

Order allocation in a multiple-vendor and quantity discount environment: A multi-objective decision making approach

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ABSTRACT

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Integrated supplier selection and order allocation is a complex problem that is important for both designing and operating supply chains. It becomes especially complicated when quantity discounts are considered at the same time. Under such circumstances, most studies often formulate the problem as a Multi-Objective Linear Programming problem (MOLP), and then transform it to a Mixed Integer Programming problem (MIP) to handle the inherited multi-objectives, simultaneously. But, objectives are not of equal importance and in this approach scaling and subjective weighting often are not considered. In addition, some of the studies that use weighting method to solve the MOLP, usually ignore to normalize the coefficients. However, as different coefficients have different units such as cost or number coefficients, so weighted summation will be meaningless. Furthermore, in most of the studies only quantitative criteria are considered in mathematical model. But, the importance of some qualitative criteria persuade decision maker to consider other affective criteria as well as cost. In this study, in order to ease the problem and to obtain a more reasonable compromised solution for order allocating among suppliers, an integration of analytical hierarchy process and linear integer and multi-objective programming is proposed. The large number of criteria and attributes are employed in this problem and they are employed in a comprehensive model to solve the multi-objective problem and to find the most preferred non dominated solutions by considering decision maker's (DM) preferences. Some illustrative examples are solved using LINGO and the results are compared. The sensitivity analysis and comparing the results with one of the well-known studies in the literature has demonstrated the flexibility and efficiency of the proposed model to deal with large sized problems and incorporate different purchasing policies, easily and in a short amount of time.

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1. Introduction

In most industries the average purchases of raw materials and component parts can account for up to 70% of the main cost of a product (Ghodsypour & O'Brien, 2001). Organizations have outsourced in a broad range of functions and activities in a supply chain during the last several decades. The purchases from suppliers can account for a large percentage of the cost of products and companies must have a strong relationship with their suppliers in a supply chain to have successful outsourcing policies. supplier

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selection is a critical issue that has a long-term impact on outsourcing decisions and enhance customer satisfaction in a value chain, practically, multi-sourcing prepares greater assurance of timely delivery, and greater upside volume flexibility (Ramasesh et al., 1991; Sawik, 2014). Dealing with more suppliers reduces the risks with respect to the business disruption in the supply chain. However, sometimes it causes more relationship management costs in terms of time and capital (Tsai & Wang, 2010). Recent trends encourage the organizations to deal with fewer but better suppliers and to establish a long-term mutually beneficial relationship with them (Prahinski & Benton, 2004; Berger et al., 2004). Therefore, determination the number of suppliers to employ is a critical issue for the design of an organization's supply chain (Lambert & Cooper, 2000). In a multiple-sourcing strategy the buying firm can split its orders between suppliers to weaken the supplier's power over the buyer. Hence, the buyer firm has an opportunity to receive lower prices and shipping costs and also to keep secure working relationship with suppliers (Kawtummachai et al., 2005). So, the order allocation is a strategic purchasing decision. On the other hand, for JIT purchasing, an ordering strategy that orders smaller quantities and more frequently as needed is more practical and feasible. So, suppliers are finding that it is meaningful to give discounts based on the total orders placed by a buyer (Xia & Wu, 2007). Although large quantity orders have the lower unit cost, but may increase inventory cost. So, it is the task of the purchasing department to evaluate all of the considerations and make profitable buying decisions under different conditions. Hence, the introduction of price discounts complicates the determination of the optimal purchasing quantities for purchasing department.

Choosing the right suppliers involves a wide range of both quantitative and qualitative factors such as cost, quality, and background of relationship (Weber et al. 1991). Furthermore, some criteria may be in conflict with each other, such as quality and on time delivery or cost and quality. Hence, it is necessary to make a tradeoff between conflicting quantitative and qualitative criteria. furthermore, as different strategies have different policies of supplier partnership, a flexible method which can reflect the corporate strategy in the supplier selection is required to consider the suppliers' and buyer's limitations.

The purpose of this paper is to present a multi-objective supplier selection and order allocation model that tries to optimize the single buyer and multiple-supplier problem by minimizing the total purchasing cost, the total number of late delivered items and the total number of defective items. In order to incorporate the qualitative criteria in the proposed model, maximizing the total purchasing value is also considered as another objective. The weight of each supplier considering 19 criteria are obtained via analytical hierarchy process (AHP) (Saaty, 1990) and used as coefficient in the function of total purchasing value. Furthermore, the solutions obtained using the integrated weighting and normalization method help the decision maker incorporate his/her purchasing policy and understand the tradeoffs between the objectives before selecting a best compromised solution. Considering both qualitative and quantitative criteria, permitting all-unit quantity or total business volume discount policy, in addition to extension of more practical constraints like buyer's limitations on budget and elimination some extra constraints and also incorporating buyer' purchasing policy via integrated normalized and weighting method make this model more practical in comparison with other previous studies. The present article is organized as follow: Mathematical formulations of the supplier selection model are presented in the following section. Then various criteria for a global supplier selection and an illustrated problem are presented. Model validation and sensitivity analysis are given in the next section. Finally conclusions are outlined in the last Section.

2. Literature review

Integrated supplier selection and order allocation is an important issue for both designing and operating supply chains. Extensive multi-criteria decision making approaches have been proposed to tackle the problem, such as the analytic hierarchy process (AHP), analytic network process (ANP), fuzzy set theory and mathematical programming. Ho et al. (2010) reviewed the literature of the multi-criteria decision making approaches for supplier evaluation and selection. Chai et al. (2013) provided a systematic

literature review on decision-making techniques of supplier selection. Furthermore, Setak et al. (2012) provided a comprehensive review of supplier selection and order allocation models. Relevant studies in the literature have been divided into two major categories. First category includes the studies which consider just one criterion, usually the cost of the procurement for the problem while suppliers offer quantity discounts on the materials. Furthermore, their models consider single-objective (usually purchasing cost) which cannot be applied for the practical cases. Considering the importance of some qualitative criteria such as quality of products and services provided by suppliers or background of relationship, persuade organizations to consider other affective criteria as well as cost. So the second category of papers of the subject declares that supplier selection problem is a multi-objective decision making problem. In this section, we only discuss the works that are important to our problem. Xia and Wu (2007) presented an integrated approach of analytical hierarchy process improved by rough sets theory and multi-objective mixed integer programming to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers. They considered the case of multiple sourcing, multiple products, with multiple criteria and supplier's capacity constraints in the presence of business volume discount. In their approach three criteria are considered for supplier selection and the multi objective functions include maximizing the total weighted quantity and the number of items delivered on time, and minimizing the total purchasing price and the number of defective items. The constraints on the capacity, discount interval, demand, quality and delivery are taken in to account. Their approach was effective in the supplier selection and order allocation problem. However, after solving the problem via optimization tool box in MATLAB, a large number of feasible solutions are obtained and their number may grow exponentially with the size of the problem. So, finding the most preferred solution between the most efficient points is difficult. But, usually the decision-maker is interested to find the most efficient solutions (non-dominated solutions or non-inferior solutions). Furthermore, the model lacks an appropriate set of weights to consider the priorities of the objective functions and to generate a more convenient objective decision and also the buyer's limitation on budget is not considered in the problem.

Tsai and Wang (2010) applied a mixed integer programming approach to solve the sourcing and order allocation problem with multiple products and in the presence of business volume discount. Three objectives are used in the model to minimize the total cost, the number of rejected products, and the number of late deliveries. Limitations on demand, capacity, the number of employed suppliers and discount intervals are considered in the model. The authors assigned relative weights to the original objectives to combine MOLP in to a single objective function without any normalization. However, as different coefficients have different units such as cost coefficient and number coefficient, so weighted summation will be meaningless. Furthermore, only quantitative criteria are considered to evaluate suppliers. Ting et al. (2007) proposed a non-linear mixed integer programming model to select suppliers and determine the order quantities. In their approach, only quantitative criteria are considered to evaluate suppliers. The model considers the single objective to minimize total purchasing cost which takes into account all-units discount, transportation cost, fixed cost, inventory cost, and the cost of receiving poor quality parts. The constraints of supplier capacity, quality and lead-time requirements are also taken into account in the model. An optimization procedure based on particle swarm optimization (PSO) was developed to determine the optimal number of sources and the lot size. Demirtas and Üstün (2008) proposed an integrated approach of ANP and multi-objective mixed integer linear programming to consider both tangible and intangible factors in choosing the best suppliers and define the optimum quantities among them. The criteria are involved in the four clusters: benefits, opportunities, costs and risks (BOCR), and the priorities are calculated for each supplier by using ANP. Three objectives are used in the model to minimize cost and defect rate and maximize total value of purchasing and subject to supplier's capacity and buyers demand constraints. Yet, discount policy and the quality and budget constraints are not considered in the model. The multi-objective problem was solved by using ϵ -constraint method and a reservation level driven Tchebycheff procedure. In this procedure one of the objective functions is selected to be optimized and the other objective functions are converted into constraints by giving an upper bound for each of them. After that these authors improved their approach to conclude multi-period inventory lot sizing scenario for a single

product and multiple suppliers (Demirtas & Ustun 2009). They suggested an integrated approach of Archimedean Goal Programming (AGP) and ANP in a two stage mathematical programming according to their previous study mentioned above and solved the model using AGP (Demirtas & Ustun, 2009). Mendoza and Ventura (2008) developed a two-phase method to integrate supplier selection and order allocation problem. Firstly, a list of potential suppliers is ranked and reduced to a manageable number using AHP. Then, a mixed-integer nonlinear programming model is used to allocate order quantities among the reduced list of suppliers. The selection process is driven by minimizing the total cost of the buyer and subject to the quality and capacity constraints of the suppliers. Kamali et al. (2011) developed a multi-objective mixed integer nonlinear programming model to coordinate the system of a single buyer and multiple vendors under an all-unit quantity discount policy. The model was used to minimize the buyer's and the supplier's cost, the number of defective items and the late delivered items and to maximize purchasing value. The model included demand, capacity, discount interval and total order quantity constraints. The authors used the compromise objective function in terms of sum of the relative deviations from the optimal values of the four objective functions to obtain a unique optimal decision. Due to the complexity of the problem Particle swarm optimization (PSO) and scatter search algorithm are used to solve the problem. Bohner and Minner (2017) recently solved a supply chain problem for simultaneous supplier selection and order allocation with supplier failure risk and various schemes of quantity discount for multiple products. The selection process is driven through a mixed-integer linear programming to minimize total expected cost. However, their approach did not considered intangible factors in the decision process.

It is obvious that in practical situation buyer faces with multiple sourcing in which suppliers may offer price discount. As it has been described above, some papers discussed the supplier selection and order allocation problem under quantity discount offered by suppliers, but we still need to develop a more appropriate model that incorporate different purchasing policies easily and in a short time with respect to buyer limitations. However, to the best of our knowledge, buyer limitation on budget has not considered in the previous papers. Also, all the previous papers considered supplier capacity constraint in the model. However, supplier capacity constraint is considered in discount interval constraints and adding this constraint to the model is not bounding, but is redundant. Moreover, integrated linear normalization and weighting method employed to solve the proposed multi-objective problem help to find the most preferred non dominated solutions by considering decision maker's (DM) preferences.

3. Model Development

Since conflicting criteria exist while attempting to configure the best possible combination of purchasing quantities among different suppliers, the basic objective function is then formulated as a multi-objective function. Also we utilize some MADM methods such as AHP to obtain a total weight for each supplier.

3.1 Model Assumptions

The proposed model is constrained by the followings:

- a. One item can be acquired from each supplier.
- b. volume discount is offered by each supplier.
- c. Shortage of each item is not allowed for each supplier.
- d. Demand of each item is determined from all retailers.
- e. Capacity of each supplier is finite.

The following notations are defined in order to describe the model:

- i index of suppliers
- k index of discount intervals
- x_{ik} the number of units purchased from supplier i at discount interval k (integer variable)
- y_{ik} 1, if the i th supplier is selected at discount interval level k ; 0, otherwise

n	the number of suppliers
l_{ik}	lower bound of the discount interval k offered by supplier i
u_{ik}	upper bound of the discount interval k offered by supplier i
p_{ik}	discounted unit price of the discount interval k offered by supplier i
q_i	the average defective rate from supplier i
h_i	the delivery lateness rate from supplier i
w_i	total weight of supplier i calculated by AHP
k_i	index of the last interval offered by supplier i
Q	the buyer's maximum acceptable defect rate
B	budget for the purchasing frame
D	the buyer's demand

3.2 Objective functions

3.2.1 Total monetary cost function

The total monetary cost of purchasing depends only on the unit price of item that is usually offered under different discount schemes; we assume that all-unit discount and total business volume discount schemes can be offered by suppliers. Under all-unit discount model, supplier i discloses discount scheme, which depends on the quantity x_i purchased from supplier i . This discount scheme divides the range of possible order quantities into intervals with progressively lower unit costs which this unit cost is applied to every unit in the order. Similarly to all-unit discount schemes, under total business volume discount scheme, supplier i offers some price intervals. In previous discount schemes, price breaks are a function of the order quantity but in total business volume discount, price discounts depend on the total value of sales volume. This discount scheme has got a different formulation when multiple items are being purchased but in the case of single item purchasing this scheme can be formulated as same as all-unit scheme (Ebrahim et al. 2009). The cost function can be modeled as follows:

$$\min z_1 = \sum_{i=1}^n \sum_{k=1}^{k_i} p_{ik} x_{ik}, \quad (1)$$

where at most one of the variables x_{ik} for $k = 1, \dots, k_i$ can be positive and the rest must be equal to zero, also it is assumed that $p_{i1} > p_{i2} > \dots > p_{ik_i}$.

3.2.2 Defective and/or rejected items function

The buyer expects to minimize the number of defective and/or rejected items for improving product quality and to reduce costs related to quality such as reworks. The objective function is stated as Eq. (2):

$$\min z_2 = \sum_{i=1}^n \sum_{k=1}^{k_i} q_i x_{ik} \quad (2)$$

3.2.3 Late delivered items function

In order to reduce the total lead time of manufacturing, the number of production breakdowns, and the holding cost, buyers desire to minimize the total late delivered items. Thus, the third objective function has been defined as Eq. (3):

$$\min z_3 = \sum_{i=1}^n \sum_{k=1}^{k_i} h_i x_{ik} \quad (3)$$

3.2.4 Total purchasing value function

In order to incorporate qualitative criteria and organization's policy while selecting the most preferred suppliers, the objective function presented in this section applies AHP with pair wise comparisons to the calculation of the weights of the each supplier. In such a manner, one of the objective functions is to calculate the total value of purchasing. The AHP is a well-known, useful and standard method introduced by Saaty (1990) to obtain weights of each alternative in multiple criteria decision making problems. This

method consists of three steps of making hierarchical structure, constructing the matrix of pair wise comparison ratios and calculating the total weights for each alternative (Kokangul & Susuz, 2009). In this method, the most preferred suppliers have the higher weights. After computing the weight of each supplier, these weights are applied as coefficients in the function of total purchasing value. Since w_i denotes the overall score of the i th supplier and x_{ik} denotes the number of purchased units from i th supplier at discount interval k , so maximizing the total purchasing value is desired. This means that suppliers with the higher priorities obtained via AHP are selected due to higher value of total purchasing value. This objective function is defined as follow:

$$\max z_4 = \sum_{i=1}^n \sum_{k=1}^{k_i} w_i x_{ik} \quad (4)$$

3.2.5 Final objective function

A single objective optimization problem terminates an optimal solution. However, due to conflicting objectives developing an optimal solution for MOLP is often difficult. For example, the supplier with high quality may have high price or high lateness. The decision-maker is interested to find the most efficient solutions (non-dominated solutions or non-inferior solutions). The purpose of MOLP approaches is to find the most preferred solution between the most efficient points. To achieve this purpose, a weighting method has been widely used (Amid et al., 2009; Weber & Current, 1993). Weber and Current (1993) showed how MOLP can create a non-inferior set of supplier selection by varying the weights of multiple criteria. However, specifying an appropriate set of weights is often difficult. Furthermore, it is impractical or impossible to generate the entire set of non-dominated solutions considering the fact that their numbers may grow exponentially with the size of the problem. However, buyers usually look for a unique decision to follow. Thus, we need only a procedure that produces an optimal compromise solution of MOLP. In this section, we present a compromised approach based on integrated MADM and linear normalization methods to solve MOLP. In the proposed approach, the multiple-objective functions of MOLP transform to a single-objective problem. Then, using the ordinary technique of linear programming, a set of optimal compromise solutions can be obtained. Hence, suppose that W_1 , W_2 , W_3 and W_4 are the weights of objective functions; in practice the weight of each objective for supplier selection problem is different, and it depends on buyer's purchasing strategy. We use the weighted sum of these objective functions as the main objective to be minimized. Therefore, the solutions obtained using the weighting method help the decision maker to incorporate his purchasing policy and to better understand the tradeoffs between the objectives before selecting a best compromised solution. In addition, since coefficients of decision variables are of different types, so they have to be normalized to have a meaningful summation. We use linear normalization method introduced in (Shih et al., 2007). In such a manner, the multi-objective functions can transform to a single objective linear function with MADM compromise as described as follows:

Linear normalization: $r_{ik} = \frac{x_{ik}}{\max_k \{x_{ik}\}}$ for benefit attributes and $r_{ik} = \frac{\min_k \{x_{ik}\}}{x_{ik}}$ for cost attributes, where i is

the index of suppliers and k is the index for intervals. Since we aim to minimize the total weighted sum of these objectives, we consider the total purchasing value as cost attribute because we aim to maximize this function and the other three objectives are considered as benefit attributes because we aim to minimize these functions. So the final objective function can be stated as follow:

$$\min \sum_{i=1}^n \sum_{k=1}^{k_i} (W_1 p_{ik} + W_2 q_i + W_3 h_i + W_4 w_i) x_{ik} \quad (5)$$

After normalizing the coefficients we have:

$$\min \sum_{i=1}^n \sum_{k=1}^{k_i} \alpha_{ik} x_{ik}, \quad (6)$$

$$\text{where } \alpha_{ik} = W_1 \frac{P_{ik}}{\max_{r,s} \{p_{rs}\}} + W_2 \frac{q_{ik}}{\max_r \{q_r\}} + W_3 \frac{h_i}{\max_r \{h_r\}} + W_4 \frac{\min_r \{w_r\}}{w_i} \quad (7)$$

3.3 Constraints

There are some constraints associated with the supplier selection problem. In the following these constraints are explained and modeled.

3.3.1 Demand constraint

Sum of the assigned order quantities from suppliers should meet the buyer’s demand. As q_i is the defective rate of the i th supplier, the total purchased quantity must be much more than buyer’s demand because its order quantity, x_i , have some defective items. This can be stated as follows:

$$\sum_{i=1}^n \sum_{k=1}^{k_i} (1 - q_i) \times x_{ik} = D \quad (8)$$

3.3.2 Discount intervals constraints

If an interval per supplier is not selected, the binary variable corresponding to that interval will be zero and the x_{ik} will be zero as well according to Eq. (9) and Eq. (10):

$$l_{ik} y_{ik} \leq x_{ik} \leq u_{ik} y_{ik} \quad \forall i = 1, \dots, n \quad \forall k = 1, \dots, k_i \quad (9)$$

$$\sum_{k=1}^{k_i} y_{ik} \leq 1 \quad \forall i = 1, \dots, n \quad (10)$$

3.3.3 Capacity constraints

As supplier i can provide up to C_i units and its order quantity x_i should be equal or less than its capacity. That is:

$$\sum_{k=1}^{k_i} x_{ik} \leq C_i \quad \forall i = 1, \dots, n$$

However, for $k = k_i$ we have $x_{ik} \leq u_{ik_i} = C_i$, that it is considered in Eq. (9) and adding the above constraint to the model is not bounding, but is redundant.

3.3.4 Quality control constraint

Since Q is the buyer’s maximum acceptable defect rate and q_i is the defect rate of the i th supplier, then we have:

$$\sum_{i=1}^n \sum_{k=1}^{k_i} q_i x_i \leq QD \quad (11)$$

3.3.5 Purchasing Budget constraint

Consider B as total allocated budget just for purchasing, the total purchasing payment cannot exceed the buyer’s budget. This constraint can be stated as follow:

$$\sum_{i=1}^n \sum_{k=1}^{k_i} x_{ik} P_{ik} \leq B \quad (12)$$

3.4 The final programming mathematical model

The final mathematical model can be shown as follows:

$$\begin{aligned} & \min \sum_{i=1}^n \sum_{k=1}^{k_i} \alpha_{ik} x_{ik} \\ & \text{subject to} \\ & \sum_{i=1}^n \sum_{k=1}^{k_i} (1 - q_i) \times x_{ik} = D \\ & \sum_{i=1}^n \sum_{k=1}^{k_i} q_i x_{ik} \leq Q D \\ & \sum_{i=1}^n \sum_{k=1}^{k_i} x_{ik} p_{ik} \leq B \\ & l_{ik} y_{ik} \leq x_{ik} \leq u_{ik} y_{ik} \quad \forall i = 1, \dots, n \quad \forall k = 1, \dots, k_i \\ & \sum_{k=1}^{k_i} y_{ik} \leq 1 \quad \forall i = 1, \dots, n \\ & x_{ik} \in z^+ \cup \{0\} \quad \forall i = 1, \dots, n \quad \forall k = 1, \dots, k_i \\ & y_{ik} \in \{0, 1\} \quad \forall i = 1, \dots, n \quad \forall k = 1, \dots, k_i \end{aligned}$$

4. Define the selection criteria for global suppliers

The main criteria and attributes system for global supplier selection have been decided based on studies in the respective fields (Min, 1994; Katsikeas et al., 2004; Hong et al., 2005). The relevant decision criteria like cost, quality, service performance, and supplier’s profile including the risk factors is involved in global supplier selection to provide an efficient approach to handle it effectively. The risk factors address some of the important and interesting issues like terrorism, government stability, economic status, and geographical location of supplier to match the supplier selection process with the current political and business scenario for greater efficiency. The criteria are denoted by C_i , attributes by A_j , and alternatives by S_k (where $i, j, k = 1, 2, \dots$); these criteria and attributes are shown in Table 1.

Table 1
Proposed criteria for global supplier selection decision making

Criteria	Sub-Criteria
Overall cost of the product (C1)	Product price (A1)
	Freight cost (A2)
	Tariff and custom duties (A3)
Quality of the product (C2)	Rejection rate of the product (A4)
	Increased lead time (A5)
	Quality assessment (A6)
	Remedy for quality problems (A7)
Service performance (C3)	Delivery schedule (A8)
	Technological and R&D support (A9)
	Response to changes (A10)
Supplier’s profile (C4)	Ease of communication (A11)
	Financial status (A12)
	Customer base (A13)
	Performance history (A14)
Risk factor (C5)	Production facility and capacity (A15)
	Geographical location (A16)
	Political stability (A17)
	Economy (A18)
	Terrorism (A19)

5. A numerical example

Suppose a supply chain consists of a single buyer and six different suppliers. The purchase manager must determine the order quantity from each supplier to satisfy an annual demand 600 units of a product. The maximum annual purchasing budget is 198000\$; also the maximum acceptable defect rate for company is 0.01. Six potential suppliers that produce the part are candidate and Table 2 shows discount intervals offered by these suppliers; and the rates of defective and late delivered items are presented in Table 3. The purchase manager wants to take into account all the possible important criteria which can affect the supply of this critical part. The main steps are explained by the following order.

Table 2

Discount intervals of example

No	Intervals	Unit price (\$)
1	1-100	400
	100-200	300
	200-300	200
2	1-150	500
	150-300	400
	300-450	300
3	1-200	450
	200-400	350
4	1-100	380
	100-250	300
	250-400	250
5	1-80	550
	80-160	450
	160-300	300
6	1-110	520
	110-220	480
	220-400	380
	400-600	310

Table 2

Data related to each supplier 6460+76650+111000

No of suppliers	Rate of defective and/or rejected items (q_i)	Rate of late delivered items (h_i)
1	0.02	0.01
2	0.008	0.007
3	0.01	0.02
4	0.05	0.03
5	0.015	0.009
6	0.03	0.04

5.1 Computation of the priorities of the criteria and the overall score of each supplier

The comparison of the importance or preference of one criterion, attribute or alternative over another can be done with the help of the questionnaire. The matrixes of pair wise comparison ratios can be constructed using scores "1" to "9" in which score "1" is used in order to show equal importance of two criteria and score "9" is applied to show the absolute superiority of one criterion to another. The consistency of the pair wise judgment of each comparison matrix is also checked during the process. First the evaluation matrix of the criteria is constructed by the pair wise comparison of the different criterion relevant to the overall objective, which is shown in Table 4. Then the different attributes are compared under each of the criterion separately by following the same procedure as discussed above. Similarly the evaluation matrixes of decision alternatives and corresponding weight vector of each alternative with respect to

corresponding attributes are determined. Finally the priority weights of each supplier can be calculated by weights per supplier multiplied by weights of the corresponding criterion. The results are shown in Table 5.

Table 4
Evaluation of criteria with respect to the overall objective

O	C1	C2	C3	C4	C5	W0
C1	1	2	2	3	3	0.359
C2	1/2	1	2	3	3	0.271
C3	1/2	1/2	1	2	2	0.172
C4	1/3	1/3	1/2	1	2	0.113
C5	1/3	1/3	1/2	1/2	1	0.085

Table 5
Summary combination of priority weight

C_i	$C1$	$C2$	$C3$	$C4$	$C5$	priority weight
S_i	(0.359)	(0.271)	(0.172)	(0.133)	(0.085)	
S1	0.298	0.231	0.259	0.204	0.204	0.253
S2	0.168	0.148	0.170	0.150	0.160	0.160
S3	0.170	0.210	0.283	0.275	0.193	0.214
S4	0.151	0.221	0.122	0.163	0.076	0.160
S5	0.085	0.083	0.104	0.109	0.169	0.098
S6	0.128	0.107	0.062	0.100	0.198	0.114

According to final priority weights, supplier 1 is the most preferred supplier and supplier 2 is the second best supplier and supplier 5 has the least priority according to defined criteria. The performance of each supplier with respect to each of the main criterion is given in Fig. 1, and the final priority weight of each supplier is given in Fig> 2.

Synthesis with respect to: Goal: Global supplier selection, Overall Inconsistency = 0.03

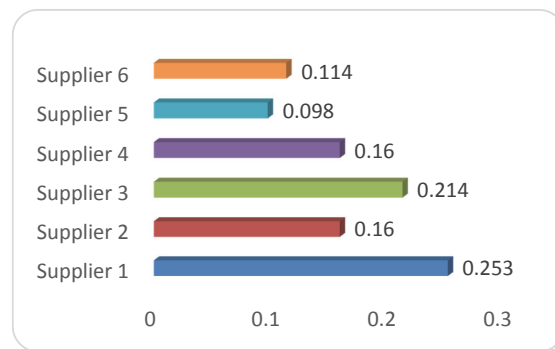
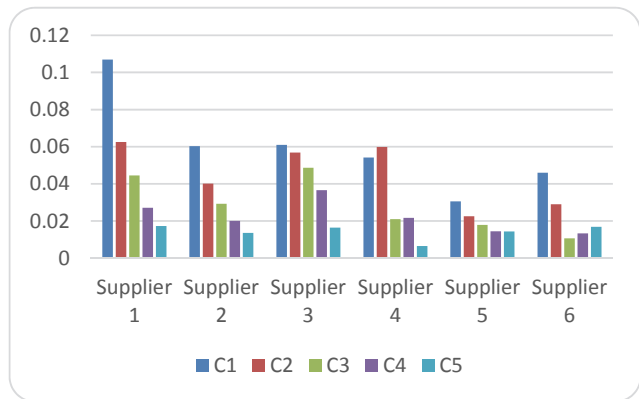


Fig. 1. Supplier performance with respect to the main criteria **Fig. 2.** Final priority weight of each supplier

Furthermore, in order to do more investigation on the effects of main criteria weights on the final supplier priority weights, we consider situation in which the weight of risk factor (C5), given in Table 4, is changed from 0.085 to 0.15 and the other criteria weights are changed accordingly. Regarding new weights of the criteria, the priority weights of suppliers 1-6 are changed by 0.25, 0.213, 0.163, 0.154, 0.12 and 0.103, respectively.

5.2 Molp Model and Solution Process

As it mentioned before, W_1, W_2, W_3 and W_4 are the weights of objective functions and it depends on the company purchasing strategies. Here we assume W_1, W_2, W_3 and W_4 are equal to 1. Now we have all required data to formulate the problem. Using Eq. (7), α_{ik} can be computed $\forall i = 1, \dots, n. \forall k = 1, \dots, k_i$. Fig. 3 shows the result of these computations. Therefore, the mathematical model of this example will be as follows:

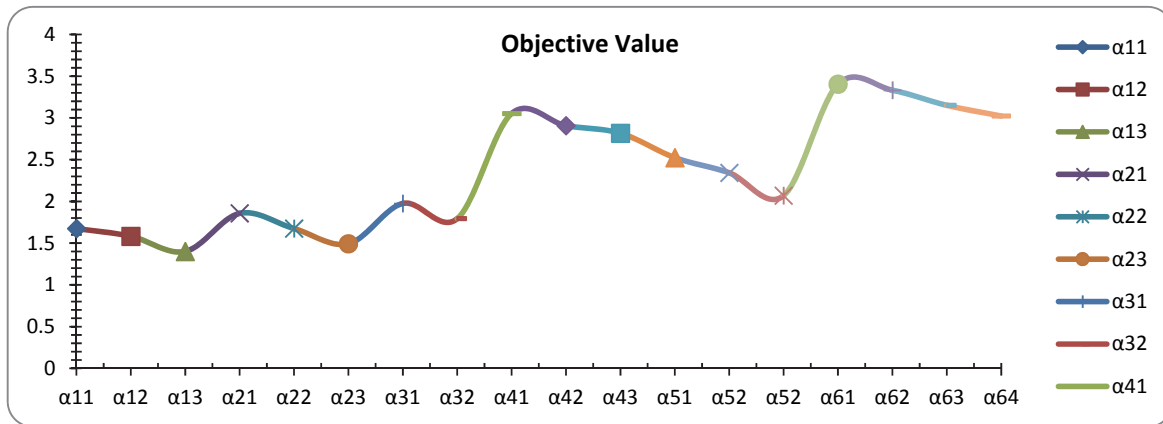


Fig. 3. Objective coefficients

Using the above objective coefficients, discount intervals and the other parameters given in Table 2 and Table 3, the mathematical formulation for this example can be written. The model can be solved using branch and bound (B&B) method via LINGO package version 8.0. The corresponding solutions are as follows: $x_{2,3} = 370$ and $x_{3,2} = 219$ and $x_{4,1} = 17$, the remaining variables are equal to zero. This means that the best purchasing strategy is to buy 370 units of total demand from supplier 2 in discount interval 3 and 219 units from supplier 3 in discount interval 2, the remaining quantity of 17 units from supplier 4 in discount interval 1. The corresponding value of objective function under this optimal policy will be 997.1970, and also total purchasing cost will be 194,110\$. Regarding to the solution, we can see the supplier 2, 3 and 4 are selected while the basic unit price of suppliers 1 is lower than the 2th and 3th supplier's; in addition the percentages of discounts for same intervals offered by supplier 1 are generally greater. Therefore, considering only the cost of purchasing, supplier 1 seems to be more preferable. The prominent answer to the question why suppliers 2, 3 and 4 are preferred is the impact of other criteria.

6. Model validation and sensitivity analysis

In the following subsections we provide a simulation study to validate the proposed model by varying the values of the weights of the objective function, eliminating the effect of the non-monetary objective functions, increasing the number of suppliers and the number of discount intervals and eliminating the budget constraint. In each subsection, we resolved the new problems and attempted to describe differences between the results with those of the primary problem, given in section 5. Furthermore, in the last subsection, we compared the proposed model with a well-known study in the literature and indicated the great efficiency of our proposed model.

6.1 Problem with different objective weights

Tradeoffs between the objectives are relatively difficult to understand when more than two objectives are used in the model. Varying the weights of the objectives can generate several efficient solutions systematically. So, it will be valuable the studying the effects of the weights (w_1, w_2, w_3 and w_4) of the objectives on the optimal decision. So, in order to do more investigation on the effects of objective priority

weights on the final output, different weights of objective functions are considered in this section. Also, this investigation helps the decision maker to better understand the tradeoffs between the objectives before selecting a best compromised solution. To this order, first we assume the previous example given in section 5, in which the objective weights of cost, defective items, late delivered items and total purchasing value are changed according to: $W_1=30\%$, $W_2=5\%$, $W_3=15\%$ and $W_4=50\%$ respectively. Considering these priority weights and after normalization, the objective coefficients (α_{ik}) are changed according to Fig. 4.

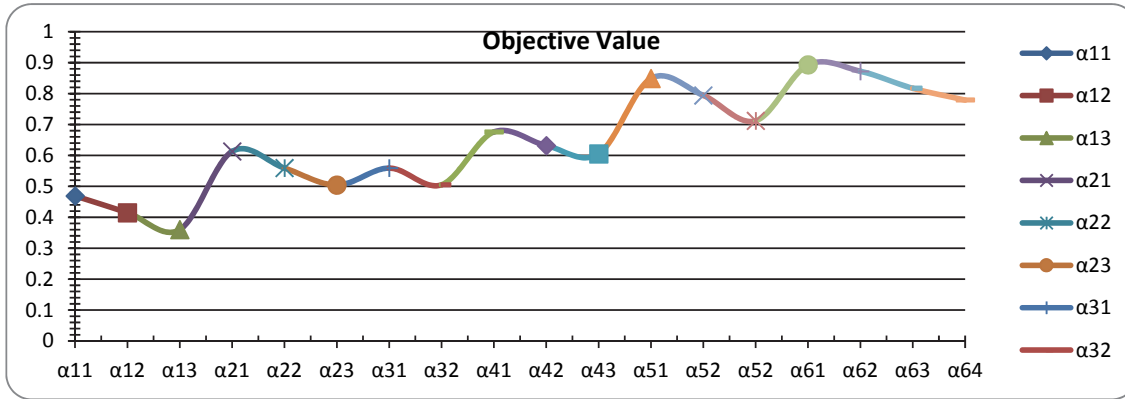


Fig. 4. Objective coefficients for the problem with different objective weights

The corresponding solutions are obtained as follows: $x_{1,3} = 291, x_{3,2} = 318$ and the remaining variables are equal to zero. This means that the best purchasing strategy is to buy 291 units of total demand from supplier 1 in discount interval 3 and 318 units from supplier 3 in discount interval 2. A review of the result reveals some observations. Selected suppliers have the highest priority weight obtained via AHP method and this reveals the high weight of the total weighted quantity of purchasing function. On the other hand as the second important objective is the total monetary cost function, so the order is allocated in the intervals with the most discounts. Furthermore, even though supplier 2 has the least rates of defective items and late delivered items, but it is not selected due to the least importance of the related objectives. Hence, the results are expected and also the model has enough flexibility to obtain optimum solution and incorporate different organization's purchasing policies easily and in a short time.

6.2 Problem in which cost is the only important factor

In order to determine the effect of the cost and discount intervals and also examine the effect of non-monetary objective functions on the final decision on the final decision, we discuss the situation where the company is just considering the cost as the main objective of decision making problem. Hence, given the same data in the first problem, we consider a situation in which “cost” is the only important factor for the buyer. Therefore the weight of this criterion (W_1) will turn to 100% and the other three objectives will be eliminated from the model. For more investigation on the effect of cost and discount intervals on the final decision, budget and quality constraints are eliminated from the model and also the rate of defective or rejected items for each supplier is assumed zero. In this way we can examine the effect of cost and discount models on the optimal decision. In this situation the coefficients of objective function will be unit costs presented in Table 2 and they can be used directly in objective function without any normalization. LINGO software is then again employed to find the compromise solution, which is $x_{1,3} = 300$ and $x_{4,3} = 300$, and the total purchasing cost will be 135,000\$. This means that the best purchasing strategy is to buy 300 units of total demand from supplier 1 in discount interval 3 and 300 units from supplier 4 in discount interval 3. A review of the result reveals some observations. As it was expected, supplier 1 and 4 are selected. That is because of percentage of discounts considered for these intervals are obviously

lower than others, and the total purchasing cost will be less than previous (194,110\$). Furthermore, since it is assumed that each supplier has no rejected items, so the total purchasing item is 600 that is equal to total demand. While in the previous problems, considering the rate of rejected items for each supplier, more than total demand must be purchased by organization.

6.3 Problem with more suppliers

Here we assume conditions with 20 suppliers that 12 suppliers of them offer discount and the others don't offer any discount. The maximum acceptable rate is 1%. Data of the problem are given in the Table 6.

Table 6
Discount intervals for the problem with 20 suppliers

No	Intervals	Unit price (\$)
1	1-100	400
	100-200	350
	200-300	300
2	1-150	500
	150-300	400
3	1-200	450
	200-300	350
	300-400	300
4	1-250	380
5	1-180	480
	180-260	420
6	1-350	480
7	1-400	350
8	1-200	400
9	200-400	300
10	1-250	400
11	1-300	380
11	1-200	420
	200-350	350
12	1-180	450
	180-350	350
13	1-450	390
14	1-230	400
	230-400	320
15	1-350	370
16	1-150	450
	150-300	360
17	1-80	410
	80-250	350
18	1-280	340
19	1-100	460
	100-340	400
20	1-320	430
	320-400	390
	400-450	310

Table 7
Data related to each supplier

No of suppliers	Rate of defective and/or rejected items (a_i)	Rate of late delivered items (h_i)	W_{AHP}
1	0.02	0.01	0.023
2	0.008	0.007	0.034
3	0.01	0.02	0.045
4	0.05	0.03	0.028
5	0.015	0.009	0.019
6	0.03	0.04	0.036
7	0.04	0.01	0.017
8	0.01	0.008	0.027
9	0.02	0.02	0.014
10	0.009	0.01	0.023
11	0.02	0.07	0.017
12	0.03	0.04	0.039
13	0.009	0.03	0.029
14	0.007	0.009	0.028
15	0.04	0.06	0.032
16	0.02	0.07	0.016
17	0.05	0.01	0.036
18	0.01	0.04	0.041
19	0.03	0.008	0.42
20	0.009	0.009	0.02

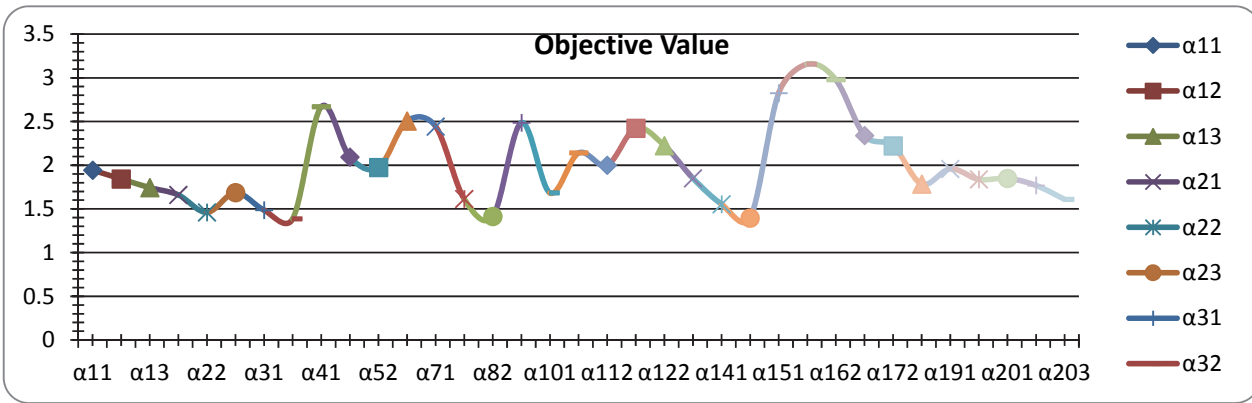


Fig. 5. Objective coefficients for problem with more suppliers

The corresponding solutions are as follows: $x_{3,3} = 386, x_{8,2} = 200, x_{14,2} = 20$ and the remaining variables are equal to zero. This means that the best purchasing strategy is to buy 386 units from supplier 3 in discount interval 3 and 200 units from supplier 8 in discount interval 2 and the remaining quantity of 20 units from supplier 14 in discount interval 2. The corresponding objective function value under this optimal policy will be 830.2360. So, the proposed model can obtain optimum order of the problem with more suppliers easily and in a short time (near to zero).

6.4 Problem with no budget constraint

The budget constraint is one of the most important factors that affects the final decision. To prove this we consider the situation in which the company doesn't have any budget constraint. So considering the same data in the first problem, we eliminate budget constraint and resolve the problem. The corresponding solutions are as follows: $x_{1,1} = 84, x_{2,3} = 450, x_{3,1} = 72$, and the remaining variables are equal to zero. This means that the best purchasing strategy is to buy 84 units from supplier 1 in discount interval 1 and 450 units from supplier 2 in discount interval 3 and the remaining quantity of 72 units from supplier 3 in discount interval 1. A review of the result reveals that the selected suppliers have the highest priority weight obtained via AHP and the least defective items and lateness rate. The total purchasing cost under this optimal policy will be 201000\$ while in the first problem considering budget constraint it was 194110\$. So, for the problems without budget constraint the best suppliers are selected even though the total purchasing cost is increased. Since in real cases organizations usually have some budget restrictions and purchasing products from the best suppliers with the best qualities and the least defective and late delivered items is not possible for them, so adding this constraint is necessary to obtain a practical optimum decision. In this situation, decision makers must define higher weights for those objectives which have the most priorities for them, to obtain the best purchasing policy considering buyer limitations.

6.5 Comparison with previous study in the literature

Burke et al. (2008) presented a non-linear integer formulation for the case of single product and multiple suppliers. The authors assumed that the suppliers have capacity limitations and analyzed the impact of supplier pricing schemes on the optimal policy of the single-buyer. Burke et al. (2008) showed that purchasing problems under quantity discount policies are NP-hard and heuristics are valuable to find near-optimal solutions for these problems. The authors used following model as the sourcing model for the case of single product and multiple suppliers under all units quantity discount:

$$\min Z_{AU} = \sum_{i=1}^n \sum_{k=1}^{k_i} v_{ik} q_{ik} y_{ik}$$

$$\text{subject to } \sum_{i=1}^n \sum_{k=1}^{k_i} q_{ik} = Q, \quad \sum_{k=1}^{k_i} y_{ik} \leq 1 \forall i, \quad l_{ik} y_{ik} \leq q_{ik} \leq u_{ik} y_{ik} \forall i, k, \quad y_{ik} \in [0, 1]$$

In which, Q is aggregate requirement and q_{ik} is the quantity purchased from supplier i in discount interval k . Here, for more investigation about accuracy of the proposed model in this paper, we use the same data in the first problem given in section 5, and resolve it using the above model presented in (Burke et al. 2008) for the case of all-unit discount. since the model is a single objective model and budget or quality constraints and also defective rate of suppliers are not considered in the model, so it is similar to the problem mentioned in section 6.2 in which $W1=100\%$ and suppliers have no defective items and also budget and quality constraints are eliminated from the model. This model is solved using branch and bound (B&B) method via LINGO package. The corresponding solution is $x_{1,3} = 300$ and $x_{4,3} = 300$, and the total purchasing cost is 135,000\$ which is exactly the same with the solution obtained in section 6.2. However computation time using Burke's model is 7s while using proposed model in this paper is zero. Furthermore, in this problem only 6 suppliers are considered but for the next problem with 20 suppliers given in section 6, finding optimal solution in a reasonable time via burke' model is impossible but computational time for the problem mentioned in section 6.3, is approximately zero. Although, our problem has some other features such as multi-objectives, defective rate, quality and budget constraints. So, nonlinearity of the objective function in Borke's model leads to a more complex problem and as the authors showed heuristics are needed to find near-optimal solutions for these problems.

7. Conclusion

Supplier selection in a quantity discount environment is usually treated as an MOLP model where the objectives are not of equal importance. As different strategies have different policies of supplier partnership, a flexible method which can reflect the corporate strategy in the supplier selection is required to consider the suppliers' and buyer's limitations. Thus, AHP was employed in this study to facilitate such decision-making; also the integrated linear normalization and weighting method has been used to develop the mathematical model to consider the importance of objectives and also to obtain a unique optimal decision for the problem. The MOLP model could determine the optimal order to be allocated to multiple suppliers under consideration of additional constraints in different aspects incorporating different purchasing policies; they can deal with both quantitative and qualitative criteria such as: costs, quality, service performance, delivery ate, cooperation, and risk factors in supplier selection problems. Obviously, organizations can include or exclude some of these criteria according to their policies. Some illustrative examples are solved via Lingo and the results are compared. Sensitivity analysis and comparing the results with one of the well-known studies in the literature have demonstrated the flexibility and efficiency of the proposed model to deal with large sized problems incorporating various purchasing policies easily and in a short time. However, some information as delivery lateness rate, defective rate, demand, etc. were assumed to be known in advance which is unrealistic and limits the application of the proposed model. Hence, this model must be adapted to consider the uncertainty associated with the problem of supplier selection and to deal with different data set and decision making as a further research. Moreover, converting the model into the multi-product and multi-period structure is a valuable research subject.

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