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Two models for the generalized assignment problem in uncertain environment

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A R T I C L E I N F O

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A B S T R A C T

The generalized assignment problem (GAP) is a unique extended form of the Knapsack problem, which is tremendously practical in optimization fields. For instance, resource allocation, sequencing, supply chain management, etc. This paper tackles the GAP in uncertain environment in which the assignment costs and capacity of agents are fuzzy numbers. Two models are presented for this problem and a novel hybrid algorithm is offered using simulated annealing (SA) method and max-min fuzzy in order to obtain near optimal solution. Computational experiments validate the efficiency of proposed method.

1. Introduction

The generalized assignment problem (GAP) is one of the most outstanding problems with wide applications in miscellaneous real-world applications such as facility location, job scheduling, supply chain management, vehicle routing, loading for flexible manufacturing systems and so forth. GAP is concerned with optimal assigning n jobs to m agents so that each job is assigned to exactly one agent, while the total resource capacity of each agent is not exceeded. Fisher et al. (1986) proved that this problem is NP-hard, so widely varied solution methods such as branch and bound algorithm, heuristics and metaheuristic methods have been proposed to achieve the near optimal solution for this class of problems.

There are many successful exact algorithms for GAP such as Branch and Bound (B&B) proposed by Nauss (2003), Max-Min ant system combined with local search and tabu search presented by Lourenc and Serra (2002) and tabu search (TS) utilized by Diaz and Fernandez (2001). Chu and Beasley (1997) offered a genetic algorithm (GA) insured the feasibility and optimality, simultaneously. Laguna et al. (1995) presented TS method, based on ejection chain approach. Ozbakir et al. (2010) modified a new bee algorithm for this problem and Woodcock and Wilson (2010) solved the problem

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optimally by a new hybrid containing TS and B&B approach. Sharkey and Romeijn (2010) presented a new model for GAP by considering a nonlinear cost function for each facility. Many other researchers have proposed various methods, including exact and approximate ones for solving the GAP and its variants like Ejection chain approach which is embedded in a neighborhood construction by Yagiura and Ibaraki (2004, 2005); and finally, a Lagrangian heuristic algorithm by Haddadi and Ouzia (2004, 2001). For more researches see Avella et al. (2010), Cohen et al. (2006), Mitrovic-Minic and punnen (2009), Rainwater et al. (2009) and Wen Zhan and Liong Ong (2007).

In real industries, it is impossible to consider constant values for some parameters in some specific conditions. For example, the assignment cost or agent capacity might be changed during the assigning process. On the other hand, it might be no historical data in order to estimate value of those parameters. In this case, fuzzy logic could be used as a suitable approach for modeling the problem. As mentioned above, in the literature of GAP, all of the surveys present the crisp models in which all the parameters are considered deterministic. In this paper, we assume that the assignment costs as well as the capacity of agents are imprecise values and considered as triangular fuzzy numbers and a new fuzzy model is generated for the GAP. In order to solve the fuzzy models, first, some approaches are accomplished to convert the parameters to equivalent crisp values and then a new hybrid is offered using simulated annealing and max-min fuzzy approach to achieve near optimal solutions.

The remaining parts of this paper are as follows. In section 2 the mathematical model is presented and parameters are labeled. In section 3, two approaches are utilized to convert the fuzzy parameters to crisp values. In addition, a hybrid algorithm is presented to solve the model. Finally, in order to evaluate efficiency of the proposed method, computational results are presented in section 4.

2. Problem formulation

In this paper the generalized assignment problem is studied with the condition that the assignment costs as well as the capacity of agents are not constant values and they are changed during the time and no historical data to estimate those imprecise parameters are available. The fuzzy logic can be used as a suitable approach to model this problem, where the parameters are considered as a triangular fuzzy numbers. Therefore, assignment costs and agent capacities in GAP problem are considered as fuzzy numbers and they are denoted by (c^p, c^m, c^o) and (b^l, b^m, b^u), respectively.

The following notations are used in formulation of the fuzzy GAP model.

Sets:

n Fixed number of agents; $i = \{1,2,...,n\}$

m Fixed number of tasks; $j = \{1,2,..., m\}$

Parameters:

$\widetilde{c}_{ij}$ fuzzy cost of task j being assigned to agent i;

r_{ij} Required resource of assigning task j to agent i;

$\widetilde{b}_i$ fuzzy resource units available to agent i;

and let define a binary decision variable as:

$$x_{ij} \begin{cases} 1 & \text{if task } j \text{ is assigned to agent } i \\ 0 & \text{else} \end{cases}$$

We aim to assign n jobs to m agents so that the total assignment cost is minimized. For this purpose this fuzzy mathematical model is presented:

$$\min \sum_{j=1}^{n} \sum_{i=1}^{m} \left(c_{ij}^{p} x_{ij}, c_{ij}^{m} x_{ij}, c_{ij}^{o} x_{ij} \right) \tag{1}$$

subject to

$$\sum_{j=1}^{m} x_{ij} r_{ij} \le (b_i^l, b_i^m, b_i^u) \quad \forall i \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \tag{3}$$

$$x_{ij} = \{0,1\} \tag{4}$$

The objective function (1) calculates overall assignment cost, which is aimed to be minimized. The constraint (2) enforces the agent resource limitations and the constraint (3) ensures that each job is assigned just to one agent and constraint (4) depicts that x_{ij} is binary variable

3. Solution approach

3.1 defazzfication of fuzzy model

In this section, it is tried to evolve the introduced fuzzy model into the crisp model. For this regards Lai and Hwang (1994) approach as well as the weighted mean method are used to modify the objective function value to crisp values. Furthermore, chance constraint programming method (CCP) is used for constraints. Finally, configuration of proposed hybrid algorithm is presented. According to the mentioned approaches, the fuzzy GAP model could be rewritten as Eqs. (5) and Eq. (6). These two models are the same in terms of their relative constraints while are different in objective functions. It is crystal clear that the Lai and Hwang approach (1994) and weighted mean method are utilized for objective function changes in Eqs. (5) and Eq. (6) respectively. On the other hand, both models used CCP method for their fuzzy constraints.

$$\max z_1 = \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{m} x_{ij} - \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{p} x_{ij}$$

$$\min z_2 = \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{m} x_{ij}$$

$$\min z_3 = \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{o} x_{ij} - \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{m} x_{ij}$$

subject to (5)

$$\sum_{j=1}^{m} x_{ij} r_{ij} \le b_i^u - \alpha \left(b_i^u - b_i^m \right) \quad \forall i$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall i$$

$$x_{ij} = \{0,1\} \quad \forall i, j$$

In addition, the second model could be written just by replacing those objective functions by Eq. (6) as mentioned before:

$$\min z_4 = \frac{1}{4} \left(\sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}^{o} x_{ij} + 2 \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}^{m} x_{ij} + \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}^{m} x_{ij} \right) \tag{6}$$

3.2. Solving the proposed models

The aim of this section is to solve two obtained deterministic models mentioned above.

For the first mathematical model, which is a multi objective problem, a novel hybrid using SA and max-min fuzzy approach are introduced whereas the second model is solved using SA directly.

3.2.1 Simulation annealing

Simulated annealing is among the most popular iterative methods applied widely to solve many combinatorial optimization problems. In fact, SA is a class of metaheuristics that performs a stochastic neighborhood search on the solution space. The immense advantage of SA over classical local search methods is its ability to avoid getting trapped in local optimal while searching for a global optimum. The underlying idea of this method arises from an analogy with certain thermodynamically processes (cooling of a melted solid). It should be noted that SA procedures could be proportionately different in alternative problems but the basic principle of the SA can be described as follow:

Start from a current solution x, another solution y is generated by taking a stochastic step in some neighborhood of x while the degree of neighborhood is optional and apparently changes from problem to another. If this new proposal improves the value of the objective function, then y replaces x as the new current solution. Otherwise, the new solution y is accepted with a probability (Note that the difference with classical descent approaches, where only improving moves are allowed and the algorithm may end up quickly in a local optimum). The probability came from a significant formula[1] in metallurgy process that is shown as Eq. (7) as follows,

$$P(\Delta E) = \exp(\Delta E / T), \tag{7}$$

where the value of T decreases in the course of iteration. Hence, the probability of accepting worse solution will decrease too. In fact, reduction in control parameter leads to maintain current solution and just accept improving solutions while the algorithm is running in the last iterations. All the mentioned procedures enable SA to avoid getting trapped in local optima. In this paper, we consider control parameter (T) equals to 2000 and cooling schedule equals to 0.995. It should be mentioned that in each temperature, SA searches the solution space five times and then the temperature decreases.

3.2.2 hybrid SA and Max-Min fuzzy

Max-min fuzzy is an applicable approach for solving the multi objective problems presented by Zimmerman (1996). The main idea of this method is to solve each objective function individually regardless of other objectives and putting the obtained solution on other objectives in order to calculate the value of all objectives. Then for each objective a membership function can be defined as below,

$$\mu_i = \frac{z_i - z_{worst}}{z_{best} - z_{worst}}, \tag{8}$$

In Eq. (8) z_{worst} and z_{best} are the worst and the best solutions obtained for each objective function, respectively. The ultimate model for max-min fuzzy could be written as Eq. (9) in which λ corresponds to the satisfaction level of the model and p is the number of objective functions.

$$\begin{aligned} &\max \lambda \\ &\lambda \le \mu_i \quad i = 1,2,\dots,p \\ &\lambda > 0 \end{aligned} \tag{9}$$

[1] Boltzmann equation

In this paper, each one of the three objective functions presented in the model (5), is solved using SA and then a membership function is calculated for each via Eq. (8). Finally, the model (9) is solved via LINGO 10 software.

4. Computational results

This problem has been coded using visual basic 6 and all the computations were done on core i7 – 1.6 GHZ system with 4 GB RAM. In random generated problems, number of tasks equals 5 or 10 and number of agents is considered 3, 4 and 5, respectively. The GAP problem is strictly sensitive to the values of parameters and selecting unsuitable range would make this problem unfeasible. Due to this fact, the required data are selected via (10) which are based on Laguna et al. (1995) while are slightly modified in some parts. This values lead to generate suitable random feasible problems.

$$
\begin{aligned}
&r_{ij} \in Uniform[1,100] \\
&c_{ij}^{m} = 111 - r_{ij} + Uniform[-10,10] \\
&b_{i}^{m} = \sum_{j=1}^{m} r_{ij} / m
\end{aligned}
\tag{10}
$$

This section is divided into two separate parts in which the computational experiments for each model are mentioned individually.

4.1 model 1

As described earlier, the first model (5) is a multi objective model and is solved using a hybrid algorithm containing SA and max-min fuzzy approach. To evaluate the efficiency of proposed method, the results are compared to the solutions that are obtained from LINGO 10 software. These comparisons are shown in Table 1 in which *m* and *n* are the number of agents and tasks as described before while λ corresponds the satisfaction level of model. In addition, Z_1, Z_2 and Z_3 demonstrate the values of objective functions.

Table 1
Comparison between lingo and hybrid results

m	*n*	Lingo					hybrid				
		λ	z_1	z_2	z_3	$\tilde{z}$	λ	z_1	z_2	z_3	$\tilde{z}$
3	5	0.5	126	173	66	(47,173,239)	0.5	126	173	66	(47,173,239)
	10	0.6254	185	268	163	(83,268,431)	0.5677	192	276	151	(84,276,424)
4	5	0.5346	136	166	107	(30,166,273)	0.5306	125	156	60	(31,156,216)
	10	0.6121	197	271	153	(74,271,424)	0.7615	182	246	167	(64,246,413)
5	5	0.5275	139	174	57	(35,174,231)	0.5292	142	185	66	(43,185,251)
	10	0.5429	278	341	136	(63,341,477)	0.5552	263	322	129	(59,322,451)

In Table 1 the parameter $\tilde{z} = \left(z^{l}, z^{m}, z^{u}\right)$ is stands for the fuzzy assignment costs that its elements calculate as Eq. (18):

$$
\begin{aligned}
&z^{l} = z_2 - z_1 \\
&z^{m} = z_2 \\
&z^{u} = z_2 + z_3
\end{aligned}
\tag{11}
$$

In order to compare the results of hybrid method with LINGO, the weighted mean of fuzzy number is employed that the results are depicted in Table 2.

Table 2
The mean comparison between hybrid and lingo

		Lingo		hybrid	
m	*n*	$\tilde{z}$	$E(\tilde{z})$	$\tilde{z}$	$E(\tilde{z})$
3	5	(47,173,239)	158	(47,173,239)	158
	10	(83,268,431)	265.2	(84,276,424)	265
4	5	(30,166,273)	158.75	(31,156,216)	139.75
	10	(74,271,424)	260	(64,246,413)	242.25
5	5	(35,174,231)	153.5	(43,185,251)	166
	10	(63,341,477)	305.5	(59,322,451)	288.5

As it can be seen in most of the cases, the fully efficient hybrid algorithm presents better results. On the other hand, the satisfaction levels of both methods to some extent are the same while hybrid gives a better result in one case. Furthermore, the problem is solved for larger scales that the results are shown in Table 3.

Table 3
The results of model 1 for larger scales

M	N	hybrid				
		λ	Z_1	Z_2	Z_3	$\tilde{Z}$
10	10	0.5835	273	306	193	(33,306,499)
	15	0.5538	394	460	405	(66,460,865)
15	10	0.5271	310	334	189	(24,334,523)
	15	0.5385	459	495	320	(36,495,815)
25	10	0.5209	401	422	238	(21,422,660)
	15	0.5380	316	327	195	(11,327,522)
50	10	0.5365	367	378	185	(11,378,563)
	15	0.5316	410	413	265	(3,413,678)

4.2 model 2

The second model (6) is a single objective and was solved using SA, directly. The achieved results obtained via SA method are depicted in Table 4.

Table 4
The solution results of model 2

m	*n*	z	Computational Time (sec)	Global optimum
3	5	100.25	2	80.75
	10	190.5	14	143.5
4	5	114.75	4	96.25
	10	233.25	15	150.25
5	5	71.25	5	52.75
	10	182	15	108.25

In our computations, m and n demonstrate the number of agents and tasks respectively and z shows the value of objective function. Furthermore, the global optimal value of assignment cost is obtained by lingo 10 that is mentioned in last column.

By comparing the results of model 2 and the optimum values it can be concluded that model 1 acts more effective than model 2. The results also are provided for larger scales that are depicted in Table 5 as follows,

Table 5
The results of model 2 for larger scales (Time is in seconds)

M	10			20			25			50			100		
N	10	15	50	10	15	50	10	15	50	10	15	50	10	15	50
Z	245	408	1547	259	398	1489	256	407	1492	288	429	1351	266	380	1451
Time	4	5	19	3	5	18	3	6	18	6	8	23	8	11	36

5. Conclusion

In this paper, we have studied the GAP in uncertain environment by considering assignment costs and capacity of agents as fuzzy numbers and a new mathematical model has been presented. In order to solve the model, the fuzzy parameters have been converted into crisp values, where two approaches were used for objective function and one approach is used for constraints conversion. A new hybrid algorithm has been offered to solve crisp models containing SA and max-min fuzzy. In computational experiments section, the results of proposed algorithm were compared to the results of LINGO software that validated the efficiency of hybrid algorithm.

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