actions that often reduces total supply chain profits. Information is unclear as it moves crossways the supply chain because the whole information is not shared among different phases. This deformation is inflated by the information that supply chains build a large range of goods. This improved variety makes it difficult to manage information exchange with various suppliers and retailers. So today's fundamental challenge is to achieve coordination in spite of multiple ownership and increased product variety.

The backlogging plays as important role in supply chain management. In many real-life situations, the practical experiences reveal that customers will wait for backlogged items during a shortage period; an important issue in the Supply chain management inventory theory is related to how to deal with the

* Corresponding author
E-mail address: vikasgaur01@gmail.com (V. Sharma)

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unfulfilled demands that occur during shortages. If the waiting time is longer then backlogging rate would be smaller. According to such phenomenon, taking the backlogging rate into account is necessary. In many cases customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. Thus concept of partial backlogging should not be ignored in supply chain management. In this paper, we have considered partially backlogging for retailers.

Managers can improve coordination within a supply chain for fulfilling the shortages for retailers by guaranteeing that every member in the chain works to maximize total supply chain profits. The proposal of combined total cost of the supplier and the customer was first presented by Goyal (1976). Later, Cohen and Lee (1988) proposed an optimal model for determining material requisite for all materials at every phase in the supply chain. Pake and Cohen (1993) extended the above study to consider stochastic sub systems to explore the supply chain system. Goyal (1995) and Hill (1997) introduced the concept which removed the limitation of identical shipments and delivering all available vendor inventories to the buyer. Rau et al. (2003) investigated a multi-echelon supply chain inventory model for a deteriorating item. An optimal joint total cost has been derived from an integrated perspective among the supplier, the producer and the buyer. Rau et al. (2004) developed a heuristic solution model for deteriorating items at the supplier connecting a supply chain between the producer and buyer.

Jaber and Osman (2006) introduced a complete effective coordination among suppliers and retailers, which has become a pertinent research issue in supply chain management. Lin and Lin (2007) focused a supportive inventory strategy between supplier and buyer. They considered the case of deteriorating items and allowed the completed back-order in the problem. Singh et al. (2007) developed a supply chain for deteriorating items with stock dependent demand rate under inflationary environment. A collaborating inventory model in a chain was introduced by Sana (2012). He developed the inventory system in form of three-layer supply chain formed by manufacturer, vendor and retailer. Imperfect production system is involved in this model. Sarkar (2013) presented an optimal policy for production inventory model with probabilistic deterioration in supply chain management. The demand rate is time dependent. Shortages are allowed in this model.

Sharma and Chaudhary (2013) introduced an optimal inventory model with Weibull deterioration with trapezoidal demand and shortage was introduced in this system. The holding cost was considered constant. Optimal ordering policy for deteriorating items was proposed with stock dependent demand under two-warehouse facility by Sharma and Chaudhary (2013). An inventory model for deteriorating items was introduced with shortages and time varying by Sharma and Chaudhary (2013).

This is a major shortcoming in the studies; the model which is formulated in that way is finally imperfect due to the confusing postulations. The backlogging is not considered in most of the previous studies. But due to shortages, all three players of supply chain lose their profits. Hence, in our present study we have considered shortages in supply chain management with a single producer and multi distributors, and multi retailers. The inventory level of producer and distributors decrease in agreement with the condition of distinct lot sizes being sent off. The deterioration rate is considered as constant at any point of time after production. In our study, the rate of production depends on the demand of the distributors. Besides this demand of the distributors is managed by the demand of the retailers. This way the demand reaching the producer is exactly as much as is required ultimately at the end of the retailers.

2. Assumptions and Notations

The proposed inventory model is developed under the following assumptions and notations:

2.1. Assumptions

The following assumptions are made for development of mathematical model:

- The model is derived for Single item inventory.
- Lead time is zero.
- Production rate is dependent on demand rate,
- Production rate is greater than the demand rate.
- Single producer, multi distributors and multi retailers are assumed in this model.
- Deterioration rate of the inventory is constant.
- Shortages are allowed in the system for distributer.
- Planning horizon is known and fixed.
- Deliveries are made at fixed interval to both the distributors and the retailers.

The following notations have been used throughout the study.

$I_p(t)$ is Inventory level of the producer at any time.
$I_d(t)$ is Inventory level of the distributor at any time.
$I_r(t)$ is Inventory level of the retailer at any time.
R is Production rate of the inventory by the producer, $R=\beta D$, $\beta>1$.
D is yearly demand rate in the whole market, D=s units/year, where's' is constant.
θ is Deterioration rate of the inventory ($\theta>0$).
H is Planning horizon of the supply chain.
A is Number of distributors.
B is Number of retailers corresponding to each distributor.
N is Total number of cycles of the producer in the complete planning horizon.
N_d is total number of deliveries from the producer to the distributor in one cycle of producer.
N_r is total number of deliveries from the distributor to the retailer in one cycle of distributor.
t_1 is time when producer stopped the production.
t_2 is time when the inventory level decrease to zero at the producer's end.
M is number of deliveries which are made by the producer before t_1.
C_r is production cost for the producer per unit item.
A_{sp} is setup cost for the producer per cycle.
Y_d is Ordering cost for the distributor per cycle.
Y_r is ordering cost of the retailer per cycle.
C_{0d} is purchasing cost per unit item for the distributor.
C_{or} is purchasing cost per unit item for the retailer.
h_{1p} is holding cost of the producer, per unit item per unit time.
h_{2d} is holding cost of the distributor, per unit item per unit time.
h_{3r} is holding cost of the retailer, per unit item per unit time.
P_{1p} is transportation cost of the producer per cycle.
P_{0d} is transportation cost of the distributor per cycle.
S_{2r} is shortage cost for the retailer per unit item per unit time.
LS_r is lost sale cost for the retailer per unit item per unit time.

3. Model Formulation

The model has a single producer who fulfills the supplies of 'A' distributors. Distributor satisfies 'B' retailers in each turn. The whole planning horizon has been separated equally into *n* cycles. The production starts at time t = 0. We assume that the producer made N_d number of deliveries in every cycle to his distributors. Time interval is equal in every cycle. The first delivery is delivered by the producer at t_2 / N_d to his distributors. Distributors forward the stocks to their retailers at once. The cycle starts at t_2 / N_d for both the distributor and the retailer.

3.1 Producer's model

The whole planning horizon has been divided equally into n cycles. The production starts at time $t = 0$. There is Nd number of deliveries made by the producer in every cycle. We consider that demand rate is constant. We suppose that every lot sent off by the producer is of equal amount. During time interval $[0,t_1]$, the inventory is reduced due to both demand and deterioration continuously. At time $t = t_1$, the production is stopped, and now the inventory decreases due to deterioration only. Hence, the producer's cycle is shown in Fig. 1 as follows.

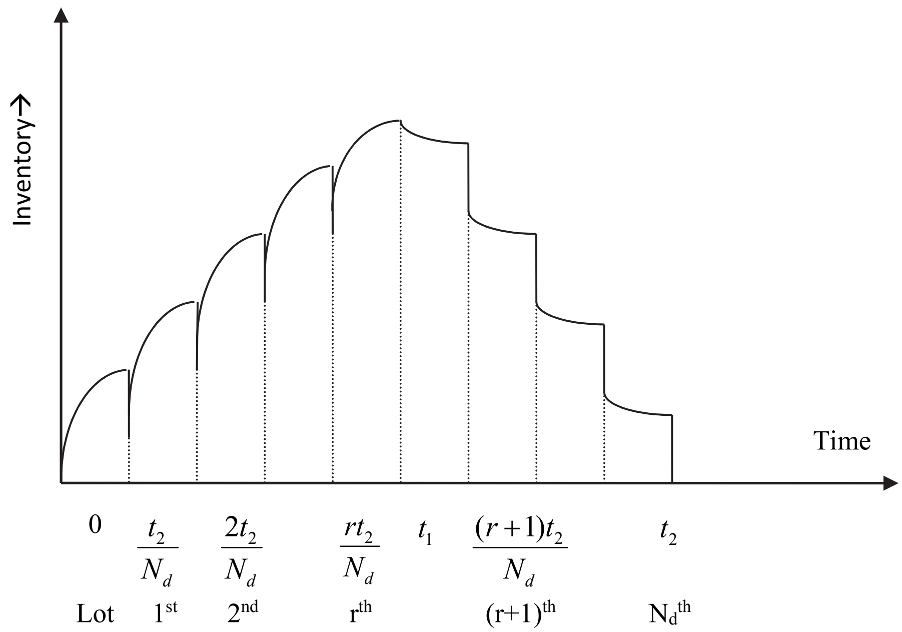

Fig. 1. Graphical representation of the cycles of the producer for production of stocks

Mathematically, we can express the above cycle as:

$$I_p'(t) - \theta I_p(t) = R - D \qquad \frac{jt_2}{N_d} \le t \le \frac{(j+1)t_2}{N_d},\ j = 0, 1, \ldots, N_d - 1. \tag{1}$$

For the time interval $0 \le t \le \frac{t_2}{N_d}$, the initial condition is $I_p(t) = 0$, when $t = 0$. The solution of the above equation is found as:

$$I_p(t) = \frac{(\beta-1)s}{\theta}\left(1 - e^{-\theta t}\right) \qquad 0 \le t \le \frac{t_2}{N_d} \tag{2}$$

The producer forwarded the first lot to distributer at time $t = \frac{t_2}{N_d}$. Hence, after the first delivery the inventory level reduces to,

$$I\left(\frac{t_2}{N_d}\right) = \frac{(\beta-1)s}{\theta}\left(1 - e^{\frac{-\theta t_2}{N_d}}\right) - AI_{0d1} \tag{3}$$

For the interval $\frac{t_2}{N_d} \le t \le \frac{2t_2}{N_d}$, the inventory level can be defined with the above Eq. (3). Initial condition is given $I_p(t) = \frac{(\beta-1)s}{\theta}\left(1-e^{\frac{-\theta t_2}{N_d}}\right) - AI_{0d1}$ at $t = \frac{t_2}{N_d}$

$$I_p(t) = \frac{(\beta-1)s}{\theta}\left(1-e^{-\theta t}\right) - AI_{0d1}e^{\frac{\theta t_2}{N_d}}e^{-\theta t} \qquad \frac{\mathrm{T}_2}{\mathrm{n}_d} \le \mathrm{t} \le \frac{2\mathrm{T}_2}{\mathrm{n}_d} \tag{4}$$

The second lot delivered by the producer at time $t = \frac{2t_2}{N_d}$, and the inventory level decrease to,

$$I\left(\frac{2t_2}{N_d}\right) = \frac{(\beta-1)s}{\theta}\left(1-e^{\frac{-2\theta t_2}{N_d}}\right) - AI_{0d1}e^{\frac{-\theta t_2}{N_d}} - AI_{0d2} \tag{5}$$

The inventory level at any time $t \le t_1$ is follows,

$$I_p(t) = \frac{(\beta-1)s}{\theta}\left(1-e^{-\theta t}\right) - e^{-\theta t}\left(AI_{0d1}e^{\frac{\theta t_2}{N_d}} + AI_{0d2}e^{\frac{2\theta t_2}{N_d}}\left(1+e^{\frac{\theta t_2}{N_d}} + e^{\frac{2\theta t_2}{N_d}} + \dots + e^{\frac{(j-1)\theta t_2}{N_d}}\right)\right) \tag{6}$$

When $\frac{(j-1)t_2}{N_d} \le t \le \frac{jt_2}{N_d}$ where j=1, 2….r.

Here the producer delivers 'r' number of deliveries before time t_1. The total number of deliveries is N_d. hence the remaining deliveries are $N_d - r$.

r^th delivery is forward at $t = \frac{rt_2}{N_d}$, So inventory level can be define as

$$I\left(\frac{rt_2}{N_d}\right) = \frac{(\beta-1)s}{\theta}\left(1-e^{\frac{-r\theta t_2}{N_d}}\right) - \left(AI_{0d1}e^{\frac{-\theta(r-1)t_2}{N_d}} + AI_{0d2}\left(1+e^{\frac{-\theta t_2}{N_d}} + e^{\frac{-2\theta t_2}{N_d}} + \dots + e^{\frac{-(r-2)\theta t_2}{N_d}}\right)\right) \tag{7}$$

Inventory level is depleting due to deterioration during the time interval $\frac{rt_2}{N_d} \le t \le t_1$ only. The Eq. (7) used as the boundary condition.

$$I_p(t) = \frac{(\beta-1)s}{\theta}\left(1-e^{-\theta t}\right) - \left(AI_{0d1}e^{\frac{\theta t_2}{N_d}} + AI_{0d2}e^{\frac{2\theta t_2}{N_d}}\left(1+e^{\frac{\theta t_2}{N_d}} + \dots + e^{\frac{(r-2)\theta t_2}{N_d}}\right)\right)e^{-\theta t} \tag{8}$$

Inventory level decreases due to deterioration during the time interval $t_1 \le t \le \frac{(r+1)t_2}{N_d}$.

The equation will be derived as for the above interval,

$$I_p'(t) + \theta I_p(t) = -D \qquad t_1 \le t \le \frac{(r+1)t_2}{N_d} \tag{9}$$

With boundary condition

$$I_p(t_1) = \frac{(\beta-1)s}{\theta}\left(1-e^{-\theta t_1}\right) - \left(AI_{0d1}e^{\frac{\theta t_2}{n_d}} + AI_{0d2}e^{\frac{2\theta t_2}{n_d}}\left(1+e^{\frac{\theta t_2}{N_d}} + \dots + e^{\frac{(r-2)\theta t_2}{N_d}}\right)\right)e^{-\theta t_1}$$

The solution of the above differential Eq. (9) can be define as

$$I_p(t) = \left(\frac{s}{\theta}\left((\beta-1)\left(1-e^{\theta t_1}\right)+1\right)e^{\theta t_1} - \frac{s}{\theta} - \left(AI_{0d1}e^{\frac{\theta t_2}{N_d}} + AI_{0d2}e^{\frac{2\theta t_2}{N_d}}\left(1+e^{\frac{\theta t_2}{N_d}} + \dots + e^{\frac{(r-2)\theta t_2}{N_d}}\right)\right)\right)e^{-\theta t} \tag{10}$$

(r+1)th lot is delivered at $t=\frac{(r+1)t_2}{N_d}$ and the inventory level decreases to,

$$I_p\frac{\left((r+1)t_2\right)}{N_d}=\frac{s}{\theta}\left((\beta-1)\left(1-e^{\theta t_1}\right)+1\right)e^{\theta t_1}e^{\frac{-\theta(r+1)t_2}{N_d}}-\frac{s}{\theta}e^{\frac{-\theta(r+1)t_2}{N_d}}-AI_{0d1}e^{\frac{-\theta r t_2}{N_d}}$$
$$-AI_{0d2}\left(1+e^{\frac{-\theta t_2}{N_d}}+\ldots+e^{\frac{-(r-1)\theta t_2}{N_d}}\right) \tag{11}$$

Similarly, we can find that for j = r,(r+1), ….N_d-2, the inventory level at time t is,

$$I_p(t)=\left(\frac{s}{\theta}\left((\beta-1)\left(1-e^{\theta t_1}\right)+1\right)e^{\theta t_1}-\frac{s}{\theta}-\left(AI_{0d1}e^{\frac{\theta t_2}{N_d}}+AI_{0d2}\left(e^{\frac{2\theta t_2}{N_d}}+e^{\frac{3\theta t_2}{N_d}}+\ldots+e^{\frac{(j+1)\theta t_2}{N_d}}\right)\right)\right)e^{-\theta t} \tag{12}$$

We now compute the different costs associated with a single cycle of the producer.

3.1.1 Expected Setup Cost

The setup cost is considered as constant hence the cost for setting up the production run is incurred for planning horizon.

$$S_P C=A_{sp} \tag{13}$$

This is the present setup cost incurred by the producer during the whole planning horizon.

3.1.2 Expected Production Cost for Producer

The production cost for a single unit item is 'C_r'. Then, the production cost during the time interval 0 to t_1.

$$PC_p=C_r\int_0^{t_1}\beta s\,dt=C_r.\beta s t_1 \tag{14}$$

3.1.3 Expected Transportation Cost for Producer

The transportation cost is assumed constant for the time up to t_2.

$$T_R C_p=P_{1p} \tag{15}$$

3.1.4 Expected Holding Cost for Producer

Inventory holding cost can be defined for the producer. Let holding cost per unit item is h_{1p}.
The holding cost during the time interval 0 to t_1 is as follows,

$$H'=\int_0^{t_1}\frac{(\beta-1)s}{\theta}\left(1-e^{-\theta t}\right)-\left(AI_{0d1}e^{\frac{\theta t_2}{N_d}}+AI_{0d2}e^{\frac{2\theta t_2}{N_d}}\left(1+e^{\frac{\theta t_2}{N_d}}+\ldots+e^{\frac{(r-2)\theta t_2}{N_d}}\right)\right)e^{-\theta t}dt$$

$$H'=\frac{(\beta-1)s}{\theta}\left(\frac{t_1\theta-1+e^{-\theta t_1}}{\theta}\right)-\left(AI_{0d1}e^{\frac{\theta t_2}{N_d}}+AI_{0d2}e^{\frac{2\theta t_2}{N_d}}\left(1+e^{\frac{\theta t_2}{N_d}}+\ldots+e^{\frac{(r-2)\theta t_2}{N_d}}\right)\right)\frac{(1-e^{-\theta t_1})}{\theta}.$$

Similarly holding cost during the time interval t_1 to t_2 is also as follows,

$$H'' = \int_{t_1}^{t_2} I_p(t) = \left(\frac{s}{\theta}\Big((\beta-1)\big(1-e^{\theta t_1}\big)+1\Big)e^{\theta t_1} - \frac{s}{\theta} - \left(AI_{0d1} e^{\frac{\theta t_2}{N_d}} + AI_{0d2}\left(e^{\frac{2\theta t_2}{N_d}} + e^{\frac{3\theta t_2}{N_d}} + \ldots + e^{\frac{(j+1)\theta t_2}{N_d}} \right)\right)\right) e^{-\theta t} dt$$

$$H'' = \left(\frac{s}{\theta}\Big((\beta-1)\big(1-e^{\theta t_1}\big)+1\Big)e^{\theta t_1} - \frac{s}{\theta} - \left(AI_{0d1} e^{\frac{\theta t_2}{N_d}} + AI_{0d2}\left(e^{\frac{2\theta t_2}{N_d}} + e^{\frac{3\theta t_2}{N_d}} + \ldots + e^{\frac{(j+1)\theta t_2}{N_d}} \right)\right)\right) \frac{\left(e^{-\theta t_2} - e^{-\theta t_1}\right)}{\theta}$$

Total holding cost for the complete planning horizon becomes as follows,

$$H_p C = h_{1p}(H' + H'') \tag{16}$$

3.1.5 Present worth Total Cost for Producer

The total cost can be evaluated for the producer is the sum total of the setup cost, the production cost, transportation cost and the holding cost. Hence, the total cost is,

$$T_p C = S_P C + P_p C + H_p C + T_R C_p \tag{17}$$

Our main aim is to minimize total cost function with respect to the producer.

3.2 Distributor's model

The cycle for the distributor starts at $t = \frac{t_2}{N_d}$. The producer has not sent even a single delivery to the distributer till this time ($t = \frac{t_2}{N_d}$). So the distributor has no inventory prior to $t = \frac{t_2}{N_d}$. The distributor takes delivery of the first lot from the producer at $t = \frac{t_2}{N_d}$. Distributer delivers the stock to the retailers at a constant interval of time. The distributor's stock level depletes due to demand and deterioration. The stock of the distributor decrease to zero at $t = \frac{2t_2}{N_d}$. Then the next delivery is delivered by the producer to distributer and this process repeats itself throughout the planning horizon. The model can be represented for distributer as following.

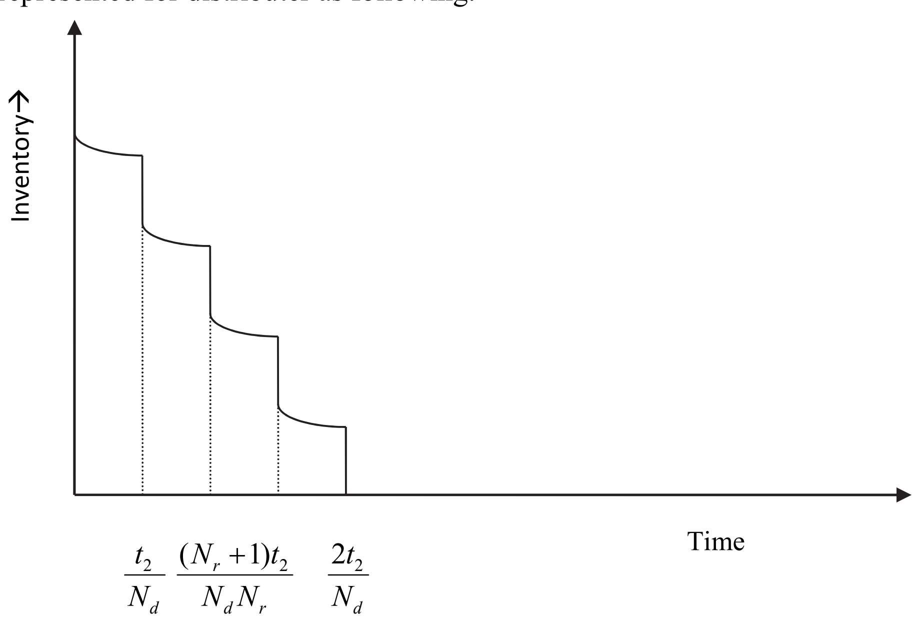

Fig. 2. Graphical representation of the cycles of the distributor for distribution of stocks

The last delivery arrives at t_2 to the distributer, which lasts up to $\frac{t_2\left(N_d+1\right)}{N_d}$. Let distributor's demand is $D_d = Z_\alpha$. Mathematically, the model can be represented by the following differential equation,

$$I_d^{'}(t) = -Z_\alpha - \theta I_d(t) \qquad \frac{jt_2}{N_d} \le t \le \frac{(j+1)t_2}{N_d}, \qquad j = 1 \ldots N_d \tag{18}$$

Let the size of stock is received by the distributor at $t = \frac{t_2}{N_d}$ be I_{0d1}. When the stock arrives to distributer, he delivers a supply of I_{0R} to every of B retailers. Hence the stock at the opening of the cycle is

$$I_d\left(\frac{t_2}{N_d}\right) = I_{0d1} - BI_{0R} \tag{19}$$

The next delivery has delivered at $t = \frac{(2N_r+1)t_2}{N_d N_r}$. The stock will be deteriorated till that time, hence the differential equation becomes,

$$I_d^{'}(t) = -\theta I_d(t) \qquad \frac{t_2}{N_d} \le t \le \frac{(N_r+1)t_2}{N_d N_r} \tag{20}$$

We obtain the solution of Eq. (10) by using Eq. (9) as boundary condition,

$$I_d(t) = \left(I_{0d1} e^{\frac{\theta t_2}{N_d}} - BI_{0r} e^{\frac{\theta t_2}{N_d}} \right) e^{-\theta t} \tag{21}$$

Next, lot of size AI_{0R} is delivered to retailer then the remaining inventory will be

$$I_d\left(\frac{\left(N_r+1\right)t_2}{N_d N_r}\right) = \left(I_{0d1} e^{\frac{\theta t_2}{N_d}} - BI_{0R} e^{\frac{\theta t_2}{N_d}} \right) e^{\frac{-\theta(N_r+1)t_2}{N_d N_r}} - BI_{0R} \tag{22}$$

Similarly, for the time interval $\frac{(N_r+1)t_2}{N_d N_r} \le t \le \frac{(N_r+2)t_2}{N_d N_r}$, we get the differential equation with boundary condition as given by Eq. (22)

$$I_d^{'}(t) = -\theta I_d(t) \qquad \frac{(N_r+1)t_2}{N_d N_r} \le t \le \frac{(N_r+2)t_2}{N_d N_r} \tag{23}$$

The solution of this equation is given by

$$I_d(t) = \left(I_{0d1} e^{\frac{\theta t_2}{N_d}} - BI_{0R} \left(e^{\frac{\theta t_2}{N_d}} + e^{\frac{\theta (N_r+1)t_2}{N_d N_r}} \right) \right) e^{-\theta t} \tag{24}$$

Similarly continuing this process we can find for $\frac{(N_r+j)t_2}{N_d N_r} \le t \le \frac{(N_r+j+1)t_2}{N_d N_r}$, where $j = 0, 1, \ldots N_r - 1$, the inventory level at any point is given by

$$I_d(t) = \left(I_{0d1} e^{\frac{\theta t_2}{N_d}} - BI_{0r} \left(e^{\frac{\theta t_2}{N_d}} + e^{\frac{\theta (N_r+1)t_2}{N_d N_r}} + \ldots + e^{\frac{\theta (N_r+i)t_2}{N_d N_r}} \right) \right) e^{-\theta t} \tag{25}$$

3.2.1 Expected Ordering Cost for Distributer

The ordering cost for distributer is considered as constant Y_d for each cycle then total ordering cost for the complete planning horizon

$$O_d C = Y_d + Y_d + \ldots + Y_d \ldots \textit{up to rth term}$$

$$O_d C = r.Y_d \tag{26}$$

3.2.2 Expected Purchase Cost Distributer

The inventory is bought by the distributor at an interval of $\frac{t_2}{N_d}$. The purchasing cost is assumed constant C_{0d} for each cycle; hence the purchasing cost for planning horizon is,

$$P_dC = C_{0d} + C_{0d} + ... + C_{0d} \quad \textit{up to rth term} \tag{27}$$
$$P_dC = r.C_{0d}$$

3.2.3 Expected Transportation Cost for distributer

The distributor has to allow the cost of getting the delivery delivered to his retailers. Transportation cost is considered as constant P_{0d} for each cycle; so transportation cost will be

$$T_dC = P_{0d} + P_{0d} + ... + P_{0d} \quad \textit{up to rth term}$$
$$T_dC = r.P_{0d} \tag{28}$$

3.2.4 Holding Cost for distributer

Inventory holding cost for carrying the stock can be evaluated for distributer as follows,

$$H_dC = h_{2d} \sum_{J=0}^{N_r - 1} \int_{\frac{(N_r + j)t_2}{N_d N_r}}^{\frac{(N_r + j + 1)t_2}{N_d N_r}} I_d(t)dt\,. \tag{29}$$

The Eq. (29) is the journal form of the inventory holding cost for the distributer's planning horizon

3.2.5 Total Cost for the distributer

The total costs for distributor are the sum of the ordering cost, purchasing cost, transportation cost and the holding cost.

$$T_dC = O_dC + P_dC + T_dC + H_dC\,.$$

This is the cost of the one distributor for the planning horizon '*H*'. But there are '*A*' distributers, and then total cost will be

$$N_dC = A.T_dC \tag{30}$$

This is the objective function from the perception of the distributors which must be minimized.

3.3 Retailer's model

The present inventory model starts at $\frac{t_2}{N_d}$ for the retailer. We have assumed that distributor has delivered N_r number of deliveries to each retailer**.** The first delivery ends at $\frac{(N_r + 1)t_2}{N_d N_r}$. At this time distributer delivered next delivery to retailers. This process replicates itself till the end.

The first delivery is executed at the time $\frac{T_2}{N_d}$ and the last delivery of the cycle happens at $\frac{2T_2}{N_d}$. During the time interval $\left[\frac{t_2}{N_d}, \frac{2t_2}{N_d}\right]$, there are $N_r + 1$ deliveries to retailer instead of n_r. But there are n_r number of deliveries delivered for all the other cycles. Graphically we can represent the inventory cycle as follows in Fig. 3.

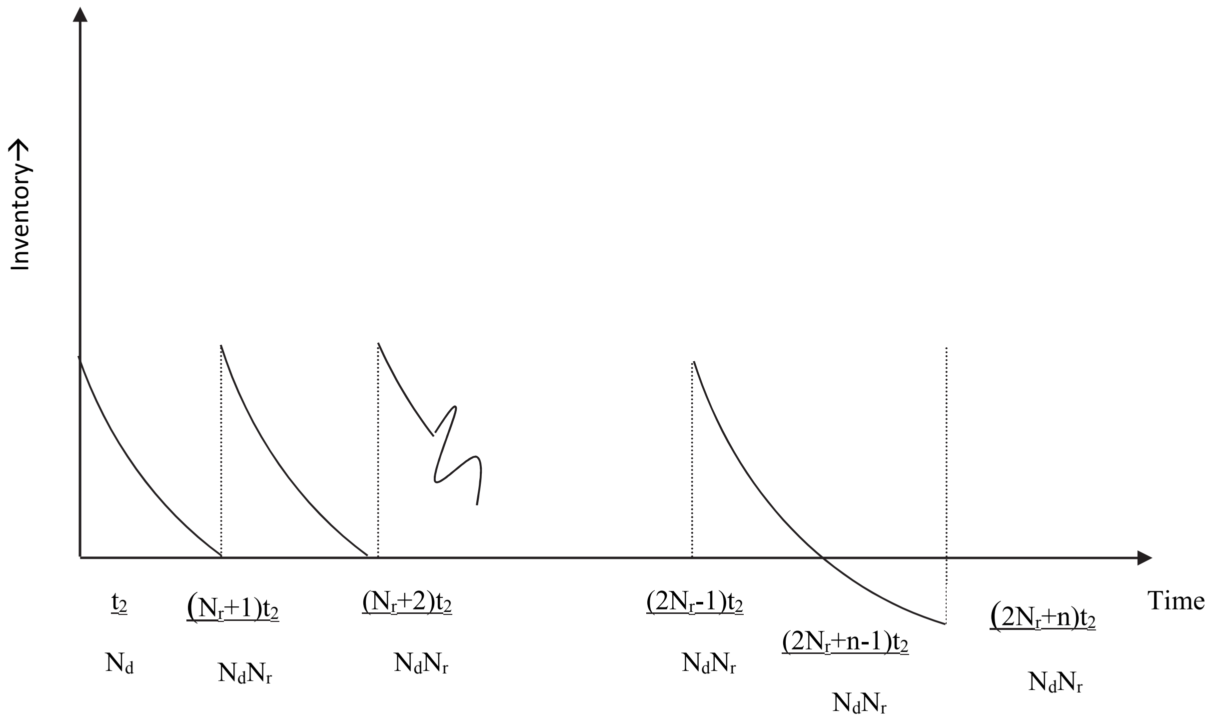

Fig. 3. Graphical representation of the inventory cycles of the retailer

The annual demand of '*X*' units is divided into '*A.B*' retailers. The annual demand for one retailer can be defined as $Y_{Dr} = \frac{X}{A.B}$ units for any interval $\frac{jt_2}{N_d N_r} \le t \le \frac{(j+1)t_2}{N_d N_r}$.

Inventory level decreases due to demand and deterioration during the time interval $\frac{jt_2}{N_d N_r} \le t \le \frac{(j+n-1)t_2}{N_d N_r}$

Mathematically, the differential equation for above can be characterized as follows,

$$I_r^{'}(t) - \theta I_r(t) = -Y_{Dr} \qquad \frac{jt_2}{N_d N_r} \le t \le \frac{(j+n-1)t_2}{N_d N_r}, \text{ j=N}_r, \text{N}_{r+1} \ldots 2\text{Nr} \tag{31}$$

But shortages are occurring during the time interval $\frac{(j+n-1)t_2}{N_d N_r} \le t \le \frac{(j+n)t_2}{N_d N_r}$, so inventory level is depleted due to only demand.

Mathematically the differential equation can be defined for this interval

$$I_r'(t) = -Y_{Dr} \qquad \frac{(j+n-1)t_2}{N_d N_r} \le t \le \frac{(j+n)t_2}{N_d N_r} \quad j=N_r, N_{r+1} \ldots 2Nr \tag{32}$$

The solution of the Eq. (32) can be derived

$$I_r(t) = \frac{X}{A.B.\theta}\left(e^{\frac{\theta(j+1)t_2}{N_d N_r}-\theta t} - 1\right) \tag{33}$$

The solution of the Eq. (33) can be derived

$$I_r(t) = \frac{X}{A.B.}\left(\frac{jt_2}{N_d.N_r} - t\right) \tag{34}$$

Let inventory level is equal to I_{0r} at the time $t = \frac{jt_2}{N_d N_r}$.

$$I_{0r} = \frac{X}{A.B.\theta}\left(e^{\frac{\theta t_2}{N_d N_r}} - 1\right) \tag{35}$$

3.3.1 Ordering cost for retailer

The ordering cost for retailer is assumed as constant Y_r for each cycle, then the total ordering cost for the complete planning horizon is as follows,

$$O_R C = (Y_R + Y_R + ... + Y_R) \ldots up\ to\ r^{th}\ term$$

$$O_d C = r.Y_d \tag{36}$$

3.3.2 Purchase Cost for retailer

The purchasing cost is assumed constant C_{0R} for each cycle; hence the purchasing cost for planning horizon is as follows,

$$P_r C = C_{0R} + C_{0R} + ... + C_{0R} \quad up\ to\ rth\ term \tag{37}$$

$$P_r C = r.C_{0R}$$

3.3.3 Holding cost for retailer

Let holding cost per unit item per unit time be h_{2r} for the retailer. Holding cost can be evaluated for the retailer as follows,

$$H_rC = h_{3r}\sum_{j=N_r+1}^{2N_r}\int_{\frac{jt_2}{N_dN_r}}^{\frac{(j+n-1)t_2}{N_dN_r}} \frac{X}{A.B.\theta}\left(e^{\frac{\theta(j+1)t_2}{N_dN_r}-\theta t}-1\right)dt \tag{38}$$

3.3.4 Shortage cost for retailer

Let S_{2r} be shortage cost per unit item per unit time for the retailer. Shortage cost will be for the retailer.

$$Sh_rC = -S_{2r}\sum_{j=N_r+1}^{2N_r}\int_{\frac{(j+n-1)t_2}{N_dN_r}}^{\frac{(j+n)t_2}{N_dN_r}} \frac{X}{A.B.}\left(\frac{jt_2}{N_d.N_r}-t\right)dt \tag{39}$$

3.3.5 Lost sale cost for retailer

Now the total lost sale cost during the time period $\left[\frac{(j+n-1)t_2}{N_dN_r},\frac{(j+n)t_2}{N_dN_r}\right]$ for the retailer will be

$$LS_{cr} = -C_{3r}\sum_{j=N_r+1}^{2N_r}\int_{\frac{(j+n-1)t_2}{N_dN_r}}^{\frac{(j+n)t_2}{N_dN_r}} (1-\gamma)\frac{X}{A.B.}dt \tag{40}$$

3.3.6 Total cost for retailer

The total cost of one retailer for the whole planning horizon can be defined as

$$T_rC = O_rC + P_rC + H_rC + Sh_rC + Ls_{cr}$$

This is the objective function with respect to one retailer. The total cost for the 'A.B' retailers will be

$$N_rC = A.B.T_rC \tag{41}$$

This is the objective function for the retailers which need minimization.

4 Numerical Illustrations

To find the optimal solution of the model, we use the following numerical data. First we evaluate the optimal solutions separately for the producer, distributer and retailer. After that a combined optimal solution has been calculated at *θ=0.02, A=3, B=5, X=1500, β=1.50, γ =0.2*. The data for Producer:

C_r=12 h_{1p}=1.5 A_{sp}=150 P_{1p}=45

Distributor data:

C_{0d}=25 h_{2d}=1.5 Y_d=55 P_{0d}=50

Retailer data:

C_{0r}=25 h_{3r}=1.5 Y_r=50

5. Observations

When we evaluate the three optimal solutions obtained in Table 1, we come to observe that

Table 1
The summary of the optimal results

T_pC	T_dC	T_rC	TC
6517.48	1284.76*	2480.24	45350.83*
12409.76	1421.45	2235.65*	50380.48
2548.15*	7839.64	4434.87	89790.32

The first optimal solution minimizes two objectives at once, one for the whole supply chain and other for the distributor. It is observed that, the retailer has to pay 5.75% more than his optimal expenses from the first solution due to shortages while the producer has to pay 133.85% more. The second solution gives that distributor has to pay 21.02% more while 587.01% extra is paid by the producer. The total cost of the whole supply chain is increased by 11.15% more. The optimal solution of the producer increased extra expenses of 556.69% on the distributor while retailer shells out 92.56% more to meet the expenses. Shortages increased the total cost for the retailer in each optimal solution.

6. Conclusion

In this paper, we have explored the condition of a single producer, multi distributors and multi retailers for a supply chain management. We have considered the shortages for retailer in this model. Most of the research papers on supply chain till date show off of a moderately simpler chain with a single producer and a single buyer. However, in present scenario every supply network was involved with a network of many distributors and retailers. The model has been used in every field such as electronics items industries, sugar industries, milk plant, manufacturing plant of different product. The demand of the producer was dependent on set forward by the distributor, while the demand of the distributor was dependent on retailer's requirement. This imparts logically a very sound construction to the whole chain. This way, every entity in the network gets what he requires and there is a minimal wastage of material and resources. Backlogging is an important issue in supply chain. Due to shortages, the optimal cost of the system will be increased. So in this paper, we have considered shortages for this system.

We have numerically shown that optimizing costs for a single article affect the supply chain. Therefore, sub optimization reduces the expenses of all participants. Everyone can benefit and can share the profits to each other. Only this kind of a solution can offer the best deals to the customer and it is quite obvious that there is only a single source of revenue for the whole supply chain and that is the end customer. Today, the study of supply chains is gaining phenomenal importance around the globe. In such a scenario we need more practical models, which can conform to real life measures. Researchers further extended this model with power demand, variable storage cost, and deterioration rate as a variable of time.

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