A revision on EOQ/JIT indifference points

Mehdi Rajabi Asadabadi*

University of San Jose-Recoletos, Cebu City, Philippines

1. Introduction

Over the last few decades, an increasing trend of implementing JIT rather than EOQ has been seen (Maiga & Jacobs, 2008) and this is mostly because of the impact of the JIT system on exposing problems and wastes in an organization (Salaheldin, 2005). Since the introduction of JIT in 1970, it has been viewed as an inventory removal system (Biggart & Gargeya, 2002; Spencer et al., 1994) while the EOQ model places orders with time intervals and consequently imposes holding costs (Fazel, 1997). Despite the imposed holding costs, numerous companies using the EOQ model are not confident enough to take the risk of replacing their inventory model with the JIT system (Wu et al., 2013). Although keeping no inventories significantly decreases the holding costs, it may increase the total cost by influencing some other costs (Min & Pheng, 2005). There are some inherent hidden costs in the JIT system, which must be taken into account in a sufficient comparison model (Schniederjans & Cao, 2001). These costs can be ranged from set up cost to stock out cost (Fazel, 1997). The limited studies dealing with such challenging decision motivates further investigations (Wu et al., 2013). A sufficient formulization of the total cost
comparison of EOQ/JIT model, considering managerial important concerns such as cost of capital used to buy the inventories or remained carrying costs in JIT has not yet been explored.

This paper aligned with the previous few studies is not capable of determining whether or not switch to the JIT system, because there are several factors as well as the total cost to be considered for making that decision, but it offers an efficient cost comparison between both of them in which some determinant and less seen costs are taken into account to show that either the EOQ model or JIT system, from cost view point, is preferred. It extends the previous proposal by exposing not only some hidden costs of JIT system, but also some less seen costs of the EOQ model. Indifference points of EOQ/JIT model regarding to various parameters are computed to show where it is costly reasonable to switch from one to another.

The remainder of this paper is organized as follows. After this, the literature review is submitted in which the previous studies dealing with the EOQ/JIT model are reviewed. Following that, the EOQ/JIT proposed model is presented and is examined through a case study.

2. Literature review

Over the last few decades, choosing a system for managing the inventories has been a challenging decision (Fazel et al., 1998). The EOQ model is to order an amount of the inventory and carry it to be used later, while JIT does not tolerate the inventory carrying cost, insists on removing any level of the inventory.

Chyr et al. (1990) present a comparison between the JIT and EOQ total cost. Their study is to investigate the damage cost and setup time and their impacts on the inventory decision. It deals with the conditions under which the total cost of the EOQ model is lower than the JIT system. Later, Fazel (1997) presents a comparison between the JIT system and the EOQ model where the EOQ/JIT model is formulated to find the indifference points with respect to the demand. In researches dealing with the EOQ/JIT total cost comparison, the points in which implementing either EOQ or JIT results the same total cost, are called indifference points (Fazel, 1997). Campbell and Joshi (1991) highlight one of strengths of the EOQ model in comparison with the JIT system. In the JIT model, there are no inventories and the orders are in small batches as organizations need, so the prices are not negotiable for some discount while in the EOQ model the amounts of orders are known and inventories are bought for current and future needs, so there are opportunities to get some discounts (Joshi & Campbell, 1991).

On the other hand, Fazel (1997) implies that there are some hidden costs for the JIT model. The study by Fazel states that these hidden costs are created by removing inventories; by employing the JIT system, the inventory relevant costs are not removed, but transferred to the suppliers. “Hidden holding costs and ordering costs are not fully eliminated, but they are transferred to suppliers and a price increase happens in purchasing price” (Fazel, 1997).

Fazel et al. (1998) present two indifference points with regard to the demands in which a discount function is considered for the EOQ model. They assume that for quantities lower than a certain level, the delivery price decreases as a linear function of the order quantity while for the amounts higher than that level, the price, which hits the lowest possible price, remains the same: “It is assumed that for quantities below $Q_{\text{max}}$ the delivery price is a decreasing, continuous, and linear function of the order quantity. Beyond $Q_{\text{max}}$, the price stays at it $P_{\text{min}}$, which is the lowest price that the supplier would charge no matter how large the order quantity is.” Schniederjans and Cao (2000) develop the indifference point of the model presented by Fazel (1997) regarding to the possible facility size reduction under the JIT system. In the JIT system, since the storages of the raw material, work in process, and finished product are aimed to be removed in the JIT system, the facility size reduces (Schniederjans & Cao, 2000). They consider an estimation of the saved cost, resulted from the facility size reductions in the JIT system, as the annual cost of maintaining those spaces of the facilities. A total cost comparison between the Economic
Manufacturing Quantity (EMQ) model and JIT system is presented by Cao and Schniederjans (2004). They come up with an EMQ/JIT model, which is extending the previous studies on the EOQ/JIT model by considering the set up cost for the JIT system.

Min and Pheng (2005) employ the discount model proposed by Fazel et al. (1998) and reformulate the EOQ/JIT model where the price discount function is considered for the EOQ model and the stock out cost is taken into account for the JIT system. Min and Pheng (2006) formulate the indifference points considering a discount function based on the order quantity. Next year, they add the stock out cost to formulate the developed indifference points considering their previous model and apply it for the ready mixed concrete industry (Min & Pheng, 2007). Wu et al. (2013) investigate the stock out cost of the JIT system. They formulize the stock out cost regarding the wasted time of the production line which is resulted from stocking out in inventories: “Let $\alpha \beta$ represents the additional stock out costs under the JIT purchasing system compared to that under the EOQ purchasing system, where $\alpha$ represents the number of additional working hours that may be affected in a JIT system than that in an EOQ system, $\beta$ represents the value created in one working hour. $\alpha \beta$ is a penalty of using JIT purchasing instead of EOQ purchasing. The total annual cost under the JIT system is, therefore, revised as: $TC_{JT} = P_J D + \alpha \beta$, where $TC_{JT}$ is the revised total annual cost under the JIT system, $P_J$ is the unit price under the JIT system, and is greater than price in EOQ model”

According to Fazel (1997) “Manufacturing companies that use either the classical EOQ model are increasingly faced with the decision of whether or not to switch to the JIT system. This is a complex decision, requiring careful examination of each system and its possible impact on a variety of cost factors.” There are successful implementations of the JIT system all over the world, which prompts organizations to switch to that system, however, there are still so many companies prefer working under the EOQ model (Min & Pheng, 2005). There are several factors to be considered before making a fundamental inventory decision, which not only influences an organization, but also all other components of the supply chain. Although the EOQ model and the JIT system have been previously studied to find the indifference points, the cost elements of both of them are in need of a revision. This study aims to develop the EOQ/JIT model considering more developed cost elements to make the model more applicable in real case studies.

3. The Model

**Nomenclature**

<table>
<thead>
<tr>
<th>$TC$</th>
<th>Total Cost</th>
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<tbody>
<tr>
<td>$D$</td>
<td>annual demand</td>
</tr>
<tr>
<td>$H$</td>
<td>carrying cost per unit per year</td>
</tr>
<tr>
<td>$Q$</td>
<td>number of units per order</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>optimum order quantity</td>
</tr>
<tr>
<td>$K$</td>
<td>ordering cost for each order</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
</tr>
<tr>
<td>$P_E$</td>
<td>purchase price per unit under the EOQ model</td>
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<tr>
<td>$r_{ind}$</td>
<td>the interest rate indifference point</td>
</tr>
<tr>
<td>$D_{ind}$</td>
<td>the demand indifference point</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>the summation of the hidden costs of the JIT system</td>
</tr>
<tr>
<td>$Z$</td>
<td>total cost difference</td>
</tr>
<tr>
<td>$p$</td>
<td>production rate</td>
</tr>
<tr>
<td>$u$</td>
<td>usage rate</td>
</tr>
<tr>
<td>$P_{EMQ}$</td>
<td>purchasing price per unit under the EMQ model</td>
</tr>
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</table>
Since the order quantity in the EOQ model is determined, it is possible to negotiate on the price with the suppliers. This helps the purchasing price in the EOQ model be lower than the JIT system. The total cost in the EOQ model is a combination of the purchasing, ordering, and holding cost. A less seen cost in the EOQ model is the cost of the invested money. Whether or not the money to buy the order quantity is borrowed, it is not free of charge. That cost should be a part of the total cost of the EOQ model, especially, when it is formulated to be compared with the JIT system. This is in favor of the JIT system, which insists on keeping no inventories. If the money is borrowed, the interest that the borrower has to pay should be considered. On the other hand, if there is enough money to buy the quantity, the interest that the company can earn by investing that money instead of buying the order quantity, such as investing in a bank, should be considered. However, if that kind of interest is not applicable or considerable, the amount of it, shown by ‘r’, is simply set equal to zero. Therefore, ‘r’ is considered to present the average of interest rate for the needed money to buy the inventory in the general form of the total cost formula of the EOQ model:

$$TC_{EOQ} = \frac{Q}{2} H + \frac{D}{Q} K + \frac{r}{2} P_E Q + P_E D,$$

where $H$ is the annual holding cost, $K$ is the cost of placing an order, $D$ is the annual demand, $D/Q$ is the annual ordering frequency, and $Q/2$ is the annual average inventory level in the storage. The first derivative of the above equation is set equal to zero and the optimum order quantity turns up as presented below.

$$Q^* = \sqrt{\frac{2KD}{H + rP_E}}$$

By placing the optimal order quantity in Eq. (1), Eqs. (3-4) are obtained as follows,

$$TC_{EOQ}^* = \frac{H}{2} \sqrt{\frac{2KD}{H + rP_E}} + \frac{H + rP_E}{2KD} DK + \frac{r}{2} P_E \sqrt{\frac{2KD}{H + rP_E}} + P_E D$$

$$TC_{EOQ}^* = P_E D + \sqrt{2KD(H + rP_E)}$$

According to the JIT system, since the holding and set up costs are assumed to be insignificant, the annual cost is computed by multiplying of the price per unit by the demand per year.

$$TC_{JIT} = P_{JIT} D.$$  

The total cost difference between the EOQ model and the JIT system is presented below.

$$Z = TC_{EOQ} - TC_{JIT},$$

$$Z = \sqrt{2KD(H + rP_E)} + (P_E - P_{JIT})D.$$  

Therefore, the demand indifference point, in which the JIT and EOQ model are equal in terms of the cost, is as below.

$$D_{ind} = \frac{2K(H + rP_E)}{(P_{JIT} - P_E)^2}$$
If the demand is below this amount, the EOQ model is more costly than the JIT system and if the demand increases above this level, the EOQ model is more cost effective. Additionally, Eq. (7) and Eq. (8) can be formulated to find the interest indifference point as below.

\[ r_{ind} = \frac{(P_{JIT} - P_E)^2D - 2KH}{2KP_E} \]  

This means that whenever the interest rate is below this amount, from the cost viewpoint, the EOQ model is better than the JIT system and vice versa. If the computed ‘r’ is negative, it implies that for any interest rate, JIT system is more cost effective. Eq. (9) is formulated with the assumption of having r as the only variable. This requires the demand to be determined, however, it seems odd to have a changing interest rate and a determined demand in the market. It is always assumed that the holding and set up costs in the JIT system are insignificant. In reality, as we see in the example, there are always small amounts of carrying and setup costs in a JIT system. Although those costs in short term are insignificant, in a year, if they are put together, the amount becomes sometimes considerable. This annual summation of the inherent hidden costs of the JIT system is regarded in the below equation as \( \alpha \).

\[ TC_{JIT} = P_{JIT}D + \alpha. \]  

Therefore, it might be of interest to know that what happens to the total cost difference of the EOQ/JIT model and also to find out that up to what amount of this annual summation of the hidden costs, \( \alpha \), is tolerable where the JIT system is being used.

\[ Z = P_E D + \sqrt{2KD(H + rP_E)} - P_{JIT} D - \alpha. \]  

Here, Z represents the difference between the EOQ and JIT system, so if this equation is set equal to zero, the indifference point turns up.

\[ \alpha^2 + \left(2(P_{JIT} - P_E)D\right)\alpha + \left(P_{JIT} - P_E\right)^2D^2 - 2KD(H + rP_E) = 0 \]  

Eq. (12) is arranged to be in the form quadratic function, so the roots are as follows,

\[ \alpha_{ind1} = \left(P_E - P_{JIT}\right)D + \sqrt{2KD(H + rP_E)}. \]  

\[ \alpha_{ind2} = \left(P_E - P_{JIT}\right)D - \sqrt{2KD(H + rP_E)}. \]  

Since \( P_E < P_{JIT} \) is expected, \( \alpha_{ind2} \) is a negative number and this root, Eq. (14), is not valid. The JIT system is preferred only where the summation of all the annual carrying and setup cost is less than \( \alpha_{ind1} \), otherwise the EOQ model is more cost effective.

4. An illustrative example

Zomorrod Sofal Co. is a manufacturer of light weight bricks located in Iran. The company uses some raw material, which is a special soil delivered by trucks. The company is currently employing the EOQ model to determine the order quantity, but wonders whether or not to buy it based on the JIT system. A cost comparison is performed to find out whether or not to switch to the JIT fundamentals.

The annual demand of the soil is 697 trucks with the purchasing price of 471$ and the annual carrying cost of 30$ for each truck. The ordering cost is 14$ and the annual interest rate is four percent. The supplier discounts the price to 469.5$ for the orders more than 5 trucks. Based on the above information and by applying Eq. (2), the economic order quantity of the raw material is 20 trucks.
\[ Q^* = \sqrt{\frac{2KD}{H + rP_E}} = \sqrt{\frac{2(14)(697)}{30 + (0.04)(469.5)}} = 20. \]

Based on Eq. (8), the demand indifference point is:

\[ D_{ind} = \frac{2K(30 + (0.04)(469.5))}{(1.5)^2} = 607. \]

Since the current demand of the company is higher than the indifference point, the EOQ model is less costly than the JIT system. Considering Eq. (7) the cost difference between employing the inventory systems is:

\[ Z = \sqrt{2KD(H + rP_E)} + (P_E - P_{JIT})D = \sqrt{2(14)(697)(30 + (0.04)(469.5))} + (1.5)(697) = 2021 \$ \]

\[ r_{ind} = \frac{(P_{JIT} - P_E)^2D - 2KH}{2KP_E} = \frac{(1.5)^2(697) - 2(14)(30)}{2(14)(469.5)} = 0.055 \]

This means that if the average interest rate for the capital used to buy the inventories increases from 0.04 to a number more than 0.055, technically, the JIT system becomes more cost effective than the EOQ model as long as the demand remains the same, but currently, as mentioned earlier, the EOQ model is more cost effective. The amount of \( \alpha_{ind1} \) is computed as follows,

\[ \alpha_{ind1} = (P_E - P_{JIT})D + \sqrt{2KD(H + rP_E)} = (-1.5)(697) + \sqrt{2(14)(697)(30 + (0.04)(469.5))} = -69.8 \]

The amount of \( \alpha_{ind1} \) as well as \( \alpha_{ind2} \) becomes negative as predicted before. For the current situation, regardless of whatever the JIT hidden costs are, the EOQ model is less costly. On the other hand, for example, if the demand drops down to 300, then applying the same formula results in the amount of $190\$, which means the JIT is less costly and can be considered as a replacement for the EOQ model for the company unless its annual hidden costs hits $190\$.

4. Discussion

Interest rate is a determinant fact of the market, and since the JIT system releases the money used to buy the inventory, it changes the equations in favor of the JIT system. In comparison with the models which do not consider the interest rates, these models are suggesting some indifference points which are more inclined toward selecting the JIT system. The JIT system releases some capital, which can be used for investments or to reduce the debts of the company, which in either way; an interest rate is possible to compute.

![Fig. 1. D_{ind}/ Interest Rate](image-url)
Fig. 1 represents the influences of the interest rate on the demand indifference point for the case study. When the interest rate is zero or when the company is not considering the interest rate (the simple EOQ model), the indifference point becomes 373. This means that the EOQ model is a more cost efficient option compared with the JIT system, unless the annual demand drops below 373 trucks. By considering the interest rate, as it is observed in the figure, the threshold for selecting the EOQ model gradually increases. If the interest rate increases to 10 percent, the threshold increases to 958 (for the interest rate of 4 percent, as it was computed in the case study, the demand indifference point is 607). This figure clearly shows how considering the interest rate can be in favor of the JIT system to be selected.

On the other hand, the inflation rate for some currencies is considerable and needs to be taken into account. Generally, the inflation rate is in favor of the EOQ model and it might be very often preferred compared with the JIT system especially where the inflation rate is high, the market is too much risky, and the company is not interested to have its money invested outside of the company. If it happens, having a certain amount of negative interest rates in the above equations can be explained. Under such circumstances, the level of inventories in the EOQ model relatively increases. Therefore, the existence of a high inflation rate may imply that keeping the inventory considering all of its carrying cost can still be more beneficial than adopting the JIT system. Besides, from the studies dealing with the EOQ model, there are some studies investigating the economic manufacturing quantity (EMQ) models such as Cao and Schniederjans (2004). This approach is extendable to present a cost comparison and proposes a developed EMQ/JIT model. There is, again, a hidden capital cost of money used to buy/produce inventory quantity (Q). The total cost of EMQ, the optimal quantity \( Q^* \), and the EMQ/JIT model are formulated where \( p \) is the production rate of the inventory and \( u \) is the usage rate of it as follows.

\[
TC_{EMQ} = H \left( \frac{(p-u)Q}{2p} + \frac{D}{Q} K + P_{EMQ} \times Q \times \frac{r}{2} + P_{EMQ}D \right)
\]

\[
Q^* = \sqrt{\frac{2KDP}{H(p-u) + rpP_{EMQ}}}
\]

\[
TC_{EMQ}^* = P_{EMQ}D + \sqrt{\frac{2KD(H(p-u) + rpP_{EMQ})}{p}}
\]

\[
Z = TC_{EMQ} - TC_{JIT}
\]

\[
Z = \sqrt{\frac{2KD(H(p-u) + rpP_{EMQ})}{p}} + (P_{EMQ} - P_{JIT})D
\]

Then, the demand indifference point is as below,

\[
D_{ind} = \frac{2K(H(p-u) + rpP_{EMQ})}{p(P_{JIT} - P_{EMQ})^2}
\]

This is the demand in which both the EMQ model and the JIT system cost the same. Only for the demand lower than this level, from cost viewpoint, the JIT system is preferred. Although numerous studies, in favor of the JIT system, highlight the hidden abilities of JIT system to remove the wastes and exposing the problems (Fullerton, 2003), those deficiencies and problems are possible to be taken care of by a discerning management system and not necessarily by changing the inventory model. However,
implementing a JIT system, without such a good managerial team capable of solving the problems, leads to a disaster. The JIT total cost is less investigated in the previous studies. It is assumed to be the price multiplied by the demand (Fazel, 1997; Fazel et al., 1998; Ming & Pheng, 2005, 2006, 2007), or a shortage with regard to the cost of production line stoppage (Wu et al., 2013). Under the JIT system, it is not possible to remove all of the inventories and their carrying costs. Although ordering cost can be significantly decreased by transferring the supplier close to the company, it is not fully eliminated and there are still some insignificant daily inventory carrying and ordering costs. These insignificant costs sometimes happen to have a considerable summation over the annual performance of the company. This model lets this annual cost be seen in the EOQ/JIT model. This paper shows that it is possible to determine a limitation for those hidden cost in which the JIT system can only be preferred as its annual hidden costs is not reaching that specific amount.

Several papers have dealt with the cost efficiency of the JIT system (Salaheldin, 2005; Maiga & Jacobs, 2008; Biggart & Gargeya, 2002; Spencer et al., 1994). A few researchers have been attempting to come up with an efficient total cost comparison of the EOQ model and the JIT system (Fazel, 1997; Wu et al., 2005; Fazel et al., 1998; Schniederjans & Cao 2000; 2001; Chyr et al., 1990; Min & Pheng, 2005; 2006; 2007). Although there are some cost elements in both models, which are determinant, especially where the inventory models are compared with each other, the previous studies have not taken them into consideration. This paper deals with both EOQ and JIT to find a formulated total cost comparison model determining when is the time to switch from JIT to EOQ or vice versa. Computing and comparing the costs are the aim of the investigation in this study, however, the total cost should not be considered as the only factor for making such a decision on inventory models because there are some other factors to be considered before making the final decision. In this paper, the classic model of EOQ is developed to take the cost of acquiring capital as a determinant factor of the total cost comparison into account. Whether a company is capable of assigning money to buy the order quantity or it has to get a loan, the interest rate should be considered, however, it can be considered regarding the lost interest opportunity of an investment or the interest payable to the lenders. The total cost of the JIT system is also developed in this paper. It develops the total cost of JIT system by making its formula observant to the inherent hidden costs of the JIT system.

This paper creates several research areas to be investigated. The correlations between the interest rate and the demand can be studied to find the interest rate indifference points. The new EOQ/JIT model can also be developed to consider the stock out risk under the JIT. A further study can also consider a discount function under the EOQ model and find the indifference points. The EOQ model can take advantage where there is a discount function such as: \( P_E = P_E^0 - \pi_E Q \) when \( Q \leq Q_{\text{max}} \) and \( P_E = P_{\text{min}}^0 \) when \( Q > Q_{\text{max}} \) (see Fazel et al., 1988). This function can be applied in the new comparison model to find the indifference point just by replacing \( P_E \) in the models with \( P_E^0 - \pi_E Q \) where \( Q \leq Q_{\text{max}} \), or with \( P_{\text{min}}^0 \) when \( Q > Q_{\text{max}} \). It results in a polynomial of third degree. If it is set equal to zero, the indifference point(s) are found by obtaining the roots.

5. Conclusion

Although the decision of switching from the EOQ model to the JIT system seems to be efficient in removing the problems and wastes, it still remains a challenging decision. There are several factors to be considered while the lack of each of them can result in a wrong decision. Some of the listed results of having a JIT system in a company can be achieved by a good management system, but the cost difference remains as one of the effective factors in inventory model decision. Over the last two decades, A few researchers have compared the EOQ model and the JIT system, but the effects of interest rate, inflation rate, and some inherent and hidden costs of JIT system have been less studied. This paper has involved some more cost elements in computing the total cost of the EOQ model as well as the JIT system, and then has evaluated these two inventory techniques to see how cost effective either of them can be and under what circumstances. The findings of the EOQ/JIT comparison model have been also applied to
develop the EMQ model and to make a comparison on EMQ/JIT model. The indifference points of the EOQ/JIT model with respect to the interest rate, demand, and annual holding costs of the JIT system have been formulated. Therefore, compared with the previous studies, the contribution of this paper is as follows. First, it developed the EOQ model to consider some new cost elements such as financial cost of consumed money for buying the inventory. Second, it developed the total cost of the JIT system by making its formula observant to the inherent hidden costs of the JIT system, and finally, it presented a developed EOQ/JIT model.

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References


