Using a metaheuristic algorithm for solving a home health care routing and scheduling problem

Neda Manavizadeh\textsuperscript{a*}, Hamed Farrokhi-Asl\textsuperscript{b} and Parya Beiraghdar\textsuperscript{c}

\textsuperscript{a}Department of Industrial Engineering, KHATAM University, Tehran, Iran
\textsuperscript{b}School of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran
\textsuperscript{c}School of Civil Engineering, College of Engineering, University of Manitoba, Winnipeg, Canada

\textbf{ABSTRACT}

The Health Care system is changing from the hospitalization to the home care, and the World Health Organization has announced that the rate of care-dependent elderly people in Europe will considerably increase within the next decades. Thus, scientific planning for this area is an essential factor to improve the community health. This paper aims to develop a mathematical modeling for Home Health Care Routing and Scheduling Problem and to solve it by means of Simulated Annealing (SA) algorithm considering real condition (staff vehicle traveling, conditions of patients and so forth). We permit interdependent services for patients in which they can order as many services as they want with any relation between them (Multiple Services) and supposed time window for each service. The mathematical formulation of the problem is coded in GMAS software, which is a well-known commercial software for solving optimization problems. In addition, for large-scale problems where GAMS is unable to solve, SA algorithm is applied to tackle the problems. Finally, sensitivity analysis on the most important parameters (number of services and number of patients with interdependent Multiple services) are conducted. The results reveal that when each patient can order infinite services with any relation between them, complexity of the problem increases, but SA algorithm can solve large instances with reasonable solution in the less computational time. Thus, SA algorithm shows a rational performance for large instances. Moreover, the most important factors that affect the objective value and the run time of the problems are number of patients, and number of patients with interdependent multiple services.

\section{1. Introduction}

The home care service was introduced in 1958 and since then, there has been a constant increase in the services offered (Rasmussen et al., 2012). The Health Care system is changing from the hospitalization to the home care (Beer et al., 2014; Mankowska et al., 2014). The World Health Organization (WHO) has announced that the rate of care-dependent elderly people in Europe will strongly increase within next decades (Rabban et al., 2017). Home care initially concentrated on nursing cares, developed complex, and technical cares such as chronic cares, rehabilitation, end-of-life pal-
liative cares, and home chemotherapy (Liu et al., 2014). Nowadays the services may include cleaning, preparing food, and help for everyday tasks (Decerle et al., 2016). The objective of the HHC organization is to provide high quality services to the patients at home in order to help them retain their health (Azadeh et al., 2015). For this purpose, they employ various qualified staff, including nurses, social workers physicians, therapists, and food couriers (Fikar & Hirsch, 2017). One of the important problems in this young field is that as the nursing companies get larger, the problem of scheduling the nursing staff arises. The challenge of this problem is to consider and combine aspects of staff and vehicle routing and scheduling. Nowadays, routing and scheduling of Home Health Care Services planners face challenging and complex optimization problems on various decision-levels, such as staff assignment, shift scheduling and, staff routing decisions (Wang & Fung, 2015). For competing in the today’s markets and lowering public expenses, the major points are increasing service quality and decreasing costs. In the vehicle routing and scheduling problem, travel times, distances and, service types have to be taken into account. The other major point in the mentioned problem is time window. Solutions with maximal patients/staff satisfaction and minimal costs are of most attraction for companies. Expenses for fuel, etc. or costs of the staff are reflected as costs in this context. While improving financial and reducing costs assets are the main goals of hospital managers, maximizing patients’ satisfaction level is also a crucial issue. HHCRSP is similar to Traveling Salesman Problem with Time Window (TSPTW) and Vehicle Routing Problem with Time Window (VRPTW) but HHCRSP has aspects that differs this from both of them.

The rest of the paper is organized as follows: Section 2 reviews briefly a related literature. In Section 3, a new mathematical modeling for HHCRSP is presented and a numerical example are provided in order to validate the model. Solution procedure for the simulated annealing algorithm is presented in Section 4. Computational experiments are conducted in Section 5 and sensitivity analysis is presented in section 6. Finally, Section 7 provides the conclusion remarks.

2. Literature review

In Home Health Care, studies are mainly divided into two major categories. The first one includes papers that investigate other related fields in HHC such as Supply Chain Management in HHC or Human Factor in HHC, and the second one consists of papers that presents mathematical modeling or using meta-heuristic algorithm for routing and assigning in HHC staff planning. In the first classification, Liu et al. (2014) compared HHC and nest elderly in Shanghai. Their purpose was assessing the home health care needs among the empty-nest elderly living. They used questionnaire for collecting data, logistic, and linear regression for analysis. They concluded that the empty-nest elderly has higher income, less social support, and higher prevalence of chronic diseases. Beer et al. (2014) stated because HHC emphasizes on major individual elements of the system-person, equipment/technology, tasks, and environments and the interaction between these components, a human-systems perspective is a fruitful approach to understand HHC. Their purpose was to apply a human-system perspective to consider the capabilities and limitations of the people, in relation to the demands of the tasks and equipment/technology in HHC. In other paper, Han et al. (2013) evaluated quality of life for patients that receive HHC services for a 60-day period. For this purpose, they studied on 100 patients in the USA and concluded that quality of life significantly improved during the start of caring and discharging (or 60 days). Axisa et al. (2005) studied on intelligent technology that can be used in HHC. They stated all these systems could provide a safe and comfortable environment for HHC, illness prevention, and citizen medicine. In the second category, Fikar and Hirsch (2017) provided a comprehensive aspect of current papers and works in the routing and scheduling problem in the field of HHC. Yağlıdağ et al. (2016) proposed a data-driven method for estimating the travel times of caregivers by using Kernel regression technique, which offered empirical modelling of the travel routes that generated by caregivers. Decerle et al. (2016) introduced a mixed-integer programming model that has focused on the various types of staff members that are employed by home care services for minimizing cost related to the transportation and the working hours. Heching et al. (2016) proposed an exact optimization method for home hospice care
staffing and scheduling by using logic-based Benders decomposition (LBBD). They used mixed integer programming (MIP) to solve this problem. Wang and Fung (2015) proposed a Markov decision process for scheduling sequential appointments to maximize patient satisfaction level and, dynamic programming for avoiding the curse of dimensionality. Xiang et al. (2015) introduced a modified ACO algorithm with a two-level ant graph model to solve the surgery-scheduling problem. They also integrated the surgery scheduling, nurse scheduling and, systematic optimization. Akjratikarl et al. (2007) presented an application of Particle Swarm Optimization (PSO) to HHC scheduling without interdependent services. Allaoua et al. (2013) presented a mathematical model that minimizes staff numbers. To solve this problem, they initially proposed an integer linear programming formulation (ILP) and tested this model on small instances. For larger problems, they developed a mat heuristic based on the decomposition of the ILP formulation into two problems. Bredstrom and Ronnqvist (2008) modeled HHC staff scheduling as a routing problem for homogeneous vehicles with precedence and synchronization constraints. Begur et al. (1997) purposed balancing the workload of employees by offering a construction and improvement heuristics. Hsieh et al. (1998) presented a mixed-integer linear programming model (MILP) for the problem. In addition, they presented a heuristic approach. Their purpose was to minimize the cost, associated with overtime and part-time work. Bertels and Fahle (2006) purpose was to combine linear programming, constraint programming, and metaheuristic for solving a Home Health Care (HHC) Routing and Scheduling Problem (HHCRSP). Trautsamwieser and Hirsch (2011) presented mathematical model and a Variable Neighborhood Search (VNS) to solve it. Interdependent services are subjects of Eveborn et al. (2006), Kergosien et al. (2009), Rasmussen et al. (2012), and Monkowska et al. (2014). Eveborn et al. (2006) presented an operational system for staff planning of HCCs called LAPS CARE. They used set-partitioning approach for assignment patients to available staff. Kergosien et al. (2009) considered an HHC problem as a multiple Traveling Salesman Problem (TSP) with additional constraints. Rasmussen et al. (2012) presented a set of partitioning approach and additional side constraints. In addition, they used a branch-and-price algorithm. In recent works, Monkowska et al. (2014) tried to present a mathematical model for HHCRSP that overcomes limitations observed in previous works. The features of their work were heterogeneous staff, different skill for staff, temporal inter-dependencies among services, and balancing cost and service objectives. In addition, respectively qualified caregivers must visit all patients.

In this paper, two groups of services are considered. The first group does not have any predecessor service and the second group has one. The second group services divides into two types. Mankowska et al. (2014) introduced these two types of services. Single services and Double services are considered. A single service includes of one service to be performed by a single staff. A double service includes of two services that are performed by two staff or one if possible; but in the paper patients are allowed to order infinite services that are called multiple services. Multiple services are divided into simultaneous services and services with a given precedence relation. In addition, time window is considered for patients to connote that the service must be started in this time window, otherwise, the service starts with delay. Staff has different skills and can only provide services tailored to these skills. For staff traveling, different vehicles are considered. Therefore, they travel between two distinct patients with different time. Furthermore, assume that the duration of a specific service by a particular staff is not always the same, it depends on the patient’s physical and environmental conditions, and it is closer to reality. In addition, each service for patients is divided into vital service and normal service. For vital services, no delay is allowed, because it threatens health of the patient, but for normal services, delays are allowed with penalty. Unlike all papers, each patient can have infinite services that may have different types of dependency relations together (Multiple services).

3. Mathematical modeling

In the following, a mathematical modeling for HHCRSP with unlimited services for patients is presented. For modeling this problem, each service of each patient is considered a node that sets nodes of each patient are linked together with binary parameters.
3.1. Notation

Indices

- $I$: Indicate node
- $J$: Indicate node
- $K$: Indicate staff
- $S$: Indicate service type

Parameters

- $C0$: Set of all services of patients and central office
- $C$: Set of all services of patients
- $C_v$: Set of vital services (nodes) of patients
- $C_n$: Set of normal services (nodes) of patients
- $C_s$: Set of single services (nodes)
- $C_m$: Set of multiple services (nodes)
- $C_{ms}$: Set of simultaneous multiple services (nodes)
- $C_{ms}$: Set of precedence Multiple services (nodes)
- $d_{ij}$: Distance between any two patients $i$ and $j$
- $V_k$: Velocity of travelling vehicle for staff $k$
- $P_{ks}$: Time to perform service type $s$ by staff $k$
- $P_{si}$: Additional time to perform service type $s$ for patient $i$
- $C_k$: Wage of staff $k$
- $t_{iks}$: Arriving time for staff $k$ in order to perform service type $s$ for patient $i$
- $[e_i, l_i]$: Time window for service $i$
- $[d_{ij}^{\min}, d_{ij}^{\max}]$: Time window for second service in set $C_m$

Decision Variables

- $r_i = \begin{cases} 
1 & \text{if patient } i \text{ requires service type } s \\
0 & \text{otherwise}
\end{cases}$
- $a_{ks} = \begin{cases} 
1 & \text{if staff } k \text{ can perform service type } s \\
0 & \text{otherwise}
\end{cases}$
- $\rho_{is} = \begin{cases} 
1 & \text{if service } i \text{ is prerequisite of service type } s \\
0 & \text{otherwise}
\end{cases}$
- $x_{iks} = \begin{cases} 
1 & \text{if staff member } k \text{ go to patient } j \text{ from patient } i \text{ for service type } s \\
0 & \text{otherwise}
\end{cases}$
- $y_k = \begin{cases} 
1 & \text{if staff member } k \text{ is selected} \\
0 & \text{otherwise}
\end{cases}$
- $z_{is}$: Time window violation (delay) for service $i$ and service type $s$

The staffs start and end their routes at the HCC organization, which refers to node 0. The set of patients’ locations and the HHC organization donated by $C0 = C \cup \{0\}$. The service of patients ($C$) is divided into two subsets: patients who require Single service donated by $C_s$ and patients require multiple service donated by $C_m$. Also $C_m$ has two types: Simultaneous services donated by $C_{ms}$ and precedence services donated by $C_{ms}$. For each node $i$ if $t_{iks}$ is greater than $l_i$, the service perform with delay that is not allowed for vital services. If $t_{iks}$ is less than $e_i$, the staff must wait until $e_i$. Also,
second service \( j \) in \( C \) has a time window \([d_{ij}^{\text{min}}, d_{ij}^{\text{max}}]\) that must be followed after the start of the first service \( i \), this means that for Multiple services, \( d_{ij}^{\text{min}} \) is a lower bound and \( d_{ij}^{\text{max}} \) is an upper bound for start time of second service after start time of first service. It is clear that for simultaneous services \( d_{ij}^{\text{min}} = d_{ij}^{\text{max}} = 0 \). \( P \) depends on the patient's physical and environmental conditions.

For formulating the problem with unlimited services for each patient, we considered each service of a patient together, we used a binary variable \( \rho_{jis} \) that is equal to one, if service \( j \) with service type \( s \) must be done after service \( i \) (service \( i \) is prerequisite of service \( j \)).

### 3.2. MILP formulation

The objective of this problem is minimizing total traveling distance (cost/time), minimizing total delay, and minimizing number of staffs. For this purpose, we present the following objective function and constraints that are similar with Mankowska et al. (2014) modeling.

\[
\min Z = w_1 \times \sum_{i \in C} \sum_{j \in C} \sum_{k \in K} d_{ij} x_{i j k s} + w_2 \times \sum_{i \in C} \sum_{s \in S} z_{is} + w_3 \times \sum_{k \in K} c_k y_k
\]  

subject to:

\[
\sum_{i \in C} \sum_{s \in S} x_{0iks} = \sum_{i \in C} \sum_{s \in S} x_{0iks} = y_k \quad \forall k \in K \quad (2)
\]

\[
\sum_{j \in C} \sum_{s \in S} x_{ijks} = \sum_{j \in C} \sum_{s \in S} x_{ijks} \quad \forall k \in K, \forall i \in C, i \neq j \quad (3)
\]

\[
\sum_{i \in C} \sum_{k \in K} \sum_{s \in S} x_{ijks} = 1 \quad \forall j \in C, i \neq j \quad (4)
\]

\[
x_{i j k s} = 0 \quad \forall s \in S, \forall i \in C_0, \forall k \in K \quad (5)
\]

\[
\sum_{j \in C} \sum_{k \in K} a_{ks} x_{jkis} = r_{is} \quad \forall s \in S, \forall i \in C, i \neq j \quad (6)
\]

\[
t_{i k s} + P_{k s i} + P_{s i} + \frac{d_{ij}}{v_k} \leq t_{j k s} + M(1 - x_{ijks}) \quad \forall s_1, s_2 \in S, \forall i, j \in C, \forall k \in K \quad (7)
\]

\[
t_{i k s} \geq e_i \quad \forall s \in S, \forall i \in C, \forall k \in K \quad (8)
\]

\[
t_{i k s} \leq l_i + z_{is} \quad \forall s \in S, \forall i \in C_n, \forall k \in K \quad (9)
\]

\[
t_{i k s} \leq l_i \quad \forall s \in S, \forall i \in C_v, \forall k \in K \quad (10)
\]

\[
t_{i k_2 s_2} \geq t_{i k_1 s_1} + d_{ij}^{\text{min}} \times \rho_{jis} \quad \forall i \in C^{mp}, k_1, k_2 \in S \quad (11)
\]

\[
t_{i k_2 s_2} \leq t_{i k_1 s_1} + d_{ij}^{\text{max}} \times \rho_{jis} + M \left(1 - \rho_{jis}\right) + z_{is_2} \quad \forall i \in C^{mp}, k_1, k_2 \in S \quad (12)
\]

\[
t_{i k_2 s_2} \geq t_{i k_1 s_1} \times \rho_{jis} \quad \forall i \in C^{ms}, k_1, k_2 \in K, s_1, s_2 \in S \quad (13)
\]
\[
t_{ik}s_{k_2} \leq t_{ik}s_{k_1} \times \rho_{j_{k_2}s_2} + M \left(1 - \rho_{j_{k_2}s_2}\right) + z_{is_2} \quad \forall i \in C_{ms}, k_1, k_2 \in S
\]

\[
K, s_1, s_2 \in S
\]

\[
\sum_{i \in C} \sum_{j \in C} \sum_{s \in S} x_{ijs} \leq M y_k \quad \forall k \in K
\]

\[
x_{ijs} \in \{0, a_{ks} \times \tau_{js}\}
\]

\[
t_{iks} \geq 0
\]

\[
z_{is} \geq 0
\]

The Eq. (1) is an objective function that consists of three terms. First term is total traveling distance. The second is delays and the last one is wage of staff. For having a reasonable proportional, \(W_1 = 2\), \(W_2 = 5\) and \(W_3 = 1\) are used as weights of objective function terms. Constraint (2) states that routes of each staff must start and end in HHC organization, if he/she has been employed by organization for this day. Constraint (3) and constraint (4) are inflow-outflow conditions, which ensure that a staff \(k\), who visits patient \(i\), has to leave this patient after the service. Constraint (5) ensures that each service must be done just once. Constraint (6) states that every required service \(s\) is assigned to exactly one qualified staff. Constraint (7) determines the start times of the service with respect to service durations and traveling times and velocity of staff. Constraint (9) to (14), are related to time windows. Constraint (15) states that staff \(k\) can do services if employed by HHC organization. Constraint (17) states that \(x_{ihs} = 1\) if staff \(k\) can perform service type \(s\). Other constraints are non-negativity constraints.

### 3.3. An illustrative example

An example with five patients, four staffs, and three types of services are illustrated in this section. Patients 1 and 2 require Single service and other patients require multiple service. Patients 3 and 5 require services 2 and 3 respectively. Patient 4 requires service 4 that two first her/his services (node 4 and node 7) have similar service type. Thus, 11 nodes are needed for problem that node 1 to 5 are related to the first service of patients, node 6 is related to the second service of patient 3. In addition, node 7, 8, and 9 are related to second, third and fourth service of patient 4 respectively, and node 10 and 11 are related to the second and third services of patient 5 respectively. Also, nodes 2 (first service to patient 2), 7 (second service of patient 4), and 11 (third service of patient 5) are vital services. All sets of patients are shown in Table 1.

<table>
<thead>
<tr>
<th>Set</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cv</td>
<td>2,7,11</td>
</tr>
<tr>
<td>Cn</td>
<td>1,3,4,5,6,8,9,10</td>
</tr>
<tr>
<td>Cs</td>
<td>1,2</td>
</tr>
<tr>
<td>Cm</td>
<td>6,7,8,9,10,11</td>
</tr>
<tr>
<td>Cmx</td>
<td>6,8,9</td>
</tr>
<tr>
<td>Cmp</td>
<td>7,10,11</td>
</tr>
</tbody>
</table>

Staff information consist of velocity of her/his vehicle, wage and skills are presented in Table 2 and patients’ information consist of time window, service type requirement and type of multiple services are tabulated in Table 3. In Table 2, skill column is set of service types that staff \(k\) can do them. As shown in Table 3, node 6 that is related to patient 3 must start with node 3 (simultaneous with 6), that it means two services of patient 3 must start simultaneously by two staff. Also, node 7 must start after node 4 (precedence with 4), node 8 must be started with node 7 (simultaneously with 7), node 9 must start with node 4 (simultaneously with 4), that it means first and forth services of patient 4 must start simultaneous and second and third services of patient 4 must start simultaneously at
least 60 minutes after the first service of patient 4. For patient 5; node 10 must start after node 5 and node 11 must start after node 10 that it means all services of patient 5 are separated. Note that time window for Multiple services in this table is $[d_{i}^{\text{min}}, d_{i}^{\text{max}}]$. All patients randomly located in an area of 500×800 distance unit. Traveling distances are assumed Euclidean. Also, service time durations are randomly generated.

Table 2
Information of staffs

<table>
<thead>
<tr>
<th>Staff number</th>
<th>Velocity per min</th>
<th>Skill(s)</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>1,3</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>2,3</td>
<td>800</td>
</tr>
</tbody>
</table>

In Table 3, time window for services 6 to 9 are denotes by $[d_{i}^{\text{min}}, d_{i}^{\text{max}}]$. Also, note that service type is related with skill of staffs and is different from service (node) number.

Table 3
Information of patients

<table>
<thead>
<tr>
<th>Node number</th>
<th>Time window</th>
<th>Service type requirement</th>
<th>Multiple service type and link</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[10,50]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>[100,150]</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>[20,100]</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>[20,40]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>[50,80]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>[0,0]</td>
<td>1</td>
<td>Simultaneous with 3</td>
</tr>
<tr>
<td>7</td>
<td>[60,90]</td>
<td>2</td>
<td>Precedence with 4</td>
</tr>
<tr>
<td>8</td>
<td>[0,0]</td>
<td>3</td>
<td>Simultaneous with 7</td>
</tr>
<tr>
<td>9</td>
<td>[0,0]</td>
<td>2</td>
<td>Simultaneous with 4</td>
</tr>
<tr>
<td>10</td>
<td>[50,90]</td>
<td>1</td>
<td>Precedence with 5</td>
</tr>
<tr>
<td>11</td>
<td>[20,50]</td>
<td>1</td>
<td>Precedence with 10</td>
</tr>
</tbody>
</table>

This example is coded in GAMS and results show all staff are employed, the rout for each one is shown in Table 4 and Fig. 1. In this figure, loop arc is used if the staff that performs one service of one patient and must perform another service of the same patient after the first service. All staff’s routs are quite reasonable and all services performed are correct. Total distance traveling is 3420, total delay is 170 minutes, and wage of all staff is 2600 dollars.

Table 4
Routs of staffs

<table>
<thead>
<tr>
<th>Staff No.</th>
<th>Visited node (virtual)</th>
<th>Visited patient (real)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-5-11-0</td>
<td>0-5-5-0</td>
</tr>
<tr>
<td>2</td>
<td>0-6-10-0</td>
<td>0-3-5-0</td>
</tr>
<tr>
<td>3</td>
<td>0-9-7-0</td>
<td>0-4-4-0</td>
</tr>
<tr>
<td>4</td>
<td>0-3-1-2-4-8-0</td>
<td>0-3-1-2-4-4-0</td>
</tr>
</tbody>
</table>
4. Metaheuristic approach

Because the runtime increases exponentially for large cases, we used Simulated Annealing (SA) algorithm for large instances. SA algorithm for the problem is coded in MATLAB software.

4.1. Coding Scheme and Initial Solution

We used a single part discrete coding scheme with delimiter as follow in order to specify services for each staff. We generated a random permutation in size \( I + J - 1 \) that \( I \) stands for a number of services and \( J \) is used as a number of staff.

For example, given a problem with \( I = 11 \) and \( J = 4 \), a random permutation is shown in Fig. 2.

![Random permutation](Fig. 2)

Elements of permutation that are bigger than \( I \) are locations of delimiters and elements between delimiters are routes for each staff respectively. This permutation shows that route of staff 1 is 2, 5 and 1, route of staff 2 is 10, 3 and 8, route of staff 3 is null and finally route of staff 4 is 6, 4, 7, 11 and 9. This is an initial solution and must repair proportional skills of staff and interdependence services. After this operation, the solution is ready to calculate violations.

4.1 Neighborhood Search Structures

After generating random permutation, we used Swap, Reversion, and Insertion operators in order to improve the initial permutation gradually. Swap operator selects two elements of permutation and substitutes their positions. This operator is shown in Fig. 3.

![Swap operator](Fig. 3)

Reversion operator selects two elements of permutation and reverses all elements between them randomly. This operator is shown in Fig. 4.

![Reversion operator](Fig. 4)

Insertion operator selects two elements of permutation and moves one after another arbitrary. This operation is shown in Fig. 5.
4.2 Tuning SA Parameters

To achieve proper value of SA parameters, we run program several times with different parameters. Best value of SA parameters is given in Table 5.

**Table 5**

<table>
<thead>
<tr>
<th>SA parameters</th>
<th>Max iteration for algorithm</th>
<th>Number of neighborhood search in each internal loop</th>
<th>Initial Temp.</th>
<th>Temperature Damping Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150</td>
<td>10</td>
<td>100</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Fig. 5.** Insertion operator

**Fig. 6.** SA algorithm steps

4.3 SA Algorithm Steps

In Fig. 6, we draw SA algorithm steps, in this figure $T_0$ is initial temperature, $S_0$ is initial solution, $f_i$ is objective function value for solution $S$, $I_1$ is max iteration for algorithm, $I_2$ is a number of neighborhood searches in each iteration and $\alpha$ is damping rate for temperature.
5. Computational results

We generate 45 instances to test our Meta-Heuristic approach (P₁ to P₄₅). For generating these instances, we generate position of patients, duration of service time, staff wage, and velocity of staff randomly as shown in Table 6.

Table 6
Probability function of instance parameters

<table>
<thead>
<tr>
<th></th>
<th>Vk ~ U (10,20)</th>
<th>P₁ ~ U (0,10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ck ~ U (200,800)</td>
<td>Xi ~ U (0,2000)</td>
</tr>
<tr>
<td></td>
<td>Pks ~ U (10,40)</td>
<td>Yi ~ U (0,1000)</td>
</tr>
</tbody>
</table>

Results of the computation to test the problems are summarized in Table 7 and Figs. (7-9).

Table 7
Result of test problems

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Number of services</th>
<th>Number of staffs</th>
<th>Number of service types</th>
<th>Objective value</th>
<th>Time (s)</th>
<th>Objective value</th>
<th>Time (s)</th>
<th>GAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>10413</td>
<td>16</td>
<td>9798</td>
<td>6</td>
<td>0.063</td>
</tr>
<tr>
<td>P₂</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>17993</td>
<td>20</td>
<td>16255</td>
<td>10</td>
<td>0.107</td>
</tr>
<tr>
<td>P₃</td>
<td>8</td>
<td>4</td>
<td>5</td>
<td>15109</td>
<td>12</td>
<td>14608</td>
<td>110</td>
<td>0.034</td>
</tr>
<tr>
<td>P₄</td>
<td>8</td>
<td>5</td>
<td>4</td>
<td>13607</td>
<td>12</td>
<td>13408</td>
<td>186</td>
<td>0.015</td>
</tr>
<tr>
<td>P₅</td>
<td>9</td>
<td>3</td>
<td>4</td>
<td>13886</td>
<td>14</td>
<td>12969</td>
<td>151</td>
<td>0.079</td>
</tr>
<tr>
<td>P₆</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>14740</td>
<td>15</td>
<td>13497</td>
<td>545</td>
<td>0.092</td>
</tr>
<tr>
<td>P₇</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>19274</td>
<td>19</td>
<td>17185</td>
<td>650</td>
<td>0.122</td>
</tr>
<tr>
<td>P₈</td>
<td>10</td>
<td>3</td>
<td>4</td>
<td>15321</td>
<td>22</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₉</td>
<td>15</td>
<td>3</td>
<td>4</td>
<td>17714</td>
<td>29</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₀</td>
<td>15</td>
<td>5</td>
<td>4</td>
<td>23339</td>
<td>29</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₁</td>
<td>15</td>
<td>5</td>
<td>5</td>
<td>25480</td>
<td>31</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₂</td>
<td>20</td>
<td>5</td>
<td>5</td>
<td>29186</td>
<td>27</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₃</td>
<td>25</td>
<td>5</td>
<td>5</td>
<td>41624</td>
<td>36</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₄</td>
<td>25</td>
<td>5</td>
<td>6</td>
<td>24614</td>
<td>48</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₅</td>
<td>30</td>
<td>5</td>
<td>5</td>
<td>42619</td>
<td>48</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₆</td>
<td>30</td>
<td>6</td>
<td>6</td>
<td>36753</td>
<td>37</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₇</td>
<td>35</td>
<td>5</td>
<td>5</td>
<td>66357</td>
<td>54</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₈</td>
<td>35</td>
<td>6</td>
<td>7</td>
<td>50546</td>
<td>64</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₁₉</td>
<td>40</td>
<td>6</td>
<td>6</td>
<td>66979</td>
<td>56</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₀</td>
<td>40</td>
<td>6</td>
<td>7</td>
<td>69953</td>
<td>107</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₁</td>
<td>45</td>
<td>6</td>
<td>6</td>
<td>72122</td>
<td>72</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₂</td>
<td>45</td>
<td>7</td>
<td>7</td>
<td>59725</td>
<td>57</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₃</td>
<td>50</td>
<td>7</td>
<td>7</td>
<td>111882</td>
<td>54</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₄</td>
<td>50</td>
<td>7</td>
<td>8</td>
<td>84904</td>
<td>43</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₅</td>
<td>60</td>
<td>7</td>
<td>7</td>
<td>123282</td>
<td>101</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₆</td>
<td>60</td>
<td>7</td>
<td>8</td>
<td>161872</td>
<td>127</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₇</td>
<td>70</td>
<td>7</td>
<td>8</td>
<td>134794</td>
<td>102</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₈</td>
<td>70</td>
<td>7</td>
<td>9</td>
<td>158870</td>
<td>128</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₂₉</td>
<td>80</td>
<td>8</td>
<td>7</td>
<td>195005</td>
<td>137</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₀</td>
<td>80</td>
<td>8</td>
<td>10</td>
<td>152887</td>
<td>133</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₁</td>
<td>90</td>
<td>8</td>
<td>8</td>
<td>233624</td>
<td>163</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₂</td>
<td>90</td>
<td>8</td>
<td>11</td>
<td>227747</td>
<td>186</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₃</td>
<td>100</td>
<td>8</td>
<td>9</td>
<td>206394</td>
<td>184</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₄</td>
<td>100</td>
<td>9</td>
<td>12</td>
<td>198412</td>
<td>246</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₅</td>
<td>120</td>
<td>9</td>
<td>10</td>
<td>288111</td>
<td>254</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₆</td>
<td>120</td>
<td>10</td>
<td>13</td>
<td>205430</td>
<td>97</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₇</td>
<td>140</td>
<td>9</td>
<td>11</td>
<td>345227</td>
<td>133</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₈</td>
<td>140</td>
<td>11</td>
<td>14</td>
<td>328155</td>
<td>130</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₃₉</td>
<td>160</td>
<td>10</td>
<td>12</td>
<td>599200</td>
<td>169</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₀</td>
<td>160</td>
<td>13</td>
<td>15</td>
<td>336811</td>
<td>178</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₁</td>
<td>180</td>
<td>11</td>
<td>13</td>
<td>613111</td>
<td>610</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₂</td>
<td>180</td>
<td>15</td>
<td>16</td>
<td>470437</td>
<td>385</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₃</td>
<td>200</td>
<td>13</td>
<td>14</td>
<td>632630</td>
<td>554</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₄</td>
<td>200</td>
<td>16</td>
<td>17</td>
<td>643163</td>
<td>475</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>P₄₅</td>
<td>250</td>
<td>15</td>
<td>15</td>
<td>988789</td>
<td>484</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{GAP} = \frac{\text{objective value(GAMS)} - \text{objective value(SA)}}{\text{objective value(GAMS)}} \]
As the results are shown in the Table 7, GAMS is not able to solve the medium and large size problems ($P_8$ to $P_{45}$).

6. Sensitivity analysis

In this section, sensitivity analysis for the parameters of the problem is conducted. To find important parameters in the problem, a specific problem with different parameter value is considered and the effect of each parameter on objective value and run time is determined. Different scenarios for sensitivity analysis are presented in Table 8.

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Different scenarios for sensitivity analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Add new patients with any relation (Cs)</td>
</tr>
<tr>
<td>2</td>
<td>Add new staffs</td>
</tr>
<tr>
<td>3</td>
<td>Add new service types</td>
</tr>
<tr>
<td>4</td>
<td>Increase vital services</td>
</tr>
<tr>
<td>5</td>
<td>Decrease Single services and increase relation between services</td>
</tr>
<tr>
<td>6</td>
<td>Add new patients with any relation and add new staffs</td>
</tr>
<tr>
<td>7</td>
<td>Add new patients with any relation and new service types</td>
</tr>
<tr>
<td>8</td>
<td>Add new staffs and new service types</td>
</tr>
</tbody>
</table>

For each scenario 10 test problems are randomly generated and results are presented with charts. For scenario 1, the results are presented in Fig. 10. The results show the increase in both objective value and run time. The Results of scenario 2 are shown in Fig. 11 and show that the objective value has been reduced, but runtime is relatively greater. Results for scenario 3 and scenario 4 are shown in Fig. 12 and Fig. 13, respectively. The results show that neither run time nor objective value have been changed much for these scenarios. For Scenario 5, the results are presented in Fig. 14. The results show that objective value decreased because total traveled distance is shorter and run time increased due to the increase in complexity of problem. Scenarios 6 to 8 are combined. The results of scenario 6 and scenario 7 are shown in Fig. 15 and Fig. 16, respectively. The results show that objective value increased because total traveled distance increased and also run time increased. The results of scenario 8 are shown in Fig. 17. The results show that neither run time nor objective value have been changed a lot.
Fig. 11. Objective value and run time for scenario 2

Fig. 12. Objective value and run time for scenario 3

Fig. 13. Objective value and run time for scenario 4

Fig. 14. Objective value and run time for scenario 5

Fig. 15. Objective value and run time for scenario 6
According to the results, it can be concluded that the most important factors that have an impact on the objective value and the run time of the problems are number of patients and number of patients with interdependent Multiple services.

7. Conclusion

Focusing on real setting, a mathematical model has been presented for home health care routing and scheduling problem with interdependent Multiple services and time windows. Each patient has been permitted to order infinite service types with any relation between her/his services. The problem with infinite services for each patient has been coded in GMAS software. In addition, for population that GAMS is unable to solve them, SA algorithm has been applied. The results showed that when each patient can order infinite services with any relation between them, complexity of problem increases, but SA algorithm could solve large instance with reasonable solution in the least time. Thus, SA algorithm showed a good performance for large instances. Results in section 5 showed the most important factors that affect the objective value and the run time of the problems, which are number of patients and number of patients with interdependent multiple services. Planning for more than a day can be developed as a future research. Considering revenue and multi-objective for the problem are other further research insights.

References


© 2020 by the authors; licensee Growing Science, Canada. This is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (http://creativecommons.org/licenses/by/4.0/).