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A sustainable supply chain network under the Stackelberg and Nash equilibrium policy in a reverse logistic model with multiple deliveries and a single distribution center

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A B S T R A C T

This paper presents a framework for reverse logistics aimed at managing reusable items within supplier-buyer relationships to promote sustainability and reduce environmental impact. In this model, the supplier produces and inspects items, shipping only perfect items to buyers, while recycling or disposing of imperfect ones. Returned items from consumers are categorized as either reusable or damaged at a collection center. The concept of a circular economy encourages the return and refilling of reusable items, while damaged items are recycled. Additionally, the model incorporates carbon emissions considerations across production, storage, transportation, and landfilling, emphasizing the importance of environmental factors. To evaluate the sustainability and economic efficiency of the supply chain network, both Stackelberg and Nash equilibrium strategies are employed. The paper provides a mathematical framework based on lemmas to analyze the impact of the network and promote sustainable supply chain practices. In this cycle, consumers use the items and eventually discard them. To support a zero-waste policy, the supplier labels the bottles with barcodes to identify used items upon collection. The supplier has two different rates at which they purchase used bottles from consumers. Refilled bottles are sent back for reuse, while damaged bottles are either repurposed as raw materials or disposed of. The research paper aims to develop a mathematical model that determines the buyer's cycle time and the number of deliveries from the supplier to the buyer, ensuring that the buyer's demand is met without shortages.

1. Introduction

Waste management concerns are critical global issues that include a range of environmental, social, and economic challenges. Effective waste management is essential for maintaining environmental sustainability, public health, and the overall well-being of communities. Waste management involves a range of activities to reduce, manage, and properly dispose of various forms of waste generated through human activities. Whether it's household waste or industrial byproducts, the various nature of waste requires creative and flexible solutions. Additionally, to resolve the challenge this issue is highlighted by the growing impacts of improper waste disposal on ecosystems, public health, and climate change. This investigation explores the shades of waste management, analyzing the difficulties, developments, and evolving approaches implemented worldwide. From conventional practices like landfilling to state-of-the-art technologies such as recycling, waste-to-energy conversion, and circular economy models, this research seeks to undo the particulars associated with waste management. In light of growing environmental challenges and the continual expansion of the global population, the effective management of waste has changed into a crucial concern. Projections from the World Bank indicate that urban waste generation is anticipated to flow by 2.2 billion tons annually by the year 2025, thereby raising the stress on ecosystems and human health (World Bank, 2018). This exploration endeavors to thoroughly examine the complex landscape of waste management, analyzing modern challenges and presenting innovative solutions that comprehensively address the environmental, social, and economic facets of this critical issue. Sustainable Supply Chain Management (SSCM) refers to the integration of environmentally and socially responsible practices throughout the various stages of a product's life cycle, from the removal of raw materials to end-of-life

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disposal. The goal of sustainable supply chain management is to create value for all stakeholders, including the environment, society, and the economy, by minimizing negative impacts and promoting positive contributions. Recent studies emphasize the importance of incorporating sustainable supply chain management practices to reduce environmental impact and improve overall organizational performance. Securing and Müller (2008) deal with the idea that a sustainable supply chain goes beyond economic considerations, integrating environmental and social dimensions to establish a comprehensive framework. This framework is designed not only to minimize adverse effects but also to make positive contributions to both society and the environment. In today's dynamic world of logistics and distribution networks, a "single setup multiple delivery system" has drawn a lot of attention as a strategic way to improve awareness and efficiency. As a result of creating a single set-up or process capable of meeting a variety of delivery requirements, this model optimizes resources and reduces costs while optimizing processes. The need for flexible and dynamic supply chain models is increasing, so it's critical to investigate these integrated systems. Jha and Shanker (2014) developed an integrated inventory model that incorporates transportation considerations for a scenario involving a single supplier and multiple deliveries. It is observed that the supplier produces and delivers products to buyers in distinct locations with similar capabilities to some vehicles. According to studies in supply chain management and logistics, a single setup and multiple delivery systems can be used to deal with the issues imposed by evolving market conditions. Li et al. (2020) propose that these kinds of systems have shown the ability to adapt to changing needs, providing a flexible and adjustable solution to organizations managing the complex conditions of modern business. The study aims to examine the difficulties of this delivery model, examining its benefits, disadvantages, and possible uses in various kinds of industries. It is essential to examine a single setup with several delivery systems and integrate sustainable practices during the production, and repair processes. After products are purchased from consumers, they are separated and sorted. After the item's separation, the supplier distributes perfect items to the buyer in N-number shipments. while imperfect items are divided, with some undergoing remanufacturing and the rest being disposed of in landfills. It is important to determine the principal being to maximize the entire life cycle cost for each participant. As a result, examinations are conducted in cooperative and non-cooperative associations. This model is solved using the Stackelberg equilibrium, a game-theoretic framework. This model represents classified decision-making scenarios where one player, the leader (supplier), makes the first decision. As a result of observing the leader's action, the follower (the buyer) responds. By incorporating this framework into their strategy, the supplier can better anticipate the buyer's logical responses to their actions. Similarly, buyers modify their decisions to maximize outcomes based on the supplier's declared strategy.

2. Literature review

Reverse logistics, an important part of modern supply chain management involves the operative and cost-effective undertaking of products from their final consumption point back to the manufacturer or a selected ability for recycling, remanufacturing or proper disposal (Rogers & Tibben-Lembke, 1999). As reverse logistics has grown in popularity over the past few years, it has become increasingly important to move goods from their final destination to the manufacturer or a specific facility. This concern extends to situations involving imperfect items, encompassing cases where returned products exhibit defects, damages or other imperfections. In contrast to traditional logistics, which focuses on the forward flow of goods from manufacturers to end-users, reverse logistics deals with the opposite flow, overseeing the return, repair and recycling of products. The attention directed towards this process is growing progressively, powered by its potential economic, environmental and regulatory implications. As the global economy undergoes continuous evolution, product returns are becoming an increasing challenge for businesses, end-of-life disposal and the imperative for sustainable practices. The economic implications of reverse logistics are substantial, offering businesses avenues to recover value from returned products, reduce overall supply chain costs and improve customer satisfaction through effective reverse logistics processes (Rogers & Tibben-Lembke, 1999). However, despite its potential benefits, the implementation of reverse logistics is not without challenges. Reverse logistics plays a key role in tackling these challenges by optimizing the management of returned goods, minimizing waste and promoting opportunities for value recovery (González-Torre et al., 2004). This significance is particularly evident in industries like electronics, automotive and retail, where considerations related to product returns and end-of-life management have become essential components of overall supply chain policy. Additionally, the influence of regulation on reverse logistics practices, especially in areas such as recycling and end-of-life product disposal, introduces complexity to the management of reverse flows (Guide & Van Wassenhove, (2009)). Reverse logistics has developed as a critical side of supply chain management, providing solutions to the challenges posed by product returns, waste management and sustainability. This literature review will investigate various dimensions of reverse logistics, exploring its environmental, technological, economic and regulatory aspects, to offer a comprehensive understanding of its role in contemporary supply chain operations. This literature review explores a variety of aspects of reverse logistics about imperfect items, including methods for managing imperfect returns, environmental effects, and economic considerations. Economic factors play a critical role in reverse logistics when dealing with defective items. Souza and Oke (2016) discussed the challenges of recovering value from damaged items and the impact of poor returns on the economy. They maintained that effective reverse logistics processes are necessary to recover parts, restore products, and reduce the costs put on by defective returns. Several optimization models have been presented to deal with the challenges of reverse logistics, including defective items. Mishra and Deshmukh (2017) developed a mathematical model to improve reverse logistics by taking imperfect items into account. This study focused on identifying the best places for collection stations, repair centers, and disposal facilities to increase the recovery value of imperfect returns. In the context of reverse logistics, environmental sustainability becomes an important concern, especially when dealing with defective items. Pishvaee et al. (2019) examined the environmental impacts of reverse logistics with defective items in a thorough study. In the study, it was emphasized that returning defective products properly and recycling them are essential to reducing their environmental impact and that using eco-friendly procedures in the reverse logistics process will help to minimize this impact. Developments in technology play a crucial role in optimizing the handling of defective returns in reverse logistics. Tan et al. (2020) investigated the potential of modern technology, including machine learning and data analysis, in improving the identification, sorting, and disposal of defective items. The study highlighted that innovative technologies might improve the overall effectiveness of reverse logistics procedures.

In recent years, environmental disasters resulting from climate change and human activities have become increasingly common. The humanitarian supply chain (HSC) aims to minimize the impact of these disasters and provide prompt response options. In supply chain management, reverse logistics is a crucial process that enhances the overall effectiveness of the supply chain. Merdivenci et al. (2024). study aims to analyze the usage area of reverse logistics in HSC operations and show where it can be applied in the future.

The Single Supply Multi-Delivery System (SSMDS) is a logistics framework developed to optimize the distribution process by integrating products from one supplier and ensuring their effective delivery to multiple locations. This approach is different from standard supply chain models in that it focuses on simplified equilibrium and centralized management to improve operational availability and productivity. Mondal et al. (2024) presents a three-echelon supply chain management system based on advertising and payment regulations that involves a single supplier, a single manufacturer and many retailers. The model reduces supply chain costs while increasing profitability by implementing a single setup, multiple delivery policy, as well as variable transportation and carbon emission costs. The SSMDS can deal with a wide range of distribution network setups, which makes it suitable for areas where supply chains are complicated and extend worldwide. It establishes an environment for managing several goods and adjusting to varying customer demand in multiple sectors, ensuring the establishment of an adaptable and rapid distribution network (Waters, 2003). The primary goal of the SSMDS is to achieve improved efficiency through careful utilization of resources. By integrating items at a single distribution location, firms can minimize transportation costs, reduce lead times for deliveries and maximize inventory levels. This strategy corresponds to sustainable logistic principles by focusing on maximizing value and reducing waste throughout the distribution system (Christopher, 2016). Effective implementation of SSMDS frequently depends on the integration of advanced technology. Major components that improve awareness, control and response in the distribution network include robotic decision-making procedures, real-time tracking and data analytics (Gunasekaran et al., 2017). Although the SSMDS has many benefits in terms of effectiveness and flexibility, it also presents difficulties in terms of cooperation, information sharing and technology use. By improving these issues, the supply chain will be able to develop and enhance products continuously while also ensuring the system operates effectively. In recent years, reverse logistics has gained increasing recognition as an important component of the SSMDS framework. The purpose of reverse logistics is to manage product returns, recycle materials and incorporate them back into the supply chain so that they can be reused (Rogers & Tibben-Lembke, 2019). Incorporating reverse logistics into SSMDS addresses environmental concerns and improves sustainability in supply chain management.

The inspection of imperfect items holds significance across various industries, influencing product performance, consumer satisfaction and overall operative effectiveness. According to a study by Garcia and Patel (2016), customer's acceptance for defective products varies depending on several factors, including the nature of the imperfection and the brand's reputation. To create effective marketing and communication strategies, organizations need to understand customer responses (Lee, 2018). In Johnson's (2017) study, which emphasizes the critical role statistical process control plays in maintaining product quality, this claim is also supported by statistical process control's claims about product quality. A major area of research is understanding how defective objects arise during manufacturing processes. In this study, Smith (2018) found that human error or machine fault frequently leads to defects in production. Moreover, Jones and Brown (2019) highlighted the importance of material faults in determining the quality of the final product. Perfect products must be identified and addressed through the effective application of quality control techniques. According to Wang et al. (2020), the implementation of strong quality control systems greatly reduces production defects. Both customers and producers incur costs when defective products are present. Smith and Davis (2021) conducted extensive economic research that showed the total cost of defective products includes warranty claims and post-sale service costs in addition to production and quality control expenses. Developments in technology give rise to promising solutions for defective goods. According to Brown et al. (2022), predictive maintenance may be improved by incorporating artificial intelligence and machine learning. This can reduce the probability of defects by recognizing any issues earlier on. The environmental impact of greenhouse gas (GHG) emissions in supply chain management has stored increased attention due to its significance. The difficult interconnections within global supply chains contribute to the heightened carbon footprint associated with products and services (Gold, 2017). This review investigates essential research discoveries concerning the various sides of GHG emissions within supply chains. A crucial area of investigation focuses on measuring and assessing GHG emissions in supply chains. According to Jones et al. (2015), using standardized methodologies is crucial for calculating emissions because inconsistent methods complicate comparisons and decision-making. Additionally, methodologies such as life cycle assessment (LCA) have gained importance for evaluating the overall environmental impact of supply chains (Green et al., 2018). Various studies investigate mitigation strategies aimed at reducing GHG emissions within supply chains. Lei (2024) explores effective supply chain network optimization techniques with a particular focus on reducing industrial carbon emissions. Themes such as carbon footprint reduction through transportation optimization (Wu & Pagell, 2011) and the adoption of green practices in logistics (Seuring & Müller, 2008) are repeated. Collaborative efforts among supply chain partners are also proposed to enhance sustainability, as emphasized by Pagell and Wu (2016). The integration of technology plays a key role in managing GHG emissions. Technologies like the Internet of Things (IoT), blockchain and data analytics are explored for their potential to improve supply chain visibility and efficiency (Christopher & Peck, 2004; Sarkis, 2019). Automation and smart transportation systems contribute to emissions reduction along the supply chain (Christopher, 2016). The regulatory landscape significantly shapes how organizations address GHG emissions in their supply chains. The regulatory environment guides the strategic decisions of organizations as they attempt to meet emission reduction targets (Carter & Rogers, 2008). The research underscores the importance of engaging suppliers and collaborating with stakeholders to achieve emission reduction goals. Collaborative initiatives across the supply chain aid in the development of joint strategies for emission reduction (Srivastava, 2007). Growing industrial responsibility towards society and the development of sustainable supply chains are the results of growing social and environmental issues. Growing industrial responsibility towards society and the development of sustainable supply chains are the results of growing social and environmental issues (Arab et al. (2024)). The goal of the model is to maximize the positive social effects while minimizing the negative environmental effects and the overall cost of every resource used.

The paper is organized as follows: Section 1 includes the introduction. Section 2 presented a brief literature review. Section 3 involves the assumptions and notations employed in the proposed model. The mathematical formulation of the model is outlined in Section 4, with the solution procedure discussed in Section 5. Section 6 incorporates a numerical example and Section 7 covers sensitivity analysis. Section 8 is dedicated to the discussion of observations, while Section 9 concludes the paper by summarizing the model.

3. Contribution

A sustainable supply chain network operating under the Stackelberg and Nash equilibrium policies, integrating a reverse logistic model with multiple deliveries and a single distribution centre, can make significant contributions. A reverse logistics approach can help to manage product returns more effectively reducing waste and impacting the environment less. The sustainability goals are achieved through recycling and proper disposal.

3.1 Sometimes it's difficult for researchers to properly restore damaged products, which leads to difficulties with utilization. In our investigation, we collect used items from a collection center to hurry up the process and separate the reusable and damaged materials at this center. The reusable items are reused again, while the damaged items are recycled to make new products or disposed of as waste. Through this approach, waste is significantly minimized by maximizing the effective reuse of previously used items.

3.2 Integrating multiple deliveries and a single distribution center allows for economies of scale and cost savings. This results in improved transportation costs and overall operational efficiency.

3.3 In this model, we attach barcodes to the bottles so that the supplier can easily identify their produced items during inspection. A labeling cost is taken into account for per-unit products in this model.

3.4 The model discusses the Stackelberg and Nash equilibrium policies. These concepts from game theory help identify the best strategies for both market players: Supplier and Buyer, while also determining the optimal solution for the supply chain model.

4. Assumptions and Notations

Notations for Buyer

$I_b(t)$ inventory of the buyer at the time t

O_b ordering cost per order (\$/order)

h_b buyer's holding cost (\$/unit/unit time)

h_{cb} carbon emission cost due to the storage of the perfect bottles at buyer's house (\$/unit)

C_b buyer's selling price for the perfect bottles(\$/unit)

N number of shipments

t_b delivery cycle time (time unit)

q delivery size per delivery (units)

Notations for Supplier

I_m(t) inventory at any time t for the manufacturer

P_m production rate (units)

D_m demand rate (units)

C_{m_1} supplier's fixed production cost

C_{m_2} supplier's production cost per unit item (\$/unit)

C_p supplier's procurement cost per unit item (\$/unit)

C_3 supplier's fixed inspection cost

C_4 supplier's variable inspection cost per unit item (\$/unit)

C_{i_1} supplier's inspection cost of the collected items

C_{i_2} supplier's inspection cost per unit item for collected return bottles

C_{l_1} fixed labeling cost

C_{l_2} supplier's labeling cost per unit item

C_s selling price for the supplier and purchasing cost for the buyer

h_m supplier's holding cost (\$/unit/unit time)

h_{cm} supplier's carbon emission cost due to storage of newly produced items

C_{ef} carbon emission cost per setup due to the material being shipped throughout the factory from one unit to another

C_{ev} carbon emission cost due to the manufacturing process (\$/unit)

C_c fixed cost associated with carbon emissions resulting from the landfill.

a warehouse of reusable bottles

b warehouse of damaged bottles

x percentage of total collected used items

y percentage of reusable collected used items

C_{ar_1} fixed transportation cost for per unit shipment of reusable bottles by vehicle 1

D_{ar_1} distance covered from warehouse 'a' of reusable bottles to the warehouse of perfect bottles by vehicle 1

CT_{ar_1} variable transportation cost per km per shipment of reusable bottles by vehicle 1

Q_{ar_1} total amount of reusable bottles transported from collection center 'a' to warehouse of perfect bottles

CP_{ar_1} total capacity of transportation vehicle used for reusable bottles per shipment

C_{br_2} fixed transportation cost by shipment of vehicle 2 for damaged bottles

D_{br_2} distance covered by vehicle 2 from warehouse 'b' of damaged bottles to warehouse of imperfect bottles

CT_{br_2} variable transportation cost per km per shipment of damaged bottles by vehicle 2

Q_{br_2} total amount of damaged bottles transported from collection center 'b' to warehouse of imperfect bottles

CP_{br_2} total capacity of transportation vehicles of damaged bottles for one shipment

C_R supplier's purchasing cost for collected reusable bottles

C_D supplier's purchasing cost for collected damaged bottles

r_1 transport vehicle 1

r_2 transport vehicle 2

4.1 Assumptions

4.1.1 Carbon emissions occur throughout the life cycle of a product, from manufacture to disposal and are therefore a major environmental issue. Pricing carbon pollution has been shown to lead to a reduction in emissions (Bai et al., 2019.). The model is developed here considering carbon emission costs throughout the model, including the costs associated with production, storage, transportation, and landfilling.

4.1.2 Typically, consumers use the product for a short time before discarding it. Under a zero-waste policy, the supplier pays the consumer for returning the waste. It's important to highlight that there are two types of returned items in this model: damaged items and reusable items. Suppliers purchase reusable items at a cost of C_D, while damaged items are bought at a lower cost of C_R. Compared to reusable items, suppliers pay less for damaged items when purchasing them from consumers ($C_R > C_D$).

4.1.3 A supplier's selling price is the same as a buyer's purchase price.

4.1.4 During the entire supply chain model, shortages are not allowed.

4.1.5 In this model, there are two types of warehouses after sorting the collected used items: a warehouse for reusable items and a warehouse for damaged items. Then, there are two types of transportation at two different locations mentioned in this model. Transportation costs depend on various factors, such as the capacity of the vehicle, the distance covered by the vehicle, fixed and variable transportation costs, and the total amount transported by the vehicle.

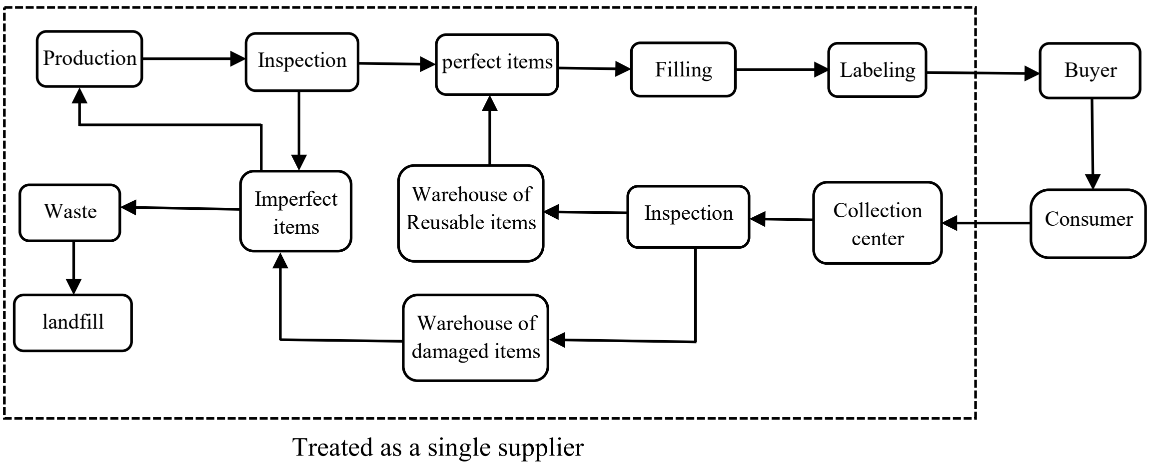

Fig. 1. Flow chart of closed-loop supply chain

5. Mathematical formulation

The model starts by increasing the supplier's stock level due to production and demand until the time . When manufacturing stops, the inventory level drops to zero at time T as a result of demand. After the supplier satisfies the buyer's demand N times, the buyer's inventory level drops until it reaches zero at time due to consumer demand. Figure 1 shows the variation in the entire system's inventory level.

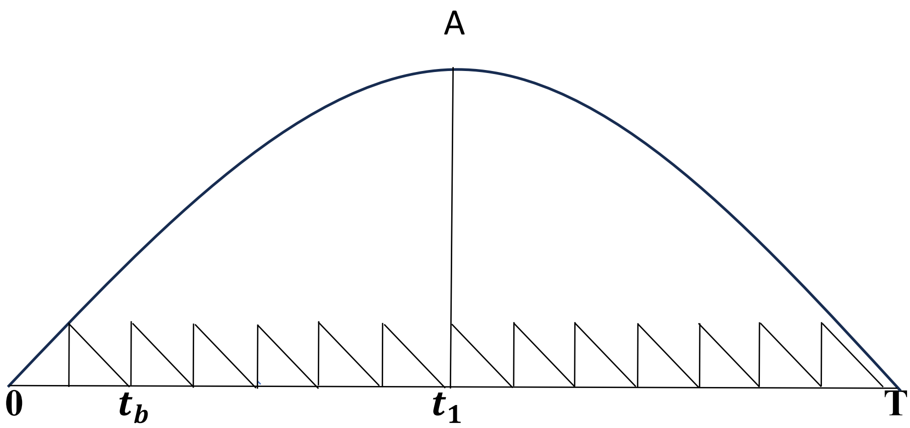

Fig. 2. Supplier-buyer inventory system

In the proposed system the inventory of the supplier and buyer's are given in terms of the following differential equations

$$\frac{dI_m(t)}{dt} = P_m - D_m \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_m(t)}{dt} = -D_m \qquad t_1 \le t \le T \tag{2}$$

$$\frac{dI_b(t)}{dt} = -D_m \qquad 0 \le t \le t_b \tag{3}$$

$$I_m(0) = 0, \qquad I_m(T) = 0,\ I_b(t_b) = 0$$

Solving these equations (1), (2), and (3) with the help of given boundary conditions, we get the following solutions.

$$I_m(t) = (P_m - D_m)t \qquad 0 \le t \le t_1 \tag{4}$$

$$I_m(t) = D_m(T - t) \qquad t_1 \le t \le T \tag{5}$$

$$I_b(t) = D_m(t_b - t) \qquad 0 \le t \le t_b \tag{6}$$

The buyer's ordering quantity is determined from the boundary condition $I_b(0) = q$ and with the help of Eq. (6) we get

$$q = D_m t_b \tag{7}$$

From Eq. (4) and Eq. (5) we get the relation

$$t_1 = \frac{D_m T}{P_m} \tag{8}$$

From Fig. 2, we can say that,

$$T = N t_b \tag{9}$$

Substituting the value of T into Eq. (8), the result is

$$t_1 = \frac{D_m N t_b}{P_m} \tag{10}$$

Supplier's profit function

The overall profit for the supplier comprises the costs associated with production, procurement, inspection, labeling, holding, transportation, carbon emission, purchasing collected used items in the primary and secondary markets along with the revenue generated from the buyer. The breakdown of each of these cost components is provided below.

Production and procurement cost

In the production of an item, there are two associated costs. The first is the cost of raw material and the second is the cost of manufacturing that item.

$$C_{m_1} + P_m t_1 (C_{m_2} + C_p)$$

Holding cost

The supplier is required to retain manufacturing items, and incurring costs for this purpose is essential. The holding cost for the supplier, encompassing both the regular holding cost h_m and the cost associated with carbon emissions h_{cm} resulting from the storage of items, is represented as $(h_m + h_{cm})$. The supplier's inventory is held in the time interval from [0, T]. Consequently, the supplier's holding cost is given by

$$H.C = (h_m + h_{cm})[\int_0^{t_1} I_m(t)dt + \int_{t_1}^{T} I_m(t)dt]$$

$$H.C = (h_m + h_{cm})[\int_0^{t_1} (P_m - D_m)t\,dt + \int_{t_1}^{T} D_m(T - t)dt]$$

$$H.C = (h_m + h_{cm})[(P_m - D_m)\frac{{t_1}^2}{2} + \frac{D_m(t_1 - T)^2}{2}]$$

Inspection cost

After production, all produced items go through the process of inspection. The inspection cost of newly produced items are

$$I.C = C_3 + C_4 P_m t_1$$

During inspection some bottles are imperfect, these imperfect bottles some bottles are used as raw material for production, and rest are collected in the form of waste

Labeling cost

After inspection, the perfect bottles are separated. For labelling barcode on perfect bottles, some cost is needed, which is given by

$$L.C = C_{l_1} + C_{l_2} P_m t_1$$

Carbon emission cost (CEC)

Carbon releases both the time during the production process and transportation of goods. So the total associated cost will be

$$CEC = C_{ev} p t_1 + N C_{ef} + C_c$$

Transportation cost

After usage, the empty bottles are collected in a collection center. These collected bottles undergo inspection, incurring an associated inspection cost denoted as C_{i_1} per unit item. After inspection, reusable and damaged bottles are sorted. Warehouse 'a' is responsible for gathering reusable bottles, while warehouse 'b' focuses on collecting damaged bottles.

For warehouse center 'a', the transportation cost of reused bottles is determined.

$$C_{i_1} + x C_{i_2} N q$$

$$\frac{(C_{ar_1} + CT_{ar_1} D_{ar_1}) Q_{ar_1}}{CP_{ar_1}}$$

For warehouse center 'b', the transportation cost of damaged bottles is determined.

$$\frac{(C_{br_2} + CT_{br_2} D_{br_2}) Q_{br_2}}{CP_{br_2}}$$

The total transportation costs are

$$TPC = \frac{(C_{ar_1} + CT_{ar_1} D_{ar_1}) Q_{ar_1}}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2} D_{br_2}) Q_{br_2}}{CP_{br_2}}$$

Supplier produces total $P_m t_1$ units, out of these items $(P_m t_1 - D_m T)$ are imperfect. After separating these imperfect units, these perfect units are sold to buyers in N shipments with order delivery size q. The revenue earned by the supplier at a price of C_s is $C_s N q$.

A percentage of used items (xNq) are being collected from the end users and the total collected units go through the inspection. The cost of inspection of these items are

After inspection, all collected items (damaged and reusable) are disassembled. Here y is the percentage of collected reusable items. As a result, the total number of reused items $xNqy$, where the damaged portion is $xNq(1-y)$. Reusable items are purchased at a cost of C_R per unit, while damaged items are acquired at a lower cost of C_D per unit.

The cost found by the supplier for these collected items.

$$C_R x N q y + C_D x N q (1 - y)$$

The supplier's profit function

$$TP_s = \frac{1}{T}\Big[C_s Nq - C_{i_1} - C_{i_2}xNq - \frac{(C_{ar_1}+CT_{ar_1}D_{ar_1})Q_{ar_1}}{CP_{ar_1}} - \frac{(C_{br_2}+CT_{br_2}D_{br_2})Q_{br_2}}{CP_{br_2}} - C_{m_1} - P_m t_1(C_{m_2} + c_p) - (h_m + h_{cm})\Big]\Big[(P_m - D_m)\frac{{t_1}^2}{2} + \frac{D_m(t_1-T)^2}{2}\Big] - C_3 - C_4 P_m t_1 - C_{l_1} - C_{l_2} P_m t_1 - C_R xNqy - C_D xNq(1-y) - C_c - C_{ev} p t_1 - Nc_{ef}] \quad (11)$$

If Q_{ar_1} denotes the total quantity of reusable items, then it can be given as $xNqy$

where $xNqy$ represents the total reusable items. Then,

$$Q_{ar_1} = Nqy$$

If Q_{br_2} denote the total quantity of reusable items, then it can be given as

$xNq(-y)$

where $xNq(1-y)$ denotes the total number of reusable items. Then

$$Q_{br_2} = xNq(1-y)$$

$$TP_s = \frac{1}{T}\Big[C_s Nq - C_{i_1} - C_{i_2}xNq - \frac{(C_{ar_1}+CT_{ar_1}D_{ar_1})xNqy}{CP_{ar_1}} - \frac{(C_{br_2}+CT_{br_2}D_{br_2})xNq(1-y)}{CP_{br_2}} - C_{m_1} - P_m t_1(C_{m_2} + c_p) - (h_m + h_{cm})(P_m - D_m)\frac{{t_1}^2}{2} + \frac{D_m(t_1-T)^2}{2}\Big] - C_3 - C_4 P_m t_1 - C_{l_1} - C_{l_2} P_m t_1 - C_R xNqy - C_D xNq(1-y) - C_c - C_{ev} p t_1 - Nc_{ef}] \quad (12)$$

After substituting the values of t_1, N, and q, the total profit function of supplier's becomes:

$$TP_s = D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2}x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{Nt_b}\Big[\frac{(C_{ar_1}+CT_{ar_1}D_{ar_1})\, xNt_b y D_m}{CP_{ar_1}} + \frac{(C_{br_2}+CT_{br_2}D_{br_2})\, xNt_b(1-y)D_m}{CP_{br_2}} + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1} + NC_{ef}\Big] - Nt_b\Big[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\Big] \quad (13)$$

Buyer's profit function

The buyer's average profit is the difference between revenue and all associated costs of the buyer's. The different associated costs are ordering costs, holding costs, and purchasing costs.

Ordering cost

To order bottles from the supplier, we need the ordering cost, then the ordering cost is O_b.

Purchasing cost

The buyer purchases q units of the perfect bottle from the supplier in N number of cycles at the cost of C_s per unit item, then the purchasing cost of the buyer $C_s Nq$

Holding cost

A supplier delivers perfect items to a buyer in N cycles; these items can be stored in a warehouse and some storage costs are associated. Carbon emission emitted due to holing the perfect items, then carbon emission cost h_{cb} is included with the holding cost of buyer's h_b. Hence the holding cost of the buyer in N number of cycles is

$$H.C = (h_b + h_{cb})N\int_0^{t_b} \mathrm{I}_b(t)dt$$

$$H.C = (h_b + h_{cb})N\int_0^{t_b} D_m(t_b - t)dt$$

$$H.C = (h_b + h_{cb})ND_m\frac{{t_b}^2}{2}$$

The buyer ordered q units from the supplier and sold in the market at the rate of C_b. The total sales revenue for the buyer in N no. of shipment is $C_b N q$.

The buyer's profit function is the result of subtracting all associated costs from the total revenue earned by the buyer. Total profit function per unit cycle time

$$TP_b = [C_b D_m - \frac{O_b}{N t_b} - C_s D_m - (h_b + h_{cb}) D_m \frac{t_b}{2}] \tag{14}$$

Total profit function of Supply Chain

The total profit function for the supply chain is the sum of the buyer's profit function and the supplier's profit function.

$$TP = TP_b + TP_v$$

$$TP = \left[C_b D_m - \frac{O_b}{N t_b} - C_s D_m - (h_b + h_{cb}) D_m \frac{t_b}{2}\right] + D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2} x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{N t_b}[\frac{(C_{ar_1} + CT_{ar_1} D_{ar_1})\, x\, y\, D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2} D_{br_2})\, x\,(1-y) D_m}{CP_{br_2}} C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1} + N C_{ef}] - N t_b \left[\frac{(h_m + h_{cm})\,(P_m - D_m)\, D_m}{2P_m}\right] \tag{15}$$

6. Solution procedure

The goal of this study is to determine the optimal values for N and t_b that lead to the maximization of overall profit. This involves identifying the optimal number of deliveries per cycle N^* and the corresponding delivery cycle time $t_b{}^*$ for optimal performance.

Case 1. To find the optimal values for N and t_b, we differentiate the total profit function with respect to N and t_b and set the resulting expressions equal to zero.

$$\frac{\partial TP}{\partial t_b} = 0, \ \frac{\partial TP}{\partial N} = 0$$

The optimization of the total profit function involves the utilization of the Hessian matrix.

To optimize the total profit TP, we differentiate the total profit function with respect to N, t_b

$$\frac{\partial TP}{\partial t_b} = \frac{O_b}{N t_b{}^2} - \frac{(h_b + h_{cb}) D_m}{2} + \frac{C_{ef}}{t_b{}^2} + \frac{1}{N t_b{}^2}\left(C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right) - N\left[\frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}\right] \tag{16}$$

$$\frac{\partial TP}{\partial N} = \frac{1}{N^2 t_b}(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}) - \frac{(h_m + h_{cm})\,(P_m - D_m) D_m t_b}{2P_m} \tag{17}$$

By setting Eq. (16) and Eq. (17) equal to zero and solving for the variables, we derive the optimal solution for $(N^*, t_b{}^*)$

$$N^* = \frac{1}{t_b}\sqrt{\frac{2P_m(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{(h_m + h_{cm})\,(P_m - D_m) D_m}} \qquad t_b{}^* = \sqrt{\frac{2P_m(O_b + NC_{ef} + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N[N(h_m + h_{cm})(P_m - D_m) D_m + 2P_m D_m (h_b + h_{cb})]}}$$

$$\frac{\partial^2 TP}{\partial N^2} = -\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^3 t_b}$$

$$\frac{\partial^2 TP}{\partial t_b{}^2} = -\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N t_b{}^3} - \frac{2c_{ef}}{t_b{}^3}$$

$$\frac{\partial^2 TP}{\partial N \partial t_b} = -\frac{(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^2 t_b{}^2} - \frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}$$

$$\frac{\partial^2 TP}{\partial t_b \partial N} = -\frac{(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^2 {t_b}^2} - \frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}$$

$$H = \begin{pmatrix} \frac{\partial^2 TP}{\partial {t_b}^2} & \frac{\partial^2 TP}{\partial t_b \partial N} \\ \frac{\partial^2 TP}{\partial N \partial t_b} & \frac{\partial^2 TP}{\partial N^2} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3} & -\frac{(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^2{t_b}^2} - \frac{(h_m + h_{cm})\,(P_m - D}{2P_m} \\ -\frac{(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^2{t_b}^2} - \frac{(h_m + h_{cm})\,(P_m - D_m)D_m}{2P_m} & -\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^3 t_b} \end{pmatrix}$$

$$-\frac{2(O_b+C_{m_1}+C_3+C_{l_1}+C_c+C_{i_1})}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3} < 0, \; -\frac{2(O_b+C_{m_1}+C_3+C_{l_1}+C_c+C_{i_1})}{N^3 t_b} < 0$$

Now, we

$$|H| = \left(-\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3}\right)\left(-\frac{2(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{N^3 t_b}\right) - \frac{\left(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right)}{N^2{t_b}^2} + \frac{(h_m + h_{cm})\,(P_m - D_m)D_m}{2P_m}$$

$$|H| = \Big[\frac{4}{N^4{t_b}^4}\left\{(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})\,(O_b + C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1} + Nc_{ef})\right\} - \left(\frac{(O_b+C_{m_1}+C_3+C_{l_1}+C_c+C_{i_1})}{N^2{t_b}^2} + \frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right)\left(\frac{(O_b+C_{m_1}+C_3+C_{l_1}+C_c+C_{i_1})}{N^2{t_b}^2} + \frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right)\Big] > 0 \tag{18}$$

Consequently, the Hessian matrix of H shows negativity definite at N and t_b. Hence, it can be inferred that the total profit function TP demonstrates concavity concerning the variables N and t_b.

Case 2. Non-Cooperative Nash equilibrium

In Nash equilibrium, no firm dominates the market, and each object is independent in making decisions without interference from others. Therefore, both the buyer and the supplier determine their optimal decision variables separately. The goal is to identify the buyer's maximum annual profit first and subsequently derive the supplier's maximum annual profit.

Lemma 1. For a given $N > 1$, the supplier's profit function

$TP_v(N, t_b)$
is as concave in t_b

Proof. To optimize the buyer's total profit, we calculate the first and second derivatives of the TP_b function with respect to t_b.

$$\frac{\partial TP_b}{\partial t_b} = \frac{O_b}{N{t_b}^2} - \,(h_b + h_{cb})\frac{D_m}{2} \tag{19}$$

By equating (19) to zero, we determine the optimal delivery cycle time for the buyer.

$$\frac{\partial TP_b}{\partial t_b} = 0$$

$$\frac{O_b}{N{t_b}^2} - (h_b + h_{cb})\frac{D_m}{2} = 0$$

$$\frac{O_b}{N{t_b}^2} = (h_b + h_{cb})\frac{D_m}{2}$$

$${t_b}^* = \sqrt{\frac{2O_b}{ND(h_b + h_{cb})}}$$

${t_b}^*$ is Buyer's optimal delivery cycle time
Differentiate again equation (19) with respect to t_b.

$$\frac{\partial^2 TP_b}{\partial {t_b}^2} = -\frac{2O_b}{N{t_b}^3} < 0 \tag{20}$$

Then $TP_b(N, t_b)$ is a concave function in t_b and maximum profit at ${t_b}^*$, for given $N > 1$

Lemma 2. For a given $t_b > 0$, the supplier's profit function $TP_v(N, t_b)$ is as concave in N

Proof. From equation (13) supplier's total profit function

$$TP_v = D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2}x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{Nt_b}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}]$$
$$- [\frac{(C_{ar_1} + CT_{ar_1}D_{ar_1})\ x\ y\ D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2}D_{br_2})\ x\ (1-y)D_m}{CP_{br_2}}]$$
$$- \quad Nt_b\left[\frac{(h_m + h_{cm})\ (P_m - D_m)D_m}{2P_m}\right] - \frac{C_{ef}}{t_b}$$

Differentiate of supplier's profit function with respect to N

$$\frac{\partial TP_v}{\partial N} = \frac{1}{N^2 t_b}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - t_b\left[\frac{(h_m+h_{cm})\ (P_m-D_m)D_m}{2P_m}\right] \tag{21}$$

Equating Eq. (21) to zero, we determined the optimal number of cycles for the supplier's

$$\frac{\partial TP_v}{\partial N} = 0$$

For given $t_b > 1$, Then TP_v is a concave function in N.

$$\frac{\partial TP_v}{\partial N} = 0$$
$$\frac{1}{N^2 t_b}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] = t_b\left[\frac{(h_m + h_{cm})\ (P_m - D_m)D_m}{2P_m}\right]$$

$$N = \frac{1}{t_b}\sqrt{\frac{2P_m\ (C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{(h_m + h_{cm})\ (P_m - D_m)D_m}}$$

Supplier's optimal number of cycles,

$$N^* = \frac{1}{t_b}\sqrt{\frac{2P_m\ (C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1})}{(h_m + h_{cm})\ (P_m - D_m)D_m}}$$

$$\frac{\partial^2 TP_v}{\partial N^2} = -\frac{2}{N^3 t_b}\left(C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right) < 0$$

Hence supplier's profit function concave in N and profit maximum at N^*, for $t_b > 1$.

Case 3. Stackelberg equilibrium

In Stackelberg equilibrium, the supplier assumes the role of the leader, and the buyer acts as the follower. Initially, the buyer determines its optimal t_b to maximize its individual profit. Subsequently, the supplier substitutes the buyer's optimal t_b into its profit function and then identifies its own optimal N to maximize its profit.

$$TP_b = \left[c_b D_m - \frac{O_b}{N t_b} - c_s D_m - (h_b + h_{cb}) D_m \frac{t_b}{2}\right]$$

$$\frac{\partial TP_b}{\partial t_b} = \frac{O_b}{N {t_b}^2} - (h_b + h_{cb})\frac{D_m}{2}$$

$$\frac{\partial TP_b}{\partial t_b} = 0$$

$${t_b}^* = \sqrt{\frac{2O_b}{ND(h_b + h_{cb})}}$$

This is the optimal value for the buyer, substituting the optimal value of buyer ${t_b}^*$ in the supplier's profit function.

$$TP_v(N, t_b) = TP_v(N, {t_b}^*)$$

$$TP_v(N, {t_b}^*) = D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2} x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{N{t_b}^*}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - \left[\frac{(C_{ar_1} + CT_{ar_1} D_{ar_1})\, x\, y\, D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2} D_{br_2})\, x\, (1-y) D_m}{CP_{br_2}}\right] - N{t_b}^*\left[\frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}\right] - \frac{C_{ef}}{{t_b}^*}$$

$$TP_v(N, {t_b}^*) = D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2} x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{N\sqrt{\frac{2O_b}{ND(h_b+h_{cb})}}}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - \left[\frac{(C_{ar_1} + CT_{ar_1} D_{ar_1})\, x\, y\, D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2} D_{br_2})\, x\, (1-y) D_m}{CP_{br_2}}\right] - N\sqrt{\frac{2O_b}{ND(h_b + h_{cb})}}\left[\frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}\right] - \frac{C_{ef}}{\sqrt{\frac{2O_b}{ND(h_b+h_{cb})}}}$$

$$TP_v(N, {t_b}^*) = D_m(C_S - C_R xy - C_D x(1-y) - C_{i_2} x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{1}{N\sqrt{\frac{2O_b}{ND(h_b+h_{cb})}}}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - \left[\frac{(C_{ar_1} + CT_{ar_1} D_{ar_1})\, x\, y\, D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2} D_{br_2})\, x\, (1-y) D_m}{CP_{br_2}}\right] - N\sqrt{\frac{2O_b}{ND(h_b + h_{cb})}}\left[\frac{(h_m + h_{cm})\,(P_m - D_m) D_m}{2P_m}\right] - \frac{C_{ef}}{\sqrt{\frac{2O_b}{ND(h_b+h_{cb})}}}$$

$$TP_v(N,t_b{}^*) = D_m(C_S - C_Rxy - C_Dx(1-y) - C_{i_2}x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \sqrt{\frac{D(h_b+h_{cb})}{2NO_b}}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - [\frac{(C_{ar_1} + CT_{ar_1}D_{ar_1})\,x\,y\,D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2}D_{br_2})\,x\,(1-y)D_m}{CP_{br_2}}] - \sqrt{\frac{2NO_b}{D(h_b+h_{cb})}}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] - C_{ef}\sqrt{\frac{ND(h_b+h_{cb})}{2O_b}}$$

$$X = \sqrt{\frac{D(h_b+h_{cb})}{2O_b}}$$

Then

$$TP_v(N,t_b{}^*) = D_m(C_S - C_Rxy - C_Dx(1-y) - C_{i_2}x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{X}{\sqrt{N}}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - [\frac{(C_{ar_1}+CT_{ar_1}D_{ar_1})\,x\,y\,D_m}{CP_{ar_1}} + \frac{(C_{br_2}+CT_{br_2}D_{br_2})\,x\,(1-y)D_m}{CP_{br_2}}] - \frac{\sqrt{N}}{X}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] - X\sqrt{N}\;C_{ef} \tag{22}$$

Deriving with the first-order derivatives of Eq. (22) with respect to N, we obtain:

$$\frac{\partial TP_v(N,t_b{}^*)}{\partial N} = \frac{X}{2N^{\frac{3}{2}}}\left(C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right) - \frac{1}{2\,X\sqrt{N}}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] - \frac{XC_{ef}}{2\sqrt{N}}$$

$$\frac{\partial TP_v(N,t_b{}^*)}{\partial N} = 0$$

$$\frac{X}{2N^{\frac{3}{2}}}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] - \frac{1}{2\,X\sqrt{N}}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] - \frac{XC_{ef}}{2\sqrt{N}} = 0$$

The optimal value of the number of shipments are

$$N^* = \frac{X^2[C_{m_1}+C_3+C_{l_1}+C_c+C_{i_1}]}{X^2C_{ef}+\frac{(h_m+h_{cm})\,(P_m-D_m)}{2P_m}} \tag{23}$$

Derive the second-order derivatives of Eq. (21)

$$\frac{\partial^2 TP_v(N,t_b{}^*)}{\partial N^2} = -\frac{3\,X}{4N^{\frac{5}{2}}}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] + \frac{1}{4XN^{\frac{3}{2}}}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] + \frac{X\,C_{ef}}{4N^{\frac{3}{2}}}$$

Lemma 3. If

$$3X\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] > \frac{1}{X}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] + XC_{ef}$$

then $TP_v(N,t_b)$ is concave in N.

Proof. From Eq. (22),

$$TP_v(N,t_b{}^*) = D_m(C_S - C_Rxy - C_Dx(1-y) - C_{i_2}x - C_{m_2} - C_p - C_4 - C_{l_2} - C_{ev}) - \frac{X}{\sqrt{N}}[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}] - [\frac{(C_{ar_1} + CT_{ar_1}D_{ar_1})\,x\,y\,D_m}{CP_{ar_1}} + \frac{(C_{br_2} + CT_{br_2}D_{br_2})\,x\,(1-y)D_m}{CP_{br_2}}] - \frac{\sqrt{N}}{X}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] - X\sqrt{N}\;C_{ef}$$

Differentiate $TP_v(N, t_b{}^*)$ for the first and second derivatives with respect to N, and we get

$$\frac{\partial TP_v(N, t_b{}^*)}{\partial N} = \frac{X}{2N^{\frac{3}{2}}}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] - \frac{1}{2\,X\sqrt{N}}\left[\frac{(h_m + h_{cm})\,(P_m - D_m)D_m}{2P_m}\right] - \frac{XC_{ef}}{2\sqrt{N}}$$

$$\frac{\partial^2 TP_v(N,t_b{}^*)}{\partial N^2} = -\frac{3\,X}{4N^{\frac{5}{2}}}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] + \frac{1}{4XN^{\frac{3}{2}}}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] + \frac{X\,C_{ef}}{4N^{\frac{3}{2}}} \tag{24}$$

For concavity of function, we prove

$$\frac{\partial^2 TP_v(N, t_b{}^*)}{\partial N^2} < 0$$

Then Eq. (24),

$$\frac{\partial^2 TP_v(N, t_b{}^*)}{\partial N^2} = -\frac{3\,X}{4N^{\frac{5}{2}}}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] + \frac{1}{4XN^{\frac{3}{2}}}\left[\frac{(h_m + h_{cm})\,(P_m - D_m)D_m}{2P_m}\right] + \frac{X\,C_{ef}}{4N^{\frac{3}{2}}}$$
$$= \frac{1}{4N^{\frac{3}{2}}}\left[-\frac{3\,X}{N}\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] + \frac{1}{X}\left[\frac{(h_m+h_{cm})\,(P_m-D_m)D_m}{2P_m}\right] + X\,C_{ef}\right] < 0$$

$$3X\left[C_{m_1} + C_3 + C_{l_1} + C_c + C_{i_1}\right] > \frac{1}{X}\left[\frac{(h_m + h_{cm})\,(P_m - D_m)D_m}{2P_m}\right] + XC_{ef}$$

For given

$$N > 1, X > 0$$

7. Numerical analysis

The input parameters considered for demonstrating the theoretical results are as follows. The parameter values were obtained from the published literature by Saxena et al. (2023).

x =0.5, y =0.8, C_b =\$ 85 per unit, C_s =\$ 65 per unit, C_p =\$ 5 per unit, C_m =\$ 15 per unit, C_i =\$ 0.5 per unit, C_3 =\$ 70 per unit, h_b =\$ 1.5 per unit per unit time, h_{cb} =\$ 0.5 per unit per unit time, h_m =\$ 1 per unit per unit time, h_{cm} =\$ 1 per unit per unit time, O_b = \$ 1500 per setup, P_m = 1500 units per unit item, D =1000 units per unit item, C_{ef} = \$ 50 per unit, C_{ev} =\$ 0.2 per unit, C_R =\$ 10 per unit, C_D =\$ 4 per unit, C_{acr_1}=\$ 1000 per unit, CT_{acr_1}=\$ 5 per unit, D_{acr_1}=\$ 20 per unit, CP_{acr_1}=\$ 1000 per unit, C_{bdr_2}=\$ 3 per unit, CT_{bdr_2}=\$ 3 per unit, D_{bdr_2}=\$ 22 per unit, CP_{bdr_2}=\$ 500 per unit, C_{m_1} =\$ 4500 per unit, C_{m_2} =\$ 10 per unit, C_3 =\$ 500 per unit, C_4 =\$ 0.3 per unit, C_{l_1} =\$ 250 per unit, C_{l_2} =\$ 0.5 per unit, C_{i_1} =\$ 500 per unit, C_{i_2} =\$ 0.6 per unit, C_c =\$ 1000 per unit.

Table 1
The optimal value for all cases

Cases	Case 1	Case 2	Case 3
t_b	0.182574	0.238307	0.2222
N	27	18	20
T	4.929498	4.289526	4.444
q	182.574	238.307	222.2
TP_b	19424.6	19285.1	19333.3
TP_v	40657.8	40729.7	40721.8
TP	60082.5	60014.7	60055.1

In Table 1, when considering the total profits across all three cases, it becomes apparent that the total profit for Case 3 exceeds that of Case 2. If we compare the buyer's and supplier's profits between the Stackelberg and Nash equilibrium, then the result is supplier's profit for the Stackelberg equilibrium is higher than the supplier's profit for the Nash equilibrium, while the profit of the buyer in the Stackelberg equilibrium less than buyer's profit in Nash equilibrium.

The concavity graphs for cases 1 and case 2 are depicted in Figs. 3-8, respectively.

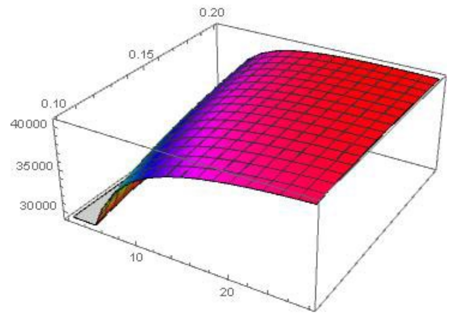

Fig 3. The concavity of the vendor's profit $\boldsymbol{TP_v}$ for case 1

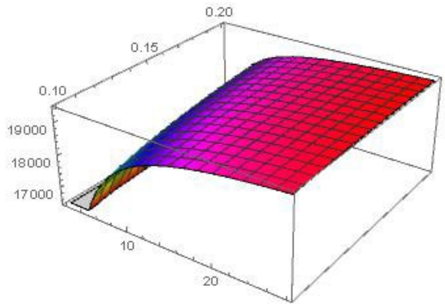

Fig 4. The concavity of the buyer's profit $\boldsymbol{TP_b}$ for case 1

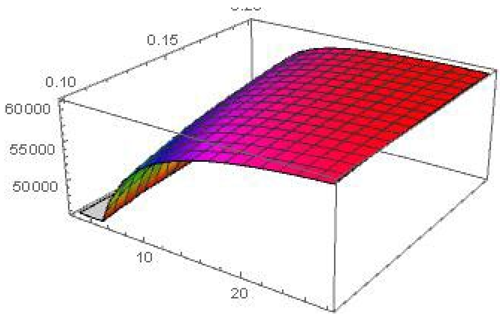

Fig 5. The concavity of the total profit $\boldsymbol{TP}$ for case 1

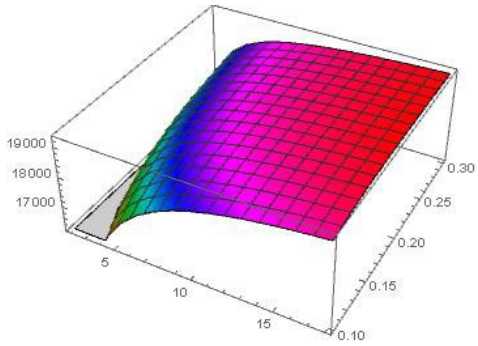

Fig 6. The concavity of the buyer's profit $\boldsymbol{TP_b}$ in the Stackelberg equilibrium

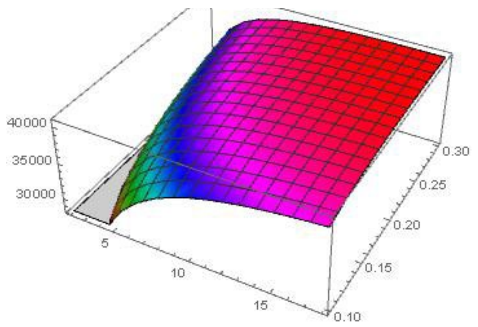

Fig 7. The concavity of the vendor's profit $\boldsymbol{TP_v}$ in the Stackelberg equilibrium

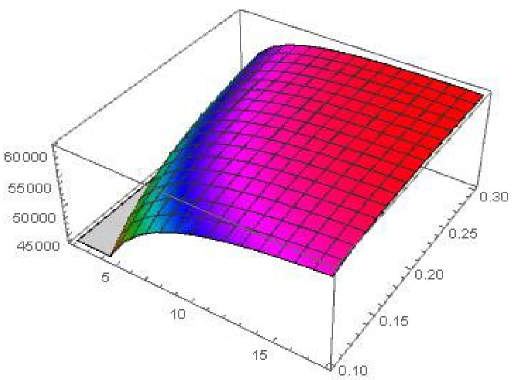

Fig 8. The concavity of the total profit $\boldsymbol{TP}$ in Stackelberg equilibrium

6. Sensitivity analysis

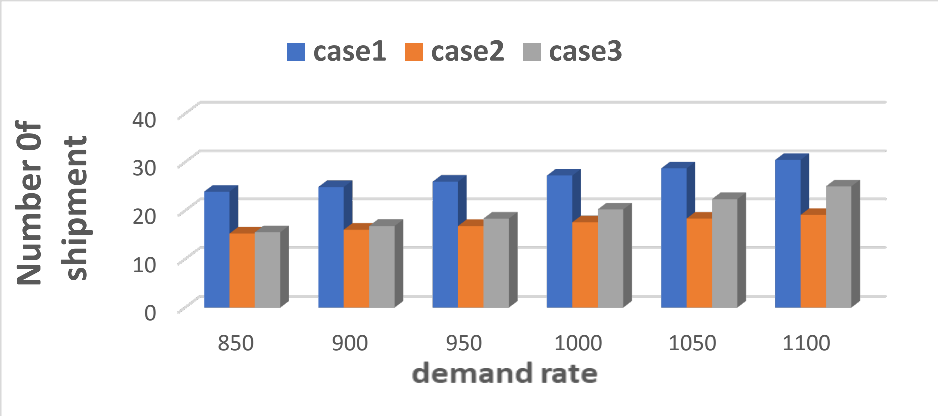

Fig. 9. The number of shipments for three distinct cases with different demand rates

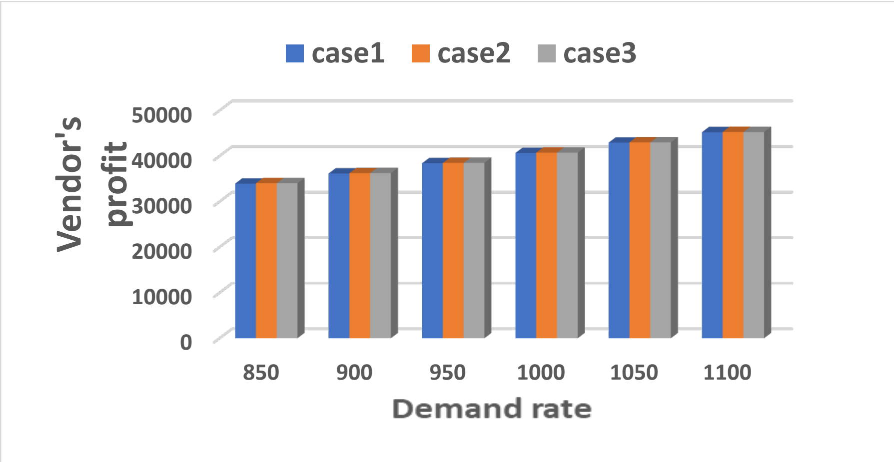

Fig. 10. The supplier's profit for three distinct cases with different demand rates

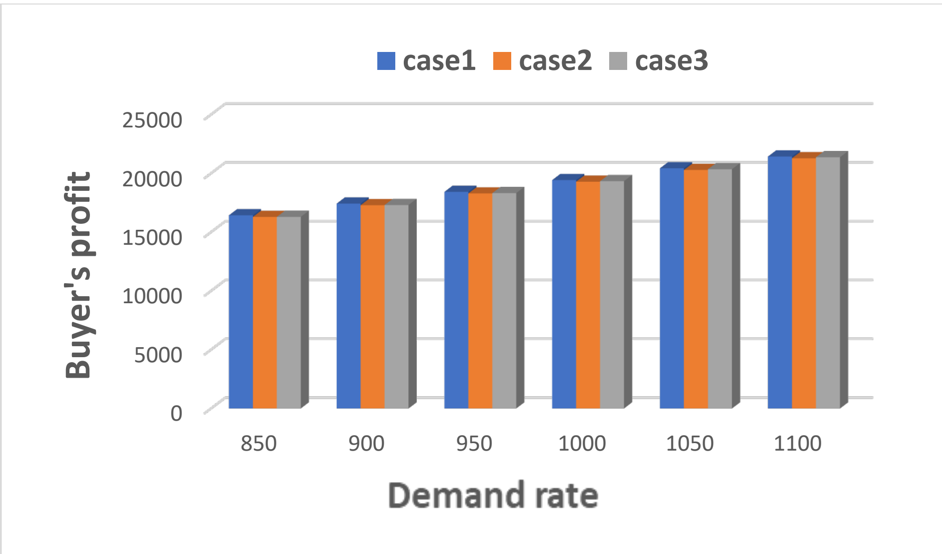

Fig. 11. The buyer's profit for three different cases with varying demand rates

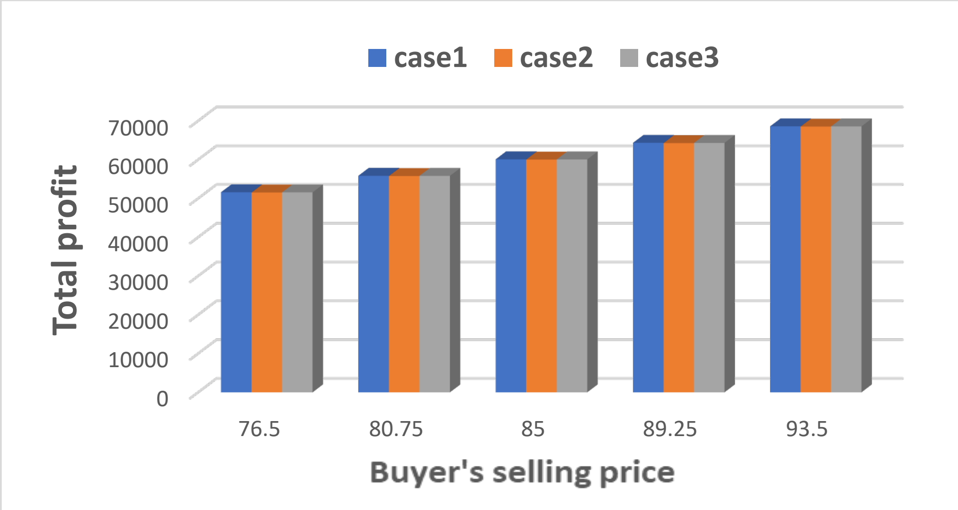

Fig. 12. The total profit for three different cases with varying buyer's selling price

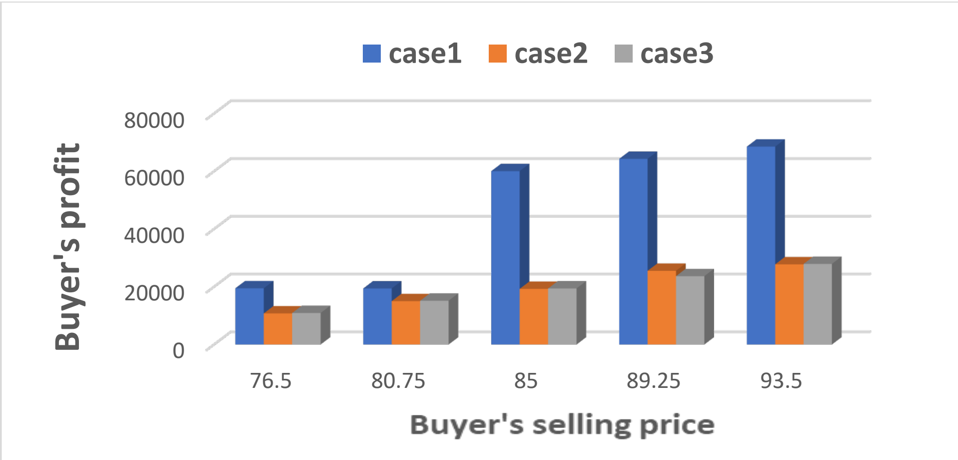

Fig. 13. The buyer's profit for three distinct cases with different buyer's selling price

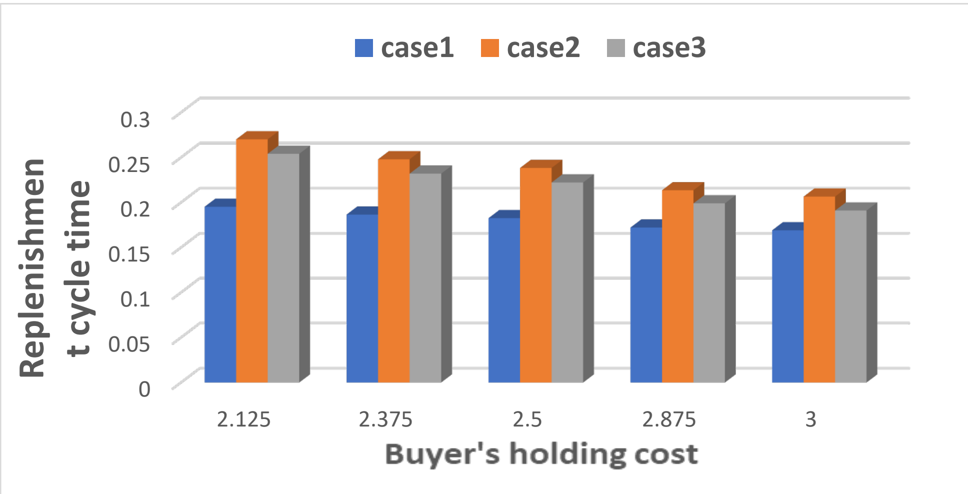

Fig. 14. The replenishment cycle time for three distinct cases with different buyer's holding cost

7. Observations from the Sensitivity analysis

The conclusions drawn from the sensitivity analysis are given as follows.

The results show that when the demand rate rises from 850 to 1100, the number of shipments increases in all three cases (depicted in Fig. 9).

In Fig. 10, The supplier's profit increases as the rate of demand increases. Similarly, the buyer's profit slightly rises in all the cases as the rate of demand increases (shown in Fig. 11).
In Fig. 12, when the selling price of buyers increases from 76.5 to 93.5, total profit strictly increases in all three cases.
The buyer's profit rapidly increases as a buyer's selling price varies from 76.5 to 93.5 (shown in Fig. 13).
In Fig. 14, Increasing the buyer's holding cost results in a decrease in cycle time in all three cases, with case 1 and 3 cycles decreasing more slowly as compared to case 2.

8. Conclusion

In this study, we have developed a sustainable supply chain management system that focuses on labeling bottles and the reuse of undamaged bottles to reduce waste. The model takes into account the carbon emission costs associated with the production, storage, transportation, and disposal of items. The procedure includes the collection of used bottles from consumers, transporting them to a collection center, and following inspection. After inspection, the bottles are sorted into two categories, reusable and damaged items. Reusable bottles undergo the processes of filling and labeling. While damaged items are either sent to landfills or utilized as raw materials for the production of new bottles. This approach aims to minimize environmental impact by promoting bottle reuse, reducing waste, and factoring in carbon emissions throughout the supply chain.

To sum up, the establishment of a sustainable supply chain network under the frameworks of both Stackelberg and Nash equilibrium policies involves the incorporation of a reverse logistics model including multiple deliveries and a centralized distribution center. This strategy aims to develop the overall efficiency and environmental sustainability of the supply chain, taking into account the strategic interactions among various decision-makers. Incorporating these elements, the supply chain aims to achieve a Cooperative equilibrium between economic sustainability, and environmental responsibility utilizing both Stackelberg and Nash equilibrium approaches.

In conclusion, an entire method to encourage environmental responsibility and cost-effectiveness is provided by integrating a reverse logistic model with multiple deliveries and a single distribution center with the implementation of a sustainable supply chain system that is determined by the Stackelberg and Nash equilibrium strategy. This well-structured framework addresses all aspects of both forward and reverse logistics. The supply chain achieves equilibrium through the strategic development of incentives using Nash and Stackelberg equilibrium strategies, which not only encourages sustainability but also minimizes waste and maximizes resource efficiency. Furthermore, the incorporation of a single distribution center and multiple deliveries in a reverse logistic model improves the supply chain's overall flexibility, contributing to long-term environmental sustainability.

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