

Supply chain production model with preservation technology under fuzzy environment

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ABSTRACT

In this paper, an attempt is made to characterize the preservation technology for deteriorating items to reduce the deterioration rate. This model assumes a single producer and single supplier and formulates a production model with a time varying rate of deterioration rate. Here production and demand are treated as a fuzzy variables and total cost is minimized for both the crisp and fuzzy model. Shortage is allowed on the supplier's part, which is partially backlogged. A solution procedure is presented to determine an optimal replenishment cycle and total cost per unit time, which is a convex function of preservation technology cost. Results have been validated with relevant example. In a way, the proposed model provides a unique theory to reduce the deterioration rate for the production model.

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1. Introduction

Deteriorating inventory were studied in the past decades (Ghare & Schrader, 1963; Covert & Philip, 1973; Dave & Patel, 1981; Kang & Kim, 1983; Goyal & Giri, 2001; Bhunia et al., 2009; Chang et al. 2010; Singh & Singh, 2011) and the focus was usually on constant or variable deterioration rate and quantity discount. Investing on preservation technology (PT) for reducing deterioration rate has received little attention in the past years. The consideration of PT is important due to rapid social changes and the fact that PT can reduce the deterioration rate, significantly. Moreover, sales, inventory and order quantities are very sensitive to the rate of deterioration, especially for fast deteriorating products. The higher rate of deterioration would result in a higher total annual relevant cost and a lower demand rate (Yang & Wee, 2006; Johnny et al., 2007). Ouyang et al. (2006) found that if the retailer can reduce effectively the deteriorating rate of items by improving the storage facility then the total annual relevant inventory cost could be reduced. Many enterprises invest on equipment to reduce the deterioration rate and extend the product expiration date. For example, refrigeration equipment is used

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to reduce the deterioration rate of fruits, flowers and sea foods in the supermarket. Murr and Morris (1975) showed that a lower temperature could increase the storage life and decrease decay. Wee et al. (2010) presented a model using PT. Huang et al. (2011) discussed the comments on PT investment for deteriorating inventory. Dye and Hsieh (2012) developed an optimal replenishment policy for deteriorating items with effective investment in PT. Dye (2013) investigated on the effect of PT investment on a non-instantaneous deteriorating inventory model. Hsieh and Dye (2013) developed a production inventory model incorporating the effect of PT investment when demand is fluctuating with time.

Generally, some customers would like to wait for backlogging during the shortage period. However, the willingness is diminished with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate is. The opportunity cost due to lost sales should be considered. Papachristos and Skouri (2000) developed an EOQ inventory model with time-dependent partial backlogging. They assumed the rate of backlogged demand increases exponentially while the waiting time for the next replenishment decreases. Teng et al. (2003) extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with partial backlogging have been performed by Teng and Yang (2004), San José et al. (2006), Chang and Lin (2010), etc.

Usually researchers consider different parameters of an inventory model either as constant, time-dependent or probabilistic nature for the development of the economic order quantity model. However, in the real life situations, these parameters may have little deviations from the exact value, which may not follow any probability distribution. In these situations, if these parameters are treated as fuzzy parameters, then it will be more realistic. These types of problems are de-fuzzified by using a suitable fuzzy technique and then the solution procedure can be obtained in the usual manner. Several authors, namely Chang et al. (2010), Lin and Yao (2000), Singh and Singh (2011), Ruoning and Xiaoyan (2010) and Singh et al. (2011) developed inventory models in fuzzy sense by considering different parameters in fuzzy sense. Yadav et al. (2013) focused on retailer's optimal policy under inflation in fuzzy environment with trade credit. Urvashi and Singh (2013) discussed a model of inventory control with fuzzy inflation and volume flexibility under random planning horizon. Dutta and Kumar (2013) discussed about the fuzzy inventory models for deteriorating items with shortages and fully backlogged condition.

In this research, an effort is accomplished to formulate a supply chain production model for deteriorating items with fuzzy demand, fuzzy production by allowing preservation technology cost. Under real life circumstances, these parameters in the production model are uncertain and imprecise. In this situation, a suitable way to model these imprecise data is to use fuzzy sets and to formulate a production model in a fuzzy environment. We use the centroid method for defuzzifying fuzzy total average cost. For the proposed model, producer and supplier use the preservation technology. The optimal solution of the proposed model not only exists but also is unique. To illustrate the theory of the proposed model, a numerical example is provided and sensitivity analysis with respect to parameters of the system is carried out in both crisp and fuzzy sense.

2. Assumptions and Notation

In developing the mathematical models of the inventory system for this study, the following common assumptions were used.

- 1) This study considered single producer and single supplier.
- 2) Multiple deliveries per order are considered.
- 3) Production rate is finite and of constant rate.
- 4) Demand rate is constant.

- 5) Reduced deterioration rate, a function of ζ such that $m(\zeta) = k(1 - e^{-a\zeta})$, $a \geq 0$, where a is the simulation coefficient representing the percentage increase in $m(\zeta)$ per dollar increase in ζ , k is the original deterioration rate, $k > 0$ and ζ is the Preservation technology cost for reducing deterioration rate in order to preserve the products, $\zeta \geq 0$.
- 6) For producer and supplier PT cost is ζ and η , respectively.
- 7) Shortages are allowed on the supplier's part. Unsatisfied demand is partially backlogged. The fraction of shortages backordered is a differentiable and decreasing function of time t , denoted by $\delta(t)$, where t is the waiting time up to the next replenishment, and $0 \leq \delta(t) \leq 1$ with $\delta(0)=1$. Note that if $\delta(t)=1$ (or 0) for all t , then shortages are completely backlogged (or lost).

The following common notations were used.

T	the length of the inventory cycle
d	the demand rate
A_p	the ordering cost per order for producer
C_p	the unit purchasing cost for producer
h_p	the unit holding cost per order
$I_{p1}(t)$	the level of inventory at time t for producer during $0 < t < T_1$
$I_{p2}(t)$	the level of inventory at time t for producer during $0 < t < T_2$
I_{mp}	the maximum inventory for Producer
T_1	the length of production time for producer (a decision variable)
T_2	the length of non-production time
p	the production rate
$I_{s1}(t)$	the level of inventory at time t for supplier during $0 < t < T_3$
$I_{s2}(t)$	the level of inventory at time t for supplier during $0 < t < T_4$
T_3	the length of inventory cycle for supplier (a decision variable)
T_4	the length of shortage time for supplier
I_{ms}	the level of maximum inventory for supplier
h_s	the unit holding cost per order for supplier
A_s	the set up cost per order for Supplier
C_s	the unit purchasing cost for the supplier
C_{s1}	the backlogging cost per unit
C_{s2}	the lost sale cost per unit
δ	the backlogging rate
n	the number of cycles
k	the original deterioration rate
ζ	the PT cost for producer
η	the PT cost for supplier
TC	the integrated cost of producer and supplier

3. Model development for producer

The inventory model is shown in Fig. 1. The planning horizon T is divided by 2 parts T_1 and T_2 , where T_1 is the production time and T_2 is the non-production time. During the time interval $[0, T_1]$, the inventory level decreases due to demand and deterioration and increases due to the production. During the time interval $[0, T_2]$, there is no production, the inventory decreases due to deterioration rate and demand. This figure also shows that if we use the PT then our production is more and we can use our stock for a long time period.

Inventory level

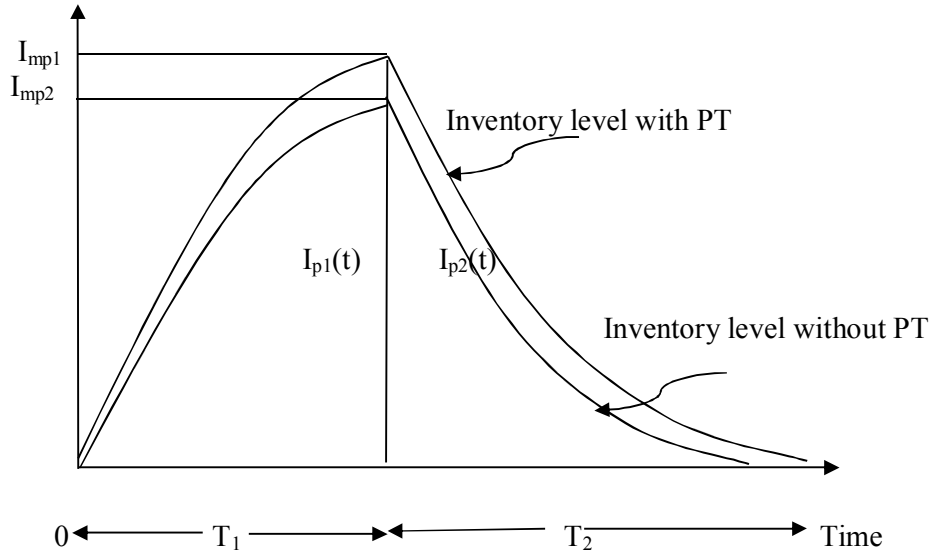


Fig. 1. The graphic representation of inventory model for producer

There are the following differential equations for the inventory level of producer.

$$I'_{p1}(t) = p - d - [k - m(\zeta)]I_{p1}(t), \quad 0 \leq t \leq T_1 \quad (1)$$

$$I'_{p2}(t) = -d - [k - m(\zeta)]I_{p2}(t), \quad 0 \leq t \leq T_2 \quad (2)$$

We have boundary conditions $I_{p1}(0) = 0$ and $I_{p2}(T_2) = 0$, the solution of the above differential equations are as follows,

$$I_{p1}(t) = \frac{(p-d)}{k-m(\zeta)} [1 - e^{[m(\zeta)-k]t}] \quad 0 \leq t \leq T_1 \quad (3)$$

$$I_{p2}(t) = \frac{d}{k-m(\zeta)} [e^{-[k-m(\zeta)](t-T_2)} - 1] \quad 0 \leq t \leq T_2 \quad (4)$$

By the boundary condition $I_{p1}(T_1) = I_{p2}(0)$, we have

$$\frac{(p-d)}{k-m(\zeta)} [1 - e^{-[k-m(\zeta)]T_1}] = \frac{d}{k-m(\zeta)} [e^{[k-m(\zeta)]T_2} - 1] \quad (5)$$

$$T_2 = \frac{p-d}{d} \left[T_1 - \frac{(k-m(\zeta))T_1^2}{2} \right]$$

$$T = T_1 + T_2 = T_1 + \frac{p-d}{d} \left[T_1 - \frac{(k-m(\zeta))T_1^2}{2} \right] \quad (6)$$

By using the boundary condition $I_{mp} = I_{p2}(0)$, we can find out the maximum inventory level with PT and without PT, which is as follows,

$$I_{mp1} = \frac{d}{k-m(\zeta)} [e^{[k-m(\zeta)]T_2} - 1], \quad (7)$$

and

$$I_{mp2} = \frac{d}{k} [e^{kT_2} - 1]. \tag{8}$$

The total relevant cost involves the following four factors,

(a). Holding cost: The present value of the holding cost per unit time is as follows,

$$\begin{aligned}
 HC_p &= \frac{h_p}{T} \left[\int_0^{T_1} I_{p1}(t)e^{-rt} dt + e^{-rT_1} \int_0^{T_2} I_{p2}(t)e^{-rt} dt \right] \\
 &= \frac{h_p}{T} \left[-\frac{(p-d)}{k-m(\zeta)} \left\{ \frac{1-e^{-[k-m(\zeta)+r]T_1}}{k-m(\zeta)+r} + \frac{e^{-rT_1}-1}{r} \right\} - \frac{de^{-rT_1}}{k-m(\zeta)} \left\{ \frac{1-e^{-rT_2}}{r} - \frac{e^{[k-m(\zeta)]T_2}-e^{-rT_2}}{k-m(\zeta)+r} \right\} \right]
 \end{aligned} \tag{9}$$

(b). Purchasing cost: The purchasing cost includes loss due to deterioration as well as the cost of the item. The present value of the purchasing cost per unit time is as follows,

$$IC_p = \frac{c_p}{T} PT_1 \tag{10}$$

(c). Ordering cost: The present value of the ordering cost per unit time is as follows,

$$SC_p = \frac{A_p}{T} \tag{11}$$

(d). Preservation Technology cost per unit time is

$$PC_p = \frac{\zeta}{T} \tag{12}$$

Hence, the present value of the total relevant inventory cost per unit time is as follows,

$$\begin{aligned}
 TC_p &= HC_p + IC_p + SC_p + PC_p \\
 &= \frac{1}{T} \left[h_p \left\{ -\frac{(p-d)}{k-m(\zeta)} \left\{ \frac{1-e^{-[k-m(\zeta)+r]T_1}}{k-m(\zeta)+r} + \frac{e^{-rT_1}-1}{r} \right\} - \frac{de^{-rT_1}}{k-m(\zeta)} \left\{ \frac{1-e^{-rT_2}}{r} - \frac{e^{[k-m(\zeta)]T_2}-e^{-rT_2}}{k-m(\zeta)+r} \right\} \right\} + c_p PT_1 + A_p + \zeta \right]
 \end{aligned} \tag{13}$$

4. Model development for supplier

The inventory model is shown in Fig.2. The planning horizon T/n is divided in 2 parts of T_3 and T_4 , where T_4 is the shortage time. Supplier's cycle starts with the maximum inventory I_{ms} at $t = 0$. During the time interval $[0, T_3]$, the inventory level decreases due to demand and deterioration with PT. During the time interval $[0, T_4]$, shortage occurs, which is partial backlogged. The inventory level at time t during the time interval $[0, T_3]$, and $[0, T_4]$, is governed by the following differential equations,

$$I_{s1}'(t) = -d - [k - m(\eta)]I_{s1}(t), \quad 0 \leq t \leq T_3 \tag{14}$$

$$I_{s2}'(t) = -\delta d \quad 0 \leq t \leq T_4 \tag{15}$$

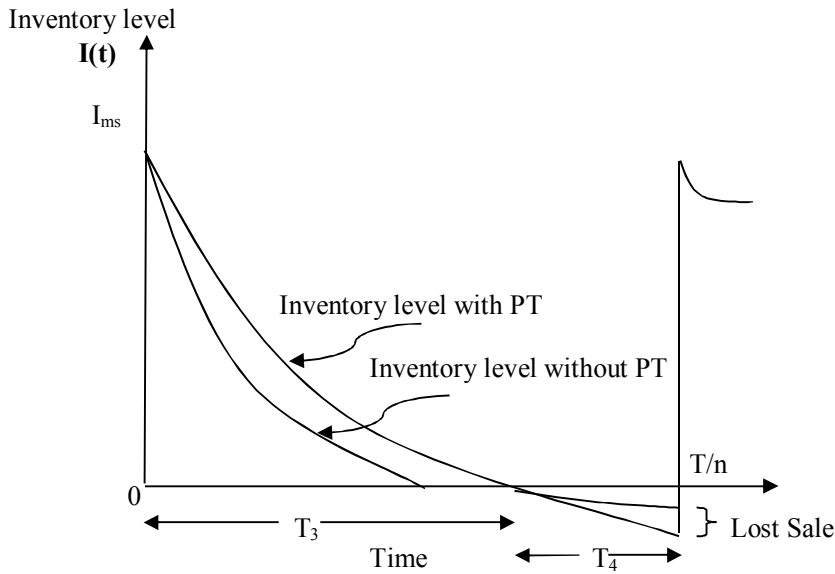


Fig. 2. The graphic representation of inventory model for supplier

With the boundary condition $I_{s1}(T_3) = 0$ and $I_{s2}(0) = 0$, the solution of the above differential equations are as follows,

$$I_{s1}(t) = \frac{d}{k - m(\eta)} \left[e^{-[k - m(\eta)](t - T_3)} - 1 \right] \tag{16}$$

$$I_{s2}(t) = -\delta dt \tag{17}$$

By using the condition $I_{ms} = I_s(0)$, we have maximum inventory for the supplier is follows,

$$I_{ms} = \frac{d}{k - m(\eta)} \left[e^{-[k - m(\eta)]T_3} - 1 \right]. \tag{18}$$

The total relevant cost involves the following six factors as follows,

(a). Holding cost: The present value of the holding cost per unit time is

$$HC_s = \frac{h_s n}{T} \int_0^{T_3} I_s(t) e^{-rt} dt = \frac{nh_s d}{(k - m(\eta))T} \left\{ \frac{e^{-rT_3} - 1}{r} + \frac{e^{[k - m(\eta)]T_3} - e^{-rT_3}}{k - m(\eta) + r} \right\} \tag{19}$$

(b). Purchasing cost: The present value of the purchasing cost per unit time is

$$IC_s = \frac{nC_s}{T} I_{Mb} = \frac{nC_s}{T} \left[\frac{d \left[e^{-[k - m(\eta)]T_3} - 1 \right]}{k - m(\eta)} \right] \tag{20}$$

(c). Ordering cost: The present value of the ordering cost per unit time is as follows,

$$SC_b = \frac{nA_s}{T} \tag{21}$$

(d). Preservation Technology cost

$$PC_s = \frac{m\eta}{T} \tag{22}$$

(e). Shortage cost: The present value of the shortage cost in the entire time horizon is as follows,

$$BA = \frac{nC_{s1}}{T} \int_0^{T_4} -I_{s2}(t)e^{-r(T_3+t)} dt = \frac{d\delta nC_{s1}e^{-rT_3}}{T} \left[\frac{-e^{-rT_4}T_4}{r} + \frac{(1-e^{-rT_4})}{r^2} \right] \tag{23}$$

(f). Lost sale cost: The present value of the lost sale cost occurs during the time period 0 to T_4 . During this time period, the complete shortage is dT_4 and the partial backlog is $d\delta T_4$. Lost sales are the difference between the complete shortage and the partial backlog. Thus, the present worth lost sale cost per unit time is as follows,

$$LS = \frac{nC_{s2}d}{T} \int_0^{T_4} (1-\delta)T_4 e^{-r(T_3+t)} dt = \frac{C_{s2}nd(1-\delta)e^{-rT_3}T_4(1-e^{-rT_4})}{rT} \tag{24}$$

Hence the present value of the total relevant inventory cost per unit time is

$$TC_s = HC_s + IC_s + SC_s + PC_s + BA + LS$$

$$= \frac{n}{T} \left[d \left\{ \frac{h_s}{(k-m(\eta))} \left\{ \frac{e^{-rT_3}-1}{r} + \frac{e^{(k-m(\eta))T_3}-e^{-rT_3}}{k-m(\eta)+r} \right\} + \frac{C_s(e^{-[k-m(\eta)]T_3}-1)}{k-m(\eta)} \right\} + \frac{C_{s1}e^{-rT_3}}{r} \left\{ \frac{-e^{-rT_4}T_4}{r} + \frac{(1-e^{-rT_4})}{r^2} \right\} + \frac{C_{s2}(1-\delta)e^{-rT_3}T_4(1-e^{-rT_4})}{r} \right\} + A_s + \eta \right] \tag{25}$$

Therefore

$$TC = \frac{1}{T} \left[\left[\left\{ h_p \left\{ \frac{(p-d)}{k-m(\zeta)} \left\{ \frac{1-e^{-[k-m(\zeta)+r]T_1}}{k-m(\zeta)+r} + \frac{e^{-rT_1}-1}{r} \right\} \right\} + c_p PT_1 + A_p + \zeta \right\} + \left\{ nd \left\{ \frac{h_s}{(k-m(\eta))} \left\{ \frac{e^{-rT_3}-1}{r} + \frac{e^{(k-m(\eta))T_3}-e^{-rT_3}}{k-m(\eta)+r} \right\} + \frac{C_s(e^{-[k-m(\eta)]T_3}-1)}{k-m(\eta)} \right\} + \frac{C_{s1}e^{-rT_3}}{r} \left\{ \frac{-e^{-rT_4}T_4}{r} + \frac{(1-e^{-rT_4})}{r^2} \right\} + \frac{C_{s2}(1-\delta)e^{-rT_3}T_4(1-e^{-rT_4})}{r} \right\} \right] + nA_s + m\eta \right]$$

There are two decision variables in the present value of the total inventory cost TC . The first part is associated with the production time period of producer, T_1 and the second part is associated with the time period of supplier T_3 . We can find out the value of T_2 and T_4 with the help of Eq. (5), Eq. (6) and the relation $T=T/n - T_3$.

5. Fuzzy model for producer and supplier

In a real situation and in a competitive market situation both the production rate and the demand rate are highly uncertain in nature. To deal with such a type of uncertainties in the super market, we consider these parameters in fuzzy form. Here p and d are not known precisely and p and d are defined by triangular fuzzy numbers such that

$$\tilde{p} = (p_1, p_0, p_2), \text{ and } \tilde{d} = (d_1, d_0, d_2),$$

where $(p_0 - \Delta_1 < p_0 < p_0 + \Delta_2)$ and $(d_0 - \Delta_3 < d_0 < d_0 + \Delta_4)$ are based on subjective judgments. We apply arithmetic operators based on fuzzy quantities and then defuzzify the same to convert them into crisp output. The membership functions for \tilde{p} and \tilde{d} is defined as follow,

$$\mu_{\tilde{p}}(p) = \begin{cases} \frac{p - p_0 + \Delta_1}{\Delta_1}, & p_0 - \Delta_1 \leq p \leq p_0 \\ \frac{p_0 + \Delta_2 - p}{\Delta_2}, & p_0 \leq p \leq p_0 + \Delta_2 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

$$\mu_{\tilde{d}}(d) = \begin{cases} \frac{d - d_0 + \Delta_3}{\Delta_3}, & d_0 - \Delta_3 \leq d \leq d_0 \\ \frac{d_0 + \Delta_4 - d}{\Delta_4}, & d_0 \leq d \leq d_0 + \Delta_4 \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

Here, $0 < \Delta_1 < p_0, 0 < \Delta_2, 0 < \Delta_3 < d_0, 0 < \Delta_4$ and p_0, d_0 are given fixed numbers. $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are determined by the decision maker based on the uncertainty of the problem. Defuzzification of \tilde{p} and \tilde{d} by the centroid method is given as follows,

$$M_p = \frac{p_0 - \Delta_1 + p_0 + p_0 + \Delta_2}{3} = p_0 + \frac{1}{3}(\Delta_2 - \Delta_1),$$

$$M_d = \frac{d_0 - \Delta_3 + d_0 + d_0 + \Delta_4}{3} = d_0 + \frac{1}{3}(\Delta_4 - \Delta_3).$$

We have

$$G(p, d) = f(T_1, T_3) + p g(T_1, T_3) + d q(T_1, T_3),$$

Where

$$f(T_1, T_3) = \frac{1}{T} \{ A_p + \zeta + n(A_s + \eta) \} \tag{28}$$

$$g(T_1, T_3) = \frac{1}{T} \left\{ c_p T_1 - \frac{h_p}{k - m(\zeta)} \left[\frac{1 - e^{-[k - m(\zeta) + r]T_1}}{k - m(\zeta) + r} + \frac{e^{-rT_1} - 1}{r} \right] \right\} \tag{29}$$

$$q(T_1, T_3) = \frac{1}{T} \left\{ \begin{aligned} & \left[\frac{nh_s}{(k-m(\eta))} \left\{ \frac{e^{-rT_3} - 1}{r} + \frac{e^{(k-m(\eta))T_3} - e^{-rT_3}}{k-m(\eta)+r} \right\} + \frac{nC_s (e^{-[k-m(\eta)]T_3} - 1)}{k-m(\eta)} + \delta nC_{s1} e^{-rT_3} \right] \\ & \left[\frac{-e^{-rT_4} T_4}{r} + \frac{(1-e^{-rT_4})}{r^2} \right] + \frac{C_{s2} n(1-\delta) e^{-rT_3} T_4 (1-e^{-rT_4})}{r} - \frac{h_p e^{-rT_1}}{k-m(\zeta)} \\ & \left[\frac{1-e^{-rT_2}}{r} - \frac{e^{[k-m(\zeta)]T_2} - e^{-rT_2}}{k-m(\zeta)+r} \right] + \frac{h_p}{k-m(\zeta)} \left[\frac{1-e^{-[k-m(\zeta)+r]T_1}}{k-m(\zeta)+r} + \frac{e^{-rT_1} - 1}{r} \right] \end{aligned} \right\} \quad (30)$$

Let $G(p, d) = y$, this implies that

$$p = \frac{y - f - dq}{g}$$

The membership of the fuzzy cost function given by the extension principle is

$$\mu_{\tilde{G}(\tilde{p}, \tilde{d})}(y) = \underset{(p,d) \in G^{-1}(y)}{\text{Sup}} [\mu_{\tilde{p}}(p) \wedge \mu_{\tilde{d}}(d)] = \underset{d_0 - \Delta_3 \leq d \leq d_0 + \Delta_4}{\text{Sup}} \left[\mu_{\tilde{p}}\left(\frac{y-f-dq}{g}\right) \wedge \mu_{\tilde{d}}(d) \right]$$

Now

$$\mu_{\tilde{p}}\left(\frac{y-f-dq}{g}\right) = \begin{cases} \frac{(p_0 + \Delta_2)g - dq - f - y}{\Delta_2 g}, & a_2 \leq d \leq a_0 \\ \frac{y - (p_0 - \Delta_1)g - dq - f}{\Delta_1 g}, & a_0 \leq d \leq a_1 \\ 0 & \text{otherwise} \end{cases}$$

Where

$$a_1 = \frac{f - y + g(p_0 - \Delta_1)}{q}, a_0 = \frac{p_0 g + f - y}{q} \text{ and } a_2 = \frac{f - y + g(p_0 + \Delta_2)}{q}$$

Defuzzification for the fuzzy total cost is thus given by centroid method is

$$\begin{aligned} M_{\tilde{TC}}(T_1, T_3) &= \frac{\int_{-\infty}^{\infty} y \mu_{\tilde{G}(\tilde{p}, \tilde{d})}(y) dy}{\int_{-\infty}^{\infty} \mu_{\tilde{G}(\tilde{p}, \tilde{d})}(y) dy} = f(T_1, T_3) + p g(T_1, T_3) + d q(T_1, T_3) + \frac{1}{3} \{ (\Delta_2 - \Delta_1) g(T_1, T_3) + (\Delta_4 - \Delta_3) q(T_1, T_3) \} \\ &= f(T_1, T_3) + \left\{ p + \frac{1}{3} (\Delta_2 - \Delta_1) \right\} g(T_1, T_3) + \left\{ d + \frac{1}{3} (\Delta_4 - \Delta_3) \right\} q(T_1, T_3), \end{aligned}$$

where $f(T_1, T_3)$, $g(T_1, T_3)$ and $q(T_1, T_3)$ are given by Eq. (28), Eq. (29) and Eq. (30), respectively. We can minimize the total average cost per unit time for the Optimal value of T_1 (say T_1^*) and T_3 (say T_3^*) as defined above and the minimum total cost is given by $[M_{\tilde{TC}}(T_1^*, T_3^*)]$.

6. Solution Procedure

To derive the optimal solution, the following classical optimization technique is used.

Step 1. Take the partial derivatives of $TC(T_1, T_3)$ with respect to T_1 and T_3 and equate the results to zero to find out the optimal value of T_1 and T_3 . The necessary conditions for optimality are

$$\frac{\partial(TC)}{\partial T_1} = 0, \quad \frac{\partial(TC)}{\partial T_3} = 0$$

Step 2. The convexity of the total cost is analyzed with the help of following conditions

$$\frac{\partial^2(TC)}{\partial T_1^2} \frac{\partial^2(TC)}{\partial T_3^2} - \left(\frac{\partial^2(TC)}{\partial T_1 \partial T_3} \right)^2 > 0$$

and $\frac{\partial^2(TC)}{\partial T_1^2} > 0$

Step 3. With T_1^* and T_3^* found in step 2, derive $TC^*(T_1^*, T_3^*)$.

7. Numerical Example

7.1. Numerical illustration for crisp model

A numerical example is considered to illustrate the model. The following values of parameters are used in this example.

$P = 360$ unit, $n=2$, $d=350$ unit, $A_p = \$0.20$, $C_p = \$0.12$, $h_p = \$0.15$, $A_s = \$0.50$, $C_s = \$0.15$, $h_s = \$0.20$, $C_{s1} = \$0.40$, $C_{s2} = \$0.30$, $\delta = 0.061$, $r = 0.001$, $\zeta = \$200$, $k = 0.007$, $\eta = \$400$, $a = 0.001$.

The software *Mathematica5.2* is used to derive the optimal solution. The optimal value of total cost is obtained as \$92.678 and production time (T_1^*) is 8.2854 days and (T_3^*) is 3.29462 days. With the help of Eq. (5) and Eq. (6), we can find out the non-production time period (T_2^*) and shortage period (T_4^*) and by using the condition of optimality we can analyze that TC is a strictly convex function, as shown below.

$$\frac{\partial^2(TC)}{\partial T_1^2} \frac{\partial^2(TC)}{\partial T_3^2} - \left(\frac{\partial^2(TC)}{\partial T_1 \partial T_3} \right)^2 = 30449 > 0 \quad \text{and} \quad \frac{\partial^2(TC)}{\partial T_1^2} = 1418.93 > 0$$

The three dimensional producer and supplier's total cost graph is shown in Fig. 3.

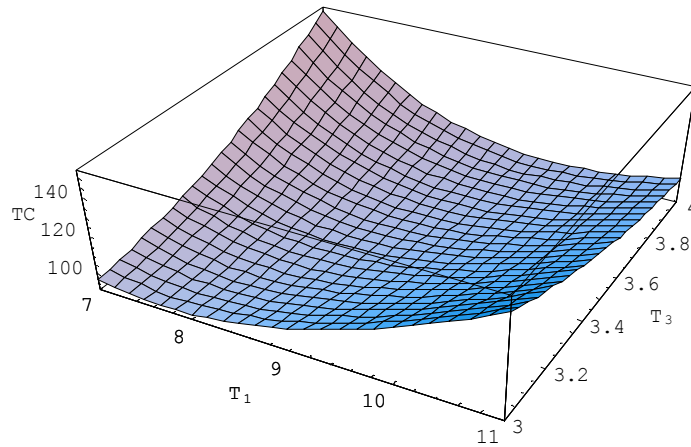


Fig. 3. The Producer and Supplier's unit time total cost in crisp model

7.2 Numerical illustration for fuzzy model

All the symbols in the fuzzy environment have the same numerical values as in the crisp environment with initial demand parameter and production with some uncertainty. The production rate and demand rate are considered as the triangular fuzzy numbers (300, 360, 400) and (310, 350, 410), respectively

$$\Delta_1 = 300 \quad \Delta_2 = 400 \quad \Delta_3 = 310 \quad \Delta_4 = 410.$$

Then optimal value of the total cost is \$102.973, production time (T_1^*) is 7.94398 days and supplier's time (T_3^*) is 3.16495 days. With the help of Eq. (5) and Eq. (6) we can find out the non-production time period (T_2^*) and shortage period (T_4^*). Total cost is more in the fuzzy environment. With the help of this data we can analyze that TC is a strictly convex function as shown as follows,

$$\frac{\partial^2(TC)}{\partial T_1^2} \frac{\partial^2(TC)}{\partial T_3^2} - \left(\frac{\partial^2(TC)}{\partial T_1 \partial T_3} \right)^2 = 706.609 > 0 \quad \text{and} \quad \frac{\partial^2(TC)}{\partial T_1^2} = 12.11618 > 0$$

The three dimensional producer and supplier's total cost graph in the fuzzy environment is shown in Fig. 4.

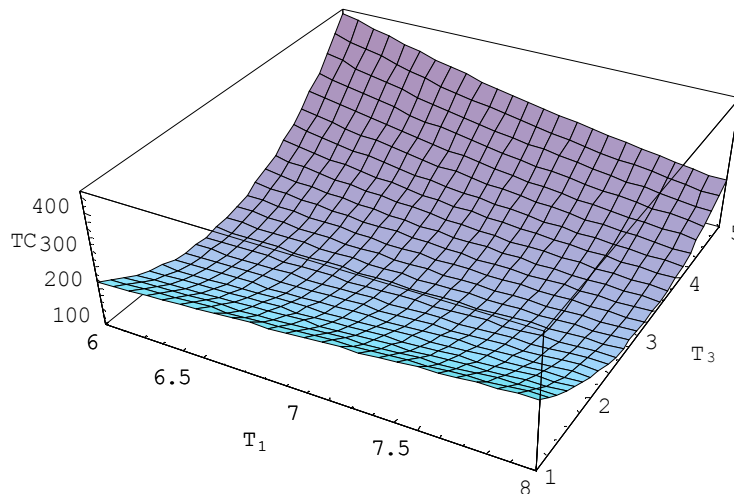


Fig. 4. The Producer and Supplier's unit time total cost in fuzzy model

8. Sensitivity analysis

Sensitivity analysis is carried out when the parameters a , ζ , η , n and r are changed. Figs. 5-9 show the changes in the total cost for variables a , n , r , ζ and k , respectively. Fig. 5(a) and 5(b) show the changes in the total cost, while changing the value of a in the crisp and fuzzy model, respectively. since $m(\zeta) = k(1 - e^{-a\zeta})$, where k is the original deterioration rate, as a increases then $m(\zeta)$ is a increasing function therefore the reduced deterioration rate ($k - m(\zeta)$) is a decreasing function then total cost decreases. This gives a conclusion that with the PT we can reduce the deterioration rate as well as the total cost. Fig. 6(a) and 6(b) show that as ζ (PT cost of producer) increases then the total cost decreases. Since we have less deterioration rate we have more sales and more profit. Therefore, this process is more profitable for producer.

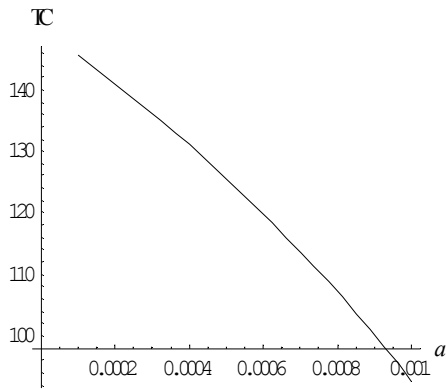


Fig. 5(a) The effect of parameter a on the total cost in the crisp model

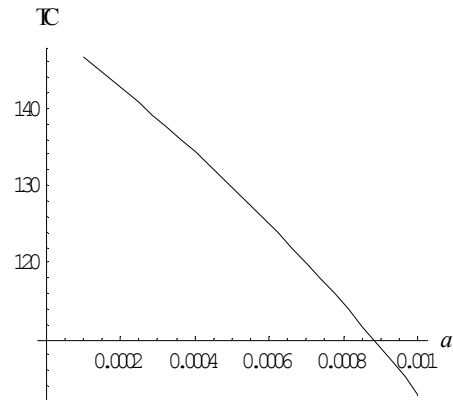


Fig. 5(b) The effect of parameter a on the total cost in the fuzzy model

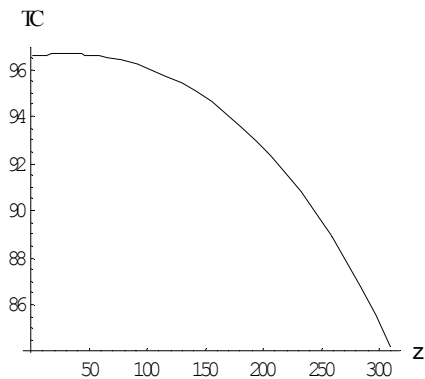


Fig.6.(a) The effect of parameter ζ on the total cost in the crisp model

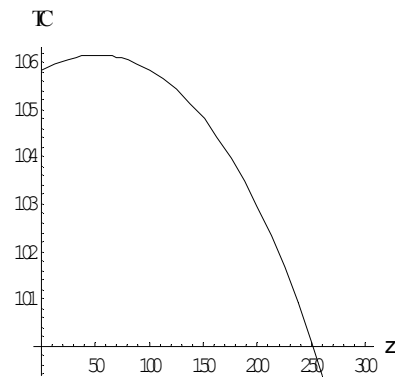


Fig.6.(b) The effect of parameter ζ on total cost in the fuzzy model

Fig. 7(a) and 7(b) show that as η (PT cost of supplier) increases, the total cost decreases. As we have less deterioration rate then more sales and more profit. Therefore, this process is more profitable for supplier.

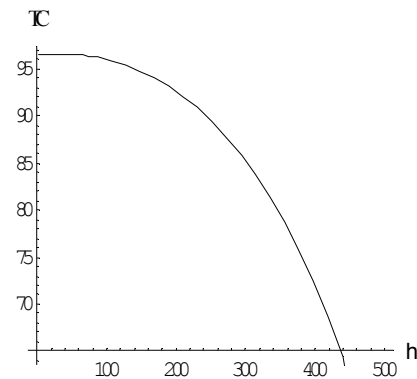


Fig 7.(a) The effect of parameter η on the total cost in the crisp model

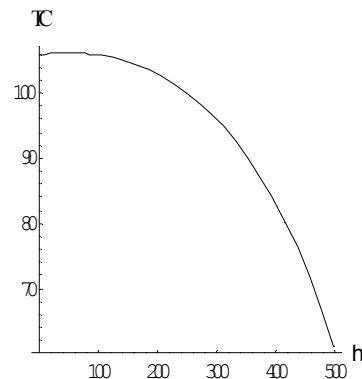


Fig 7.(b) The effect of parameter η on the total cost in the fuzzy model

Fig. 8(a) and Fig. 8(b) show the effect of n (number of deliveries) on the total cost of the model in the crisp and fuzzy model. As we increase the n then total cost decreases up to a point and then it again increases. Therefore, we have to take an optimal value of the number of cycles to minimize the total cost.

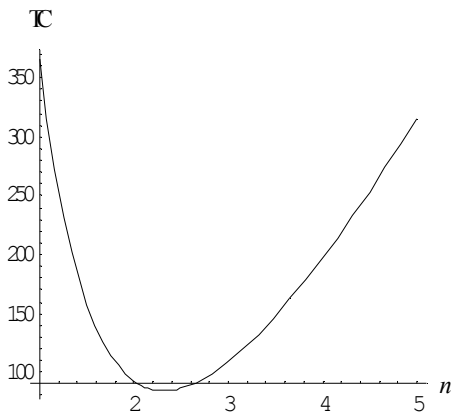


Fig 8.(a) The effect of parameter n on the total cost in the crisp model

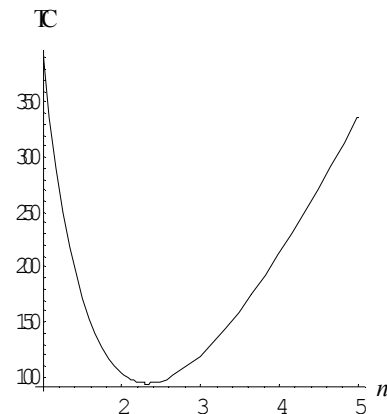


Fig 8.(b) The effect of parameter n on the total cost in the fuzzy model

According to Fig. 9(a) and 9(b), as r (inflation rate) increases the total cost increases in crisp and fuzzy model both.

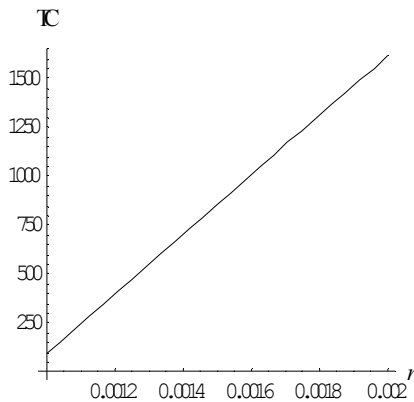


Fig. 9. (a) The effect of parameter r on the total cost in the crisp model

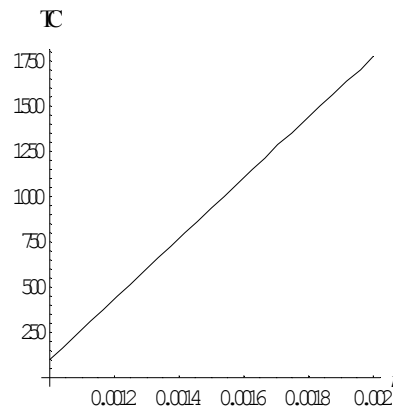


Fig. 9. (b) The effect of parameter r on the total cost in the fuzzy model

The following inferences can be observed from the sensitivity analysis based on Table 1.

- 1) The total cost per unit time is less sensitive on the changes in set up cost parameter.
- 2) The holding cost and purchasing cost has stronger effect on the optimal value of total cost than the other parameters.
- 3) In the fuzzy sense, optimal production time and supplier's time period is lower than the time period obtained in the crisp model.
- 4) Total cost obtained in the fuzzy sense is higher than the crisp total cost.

Table 1
Sensitivity analysis of the proposed model

Parameters		Percentage of under-estimation and over-estimation parameters				
		-50	-20	0	+20	+50
h_p	T_1	8.39818	8.33003	8.2854	8.24164	8.17714
	T_3	3.33672	3.31126	3.29462	3.27824	3.25414
	TC	163.104	120.853	92.678	64.4959	22.2102
c_p	T_1	8.285635	8.2855	8.2854	8.28543	8.28534
	T_3	3.2946	3.2946	3.29462	3.2946	3.29457
	TC	71.2539	84.1083	92.678	101.248	114.102
A_p	T_1	8.28507	8.28532	8.2854	8.28565	8.28590
	T_3	3.29447	3.29456	3.29462	3.29468	3.29478
	TC	92.666	92.6732	92.678	92.6828	92.69
h_s	T_1	10.6858	8.96774	8.2854	7.78765	7.24681
	T_3	4.80537	3.74045	3.29462	2.95779	2.57807
	TC	33.2215	72.3353	92.678	109.935	131.577
c_s	T_1	8.32514	8.3039	8.2854	8.26368	8.22452
	T_3	3.21619	3.26421	3.29462	3.32375	3.36501
	TC	113.308	101.001	92.678	84.2597	71.452
A_s	T_1	8.2834	8.28466	8.2854	8.28633	8.28754
	T_3	3.29384	3.29431	3.29462	3.29493	3.2954
	TC	92.6181	92.6541	92.678	92.7019	92.7378
c_{s1}	T_1	8.30627	8.29371	8.2854	8.2774	8.26549
	T_3	3.28856	3.29222	3.29462	3.29699	3.30049
	TC	92.2668	92.5152	92.678	92.8386	93.0753
c_{s2}	T_1	9.13466	8.51449	8.2854	8.12562	7.95948
	T_3	3.06456	3.22907	3.29462	3.34207	3.39294
	TC	77.1776	88.2402	92.678	95.9005	99.3641

9. Concluding Remarks

In this paper, we have investigated a model where producer and supplier use the PT to reduce the deterioration rate. This model is analyzed under two circumstances of the crisp and the fuzzy forms. Here, production and demand is treated as a fuzzy variable. Producer and supplier have different PT cost, which depends on the type of technology used. A solution procedure is also derived to optimize the total cost of producer and supplier. The main emphasis of this paper is on cost reduction by making effective investment in PT. Sensitivity analysis shows that while PT applies in the model, then the total cost is reduced due to the decrement in the deterioration rate. These changes can also be seen in the fuzzy sense but total cost is slightly higher than the crisp model. Effect of cost parameters on the total cost is also analyzed. Finally, the proposed model can be extended in several ways. For example, we could extend the model to allow for multi supplier.

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