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An adaptive large neighborhood search heuristic for solving the reliable multiple allocation hub location problem under hub disruptions

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ABSTRACT

The hub location problem (HLP) is one of the strategic planning problems encountered in different contexts such as supply chain management, passenger and cargo transportation industries, and telecommunications. In this paper, we consider a reliable uncapacitated multiple allocation hub location problem under hub disruptions. It is assumed that every open hub facility can fail during its use and in such a case, the customers originally assigned to that hub, are either reassigned to other operational hubs or they do not receive service in which case a penalty must be paid. The problem is modeled as two-stage stochastic program and a metaheuristic algorithm based on the adaptive large neighborhood search (ALNS) is proposed. Extensive computational experiments based on the CAB and TR data sets are conducted. Results show the high efficiency of the proposed solution method.

1. Introduction

Hubs are intermediate facilities that perform a set of tasks such as consolidation, break-bulk, sorting, etc. in transportation and telecommunication networks. In other words, the traffic flows (cargo, passengers, or data) in the network rather than being sent directly from their origins to their destinations, are routed via these intermediate facilities. Therefore, smaller number of connections with large flow volumes are used in the network which, in turn, makes it possible to exploit economies of scale in transportation costs, especially on the inter-hub connections.

Hub location problem (HLP) deals with locating the hub facilities in the network and determine the pattern based on the non-hub nodes assignment to each hub so that a specific objective function is optimized. Regarding the non-hub nodes assignment to hubs in the HLP, we have two types of allocations. First type, called single allocation, each non-hub node can be allocated to exactly one hub in the network, whereas in the second type, called multiple allocation, each non-hub node can simultaneously be allocated to more than one hub in the network. In both mentioned schemes, the hub

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nodes as well as the network links can have limited or no capacity. Therefore, HLPs are divided into four main categories in the literature:

I. Capacitated single allocation hub location problem
II. Uncapacitated single allocation hub location problem
III. Capacitated multiple allocation hub location problem
IV. Uncapacitated multiple allocation hub location problem

HLPs are frequently used in some industries such as transportation, communication and computer network design. Most of the HLP studies, assume that all of the established hubs in network function perfectly well throughout the planning horizon and hence they will be accessible to all the customers. However, the infrastructures of supply chains are always under risk of disruption due to environmental, technological and international damages. Natural disasters like flood, hurricane and earthquake can affect extensive geographical areas and make transportation and other network elements un-operational (Matisziw et al., 2010).

As the functions such as consolidation, break-bulk, and sorting are executed at hub nodes, disruption at hubs can result in high costs for the customers as well as the network operators. Therefore, reassignment of the customers from disrupted hubs to the non-affected hubs is of crucial importance for reducing network costs. In this paper, we have considered the reliable uncapacitated multiple hub location problem under hub disruption. It is assumed that when a disruption happens at each hub and makes it un-operational, all the assigned non-hub nodes to the disrupted hub should be reassigned to other operational hubs or if the costs for serving these nodes via the operational hubs are too high, then serving these nodes can be cancelled and a penalty is paid for each unit not served. We have modeled different possibilities of hub disruption using a group of scenarios in which a random subset of hubs is un-operational due to disruptions. The problem has been modeled as a two-stage stochastic program in which the decisions on hub locations are made in the first phase. In second phase when disruption scenario has occurred, the allocation of non-hub nodes to hubs takes place in second phase with regard to the operational hubs. To solve the proposed model, a metaheuristic algorithm which is based on the Adaptive Large Neighborhood Search (ALNS) is presented and the effectiveness of this algorithm is tested by solving large set of instances from the CAB and TR data sets.

The remainder of this paper is organized as follows. The literature review is presented in the next section. The mathematical models are presented in third section. Section four describes the proposed solution algorithm in detail. Numerical results are presented in section five and finally, conclusions and some directions for future research are presented in last section.

2. Literature

O'Kelly (1986) presented first mathematical formulation for the single allocation p-hub median problem as quadratic model. Mixed integer linear models for different versions of the single and multiple allocation hub location problem such as p-hub median, p-hub center and hub covering problems were proposed for the first time by Campbell (1994). Later, other mathematical formulation for hub location problem were proposed by Ernst and Krishnamoorthy (1994), Skorin-Kapov et al. (1996) and Ebery (2001).

Many studies on HLP assumed that the established hubs would always be operational. Nevertheless, these facilities may fail due to different reasons in practice. As an example, unexpected weather conditions can adversely affect the availability of an airport serving as a hub in air transportation industry. The same problem can occur in supply chain and logistics systems, where facilities, same as hubs, play the central role and their locations are derived using facility location models (An et al., 2014). Therefore, considering reliability in HLP is of utmost importance. Snyder and Daskin (2005) study facility location in which some of cases with definite probability become unusable and assume that customers would be served by facilities which are not affected by disruption. Berman et al. (2009) and Shen et al. (2011) who are inspired by this model developed new location problem models with disruption consideration. They supposed that facilities are not completely reliable and customers do not have any information about a facility being operational or not and it is supposed that every facility may be non-operation by a definite probability. Shen et al. (2011) present reliability subject in this area where some facilities are disrupted temporarily. If a facility becomes defected, other allocated customers shall be reassigned to other operational facility. Authors develop 2 step stochastic program in a non-linear integer model. Wang and Ouyang (2013) present continuous probability approach in order to identify competitive facility location at risk of disruption condition. They use models related to games theory in order to optimize location of facility services in condition of facilities competition and facility disruption risk. They believe that customer demand share in market depends on server facilities performance and competitor presence in closed place because customers are usually following up nearest way. Author's model which are based on game's theory, merge these complicated factors in an integrated framework. They use experimental and hypothesis data in order to evaluate their suggestive models and monitored impact of competition, disruption risk in facilities and transportation cost on optimized plan.

Medal et al. (2014) present a multi-objective model for the facility problem. They use two methods for decreasing the risk of disruption: a) identifying facility location strategically, and b) using the rigid and reliable facilities. Authors merge these two theories in their suggestive model decline farthest distance from demand points to nearest available facility after disruption in facilities. It is supposed that decider is reluctant to the risk and eager to decrease disruption of facilities with maximum output therefore a multi objective mixed integer model is suggested. Matisziw et al. (2010) present a multi objective optimized model for the first time in order to restoration network after disruption and time scheduling of possible restoration scenario when network nodes and arcs are lost and disruption takes place.

In order to enhance the reliability of hub location problems, Kim (2008) proposes a single allocation p-hub protection with primary and secondary routes. Kim and O'Kelly (2009) propose single and multiple allocation models to derive an optimal network structure that maximizes the expected network flow given that each arc or hub has a given reliability. Their work does not consider backup hubs and alternative routes and a tabu search heuristic is utilized to solve the real instances with up 20 nodes. Zeng et al. (2010) address the reliable single and multiple allocation hub location models by considering hub unavailability where alternative routes have been developed and a heuristic algorithm has been proposed. In another study, Taghipourian et al. (2012) studied hub services to non-hub nodes in forecasted disruption. In this research they considered some non-hub nodes as a virtual hub, so as main disrupted hubs should be closed and virtual ones open and serves other hubs in forecastable inappropriate weather condition and other forecastable disruptions. Next, they proceed to present a nonlinear fuzzy mathematical model in order to decrease costs. Parvaresh et al. (2013) study imperative disruption of hubs. They model their study as Steklberg game that includes a leader and follower as two steps with consideration of bi-objective model. First objective function is used in order to minimize means of transportation cost and second objective function is used to maximize cost of disruption imposed on network. Parvaresh et al. (2014) develop model presented in 2013 and present it in two-level with three objective functions and add a decision variable with two other constraints to modify solution algorithm. In a newer research, Azizi et al. (2014) propose a new formulation with conservation of different theories. In this research one backup hub is selected from the available hub of network for disrupted hub and all of allocated flows of defected hub are allocated to backup hub. An et al. (2015) propose a set of reliable hub-and-spoke network design models where the selection of backup hubs and alternative routes are taken into consideration to proactively handle hub disruptions. They develop Lagrangian relaxation and Branch-and-Bound methods to solve these nonlinear mixed integer formulation. More recently, Mohammadi et al. (2016) introduce a different perspective to reliable hub-and-spoke network design. The authors categorize the disruption into two classes: a) complete disruption (accessibility disruption), and b) partial disruption (capacity disruption). They also assume that hub network is incomplete and the connections between the hubs is tree.

As hub location problem is an NP-hard problem, exact solutions for the large and real-sized instances are very time consuming and even sometimes impossible to reach. O'Kelly (1985) propose two solution methods for solving the p-hub median problems. Both of these algorithms consider all of possible scenarios for choosing p-hub locations. In first algorithm, demand nodes are allocated to nearest hub and in second method, allocations are determined based on the value of the objective function between the first nearest and the second hub. He used CAB data set to test his solution method. Metaheuristic methods have successfully been implemented for solving hub location problems by many researches in this filed. Skorin-Kapov (1994) uses taboo search (TS) algorithm for solving the single allocation p-hub median problem. In another research, Abdinnour-Helm (2001) propose a simulated annealing (SA) solution method for the single allocation p-hub median problem. Later, Perez et al. (2007) present heuristic algorithm as hybrid of two metaheuristics: variable neighborhood search (VNS) and path relinking (PR) for the uncapacitated single allocation HLP. Their algorithm shows a better performance in comparison to the simulated annealing and taboo search described above. Lin et al. (2012) uses genetic algorithm (GA) method for solving the p-hub median problem with integral constraints. Marti et al. (2014) use scatter search in order to solve the uncapacitated p-hub median problem. They also strengthened their algorithm by hybridization with path relinking.

Although every metaheuristic algorithm has its own characteristics but recently adaptive large neighborhood search (ALNS) has been used extensively in routing and allocation problems and in many cases higher quality solutions are obtained in comparison to other metaheuristics on the same problems. The ALNS heuristic which is generalization of the Large Neighborhood Search (LNS) algorithm has been presented for the first time by Ropke and Pisinger (2006) for solving pickup and delivery vehicle routing problem (VRP). Hemmelmayr et al. (2012) use the ALNS for solving two-echelon VRP and the location routing problem (LRP). They show that the solution obtained by the ALNS are better than other solution methods for the two-echelon VRP and excellent results have been obtained in case of the LRP. In another study, Demir et al. (2012) use the ALNS for solving the pollution-routing problem (PRP) and effectiveness of their applied algorithm is demonstrated on a large set of test instances. Mauri (2012) uses the ALNS algorithm for solving the berth allocation problem (BAP). Results show that the ALNS improve the best known solutions in so many cases in comparison with other algorithms which are used for solving the same problem. More recently, Grangier et al. (2016) use this algorithm for solving the two-echelon multiple-trip vehicle routing problem and obtained superior solutions.

3. Mathematical formulations

Let $G=(N,E)$ be a graph, where N is the set of nodes and E is the set of edges such that $E \subseteq N \times N$. A subset $J \subseteq N$ of nodes would be selected as the hubs with remaining $|N|-|J|$ spokes being allocated to these hubs. The following parameters are used in our model:

f_k: fixed cost of establishing a hub at node $k \in N$

w_{ij}: amount of flow originated at node $i \in N$ and destined to node $j \in N$

c_{ij}: transportation cost per unit of traffic between nodes $i \in N$ and $j \in N$

α : discount factor ($0 \le \alpha \le 1$) representing the scale economies on the inter-hub connections

c_{ijkm}: unit transportation cost between nodes $i \in N$ and $j \in N$ that is routed via hubs $k \in N$ and $m \in N$ calculated as:

$$C_{ijkm} = c_{ik} + \alpha c_{km} + c_{mj}$$

θ : unit penalty cost for the traffic that is not routed because of hub disruptions.

$I_{k(\xi)}$: binary parameter representing the operational status of hubs equal to 1 if hub k is operational and equal to 0 if the hub is disrupted.

We define the following sets of decision variables:

- $Z_k \in \{0,1\}$ is 1 if a hub is opened at node $k \in N$ and 0, otherwise;
- $Y_{km} \in \{0,1\}$ is 1 if node $i \in N$ is assigned to the hub node $k \in N$ and 0, otherwise
- $X_{ijkm}(\xi) \geq 0$ is the fraction of flow originated from origin i and destination to node j $(i, j \in N)$ that is routed through hubs located at nodes k and $m\,(k, m \in N)$ in that order.
- $V_{ij}(\xi) \geq 0$ is the fraction of flow from origin i to destination j $(i, j \in N)$ that is not routed (for which penalty cost is incurred).

Based on the parameters and the variables define above, the two-stage stochastic programming model for our problem can be written as follows:

$$\min \sum_k f_k Z_k + E[Q(Z,\xi)] \tag{1}$$

subject to:

$$Z_k \in \{0,1\} \qquad \forall\, k \tag{2}$$

where

$$Q(Z,\xi) = \min \sum_i \sum_j \sum_k \sum_m w_{ij} c_{ijkm.} X_{ijkm}(\xi) + \sum_i \sum_j w_{ij} V_{ij}(\xi) \tag{3}$$

$$\sum_k \sum_m X_{ijkm}(\xi) + V_{ij}(\xi) = 1 \qquad \forall\, i, j \tag{4}$$

$$\sum_m X_{ijkm}(\xi) + \sum_{m|m\neq k} X_{ijmk}(\xi) \leq Z_k \qquad \forall\, i, j, k \tag{5}$$

$$\sum_m X_{ijkm}(\xi) + \sum_{m|m\neq k} X_{ijmk}(\xi) \leq I_k(\xi) \qquad \forall\, i, j, k \tag{6}$$

$$X_{ijkm}(\xi) \geq 0 \qquad \forall\, i, j, k, m \tag{7}$$

$$V_{ij}(\xi) \geq 0 \qquad \forall\, i, j \tag{8}$$

In above the formulation E_ξ denotes the mathematical expectation with respect to ξ and Ξ is the support of ξ. The objective function (1) minimizes the sum of the first-stage cost of opening hub facilities and the transportation cost and penalty (as calculated in equation (3)). Constraint (4) state that each origin-destination flow must either be routed via some pair of hubs or a penalty must be occurred if it is not routed. Constraints (5) and (6) prohibit commodities from being routed via an unopened hub or a disrupted hub, respectively. Finally Eq. (2), Eq. (7) and Eq. (8) are domain constraint for associated decision variables.

Let the uncertainty associated with operational status of the hub facilities be described by a finite set of scenarios $(s \in S)$ each of which having a probability (p_s) that is assumed to be known. Under each scenario s, denote the realized value of the random variable $I_k(\xi)$ as $I_{k,s}$. We can now write the so-called extensive from or the deterministic equivalents of the above two-stage stochastic problem as follows.

$$\min \sum_k f_k Z_k + \sum_s p_s \left(\sum_i \sum_j \sum_k \sum_m w_{ij}.C_{ijkm.} X^s_{ijkm} + \sum_i \sum_j \theta.w_{ij}.V^s_{ij} \right) \tag{9}$$

$$\sum_k \sum_m X_{ijkm}^s + V_{ij}^s = 1 \qquad \forall\, i,j,s \qquad (10)$$

$$\sum_m X_{ijkm}^s + \sum_{m|m\neq k} X_{ijkm}^s \leq Z_k I_{k,s} \qquad \forall i,j,k,s \qquad (11)$$

$$X_{ijkm}^s, V_{ij}^s \geq 0 \qquad \forall i,j,k,m,s \qquad (12)$$

$$Z_k \in \{0,1\} \qquad \forall\, k \qquad (13)$$

4. Solution Method

As mentioned earlier, the ALNS metaheuristic is a generalized version of the LNS algorithm and was first presented by Ropke and Pisinger (2006) for solving the pickup and delivery VRP. The LNS method which has been presented first time by Shaw (1997) for solving the VRP, try to improve initial solutions of a combinatorial optimization problem by changing the solutions locally one at a time. Since the selection of neighborhood directly affects the process of generating new solutions within the search space, it should be handled carefully and in a smart manner. Let x be a feasible solution to our reliable hub location problem and X be the set of all feasible solutions to this problem. For each solution $x \in X$ we define a neighborhood $N(x) \subseteq X$ as a function $N : X \rightarrow P(X)$. In neighborhood search method of function N is created by combination of destroy and repair operators. The basic idea behind this method is that some part of solution is destroyed and then is repaired in the following steps. The main purpose of the destroy operator is to remove a part of a given solution so that the repair operator could rebuild that part resulting in a new solution (Lutz, 2015). Unlike the LNS algorithm in which only one destroy and one repair operator is used, the ALNS is able to use several operators for the repair and several operators for the destroy functions, simultaneously. Then algorithm will allocate a weight for each operator that reflects success level of related function in the previous steps. Operator selection is random in each stage and will be according to related weights. If $D = \{ d_i | i = 1,...,k \}$ is a group of "*k*" destroy operators and $R = \{ r_i | i = 1,...,l \}$ is a group of "*l*" repaired operators and primary weights of operators are defined as $w(d_i)$ and $w(r_i)$, so operator selecting probability is as below:

$$P(d_i) = \frac{w(d_i)}{\sum_{j=1}^{k} w(d_j)} \qquad P(r_i) = \frac{w(r_i)}{\sum_{j=1}^{l} w(r_j)}$$

Adjusting the operators weights plays essential role for increasing the probability of using more successful functions in comparison with less successful ones. Success of a given operator varies for different problems. Other factors such as instance size can also affect the usefulness of an operator in the same problem.

In order to solve the reliable hub location problem using the ALNS algorithm, we developed and used four destroy and three repair operators. For the proposed algorithm, destroy 1, 2 and 3 operators are able to be used in combination with any of the repair 1 and 2 operators. However, the destroy 4 and repair 3 operators are used together.

4.1 Destroy 1

Destroy 1 operator randomly selects 40% of the opened hubs in the solution and changes them to non-hub nodes, the goal of this operator is to destroy the hub location part of the solutions.

4.2 Destroy 2

In destroy 2 operator, 40% of the non-hub nodes are randomly selected and then are changed to hub nodes. The goal of this operator is to destroy the part of solutions which determines the non-hub nodes as well as their allocation to the hub nodes.

4.3 Destroy 3

Destroy 3 operator randomly selects 60% of entire nodes in the netwok and changes their status randomly. In other words, the selected hub nodes are changed to non-hubs or stay as hub with equal probabilities (0.5). Also the status of a selected non-hub node is changed to or still stay as a non-hub node with equal probability.

4.4 Repair 1

In repair 1 operator, the average failure probability of each node in the network is estimated based on the realized scenario matrix. Then, 10% of hubs in the solution obtained from the destroy heuristics are randomly selected. For every selected hub if the failure probability of that hub is more than the corresponding probability for the nearest node to that hub, then the hub becomes non-hub and the nearest node becomes hub.

4.5 Repair 2

In repair 2 operator, the nodes in the network are sorted based on the total distance from other nodes in non-decreasing order. Then 10% of the hub nodes in the solution (obtained from destroy heuristic) that have the largest total distances are turned to non-hub nodes. On the other hand, the same number of nodes which have the least total distances are turned to hubs.

4.6 Destroy 4 and repair 3

In destroy 4 operator, 20% of hub nodes are randomly selected and for each of these hub nodes, the repair 3 operator calculates the network cost in two cases: a) network cost assuming the considered hub stays as hub, b) network cost assuming that the considered hub becomes a non-hub node. If the cost of case (b) is smaller than case (a), then the node stays as hub, otherwise it is turned into a non-hub node. Results show that the use of destroy 4 operator in combination with the repair 3 operator makes more qualified and successful solutions compared with combination of other destroy and repair heuristics. However, this combination needs more time to be performed compared with the other operator combinations. The pseudo-code for the proposed ALNS algorithm for the reliable hub location problem under hub disruptions is shown in Fig. 1.

Algorithm: Adaptive Large Neighborhood Search (ALNS)

Input: Initial solution $x_0 \in X$, Maximum iterations *MaxIt*,
Current solution $x = x_0$, best solution $x_{best} = x_0$;
while stopping criteria not met **do**
    for $Iter = 1, \ldots, MaxIt$ **do**
        select $r \in R$, $d \in D$ according to probabilities p
        $x = r(d(x))$
        if accept(x, x') **then**
            $x = x'$
            if $f(x) < f(x_{best})$ **then**
                    $x_{best} = x$
        adjust the weights w and probabilities p for the heuristics
return x_{best}

Fig. 1. Pseudo-code for the ALNS algorithm for the reliable hub location problem under hub disruptions

The stopping criterion set for the proposed algorithm is the number of iterations performed. Based on a set of preliminary experiments, it was shown that in most cases setting the maximum of 500 iterations provide a good trade-off between the solution quality and the run time of the algorithm.

5. Numerical experiment

To test the efficiency of the proposed solution algorithm as well as the validity of the proposed mathematical formulation, we have conducted a set of computational experiments. For this purpose, we have employed two famous data sets from the literature of HLP, namely the CAB and TR data sets which have extensively been used in the literature of HLP. The CAB data set is based on the airline passenger interactions between 25 US cities in 1970 evaluated by the Civil Aeronautics Board (CAB). Since the CAB data set does not contain fixed hub establishment, we have used four different values for this parameter as: 100, 150, 200, and 250 like most of the works in the literature. The second data set that is used in our computational experiments is the TR data set which is based on the cargo flows between 81 cities of Turkey. The fixed hub establishment costs in the TR data set are scaled by three different scaling factors (CF) as 0.1, 0.3, and 0.5. For both the data sets, the parameter α is considered at five levels: 0.2, 0.4, 0.6, 0.8, and 1. Mathematical models are solved using ILOG CPLEX 12.6 optimization software and the ALNS algorithm is coded in MATLAB R2013a. All the experiments are conducted on a computer with 3.3-GHz Intel Core i3 CPU and 4-GB of RAM under Windows 7 operating system.

The results obtained by solving the problem on CAB data set using the proposed ALNS heuristics as well as CPLEX for penalty coefficient θ=2000 are presented in Table 1. The first column in this table includes applied discount factor for transportation cost on inter-hub connections. The second column shows fixed cost of hub establishment in the network. Next three columns present the solution results which are obtained by CPLEX. These three columns include the optimum value of objective function, the opened hubs in the optimum solutions, and the CPU time (in seconds) it took to reach the optimum solution. Result for solving the problem by ALNS are shown in next three columns and the last column indicates the optimality gap between the objective value obtained by the ALNS and the corresponding optimal value obtained by CPLEX.

Table 1
Results for the CAB data set with $\theta = 2000$

α	F	CPLEX			ALNS			%GAP
		Opt	Hubs	CPU (s)	Opt	Hubs	CPU (s)	
0.2	100	1251.896	4,12,16,18,22	28.985	1251.896	4,12,16,18,22	5.389	0.00
	150	1434.326	4,18,22	31.394	1434.326	4,18,22	3.262	0.00
	200	1493.734	18	25.021	1493.734	18	2.427	0.00
	250	1543.734	18	25.437	1543.73	18	2.404	0.00
0.4	100	1347.617	4,12,18	26.274	1347.617	4,12,18	3.642	0.00
	150	1443.734	18	23.389	1443.734	18	2.673	0.00
	200	1493.734	18	23.699	1493.734	18	2.283	0.00
	250	1543.734	18	23.10	1543.734	18	2.449	0.00
0.6	100	1371.645	4,18	22.324	1371.645	4,18	2.916	0.00
	150	1443.734	18	22.138	1443.734	18	2.350	0.00
	200	1493.734	18	22.399	1493.734	18	2.187	0.00
	250	1543.734	18	22.621	1543.734	18	2.257	0.00
0.8	100	1380.987	4,18	22.280	1380.987	4,18	2.869	0.00
	150	1443.734	18	21.853	1443.734	18	2.466	0.00
	200	1493.734	18	22.168	1493.734	18	2.497	0.00
	250	1543.734	18	22.00	1543.734	18	2.288	0.00
1	100	1385.640	4,18	22.191	1385.640	4,18	2.718	0.00
	150	1443.734	18	22.432	1443.734	18	2.557	0.00
	200	1493.734	18	23.270	1493.734	18	2.270	0.00
	250	1543.734	18	23.169	1543.734	18	2.270	0.00

We observe from Table 1 that the gap percentage between objective function of two methods is equal to zero for all the instances which shows that the proposed ALNS algorithm is capable of obtaining the optimum solutions. From solution times perspective, it is also seen that all the instances are solved within a time span of maximum five seconds which is an indication of the efficiency of the proposed solution algorithm. Results also indicate that if fixed cost of opening hubs increases, the number of opened hubs in optimum solution will decrease. In the meantime, lower values of discount factor results in increased number of opened hubs in the optimum solution. Numerical results for the CAB dataset based on penalty coefficient values as θ=3000 and θ=4000 are shown respectively in Tables 2 and Table 3.

Table 2
Results for the CAB data set with θ = 3000

α	F	CPLEX			ALNS			%GAP
		Opt	Hubs	CPU (s)	Opt	Hubs	CPU (s)	
0.2	100	1252.452	4,12,16,18,22	28.938	1252.452	4,12,16,18,22	5.590	0.00
	150	1479.197	4,12,18	58.478	1479.197	4,12,18	3.340	0.00
	200	1629.197	4,12,18	79.813	1629.197	4,12,18	3.432	0.00
	250	1753.000	12,18	57.447	1753.000	12,18	2.854	0.00
0.4	100	1374.248	4,12,16,18,22	46.887	1374.248	4,12,16,18,22	5.037	0.00
	150	1548.254	4,12,18	26.951	1548.254	4,12,18	3.476	0.00
	200	1698.254	4,12,18	45.095	1698.254	4,12,18	3.305	0.00
	250	1788.190	18	30.934	1788.190	18	2.216	0.00
0.6	100	1452.102	4,12,18	31.697	1452.102	4,12,18	3.844	0.00
	150	1602.102	4,12,18	24.386	1602.102	4,12,18	3.371	0.00
	200	1724.165	11,18	23.123	1724.165	11,18	2.771	0.00
	250	1788.190	18	22.715	1788.190	18	2.200	0.00
0.8	100	1490.242	4,8,18	36.496	1490.242	4,8,18	3.560	0.00
	150	1638.979	11,18	33.271	1638.979	11,18	3.152	0.00
	200	1738.190	18	22.812	1738.190	18	2.554	0.00
	250	1788.190	18	22.303	1788.190	18	2.154	0.00
1	100	1512.400	4,8,18	24.543	1512.400	4,8,18	3.184	0.00
	150	1647.549	11,18	22.764	1647.549	11,18	2.854	0.00
	200	1738.190	18	22.286	1738.190	18	2.108	0.00
	250	1788.190	18	21.955	1788.190	18	2.273	0.00

Table 3
Results for the CAB data set with θ = 4000

α	F	CPLEX			ALNS			%GAP
		Opt	Hubs	CPU (s)	Opt	Hubs	CPU (s)	
0.2	100	1252.452	4,12,16,18,22	26.168	1252.452	4,12,16,18,22	5.572	0.00
	150	1479.499	4,12,18,22	48.115	1479.499	4,12,18,22	4.351	0.00
	200	1638.692	4,12,18	73.779	1638.692	4,12,18	3.363	0.00
	250	1788.692	4,12,18	91.573	1788.692	4,12,18	2.919	0.00
0.4	100	1374.248	4,12,16,18,22	37.323	1374.248	4,12,16,18,22	5.00	0.00
	150	1557.772	4,12,18	41.788	1557.772	4,12,18	3.173	0.00
	200	1707.772	4,12,18	50.570	1707.772	4,12,18	3.387	0.00
	250	1840.891	12,18	51.327	1840.891	12,18	2.636	0.00
0.6	100	1454.257	4,8,18,22	38.631	1454.257	4,8,18,22	4.442	0.00
	150	1611.688	4,12,18	30.354	1611.688	4,12,18	3.439	0.00
	200	1761.688	4,12,18	30.708	1761.688	4,12,18	3.379	0.00
	250	1862.172	11,18	23.920	1862.172	11,18	2.66	0.00
0.8	100	1499.759	4,8,18	37.602	1499.759	4,8,18	3.587	0.00
	150	1649.759	4,8,18	24.856	1649.759	4,8,18	3.257	0.00
	200	1776.986	11,18	24.956	1776.986	11,18	2.759	0.00
	250	1876.986	11,18	22.717	1876.986	11,18	2.642	0.00
1	100	1521.918	4,8,18	33.902	1521.918	4,8,18	3.332	0.00
	150	1671.918	4,8,18	33.144	1671.918	4,8,18	3.119	0.00
	200	1785.556	11,18	22.776	1785.556	11,18	2.706	0.00
	250	1883.082	18	22.531	1883.082	18	2.075	0.00

As observed in Table 2 and Table 3, the proposed algorithm could get optimal solutions for all the instances in a very short computational times. Numerical experiment results based on the TR data set with different penalty coefficient values (1000, 1500 and 2000) are shown in Table 4. Since it is not possible to solve the large-sized instances of the TR data set by CLPEX in our computer, we only solved these instances using the proposed ALNS algorithm. The first column in Table 4 indicates applied coefficient discount for transportation cost on the inter-hub connections. Second column shows scaling factor for the fixed hub establishment costs. Next columns show the results of solving the problem based on different penalty coefficients. For each value of penalty coefficient (θ), three columns include respectively the optimum value of objective function, the optimum set of opened hubs, and the corresponding CPU times required for solving each instance.

Table 4
Results for the TR data set

α	CF	θ = 1000			θ = 1500			θ = 2000		
		Opt	Hubs	CPU (s)	Opt	Hubs	CPU (s)	Opt	Hubs	PU (s)
0.2	0.1	748.275	6,25,33,34,35	85.70	748.2750	6,25,33,34,35	171.71	748.275	6,25,33,34,35	179.746
	0.3	966.6485	6,34	95.20	1060.564	6,33,34,35	138.64	1089.487	6,33,34,35	138.64
	0.5	1000	-	372.03	1218.558	6,34	372.03	1260.419	6,34	87.14
0.4	0.1	820.55	5,33,34,35	126.69	842.905	6,25,33,34,35	175.04	842.912	6,25,33,34,35	182.74
	0.3	979.297	34	87.31	1120.419	34,35,38	89.78	1130.039	6,33,34,35	98.13
	0.5	1000	-	63.82	1234.561	6,34	89.73	1276.659	6,34	94.21
0.6	0.1	855.031	6,33,34,35	147.61	916.338	6,25,33,34,35	154.78	927.523	6,33,34,35	165.31
	0.3	979.297	34	87.98	1138.133	6,34	77.50	1168.968	6,33,34	101.48
	0.5	1000	-	84.32	1246.166	6,34	83.93	1288.489	6,34	79.06
0.8	0.1	872.19	6,34,35	130.46	966.42	6,33,34,35,44	150.77	969.15	6,25,33,34,35	152.72
	0.3	979.29	34	109.16	1147.745	6,34	118.87	1206.580	38,41	108.80
	0.5	1000	-	74.05	1255.778	6,34	88.88	1298.611	6,34	93.61
1	0.1	880.32	6,34,35	118.17	998.71	33,35,38,41	152.74	997.12	6,25,33,34,35	161.76
	0.3	979.29	34	99.41	1193.884	34,35,38	103.07	1215.246	38,41	90.63
	0.5	1000	-	68.62	1256..859	6	73.68	1305.907	6,34	92.10

We can observe from Table 4 that the proposed algorithm solves all the problem instances for the large-sized TR data set in less than 3 minutes. Based on the obtained results we can conclude that the proposed ALNS algorithm has a high efficiency for solving large size problems. Significant and interesting point is based on the value of penalty coefficient which can be seen from these tables. No hub is established in network in optimum solution when fixed cost of hub establishment is high therefore none of available flows in network has been routed instead penalty of non-transporting has been paid. Also as observed from these tables, the values of the expected transportation cost have been increased by increasing of penalty coefficient and fixed cost of hub establishment.

6. Conclusion

In this paper, we have proposed formulation for the reliable uncapacitated multiple allocation hub location problem under hub disruptions. It was assumed that every open hub facility can fail after installation. If a hub fails, customers originally assigned to that hub, are either reassigned to other hubs that are still operational or they do not receive service in which case a penalty should be paid because of high expenses of reallocation. The problem was modeled as two-stage stochastic program and then transformed into its deterministic equivalents (extended forms) by defining a set of scenarios and associating with each scenario, the corresponding probability of occurrence. As our proposed problem is a NP-hard problem, a metaheuristic algorithm based on the was developed for solving it. Computational experiments are conducted to show the efficiency of our solution method. It was shown that the proposed algorithm obtains optimal solutions for all instances of the CAB data set in short computational times. Our results show that the structure of the solution changes when uncertainty is considered. In general, when the uncertainty in the operational status of hubs is considered, the number of hubs in optimal solution is greater than the classical counterpart in which it is assumed that the hubs are not subject to failure. Also results indicate that if fixed cost value for opening hubs increases, the number of opened hub in optimum solution will decrease and also if value of discount factor between hub connections increases, the number of opened hubs will drop.

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