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A hybrid mathematical programming model and statistical approach for bidding price decision in construction projects

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A B S T R A C T

Bidding price decision is a key issue for the contractors and construction companies. The performance of the contractors in competitive biddings is directly dependent on their bidding strategy. This paper aims to present a hybrid statistical and mathematical modeling approach for calculating the optimum bidding price in competitive biddings. By statistical analysis of the previous data, some uncertain parameters like the number of bidders and the cost of the project are appraised. Then, a scenario-based mathematical programming model to determine the best bid price is designed. For making the model in more accordance with the real-world, factors like risk, minimum acceptable rate of return (MARR) and opportunistic behavior are taken into account. In order to achieve an insensitive solution to the change in the realization of the input data from the scenarios, a robust mathematical model is used. The performance of the model is analyzed through some numerical tests. Furthermore, sensitivity analysis of important factors and robustness analyses of the model facing uncertain parameters are performed. Finally, a case study is presented to show the model's performance in facing the real-world problems. Numerical results demonstrate the good performance of the model in decreasing estimation faults and improving the profit for the contractor. These findings show the good performance of the model facing real-world situation.

1. Introduction

Competitive bidding has been considered as one of the most popular approaches to select contractors to carry out construction projects. Many national and international construction companies may try to win a bid through a competitive process. In order to be successful in such a competitive environment, the contractors should do their best to win the bids and acquire enough profit from each contract. Each invitation to tender is potentially a business opportunity for the contractors. In competitive biddings, a contractor who has received the invitation from the owner has to take two main sequential decisions. A decision is needed whether to participate in the bidding or decline it in search of better options. Assuming that the invitation to tender was accepted considering the analysis of different factors, a decision on bid price is needed to be taken. The contractor should set the bid price by adding a markup on the estimated cost. The contractor with the least offer between the other bidders will win the biding. Since the price is usually the most important factor in the owner's assessment of bids, it strongly affects the chance of winning a contract. Therefore, a suitable bid strategy is crucial for the contractor's success in competitive biddings. However, a wrong strategy in bidding may result in the defeat of the contractor in the bidding. A suitable bidding strategy needs a balance between being low enough to increase the winning chance and high enough to lead to a satisfying profit. Many factors are effective on the bidding decisions, ignoring of which may have negative effects on the contractor's bidding decision. Therefore, bid price determination is a very hard task needing a subtly equivalence between the winning probability, profit and uncertainty (Hosny & Elhakeem, 2012; Liu et al., 2018). Considering the important role of the bid price in the contractor selection, auction decisions have been a important subject for the construction companies.

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With the help of scientific tools and methods, the contractor can find the best bidding price to win the bidding. That is the main reason of the studies about auction decisions, which are mainly about bidding skills and strategies (Lorentziadis, 2016). However, these skills and strategies mostly tried to improve the winning chance (or financial gain) and neglected some possible consequences and real parameters. A universal bidding model most include the real-world situation and related parameters. This paper aims to propose a robust approach for determining the optimum amount of bidding price which includes some of the real-world parameters and factors.

The rest of the paper is organized as follows: In section 2, a brief review of the relevant literature of the bidding price decision is presented and the description of the research gap and our contribution are explained. The research methodology is described in section 3. In section 4, the hybrid approach for bid price decisions is presented. Numerical tests and related discussion are presented in section 5. A case study and related analysis are presented in section 6. Managerial insights are presented in section 7 and finally, conclusion and future works are explained in section 8.

2. Literature review

2.1. Bidding price decision

After the initial study by Friedman (1956), many researchers have studied the bid price decision problem (Rothkopf & Harstad, 1994; Lorentziadis, 2016). These researches can be categorized into four classes (Marzouk & Moselhi, 2003; Wang et al., 2007): statistical models, artificial intelligence-based (AI-based) models, MCDM models, and operations research (OR) methods. Statistical models calculate the bid markup based on the statistical bidding behavior of the competitors (Stark and Rothkopf, 1979; Zhang et al., 2011). Statistical models proposed by Friedman (1956) and Gates (1967) have been the most relevant approaches for estimating the winning chance. In the last years, entropy index (Christodoulou, 2010); simulation (Hosny & Elhakeem, 2012) and multivariate statistic approaches (Skitmore & Pemberton, 1994; Liu & Ling, 2005) were applied in bidding price decisions. AI-based methods apply case-based logic (Chua et al., 2001; Dikmen et al., 2007) and artificial neural networks (Li, 1996; Liu and Ling, 2005). AI-based approaches are younger than the other methods, since, the primary tries to apply AI in auctions were in the 1990s (Moselhi et l., 1993; Li, 1996). The hybrid case-based logic with the GA is one of the newest usages of AI in auctions (Kim & Shim, 2013). MCDM approaches study the effect of many factors in addition to price in bi decisions (Cheng et al., 2011; Chou et al., 2013; Jato-Espino et al., 2014). Uncertainties (Wang et al., 2007), the number of bidders (Carr, 2005) and opportunistic behavior (Lo et al., 2007) are some of the most important parameters affecting bid decisions. In this way, Chou et al (2013) proposed a combination of AHP and simulation to help the construction companies in their decisions. OR methods employs mathematical programming models for finding the best bidding price decisions. Recently, game theory (Dong-hong et al., 2009a, b; Huang, 2016; Han and Liu, 2014), consecutive model (Takano et al., 2014) and stochastic optimization (Davatgaran et al., 2018) have been applied in such OR methods. Table 1 shows a comparative investigation of the study gap in the literature.

Table 1
A comparative investigation of the study gap in the literature.

Authors	Uncertainty				Risk	Competition	Opportunistic behavior	Base price	MARR	Model type		
	Scenario	Robust	Probabilistic	Fuzzy						Statistical	AI	OR
Friedman (1956)			✓			✓				✓		
Liu and Ling (2003)				✓							✓	
Liu and Ling (2005)			✓			✓				✓		
Skitmore and Pemberton (2006)			✓			✓				✓		
Dong-hong and Xi-yan (2009b)						✓		✓				✓
Bigdeli et al. (2010)			✓								✓	
Christodoulou (2010)			✓		✓	✓					✓	
Soleymani (2011)						✓						✓
Hosny and Elhakeem (2012)			✓			✓				✓		
Kumar et al. (2013)				✓								✓
Chou et al. (2013)			✓	✓	✓					✓		
Takano et al. (2014)			✓		✓	✓						✓
Takano et al. (2018)			✓			✓				✓		✓
Davatgaran et al. (2018)	✓											✓
This paper	✓	✓	✓		✓	✓	✓	✓	✓	✓		✓

2.2. Research gaps

Although many researches have focused on the bid price decision, some study gaps prepared the motivation for this research. Various factors are effective on bid decisions. However, many researches have designed models to improve the winning chance (or financial gain) and neglected some possible consequences and real parameters. Uncertainty, competition, risk, MARR, and upper/lower can be effective on bidding price determination (Wang et al., 2017; Kauppinen et al., 2018; Yan et al., 2018). None of the existing studies has considered the concurrent role of uncertainty, competition, MARR, opportunistic behavior, base price and risk on the problem. In other words, there is unfulfilled research regarding more effective factors in which concurrent application of the above parameters simultaneously can lead to a more comprehensive and useful model.

Thus, the main theoretical contribution of this study is the consideration of more of the real-world parameters in the bid decision model. To the best of our knowledge, this research presented the first bidding price decision model which considers the concurrent effect of the above-mentioned parameters on the bidding price determination. Furthermore, the manuscript presents a hybrid statistical and robust mathematical programming model which is the first kind in the literature based on the best of the authors' knowledge.

Fig. 1 shows the conceptual framework of the paper for the bidding price decisions.

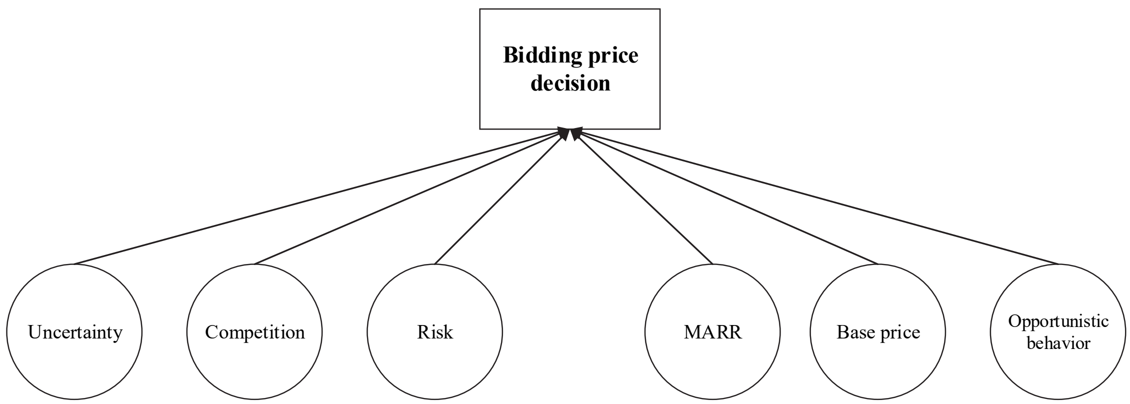


Fig. 1. The conceptual framework for bidding price decision.

3. Research methodology

After a short literature review, the manuscript is organized through a four-step research methodology which includes: 1. designing a primitive objective function and the model's constraints; 2. proposing the winning chance function for completing the model; 3. Numerical tests and analysis and, 4. Appraising the model's performance in some real problems as case studies.

As mentioned before, the main motivation for this research in to propose a robust mathematical model for bidding price decision considering some of the effective real-world parameters. There are some mathematical constraints in the proposed model, which are:

- MARR constraint: the first of the model's constraints compels the contractor to present a bid price which yields the MARR with a pre-determined probability. In civil engineering, MARR is the least rate of return on an investment which a construction company tends to admit prior to perform a project and accept its risks (Park, 2001). Thus, a MARR constraint as the goal rate for analyzing the project investment is proposed and the project should fulfill the MARR for the construction company with a pre-determined probability.

- Maximum allowable risk constraint: By considering this constraint, the maximum allowable risk for the contractor is limited. The mentioned constraint decreases the risk of attacking by a considerable amount of loss for the construction companies.
- Base price (upper bound) constraint: This constraint compels the construction companies to bring up upper bound on bid price. To hinder abnormally high bid price, the owners generally state an upper bound entitled base price on the bidding price. Therefore, for owning a good winning probability, the contractors have to consider this parameter and determine the bid price under or ultimately equal to this announced limit.
- Lower bound constraint: This constraint forces the construction companies to consider the lower limit on bid decisions. To hamper abnormally low prices, the owners generally state lower limit on bid prices which have an important role in hindering opportunistic behavior (Abdoli and Khirandish, 2010). If the construction company neglect from this lower bound, he/she will lose the chance of winning the bid.

4. Mathematical model

4.1. Model notation

Table 2 shows the notations used in the model.

Table 2
Notations used in the model.

Indexes	
s	scenarios of the project's cost; $s = \{1,2,\dots,S\}$
i	set of random samples of the project's cost; $i = \{1,2,\dots,I\}$
k	the number of competitors; $k = \{1,2,\dots,K\}$
Decision variable	
m	bid markup
δ^s	unsatisfied amount of MARR for the bidder under scenario s
Functions	
$P(b)$	Winning chance by the bid price of b
$F(y)$	probability density function when the competitors' bid price is y
$G(k)$	probability mass function when the number of competitors is k
$\Phi^s(x)$	the cumulative amount of $\tilde{E}(s)$ in x $(a^S \le x \le b^S)$.
Parameters	
pr^S	probability of scenario s, $s \in S$
p_i^s	occurrence probability of sample i at scenario s, $s \in S$
$\tilde{E}(s) \approx TruncNormal\left(\mu^s, (\sigma^2)^s, a^s, b^s\right)$	Project's estimated cost under scenario s which is a truncated Normal random variable in $[a^s, b^s]$ with mean and variance of μ^s and $(\sigma^2)^s$, respectively
E_i^s	ith sample of project's cost under scenario s, $s \in S$
$C(s) = \mu^s$	estimation of the real cost of the project under scenario s, $s \in S$
W	variance cost parameter
BP	base price or upper limit on the bidding price announced by the owner
α	MARR
β	confidence level of achieving the MARR
φ	maximum allowable risk (loss) for the contractor
γ	confidence level of maximum allowable risk (loss)
L	lower bound on the bidding price announced by the owner
η	minimum winning chance which is requested by the contractor
ω	trade-off parameter between the cots and solution robustness (shortage cost)
Objective function	
AVG	the contractor's expected profit
avg^s	the expected profit for the contractor at scenario s, $s \in S$

4.2. Model assumptions

The following assumptions are considered for mathematical formulation:

- The owner applies a constant-price contract: It should be mentioned that there are many project contracts from the payment point of view (e.g., cost-plus contract, buyback and lump sum). The proposed model is suitable for the fixed-price contracts, in which the contractor should first estimate the project's cost and then determine the bid price by adding a markup to the estimated cost. Thus, the proposed model is only suitable for fixed-price contracts and other contract types, some other approaches such as overhead costs' analysis should be used.

- The project's estimated cost under scenario s, $\tilde{E}(s)$, is a random variable with a $[a^s, b^s]$ truncated Normal distribution with parameters of μ^s and σ^s.
- The project's true cost is approximated by the mean of $\tilde{E}(s)$. In other words, C(s) is appraised by μ^s.
- The number of competitors follows a Poisson distribution with mean and variance of λ. (Please refer to section 4.3 [Winning probability section] to find more details).
- The competitors' bid price for the auction follows an Erlang distribution with a shape parameter of κ and a scale parameter of θ. (Please refer to section 4.3 [Winning probability section] to find more details).

4.3. Formulation

The contractors generally follow a two-step approach for bid price decisions. He/she starts with estimation of the costs of the project and then determines a markup, m, on the estimated cost. Therefore, the contractor's bidding price under scenario s can be calculated by $B(s)=(1+m)\tilde{E}(s) \quad ;\forall s=1,\dots,S$. The bidder who submits the lowest bid price is the winner.

In order to face uncertainty exists in real-world situation; a scenario-based approach is used for modeling the problem. Therefore, a set of scenarios of the project's cost, $s=\{1,2,\dots,S\}$ are defined. Each scenario is representative of the situation that a project is performed in (from the cost point of view) and has its occurrence probability (pr^S), estimated cost ($\tilde{E}(s)$) and standard deviation $((\sigma)^s)$. The number of scenarios and their corresponding occurrence probability should be determined by the contractor. The contractor can use some approaches such as consulting with the experts, using his/her knowledge or using his/her experiences on the past performed projects.

After determining the number and the occurrence probability of the scenarios, μ^s and σ^s should be estimated. In order to do so, the data of similar projects which were outsourced previously for each scenario should be prepared at first. E_i^s shows the ith sample of the project's cost under scenario s. Then, using statistical analysis of the prepared data (or samples which are randomly produced from $\tilde{E}(s)$), the mean and standard deviation of costs at each scenario are appraised. In other words, by analyzing all of the E_i^s s in each scenario, the parameters of $\tilde{E}(s)$ such as μ^s and $(\sigma^2)^s$ can be determined.

Since $\tilde{E}(s)$ is a positive random variable, defining it as a Normal distribution including negative numbers is one of the defects of some of the studies like Friedman (1956). To eliminate this defect, $\tilde{E}(s)$ is considered as a truncated Normal distribution. Furthermore, the true cost of the project in each scenario, $C(s)$ is estimated by the $\tilde{E}(s)$.

Objective function: Defining $\mathrm{P}((1+m)\tilde{E}(s))$ as the contractor's winning probability when the bid price is $(1+m)\tilde{E}(s)$ under scenario s and $(1+m)\tilde{E}(s)-C(s)$ as the winner's profit from the contract, the expected benefit for the bidder at scenario s can be calculated as:

$$avg^s=\left[(1+m)\tilde{E}(s)-C(s)\right].\mathrm{P}((1+m)\tilde{E}(s)) \quad ; \forall s=1,\dots,S \tag{1}$$

As mentioned, $\tilde{E}(s)$ has a truncated Normal distribution withcan be estimated from $\tilde{E}(s).\left(\mu^s,(\sigma^2)^s,a^s,b^s\right)$ previously outsourced similar projects. Samples of the $\tilde{E}(s)$ are randomly produced by drawing samples from the proposed distribution function. These samples can be prepared in two methods: one is to gather data of homogenous projects outsourced by competitive bidding for each of the specified scenarios. Another is to produce random samples for each of the scenarios which are produed by drawing samples from $\tilde{E}(s)$. In this method, the proposed distribution function for each scenario is defined for the statistical software (e.g., Minitab). Then, a set of samples from the specified distribution function are randomly produced for each scenario by the software. Ith sample of project's cost under scenario s and occurrence probability of sample i at scenario s are E_i^s and p_i^s , respectively. So, (1) can be rewritten as:

$$\max \; avg^s = \sum_{i=1}^{I} p_i^s . \left[\left(1+m\right) E_i^s - C\left(s\right) \right] . \mathrm{P} \left[\left(1+m\right) E_i^s \right] \quad ; \forall s = 1,...,S \tag{2}$$

MARR constraint: Some contractors may deliberately decrease their bid markup to improve their chance for winning the contract. However, this study has this opinion that some other factors, such as MARR, in addition to the winning chance should be considered in price determination process in competitive biddings. MARR constraint limits the bid price in such manner that warranties this amount in each scenario. By defining β as the confidence rate of obtaining the MARR α and δ^s is the unsatisfied amount of MARR for the contractor under scenario s, the following is the MARR constraint:

$$\mathrm{P}\left(\frac{\left(1+m\right).\tilde{E}\left(s\right) - C\left(s\right)}{C\left(s\right)} + \delta^s \geq \alpha \right) \geq \beta \quad ; \forall s = 1,...,S \tag{3}$$

$\Phi^s\left(x\right)$ is the cumulative value of $\tilde{E}\left(s\right)$ in $x\left(a^s \leq x \leq b^s\right)$. Considering the attributes of truncated Normal distribution (Burkardt, 2014), (3) can be described in another manner as below:

$$\frac{\Phi^s\left(\frac{C\left(s\right).\left(\alpha+1+\delta^s\right)}{\left(m+1\right)} \right) - \Phi^s\left(a^s\right)}{\Phi^s\left(b^s\right) - \Phi^s\left(a^s\right)} \leq 1-\beta \quad ; \forall s = 1,...,S \tag{4}$$

Defining $Z \sim N(0,1)$, Z_x as the cumulative value of Normal distribution function at x and $\tilde{F}\left(s\right) \approx Normal\left(\mu^s, \left(\sigma^2\right)^s\right) \quad ; \forall s = 1,...,S$, (5) is evident:

$$\Phi^s\left(x\right) = \mathrm{P}\left(\tilde{F}\left(s\right) \leq x\right) = \mathrm{P}\left(\frac{\tilde{F}\left(s\right) - \mu^s}{\sigma^s} \leq \frac{x-\mu^s}{\sigma^s} \right) = Z_{\left(\frac{x-\mu^s}{\sigma^s}\right)} \tag{5}$$

Due to (5), (4) can be presented in another style:

$$\frac{Z_{\left(\frac{\frac{C\left(s\right).\left(\alpha+1+\delta^s\right)}{\left(m+1\right)} - \mu^s}{\sigma^s} \right)} - Z_{\left(\frac{a^s-\mu^s}{\sigma^s}\right)}}{Z_{\left(\frac{b^s-\mu^s}{\sigma^s}\right)} - Z_{\left(\frac{a^s-\mu^s}{\sigma^s}\right)}} \leq 1-\beta \quad ; \forall s = 1,...,S \tag{6}$$

By defining Z_x^{-1} , (6) can be presented in another style of (7):

$$\frac{\frac{C\left(s\right).\left(\alpha+1\right)}{\left(m+1\right)} - \mu^s}{\sigma^s} \leq Z^{-1}_{\left[(1-\beta).Z_{\frac{b^s-\mu^s}{\sigma^s}} + \beta.Z_{\frac{a^s-\mu^s}{\sigma^s}} \right]} \quad ; \forall s = 1,...,S \tag{7}$$

If $A^s = \left(1-\beta\right).Z_{\frac{b^s-\mu^s}{\sigma^s}} + \beta.Z_{\frac{a^s-\mu^s}{\sigma^s}}$ and $LB^1 = \underset{s=1,...,S}{Max}\left(\frac{C\left(s\right).\left(\alpha+1+\delta^s\right)}{\mu^s + \sigma^s.Z^{-1}_{A^s}} - 1 \right)$, (3) can be finally presented as:

$$m \geq LB^1 \tag{8}$$

Maximum acceptable risk constraint: Overtaking the project's expenses from the bid price results in an economical damage for the construction companies (Love et al., 2010). Since the project's scope is mixed with uncertainty, construction companies admit a logical rate of risk. However, financial damages which are more than the logical rate can force the companies to claims from the owner to offset or at least decrease their financial damages (Abdoli and Khirandish, 2010). Therefore, a maximum allowable risk constraint is necessary for preventing considerable damages for the contractor. Defining γ as the γ-quantile of a random financial loss and since the random loss in each scenario is $C(s)-(1+m)\tilde{E}(s)$, this constraint can be presented as:

$$\mathrm{P}\left(\frac{C(s)-(1+m)\tilde{E}(s)}{C(s)} \leq \varphi\right) \geq \gamma \quad ;\forall s = 1,\ldots,S \tag{9}$$

If $B^s = (1-\gamma).Z_{\frac{b^s-\mu^s}{\sigma^s}} + \gamma.Z_{\frac{a^s-\mu^s}{\sigma^s}} \quad ;\forall s = 1,\ldots,S$ and $LB^2 = Max\left(\frac{C(s).(\varphi+1)}{\mu^s+\sigma^s.Z_{B^s}^{-1}} - 1\right);\ \forall s = 1,\ldots,S$ and by applying the same approach that was applied for MARR constraint, maximum allowable risk constraint can be written as:

$$m \geq LB^2 \tag{10}$$

Base price constraint: To hamper abnormally high bid prices, generally an upper bound entitled base price is stated by the owner. Proposing the prices more than the base price will result in the elimination of the contractor from the competition in the bidding. Defining BP as the base price, this constraint can be presented as below:

$$(1+m).E\left(\tilde{E}(s)\right) \leq BP \quad ;\forall s = 1,\ldots,S \tag{11}$$

In (11), $E\left(\tilde{E}(s)\right)$ is the expected value of the appraised cost of project in scenario s. Since $E\left(\tilde{E}(s)\right)$ is corresponding to μ^s and by defining $UB = Min\left(\frac{BP}{\mu^s} - 1\right);\ \forall s = 1,\ldots,S$, (11) can be presented as:

$$m \leq UB \tag{12}$$

Lower bound constraint: A logical lower bound on the bid price let the owners hamper or specify unusually low bid offers. This is a good approach for hampering the opportunistic behavior in which a bidder may deliberately offer a low price to improve his/her winning chance. After winning the bid, they usually claim from the owner to obtain their expected profit (Ho and Liu, 2004; Crowley and Hancher, 1995). This constraint can be present as:

$$(1+m).E\left(\tilde{E}(s)\right) \geq L \quad ;\forall s = 1,\ldots,S \tag{13}$$

By defining $LB^3 = Max\left(\frac{L}{\mu^s} - 1\right);\ \forall s = 1,\ldots,S$, (13) is equivalent to:

$$m \geq LB^3 \tag{14}$$

4.4. Winning chance

The last phase of model design is to specify $P[b]$ that is the contractor's chance of winning the bid who offers b in bidding. In order to calculate $P[b]$, the number of bidders and their corresponding bid prices should be appraised.

In probability theory, the Poisson distribution is a discrete probability distribution that shows the probability of a given number of events occurring in a fixed interval of time or space (Haight, 1967). Using the Poisson distribution in estimating the number of competitors in competitive biddings is often justified on theoretical concepts. If there are some similar independent auctions, it may be reasonably assumed that each auction has the same probability distribution of the number of bids serving as an estimation of the common distribution (Song, 2004). In the case of several similar alternative contractors, each independently decides whether to bid or not, the number of bidders has a binomial distribution (Hickman et al., 2017). As the number of individuals becomes larger and the probability that an individual bid becomes smaller, the binomial distribution approaches a Poisson distribution (Haight, 1967; Engelbrecht-Wiggans, 1978). Furthermore, there are numerous data examinations for several similar construction contracts in different countries showing that the Poisson distribution is a reasonable approximation of the number of bidders in competitive biddings (Keller and Bor, 1978; Engelbrecht-Wiggans, 1978; Hickman et al., 2017). Furthermore, many studies have used the Poisson distribution in order to estimate the number of bidders in competitive biddings (Friedman, 1956; Mohlin et al., 2015; Ballesteros-Pérez and Skitmore, 2016). Engelbrecht-Wiggans (1978) observed that the Poisson model is essentially the only alternative to the common fixed bidder model that has ever been considered. Thus, there are both theoretical and empirical supports for the claim that the Poisson model appraise the number of competitors in auctions in a good manner.

In statistics, the Gamma distribution is a two-parameter family of the continuous probability distribution. The exponential distribution, Erlang distribution, and Chi-squared distribution are special cases of the Gamma distribution (Rohatgi and Saleh, 2015). In many studies concerning bid decisions in competitive biddings, it was assumed that bid values follow a Gamma distribution (Friedman, 1956; Skitmore, 2014; Dougherty & Nozaki, 1975; Weverbergh, 1982). The main reason for the prevalence use of Gamma distribution in bid models is that the Gamma distribution includes many of the common bidding distributions in the literature such as exponential, Erlang and chi-squared distribution. In addition to theoretical evidence, empirical data analysis indicates that Gamma distribution offers a generally suitable fit in estimation for the competitors' bid values in competitive biddings (Hossein, 1977; Skitmore, 2014; Dougherty and Nozaki, 1975). Thus, there are both scientific evidence and empirical experiments that Gamma distribution is a suitable fit in estimating the bid values of the competitors in competitive biddings. Thus, the number of contractors follows a Poisson distribution with the following function:

$$G(k)=\frac{\lambda^{k}}{k!}\exp(-\lambda)\ \ ;k=1,2,... \tag{15}$$

Furthermore, the competitors' price offer follows an Erlang function with the following function:

$$F(y)=y^{\kappa-1}\frac{\exp\left(\frac{-y}{\theta}\right)}{(\kappa-1)!\theta^{\kappa}}\ \ ;y\geq 0 \tag{16}$$

where y is the offered price, $\kappa\ (\geq 1)$ and $\theta(\succ 0)$ are shape and scale parameters, respectively. The mean and variance of the distribution (16) are $\kappa\theta$ and $\kappa\theta^2$, respectively.

Since the contractor who submits the minimum price is the winner of the contract, the winning chance in the case of proposing $(1+m)E_i^s$ as the bidding price is:

$$\mathrm{P}\left[(1+m)E_i^s\right]=\sum_{k=0}^{\infty}G(k)\left(\int_{(1+m)E_i^s}^{\infty}F(y)dy\right)^{k} \tag{17}$$

Since Erlang is a special form of the Gamma distribution in which its first parameter (κ) is a positive integer, $\int_{(1+m)E_i^s}^{\infty}F(y)dy$ can be determined by (18):

$$\begin{aligned}\int_{(1+m)E_i^s}^{\infty}F(y)dy &=1-CDF\left[(1+m)E_i^s\right]=\sum_{l=0}^{\kappa-1}\frac{1}{l!}\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\left(\frac{(1+m)E_i^s}{\theta}\right)^{l}\\ &=1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^{l}\end{aligned} \tag{18}$$

In this equivalent, $CDF\left[(1+m)E_i^s\right]$ is the cumulative value of the Erlang in $(1+m)E_i^s$. Thus, (17) can be presented in another style:

$$\mathrm{P}\left[(1+m)E_i^s\right]=\exp(-\lambda)\left[\sum_{k=0}^{\infty}\frac{\lambda^{k}}{k!}\left(1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^{l}\right)^{k}\right] \tag{19}$$

Based on (20) and (21):

$$\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}=\exp(\lambda) \tag{20}$$

$$\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}\left(1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^l\right)^k=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\lambda\left(1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^l\right)\right]^k \tag{21}$$

$$=\exp\left(\lambda\left(1\text{-}\exp\left(\frac{-(1+\mathrm{m})E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+\mathrm{m})E_i^s}{\theta}\right)^l\right)\right)$$

The chance of winning in the case of $(1+m)E_i^s$ as bid price is: can be calculated as:

$$\mathrm{P}\left[(1+m)E_i^s\right]=\exp\left(-\lambda\left(1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^l\right)\right) \tag{22}$$

Therefore, the expected benefit for the contractor under scenario s is:

$$avg^s=\sum_{i=1}^{I}p_i^s.\left[(1+m)E_i^s-C(s)\right].\exp\left(-\lambda\left(1-\exp\left(\frac{-(1+m)E_i^s}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l!}\left(\frac{(1+m)E_i^s}{\theta}\right)^l\right)\right);\forall s=1,...,S \tag{23}$$

Thus, the objective function which is maximizing the contractor's expected profit is:

$$AVG=\max\ \mathrm{Z}=\sum_{s=1}^{S}pr^s.avg^s \tag{24}$$

4.5. Robust model

4.5.1 Robust mathematical programming

This research uses a robust optimization approach to face the uncertainty of the parameters. Robust optimization is a special optimization method in which a certain measure of robustness is desirable against uncertainty. This measure can be illustrated as a definite variance in the values of the model's parameters or its solution (Bertsimas & Sim, 2004). Mulvey et al. (1995) introduced a robust optimization model at first. They presented an augmented scenario-based stochastic model as robust optimization. In Mulvey's model, the robust optimization includes two types of robustness: solution robustness and model robustness, which means that an optimal solution should remain near-optimal in the case of turbulence in the input value and a solution must be closely feasible in each of the scenarios in order to be robust (Habibi-Kouchaksaraei et al., 2018). The variables of the robust model are classified in two fields: the first stage and second stage variables. First stage variables (design variables) are resistant against the variations in the input data and second stage variables (control variables) are sensitive to the changes in the input data (Alimian et al., 2019). Furthermore, a robust optimization model has two kinds of constraints: structural constraints that all of its parameters are certain and control constraints as assistant constraints that are sensitive to variable data (Pouriani et al., 2019).

The structure of the Mulvey's robust mathematical model is as (25) – (28) (Mulvey et al., 1995):

$$min\ \xi = c^T x + d^T y \tag{25}$$

subject to

$$Ax=b \tag{26}$$

$$Bx+cy=e \tag{27}$$

$$x\ ,\ y\geq 0 \tag{28}$$

In this model, constraint (26) has only fixed coefficients that are not sensitive to the variations of the input values and therefore is a structural constraint. On the other hand, (27) has uncertain coefficients and therefore is a control constraint. The model designer should consider a set of scenarios Ω = {0, 1, …, s} in order to face the uncertainty in the problem. The model designer should also estimate the occurrence probability of scenarios, p_s. (d_s^T, B_s, C_s, e_s) are the uncertain parameters of the model in each scenario. An important goal of robust optimization is to serve the feasibility of the model in all of the scenarios. This object is realized in the model by using the parameter δ_s, which is the amount that each solution is in the infeasibility area. This parameter is positive when the model is infeasible and is zero, in the case of feasibility of the model (Alimian et al., 2019).

Now, the equivalent robust model can be presented as below (Mulvey et al., 1995):

$$min\ \Upsilon = \sigma(x, y_1, y_2, \dots, y_s) + \varpi\rho(\delta_1, \delta_2, \dots, \delta_s) \tag{29}$$

subject to

$$Ax=b \tag{30}$$

$$B_s x + C_s y_s + \delta_s = e_s \ \ \forall s \in \Omega \tag{31}$$

$$x\ , y_s \geq 0 \ \ \forall s \in \Omega \tag{32}$$

The objective function has two sections. The first is for the robustness of the solution and the second is the model's robustness. ϖ is the balance parameter for the control and cost constraints. To keep the model's robustness, an average of the events and variance is used to analyze the infeasibility of the model. In order to show the robustness of solutions, Eq. (33) is used:

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_s p_s \xi_s + \lambda \sum_s p_s (\xi_s - \sum_{s'} p_{s'} \xi_{s'})^2 \tag{33}$$

In this equation, λ is variance cost which specified the effect of variance on objective function. Furthermore, ξ_s= f(x,y_s) is cost function of scenario s and is calculated as below:

$$\xi_s = c^T x + d_s^T y_s \tag{34}$$

It is clear that Eq. (33) is a nonlinear equation, solving of which needs more time-consuming. Therefore, it can be converted to a linear equation by replacing Eq. (33) with Eq. (35) (Yu & Li, 2000):

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_s p_s \xi_s + \lambda \sum_s p_s \left| \xi_s - \sum_{s'} p_{s'} \xi_{s'} \right| \tag{35}$$

4.5.2 Robust model

Uncertainty is a critical and unavoidable factor in bidding price decisions. There are many uncertain parameters in the bidding price decision model such as the number of competitors, the bidding price of each contractor and the project costs (Takano et al., 2018; Hosny & Elhakeem, 2012; Kim, 2015). Many of the studies in the literature have proposed models to increase the profit or winning probability for the contractors. However, this manuscript searches for the solution to face the uncertainty and to make the model more reliable. In order to face the uncertainty and keep the model feasible against the variation in input parameters, the robust optimization is used in this manuscript. Mulvey et al. (1995) proposed a robust optimization method to approach the suitable risk aversion by showing the values of essential input data in a set of scenarios. This method results in a series of solutions which are less sensitive to the model date from a scenario set. Since the robust approach presented by Mulvey et al. (1995) is more suitable for the scenario-based models, Mulvey's approach is used to make the proposed model robust. According to (29) and (35), the robust equivalent of objective function of the model (24) is:

$$\max\ Z = AVG - W.\sum_{s=1}^{S} pr^s.\left|avg^s - AVG\right| - \omega \sum_{s=1}^{S} pr^s\, avg^s \tag{36}$$

where W is variance cost and ϖ is trade-off parameter between the costs and solution robustness (shortage cost). In more detail, the variance cost parameter (W) is a goal programming parameter that is used for determining the effect of variance on the robust objective function. A suitable value of W would help to reach full robustness in terms of solution and along with ϖ will result in an equivalency between solution and model robustness. Finally, the robust model for bidding price decision for the contractors in construction projects is presented as Eqs.(37-40):

$$min\ \mathrm{Z} = W.\left\{\sum_{s=1}^{S} pr^{s}.\left|avg^{s} - AVG\right|\right\} + \omega\sum_{s=1}^{S} pr^{s}.avg^{s} - AVG \tag{37}$$

$$\max\left(LB^1, LB^2, LB^3\right) \le m \le UB \tag{38}$$

$$\sum_{s=1}^{S} pr^{s} \sum_{i=1}^{I} p_{i}^{s}.\exp\left(-\lambda\left(1-\exp\left(\frac{-(1+m)E_{i}^{s}}{\theta}\right)\sum_{l=0}^{\kappa-1}\frac{1}{l\,!}\left(\frac{(1+m)E_{i}^{s}}{\theta}\right)^{l}\right)\right) \ge \eta \tag{39}$$

$$\delta^{s} \ge 0 \quad ;\forall s = 1,\ldots,S \tag{40}$$

5. Numerical tests

The performance of the model is appraised in this section by numerical tests. Computations were performed on a Windows 10 personal computers and a Core i7 processor. MATLAB (R2016a) and a MATLAB toolbox, *fmincon*, were used to implement the computations.

5.1. Problem design

To find critical points from the model, the problems were designed simple. Thirty problems listed in Table 3 are set. Three scenarios of the project's cost including optimistic, likely and pessimistic are considered in the numerical problems. The trade-off parameter ω is set to 1 for all the numerical problems. The occurrence probability of each scenario is 0.3, 0.4 and 0.3, respectively. The number of random samples of the project's cost, I, in the training set is 100 and the BP is 1.5. The variance cost parameter, W, is set to 1 for all the numerical problems. The E_i^s s are randomly created by selecting samples from a $N\left(\mu^s, \left(\sigma^2\right)^s\right)$ function.

Table 3
Numerical problems

No.	(\$)$\left(\mu^1,\mu^2,\mu^3\right)$	α	$\left(\sigma^1,\sigma^2,\sigma^3\right)$	No.	(\$)$\left(\mu^1,\mu^2,\mu^3\right)$	α	$\left(\sigma^1,\sigma^2,\sigma^3\right)$
1	(1.8,1.9,2.0)	0.12	(0.10, 0.10,0.10)	16	(1.4,1.6,1.8)	0.16	(0.10, 0.10,0.10)
2	(1.8,1.9,2.0)	0.15	(0.12, 0.12,0.12)	17	(1.4,1.6,1.8)	0.17	(0.12, 0.11,0.13)
3	(1.8,1.9,2.0)	0.18	(0.10, 0.11,0.11)	18	(1.4,1.6,1.8)	0.18	(0.11, 0.11,0.11)
4	(1.2,1.4,1.6)	0.20	(0.09, 0.09,0.09)	19	(2.4,2.6,2.8)	0.12	(0.10, 0.10,0.10)
5	(1.2,1.4,1.6)	0.15	(0.10, 0.10,0.10)	20	(2.4,2.6,2.8)	0.13	(0.11, 0.11,0.11)
6	(1.2,1.4.1.6)	0.18	(0.11, 0.12,0.11)	21	(2.4,2.6,2.8)	0.15	(0.12, 0.13,0.13)
7	(1.8,2.0,2.2)	0.16	(0.12, 0.12,0.12)	22	(1.6,1.8,2.0)	0.16	(0.12, 0.13,0.14)
8	(1.8,2.0,2.2)	0.17	(0.08, 0.09,0.10)	23	(1.6,1.8,2.0)	0.17	(0.11, 0.12,0.12)
9	(1.8,2.0,2.2)	0.12	(0.09, 0.10,0.10)	24	(1.6,1.8,2.0)	0.18	(0.13, 0.13,0.13)
10	(1.8,1.9,2.0)	0.13	(0.09, 0.09,0.09)	25	(2.9,3.1,3.3)	0.12	(0.12, 0.12,0.13)
11	(1.8,1.9,2.0)	0.15	(0.10, 0.10,0.11)	26	(2.9,3.1,3.3)	0.15	(0.13, 0.14,0.14)
12	(1.8,1.9,2.0)	0.12	(0.11, 0.10,0.12)	27	(2.9,3.1,3.3)	0.16	(0.10, 0.11,0.11)
13	(1.2,1.4,1.6)	0.18	(0.13, 0.13,0.13)	28	(2.0,2.2,2.4)	0.17	(0.09, 0.09,0.09)
14	(1.2,1.4,1.6)	0.18	(0.12, 0.12,0.11)	29	(2.0,2.2,2.4)	0.18	(0.10, 0.10,0.10)
15	(1.2,1.4,1.6)	0.18	(0.13, 0.13,0.13)	30	(2.0,2.2.2,4)	0.20	(0.09, 0.10,0.10)

5.2. Numerical findings

Numerical sets in Table 3 were analyzed by optimization toolbox, *fmincon*, and Table 4 demonstrates the findings. In this table, bid markup (m) for each of the thirty numerical problems is presented in columns two and six. The objective function of the model (Z or equation 37) is presented in columns three and seven. Furthermore, the expected profit (AVG or equation 24) is presented in columns four and eight.

Table 4
Numerical results

No.	m	Objective function (Z)	Expected profit (AVG)	No.	m	Objective function (Z)	Expected profit (AVG)
1	0.1931	-0.0176	0.0183	16	0.2672	-0.0004	0.0004
2	0.2100	-0.0171	0.0185	17	0.3535	-0.0004	0.0005
3	0.2632	-0.0004	0.0005	18	0.3629	-0.0035	0.0038
4	0.2780	-0.0068	0.0139	19	0.3357	-0.0052	0.0055
5	0.3108	-0.0079	0.0131	20	0.3360	-0.0007	0.0008
6	0.2910	-0.0028	0.0031	21	0.3671	-0.0056	0.0058
7	0.2507	-0.0030	0.0033	22	0.2631	-0.0004	0.0004
8	0.2591	-0.0280	0.0283	23	0.3257	-0.0036	0.0038
9	0.1788	-0.0177	0.0182	24	0.3401	-0.0263	0.0284
10	0.1779	-0.0061	0.0067	25	0.3321	-0.0008	0.0009
11	0.3555	-0.0130	0.0135	26	0.3463	-0.0065	0.0068
12	0.3483	-0.0120	0.0129	27	0.3564	0.0013	-0.0009
13	0.2791	-0.0167	0.0178	28	0.3787	-0.0008	0.0009
14	0.3241	-0.0186	0.0211	29	0.3703	-0.0007	0.0008
15	0.3021	-0.0170	0.0198	30	0.3859	-0.0056	0.0059

As shown in Table 4, the objective function for all of the numerical problems is negative except for case number 27. Since the objective function is from the minimization type, the positivity of Z shows that the penalty costs for the robust model ($W.\left\{\sum_{s=1}^{S} pr^s.\left|avg^s - AVG\right|\right\} + \omega\sum_{s=1}^{S} pr^s\, avg^s$) is greater than the expected profit (AVG). For case 27, Z is positive and AVG is negative. Therefore, the best decision for the contractor is not to bid and hamper potential damages and subsequent problems.

5.3. Sensitivity analysis

In this section, the effect of the MARR and the variance cost on the bid markup, the expected profit and objective function is analyzed. In order to analyze the effect of MARR on the objective function and bid markup, sensitivity analysis of this parameter is conducted. Therefore α is increased by 50% (or 1.5α) and then reduced by 50% (or 0.5α), and the impact of these variations is determined. The outcomes are presented in Fig. 2 and Fig. 3.

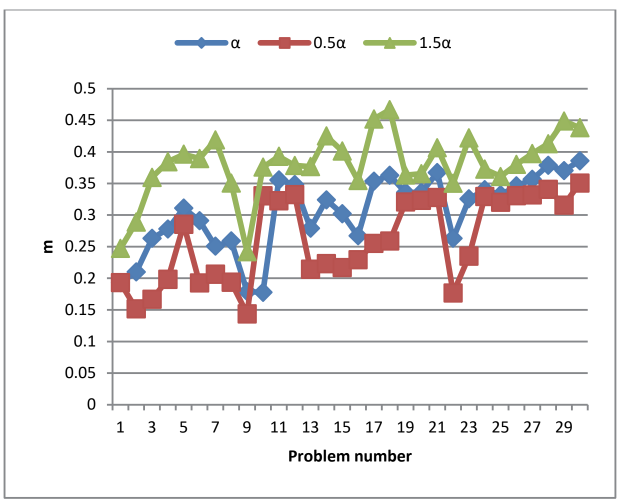


Fig. 2. Effect of MARR on the bid markup

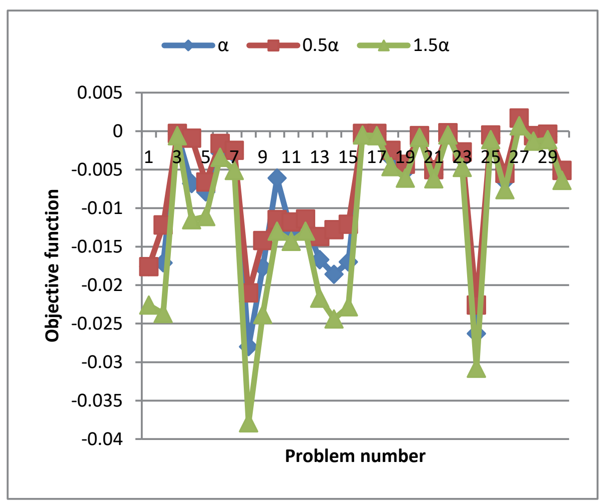


Fig. 3. Effect of MARR on the objective function

Considering Fig. 2 and Fig. 3, increasing α by 50% results in an increase in m and a reduction in the objective function. The reason is that as the MARR increases, the contractors generally tend to increase the markup to fulfill the MARR. Bt the same argument, reducing α result in fulfilling the MARR constraint by a lower amount of m.

The analysis of the effect of MARR on the objective function is a little more complicated. The objective function of the model has two main sections: the expected profit and deviation cost. Sensitivity analysis shows that increasing α leads to an increase in the expected profit and the deviation cost. More specifically, the average expected profit for 1-30 is 0.0091, 0.0082 and 0.0138 for α , 0.5α and 1.5α , respectively. On the other hand, the average deviation cost for the problems 1-30 is 0.0016, 0.0011 and 0.0019 for α , 0.5α and 1.5α , respectively. So, increasing the MARR increase both the expected profit and the deviation cost; however, the effect of MARR on the expected profit is much more considerable which leads to improvement (decrease) in the objective function. In summary, the effect of changes in the MARR on the important elements of the model is shown in Table 5.

Table 5
The effect of MARR on the model's elements

α	m	Expected profit	Deviation cost	Objective function
↓	↓	↓	↓	↑
↑	↑	↑	↑	↓

As mentioned before, the variance cost parameter (W) determines the effect of the variance term in the robust objective function (Mulvey et al., 1995). Since W is a critical parameter in Mulvey's robust optimization model, a sensitivity analysis of this factor and its effect on the objective function is performed in this section. Thus, problems 1 to 30 are solved by different values of W from 0 to 1. Fig. 4 shows the different values of the average objective function (for problems 1 to 30) correspond to different values of W. According to Fig. 4, increasing the value of W leads to the tendency of the model to employ more robustness in solutions to reduce the variance's effect. This leads to a decrease in the objective function (Alimian et al., 2019). Since the objective function is from minimization type, W that results in the least objective function is the optimal value for W. As depicted in Fig. 4, the lowest objective function is achieved by the value of 1 for W. This value for W results in a considerable decrease in the deviation cost among the related solutions. Thus, the value of 1 for W is the best value for this particular example.

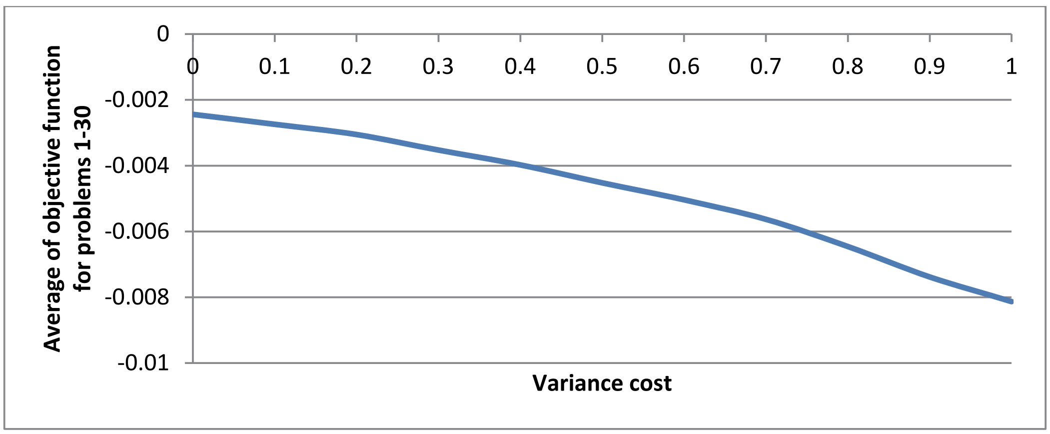


Fig. 4. Sensitivity analysis for the variance cost parameter.

5.4. Robustness analysis

As mentioned before, there are some uncertain factors in the model such as λ, κ, θ, μ^s and σ^s . Errors in estimating these parameters can have important role in the model's outcomes. Therefore, it is essential to appraise the robustness of the model facing potential errors in the estimated parameters. To do so, the testing sample set containing random errors is used. After producing the training data, samples ξ from the function are selected. Afterward, the amount of a parameter is altered. For example, θ would change to θ(1+Δξ), where Δ is error and will be 0,1 or 0.2. Then, the training samples are produced for each scenario and calculations are conducted for each testing sample set. Table 6 demonstrates the effects of errors in each parameter on the mean awaited benefit.

Table 6
The mean awaited benefit by the impact of errors in parameters.

Parameter	---	λ		κ		θ		$\mu^s(\tilde{E})$		$\sigma^s(\tilde{E})$	
Δ	0	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
Mean awaited benefit	0.0091	0.0061	0.0018	0.0125	0.0164	0.0119	0.0159	0.0059	0.0020	0.0081	0.0042

Considering Table 6, errors in λ, μ^s and σ^s leads to a reduction in the mean awaited benefit and errors in κ and θ increase it. To present the reason more clearly, the model's uncertain parameters are categorized into two classes: 1. Parameters which affect the competitors' decisions (κ and θ) and 2. Parameters affecting our (as a specified contractor) bid prices (λ, μ^s and σ^s) (Jank and Zhang, 2011). Obviously, error in the second set parameters leads to a wrong decision by us which decrease our mean awaited benefit. On the other hand, errors in the first class parameters result in a wrong decision by other bidders. This false decision by the competitors has an important effect on increasing our success chance and the expected profit, subsequently. Fig. 5 depicts the categorization of the uncertain parameters and the impact of their estimation fault on the success chance and mean awaited benefit.

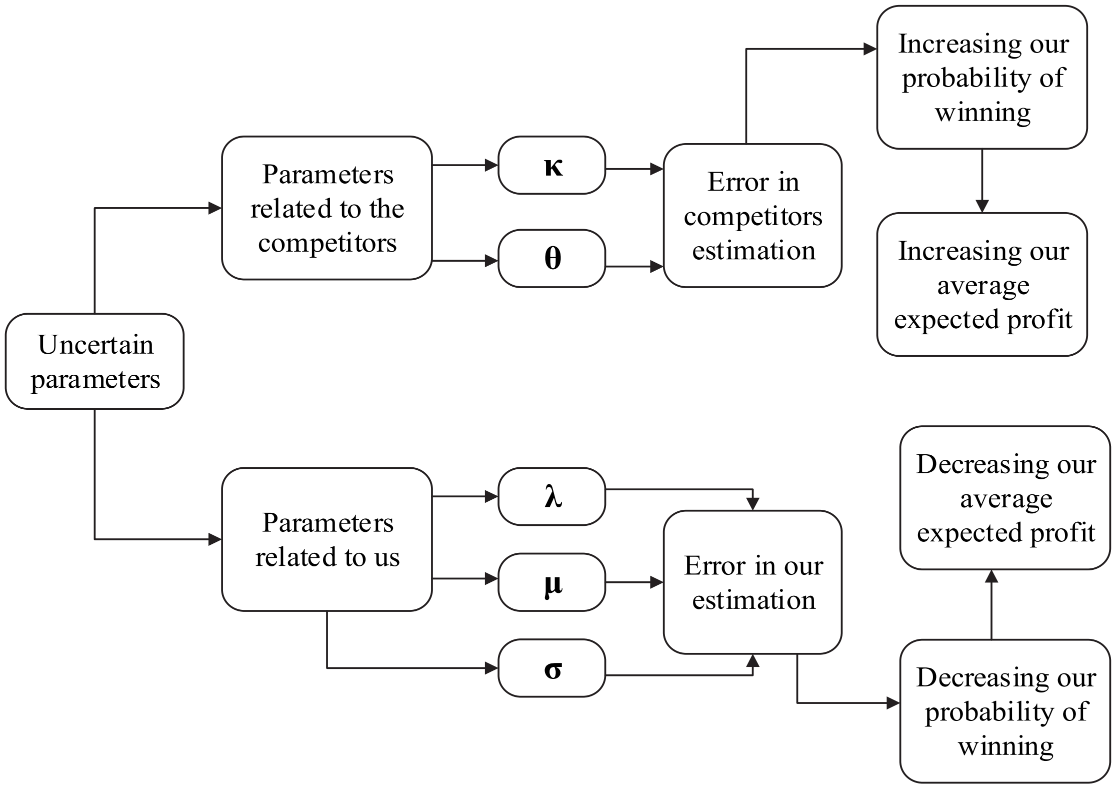


Fig. 5. Uncertain parameters and the effect of their estimation error on the success chance and the awaited benefit.

6. Case study

To illustrate the suitable performance of the method in facing real issues, some case studies from Iran National Iranian Oil Company (NiOC) are solved in the followings. Thus, the proposed approach is implemented step by step and the results are reported. NiOC used Design-Bid-Build agreement kind for all of these six cases and also the projects that are used as data. NOC applies a fixed price payment method and the construction company should fulfill a project at a fixed cost which is determined in the contract. The mentioned price covers whole costs and benefits.

A pipeline construction project in Gachsaran suburbs was intended to be outsourced to a contractor through a competitive auction. Therefore, a public invitation in newspapers was published and some general specifications of the project were announced there. In more detail, the contactors should had some criteria such as warrant in the database of Iranian companies, capability of paying a specified amount of contract's price as warranty and possession of the professional license in oil-gas branch. Fig. 6. Show the phases of implementing our approach for the case studies.

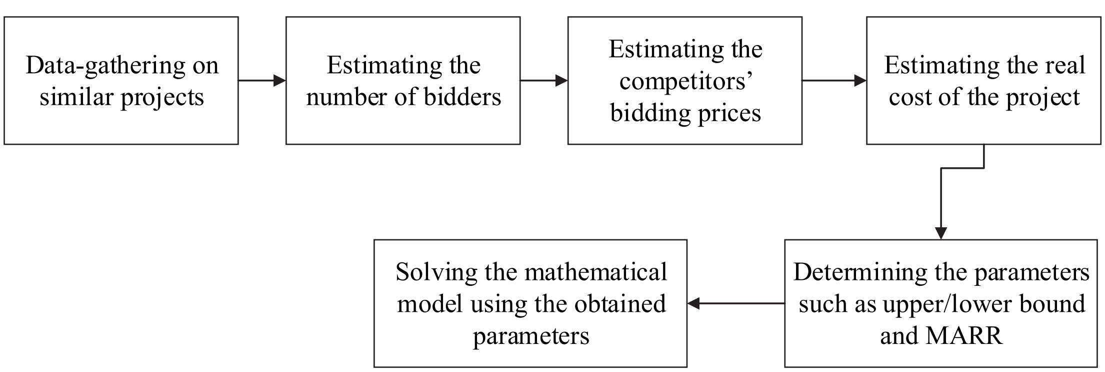


Fig. 6. Steps of calculating the bidding price in the proposed approach

1. Data-gathering phase: The first phase of bid price calculation is date-gathering. Thus, homogenous projects which are similar to the specified project fro, field, type, scale, and time points of view are gathered (Dikmen et al., 2007). So, many of projects outsourced by NIOC were considered and thirty-five of them in the past two years were gathered.

2. Estimating the number of competitors: The second phase is to evaluate the data and appraise the uncertain parameters of the model. In this way, factor λ is approximated by the data of the number of competitors. As mentioned in the last sections, we assume that the Poisson distribution has good performance in estimating the number of competitors in auctions. Thus, it is assumed that:

H_0 : The number of competitors in the bid follows a Poisson distribution.

H_1: The number of competitors in the bid does not follow a Poisson distribution.

To analyze the above-mentioned hypo-thesis, the Chi-squared fitting test is applied. Table 7 illustrates the outcomes of the Poisson fitting test for the number of competitors.

Table 7
Outcome of the Chi-squared fitting test for the number of competitors.

Number of contractors	Observed	Poisson probability	Awaited	Contribution to Chi-Sq
≤3	2	0.084654	2.96698	0.315557
4	2	0.093523	3.27212	0.494302
5	6	0.129801	4.54304	0.467402
6	4	0.150190	5.25684	0.300517
7	7	0.148889	5.21411	0.611904
8	5	0.129302	4.52510	0.049911
9	4	0.099688	3.49102	0.074296
≥10	5	0.163688	5.73147	0.093354
Poisson average for the number of competitors = 6.939				
N	N*	DF	Chi-Sq	P_Value
35	0	6	2.40709	0.875

Since the P_Value is more than the significance level (0.05), H_0 can't be ignored and it can be stated that the number of competitors follows a Poisson distribution with λ = 6.939.

3. Estimating the competitors' bidding prices: It was stated that the Gamma distribution fits the bidding price of the bidders in good manner. Therefore, the following hypothesis is presented:

H_0 : bidding price of the bidders follows a Gamma distribution.

H_1: bidding price of the bidders does not follow a Gamma distribution.

Fig. 7 shows the results of evaluating the above-mentioned hypo-thesis. Since the P_Value is more than the significance level of 0.05, H_0 cannot be ignored and the proposed parameter follows a Gamma distribution with shape and scale parameter of 1031 and 0.01729, respectively.

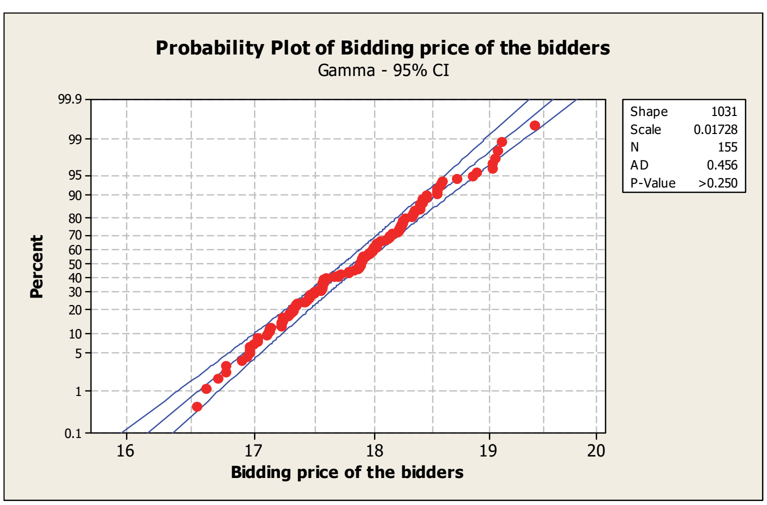


Fig. 7. Probability plot for the bidders' bid price.

4. Estimating the true cost of the projects: The last phase for data analysis step is determining the function that best fits the project's cost. Minitab's *Distribution Identification* module is applied. The findings demonstrate that Normal distribution is the best function fitting the true costs. In fact, Normal distribution fits the true costs by a P_Value of 0.519 and an AD of 0.298. AD is Anderson-Darling statistics which lower AD shows the more suitability of fitting.

5. Solving the mathematical model: By gathering the needed data, the model (37-40) can be run. Findings show that m=0.10326 is the best value for m and based on the cost which is approximated, the optimum bid price is 18.363×10^9 . In reality, a bidder who offered 17.538×10^9 Rials won the contract and the true cost of the project was 16.993×10^9 Rials. After determining the best bidding price for case 1, the same approach is implemented for five of the other construction projects which were outsourced by the company. The phases needed for the optimum bid decision for the other cases are presented as below:

1. Data-gathering phase: As mentioned before, the first phase of bid price calculation is date-gathering. In order to do so, many projects that were outsourced by NIOC were monitored and some of them that were compatible with the mentioned criteria in the last three years were selected. Table 8 demonstrates the elements of the five cases.

Table 8
Elements of the other five case studies.

Code*	Fields	Duration (days)	Cost interval (10^9Rials)	Number of homogenous projects for data analyses
2	Oil-pipeline	252	26-30	42
3	Road	117	46-50	38
4	Well-digging	103	56-60	41
5	Road	126	71-75	43
6	Oil-pipeline	277	76-80	44

** The oil pipeline construction project is case 1.*

In order to approximate the uncertain parameters, the gathered data is evaluated and the function best fitting the parameters are determined.

2. Estimating the number of competitors: Table 9 illustrates the outcomes of the Poisson fitting test for the number of competitors.

Table 9
Outcome of the Chi-squared fitting test for the number of competitors.

		Case 2	Case 3	Case 4	Case 5	Case 6
Number of bidders	≤ 3	2	4	4	0	0
	4	3	2	3	4	7
	5	4	7	3	4	8
	6	5	7	6	4	6
	7	6	4	9	6	8
	8	4	6	2	7	5
	9	6	4	6	7	5
	10	3	3	5	11	3
	≥11	8	0	4	0	2
N		41	37	42	43	44
DF		8	6	7	5	6
Chi-Sq		6.91507	3.94689	6.46645	2.43158	2.35039
P_Value		0.545	0.685	0.485	0.789	0.887
Poisson average		7.7559	6.5407	7.2139	7.7211	6.7489

3. Estimating the bidders' bid prices: Table 10 demonstrates the outcomes of the Gamma fitting test for the bidders' bid price.

Table 10
Outcomes of the Gamma fitting test for the competitors' bid prices.

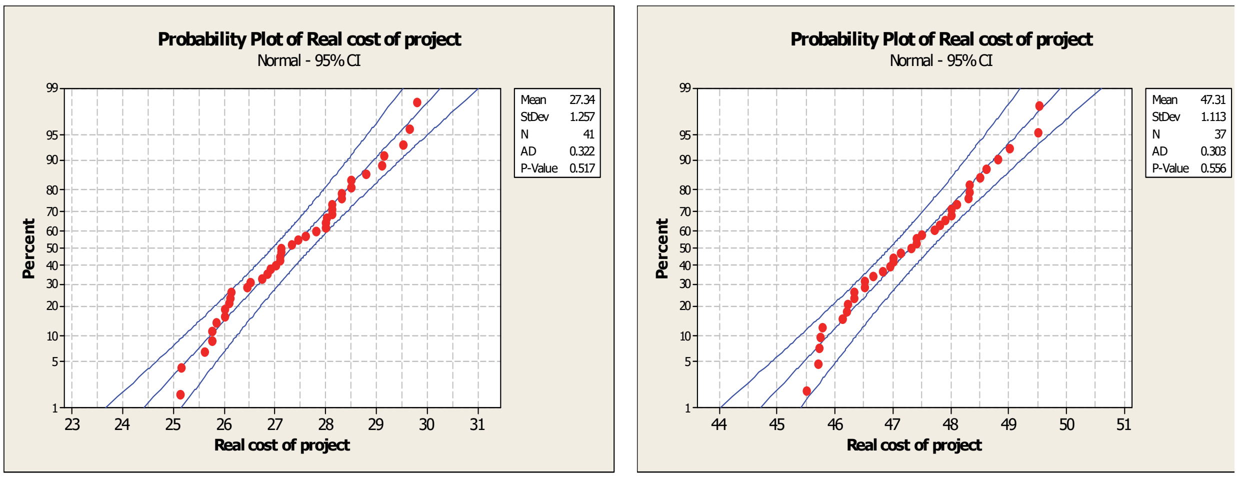
Case number	N	P_Value	Shape	Scale
2	317	0.228	716	0.039531
3	241	0.217	1191	0.040720
4	302	>0.25	1078	0.055309
5	331	>0.25	6263	0.012081
6	294	0.231	2951	0.028502

4. Estimating the true cost of the projects: The last phase for data analysis step is determining the function that best fits the project's cost. Minitab's *Distribution Identification* module is applied. Table 11 demonstrates the outcomes of the test for the true cost of the project.

Table 11
Outcomes of fitting test for the true cost of the projects.

Case number	2		3		4		5		6	
Distribution	**AD**	**P_Value**	**AD**	**P_Value**	**AD**	**P_Value**	**AD**	**P_Value**	**AD**	**P_Value**
Normal	0.31	0.518	0.30	0.557	0.31	0.528	0.31	0.551	0.327	0.510
LogNormal	0.37	0.520	0.30	0.552	0.31	0.548	0.31	0.539	0.357	0.440
Exponential	17.4	0.002	17.2	0.001	17.7	0.002	17.3	0.002	18.20	0.002
Weibull	0.57	0.134	0.567	0.150	0.66	0.083	0.47	0.241	0.379	0.292
Gamma	0.33	0.302	0.32	0.308	0.33	0.317	0.34	0.313	0.362	0.295
Logistic	0.42	0.312	0.37	0.315	0.41	0.327	0.39	0.324	0.336	0.314

Fig. 8 demonstrates the probability plot for the data of true cost fitting on Normal distribution for two cases (to decrease the paper's size, the plot is shown for just two cases).



a) Case 2 b) Case 3

Fig. 8. Probability plot for the true cost of the project fitted by Norman function for projects 2 and 3

5. Solving the model: Now, the bid price determination for cases 2-6 can be solved by our approach. The outcomes of the cases 2-6 and also the data of what happened in practice are presented in Table 12.

Table 12
Findings of solving 2-6 cases and the data of what happened in practice

Case number	m	Estimated cost (10^9 Rials)	Bid price (10^9 Rials)	True cost of the project (10^9 Rials)	Winner's bid (10^9 Rials)
2	0.0918	27.342	29.854	26.855	27.224
3	0.1109	47.313	52.558	48.018	50.105
4	0.0867	57.555	62.543	58.888	60.756
5	0.1147	72.530	80.851	73.280	76.048
6	0.1211	78.925	88.485	79.580	81.113

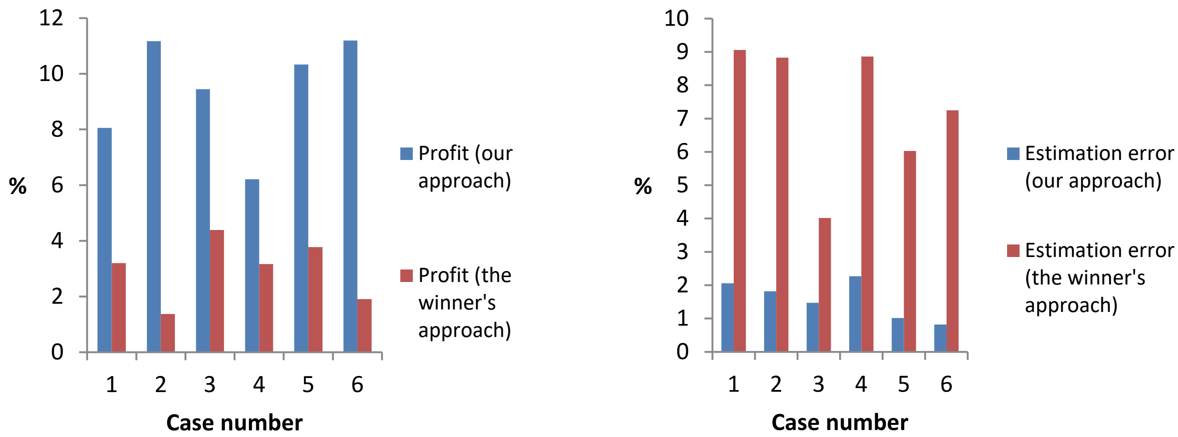
To analyze the performance of the model better, cost estimation error and the mean benefit for the contractors are evaluated. Fig. 9 illustrates the cost approximation error and the mean benefit achieved by our method vs. the methods applied by the winners.

The effect of our approach on cost estimation accuracy: Based on Fig.8, it can be concluded that our method results in less error in cost approximation and more benefit for the bidders. To evaluate the correctness of this sentence, statistically significance of the impact of our approach on cost approximation error and benefit should be analyzed. In this way, the Paired-samples T-test is applied. The below hypothesis is designed about the impact of the approach on cost approximation accuracy:

H_0: Our approach has no considerable impact on reducing cost approximation error ($\mu = \mu_0$).

H_1: Our approach has a considerable impact on reducing cost approximation error ($\mu \succ \mu_0$).

Firstly, the margin between cost approximation errors in the two situations (with and without applying the method) should be calculated. The findings are demonstrated in Table 13.



(a) (b)

Fig. 9. Comparing our vs. the winners method'. (a) Benefit, (b) Cost approximation error achieved from

Table 13
Margin between cost approximation errors with vs. without applying the proposed method.

Case number	Approximation error (our method) (%)	Approximation error (winner) (%)	D (%)
1	2.061	9.081	7.020
2	1.802	8.840	7.041
3	1.451	4.021	2.572
4	2.272	8.870	6.600
5	1.021	6.051	5.031
6	0.811	7.261	6.453

As the sample numbers is low, the D_i samples must follow a Normal function to apply the Paired-samples T-test. Applying Minitab, the probability plot for D_i samples which are fitted to Normal function was prepared and their Normality was validated by a P_Value of 0.0781. The following phase is to determine the test statistic that is computed as follow. In this formula, $\overline{D}$ is the mean of D, S_D is standard deviation and n is the amount of samples:

$$\frac{\bar{D}-\mu_0}{\frac{S_D}{\sqrt{n}}} = \frac{0.05815}{\frac{0.01765}{\sqrt{6}}} = 8.070$$

The rejection area is $[t_{0.05,5} = 2.015, \infty)$ and as the statistic is in, H_0 is rejected at 0.05 and H_1 is approved. Thus, "the proposed approach has a considerable impact on reducing cost approximation fault".

The effect of our method on the benefit for the bidders: The next phase is to analyze the approach in improving the benefit for the bidder. The below hypothesis is stated about the impact of our approach on improving the benefit for the bidders:

H_0 : Our approach has no considerable impact on improving the benefit for the bidder ($\mu = \mu_0$).

H_1: Our approach has considerable impact on improving the benefit for the bidder ($\mu \prec \mu_0$).

First, the margin between benefit achieved by the bidder in the two situations (with and without applying our method) is determined and the findings are demonstrated in Table 14.

Table 14
Margin between benefits achieved by the contractor with vs. without applying our method.

Case number	Benefit (our method) (%)	Winner's benefit (%)	D (%)
1	8.0563	3.2011	-4.8551
2	11.171	1.3740	-9.7962
3	9.450	4.3882	-5.0620
4	6.212	3.1682	-3.0423
5	10.332	3.7773	-6.5534
6	11.190	1.910	-9.2801

As the sample numbers is low, the D_i samples must follow a Normal function to apply the Paired-samples T-test. Applying Minitab, the probability plot for D_i samples which are fitted to Normal function was prepared and their Normality was validated by a P_Value of 0.5088. The following phase is to determine the test statistic that is determined as follow:

$$\frac{\overline{D}-\mu_0}{\frac{S_D}{\sqrt{n}}} = \frac{-0.0643}{\frac{0.0266}{\sqrt{6}}} = -5.9211$$

The rejection area is $(-\infty, t_{0.05,5} = 2.015]$ and since the statistics is in, H_0 is declined at 0.05 and H_1 is validated. Thus, "the approach has a considerable impact on improving the benefit for the bidder".

Therefore, the computations approved the considerable impact of our approach on reducing cost approximation error and improving benefit achieved by the bidders. In more detail, our method results in a 5.820% reduction in the cost approximation error on average. This robust cost approximation is the foundation of a good bidding price decision and helps the bidders to improve their gains. In addition, applying our method, the bidders can achieve more benefit (almost 6.43% on average). Thus, a good cost approximation and a good bid price decision can have positive effects in the construction companies' performance in the competitive biddings.

7. Managerial facts and research limitations

To ease the application of the approach for its potential users, a set of useful facts and insights are proposed in the following. Thus, it counseled that the bidders study these facts previous to using the approach:

1. The approach is directly dependent on the quality and quantity of data. If the bidder cannot prepare the needed data, our approach would not be useful for him/her.

2. The data preparation is generally need considerable time and cost. Thus, based on the scale and approximated cost of the project, the contractor must evaluate whether our approach is useful or not.

3. The historical data must be similar from kind, class, scale and time.

4. The approach cannot specify the bid price for more than a project at the same time. In the case of inviting for more than a project simultaneously, the contractor must implement the approach for one by one of the cases. This process needs considerable time and is hard for many of the contractors.

6. Previous to performing the model, items such as MARR, maximum allowable risk, and base price must be specified using the experts.

7. The outcomes and subsequent decisions after bidding price determination by the approach can be categorized into three fields:

- Red situation: when the awaited gain for the bidder is negative, he/she must not participation in the auction and hampers financial damage.
- Yellow situation: If the awaited benefit for the bidder is 0 or very low more than 0, the contractor must be cautious and achieve more data, consults more and apply more resolution for bid/no bid decision.
- Green situation: when the awaited benefit for the bidder is completely positive, the contractor can bid with confidence and propose the price which is determined by the model.

8. Conclusion

This paper devised a new framework for bid price determination in competitive auctions of construction projects. Since the cost approximation error extremely decreases the bidder's chance for winning the contract, a statistical scenario-based method was used to decrease the impact of cost approximation error. In order to stabilize the solution against the parameter's variation and keep the solution's feasibility against noises, the robust mathematical modeling approach was used. Therefore, a robust model with the objective function of maximizing the mean awaited benefit under scenarios and some constraints like the MARR, maximum allowable risk, lower/upper limit and minimum winning chance constraints was presented. After preliminary numerical tests, sensitivity analysis for the MARR and variance cost parameter was conducted and the effect of these two parameters on the objective function, bid price and awaited benefit were evaluated. Then, a robustness analysis facing uncertain factors was executed. The findings show that errors in the estimation of the parameters impacting the competitors' decisions significantly increase our (as a contractor) winning chance and wrong approximation of the parameter controlling our decisions reduces our winning probability. Then, our method was implemented on six of NIOC's projects. The findings show the excellence of our method comparing other approaches from the contractor's benefit and the cost approximation error points of view. These results demonstrate the good performance of the method in real problems.

This paper can be developed in various directions. Other approaches such as fuzzy method can be used to face the uncertainty of some parameters. In addition, a model which calculates the bid price for more than a case at the same time will be helpful for the bidders. In addition, more of the real-world parameters such as resource limitations may be considered in the model.

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