Analysis and modeling of rail maintenance costs

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\textbf{ABSTRACT}

Railroad maintenance engineering plays an important role on availability of roads and reducing the cost of railroad incidents. Rail is of the most important parts of railroad industry, which needs regular maintenance since it covers a significant part of total maintenance cost. Any attempt on optimizing total cost of maintenance could substantially reduce the cost of railroad system and it can reduce total cost of the industry. The paper presents a new method to estimate the cost of rail failure using different cost components such as cost of inspection and cost of risk associated with possible accidents. The proposed model of this paper is used for a real-world case study of railroad transportation of Tehran region and the results have been analyzed.

\section{1. Introduction}

Railroad plays an important role on economy of any country especially for developing countries. A serious disruption on rail could create serious accidents such as derailment. Therefore, we need to monitor rail conditions by inspecting different components, lubricating necessary parts or grinding specific portions of rail, carefully. Obviously, there are some costs associated with each part of these components and we need to have a fair balance among them to optimize the cost of maintenance. Inspection cost is a major portion of maintenance, for instance, in European commission; inspection cost is estimated to be between 375 to 850 million dollars per year (Cannon et al., 2003). During the past few decades, there have been many attempts to reduce the cost of maintenance. Hargrove (1985) proposed a model for track degradation based on the failure rate of each component, separately. Ebersohn (1997) introduced a comprehensive method as total productive maintenance by suggesting traffic information, cost, assets and historical data of rail failure. Esveld (2001) investigated various factors influencing rail failure. Cannon et al. (2003) considered hidden effects of using ultrasonic sound waves for detecting rail failure. They showed there are many subsurface of detects on rail, which cannot be detected through regular use of ultrasonic sound waves but they can easily create...

In this paper, we present a new mathematical model to optimize the cost of rail maintenance. The organization of this paper first explains details of the proposed model in section 2 and the results of the implementation of our proposed model are given in section 3. Finally, concluding remarks are given in section 4 to summarize the contribution of this paper.

2. The proposed model

In this section, we first explain different methods to determine the rate of failure. There are different factors rate of failure either directly or indirectly such as million gross tones (MGT), axle load, speed, traffic type and density, track curvature and elevation, etc. As the age of rail increases, the rate of failure on rail will also increase proportion to load passes the rail. The rate of failure can be expressed as a Weibull distribution with $\Lambda(m)$ where $m$ represent MGT. Based on this function, there is a higher chance of having crack on a rail when the rail is getting older and MGT increases. In the event we do not see any crack, we will have a higher chance of broken rail. Let $f(m)$ and $F(m)$ be density function and cumulative rail failure distribution, respectively. Therefore, we have,

$$F(m) = 1 - \exp(-\lambda m^\beta),$$

$$f(m) = \frac{dF(m)}{dm} = \lambda \beta m^{\beta-1} \exp(-\lambda m^\beta),$$

where $\beta$ is the shape parameter distribution and $\lambda$ is the inverse of characteristic function for the Weibull distribution with $\lambda > 0$ and $\beta > 1$. Using Eq. (1) and Eq. (2), $\Lambda(m)$ can be defined as follows,

$$\Lambda(m) = \frac{f(m)}{1-F(m)} = \frac{\lambda \beta m^{\beta-1} \exp(-\lambda m^\beta)}{1-(1-\exp(-\lambda m^\beta))} = \lambda \beta m^{\beta-1}. \quad (3)$$

When there is a failure in some part of a rail, we may either replace or repair the part. Since the replaced part is only a small part of total rail, the replaced part can be ignored in Weibull function. The proposed model of this paper considers three cost components of inspection, preventive and corrective maintenance and the cycle of maintenance is terminated when the usage of rail reaches to $L$ in terms of MGT. The proposed model of this paper considers preventive maintenance and it looks for any possible failure. Fig 1 shows the interval of preventive maintenance, which increases the reliability and reduces the rate of failure. We may have some corrective maintenance between each preventive maintenance.

![Fig. 1. Proposed maintenance model](image)
In Fig. 1 $\tau$ represents age restoration after each preventive maintenance, which can defined as follows,

$$\tau = \alpha \times x,$$  \hspace{1cm} (4)

where $\alpha$ is a quality of preventive maintenance, which is between zero and one. A value of zero means preventive maintenance has no impact on the rail and it is as bad as old. A value of one represents a perfect maintenance operation, which brings the rail to a good condition or as good as new. The following assumptions are made to simplifies the proposed model of this paper,

- The rate of failure is a non-decreasing function of time,
- Corrective maintenance covers rail replacement and minimal repairs,
- Preventive maintenances are executed in constant time cycles in every $x$ MGT,
- Any preventive maintenance improves overall conditions of rail,
- Any increase in level of restoration depends on the quality of maintenance,
- All cost components during the preventive maintenance is constant and it is assumed to be $L$.

The proposed model of this paper consider the total cost of maintenance ($C_T$) is as follows,

$$C_T = C_m + C_i + C_r,$$  \hspace{1cm} (5)

where $C_i$ and $C_r$ are the costs of inspection and risk associated with unexpected accidents, respectively and $C_m$ is the cost of maintenance, which is defined as follows,

$$C_m = C_{pm} + C_{cm},$$  \hspace{1cm} (6)

where $C_{pm}$ is the cost of preventive maintenance, which consists of grinding and lubrication. In addition, $C_{cm}$ is associated with corrective cost items such as welding process, grinding, etc. The failure intensity considering preventive maintenance is defined as follows,

$$\Lambda_{pm}(m) = \Lambda(m - k\tau),$$  \hspace{1cm} (7)

where $k$ is the number of maintenance operations until time $m$. Therefore the rate of failure ($FR$) is as follows,

$$FR = \int_{kx}^{(k+1)x} \Lambda(m - k\tau)dm,$$  \hspace{1cm} (8)

In order to calculate total number of failure we have,

$$FR_T = \sum_{k=0}^{N+1} \int_{kx}^{(k+1)x} \Lambda(m - k\tau)dm,$$  \hspace{1cm} (9)

where $N$ is the number of preventive maintenance. Let $C_{1cm}$ be the cost of corrective maintenance per meter and $Le$ be the length of rail. Total cost of corrective maintenance is calculated as follows,

$$C_{cm} = C_{1cm} \times Le \times \sum_{k=0}^{N+1} \int_{kx}^{(k+1)x} \Lambda(m - k\tau)dm.$$  \hspace{1cm} (10)

The total cost of preventive maintenance is calculated as follows,
where $C_{pm}$ is the cost of preventive maintenance per meter. So, total inspection cost is defined as follows,

$$C_i = N_i \times L \times C_{1i},$$

(12)

where $C_{1i}$ is the cost of inspection per meter and $N_i$ is the number of preventive maintenance, which is forecasted and it is calculated as follows,

$$N_i = \lfloor \frac{L}{I_f} \rfloor,$$

(13)

where $I_f$ is the optimal inspection interval. The cost associated with the risk of unexpected incidents is as follows,

$$C_r = E[N(L)] \times [P_n(B) \times b + (1 - P_n(B)) \times (P_n(A) \times a)],$$

(14)

where $a$ is the cost of each unexpected accident, $b$ is the cost of repairing potential failure using ultrasonic sound waves, $P_n(A)$ is the probability of potential failure leading to accidents, $P_n(B)$ probability of detecting potential failures using ultrasonic sound waves and $E[N(L)]$ is the expected number of failures. Therefore, the total cost is calculated as follows,

$$C_T = \left\{ C_{1cm} \sum_{k=0}^{N+1} \int_{k-1}^{k+1} L(m - k\tau) dm \right\} \times N \cdot C_{pm} \times L \times \left( \frac{L}{I_f} \right) \times C_{1i} \times L + \left( E[N(L)] \times [P_n(B) \times b + (1 - P_n(B)) \times (P_n(A) \times a)] \right),$$

(15)

3. The results

In this section, we present the details of the implementation of our proposed model on railroad transportation of Iran. We gathered statistical information of railroad activities between 2006 and 2007. MATLAB toolbox optimization is used to generate maximum Likelihood (MLE) estimator to measure the values of $\lambda$ and $\beta$. Fig. 2. Shows details of the operational MGT generated by MATLAB.
Table 1 shows details of the costs and MGT in two years of 2006 and 2007.

**Table 1**

<table>
<thead>
<tr>
<th>Year</th>
<th>Track length (KM)</th>
<th>MGT</th>
<th>Maintenance cost (Rials × 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>901</td>
<td>42.8</td>
<td>11736.75</td>
</tr>
<tr>
<td>2007</td>
<td>930</td>
<td>43.3</td>
<td>12562.50</td>
</tr>
</tbody>
</table>

The cost includes the necessary surface welding, replacement of rail, inspection of rail, lubrication of rail and other overhead expenditures. Fig. 3 shows details of the results of our proposed model compared with what real-world case study.

The reason for having errors in our computation could be because of the existence of the cost of risk associated with the proposed model. Note that there was no incident during the years of our investigation. The difference between the errors in two consecutive years can also be described for the nonlinear behavior of the proposed model.

**4. Conclusion**

In this paper, we have presented an improved mathematical model for managing the cost of rail maintenance. The proposed model of this paper considers a Weibull distribution for failure intensity function and different expenditure of inspection, corrective, etc. The proposed model has been implemented for a real-world case study of railroad industry of Iran using the data of the years of 2006 and 2007. The preliminary results indicate that the proposed model of this paper can predict the cost of maintenance with an acceptable error. The proposed model of this paper can be extended with some additional real-world assumptions such as considering the cost of interruption of a traffic for the failure and we leave it as a future work for interested researchers.

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