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An economic production model for time dependent demand with rework and multiple production setups

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A B S T R A C T

In this paper, we present a model for time dependent demand with multiple productions and rework setups. Production is demand dependent and greater than the demand rate. Production facility produces items in *m* production setups and one rework setup (*m*, 1) policy. The major reason of reverse logistic and green supply chain is rework, so it reduces the cost of production and other ecological problems. Most of the researchers developed a rework model without deteriorating items. A numerical example and sensitivity analysis is shown to describe the model.

1. Introduction

The inventory lot-sizing problem for deteriorating items is prominent in the literatures due to its important connection with commonly used items in daily life. Fruits, vegetables, meats, photographic films, electronic products etc. are instances of deteriorating products. Deteriorating items are often classified in terms of their lifetime or utility as a function of time while in stock. The study of this paper concentrates on the deteriorating items classified as decreasing-utility with random lifetime. Fruits, vegetables, fish, etc. are some of the examples, which are classified in this category. Since the utility of such items are time dependent, their demand is more likely to be time dependent as the customers may be willing to buy more when the utility is high and less when the utility is low.

Schrady (1967) was the first researcher who concentrated on rework and remanufacturing process. Researches on rework have attracted many researchers. Khouja (2000) considered direct rework for economic lot sizing and delivery scheduling problem (ELDSP). Taleizadeh et al. (2011) studied

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production inventory models of two joint systems with and without rework. Khan et al. (2011) reviewed some research of EOQ model, which include imperfect items. Dobos and Richter (2004) developed a production and recycling inventory model with *n* recycling lots and *m* production lots. Teunter (2004) developed EPQ models with rework in two policies. Later, Widyadana and Wee (2010) introduced an algebraic approach to solve Teunter (2004) models. Misra (1975) introduced an optimal production quantity model for deteriorating item. Wee (1993) considered an EPQ deteriorating inventory with partial backordering. Goyal and Gunasekaran (1995) developed an EPQ model with marketing policies and deteriorating items. Widyadana et al. (2011) presented a simple method to solve a deteriorating item inventory problem. Widyadana and Wee (2012) developed an EPQ model with multiple production setups and rework. Singh et al. (2011) developed a multi item production model with reliability and flexibility under fuzzy environment. Singh et al. (2013) presented three-stage supply chain coordination under fuzzy random demand and production rate with imperfect production process. Singh et al. (2012, 2013) developed a warehouse production model with imperfect quality items. Singh et al. (2012) considered an economic production lot size model with rework and flexibility under shortages.

In this paper, we develop an economic production model for time dependent demand with multiple production and rework setups. The rework production system is shown in Fig. 1. In this system, items are inspected after production. Good quality items are stocked and sold to customer instantaneously. Defective items planned for rework.

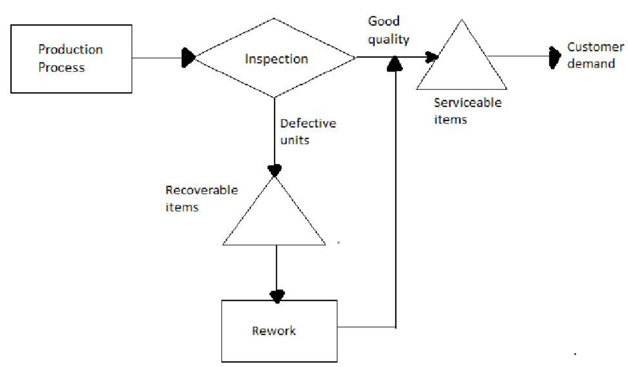

Fig. 1. The production system with rework

Assumptions:

- Demand rate is time dependent, i.e. $d = ae^{bt}$, where a and b are constants and $a > b > 0$.
- Production rate is demand dependent, i.e. $p = \lambda d = \lambda ae^{bt}$, where $\lambda > 1$.
- Rework and deterioration rate is constant.
- There is a replacement for deteriorated items.
- Shortages are not allowed.
- The rate of producing good quality items and rework must be greater than the demand rate.
- No machine breakdown occurs in the production run and rework period.
- Defective items are generated only during production period. Rework process results in only good quality items.

Notations:

d demand rate (unit/year)

p production rate (unit/year)

p_r rework process rate (unit/year)

θ	deterioration rate (unit/year)
α	percentage of good quality items
m	number of production setup in one cycle
D_i	total deteriorating units (unit)
K_s	production setup cost ($/setup)
K_r	rework setup cost ($/setup)
h_s	serviceable items holding cost ($/unit/year)
h_r	recoverable items holding cost ($/unit/year)
D_c	deteriorating cost ($/unit)
I_1	serviceable inventory level in a production period
I_2	serviceable inventory level in a non -production period
I_{r1}	recoverable inventory level in a production period
I_{r2}	recoverable inventory level in a non - production period
I_{r1}	recoverable inventory level in a rework production period
I_{t1}	total serviceable inventory in a production period
I_{t2}	total serviceable inventory in a non - production period
I_{t3}	total serviceable inventory in a rework production period
I_{t4}	total serviceable inventory in a rework non - production period
I_{v1}	total recoverable inventory in *m* production period
I_{v2}	total recoverable inventory in non - production period
I_{v3}	total recoverable inventory in a rework production period
TTI	total recoverable inventory in a production period
TRI	total recoverable inventory
I_{Mr}	maximum inventory level of recoverable items in a production setups
I_{Er}	maximum inventory level of recoverable items when rework process started
T_1	production period
T_2	non production period
T_3	rework process period
T_4	non rework process period
TCT	total cost per unit time

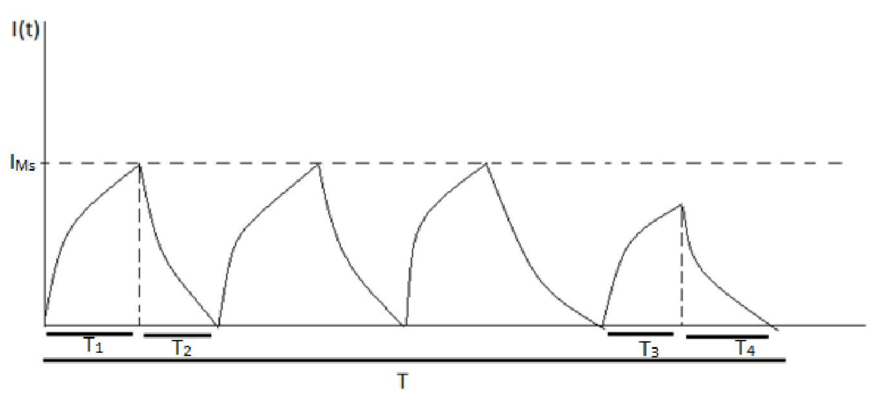

Fig. 2. Serviceable inventory level of 3 production setups and 1 rework setup

2. Formulation of the model

The inventory level of serviceable items in three production setups is shown in Fig. 2. Production is performed during T_1 time period. When production is produced, there are $(1-\alpha)p$ products defect per unit time. The work process starts after a predestined production up time and production setups. The rework process is performed in T_3 time period. Since production processes of material and product defect are different, rework rate is not the same as the production rate. The inventory level in a production period from the serviceable items can be formulated as:

$$\frac{dI_1(t_1)}{dt_1}+\theta I_1(t_1)=\alpha(\lambda-1)ae^{bt}. \qquad 0\le t_1\le T_1 \tag{1}$$

The inventory level in a non-production period can be formulated as:

$$\frac{dI_2(t_2)}{dt_1}+\theta I_2(t_2)=-\alpha ae^{bt}. \qquad 0\le t_2\le T_2 \tag{2}$$

Since $I_1(0)=0$, the inventory level in a production period is:

$$I_1(t_1)=\frac{\alpha(\lambda-1)a}{(b+\theta)}(e^{bt_1}-e^{-\theta t_1}), \qquad 0\le t_1\le T_1 \tag{3}$$

The total inventory in a production up time can be modeled as:

$$I_{t1}(t_1)=\int_0^{T_1}\frac{\alpha(\lambda-1)a}{(b+\theta)}(e^{bt_1}-e^{-\theta t_1})dt_1, \tag{4}$$

$$I_{t1}=\frac{\alpha(\lambda-1)a}{(b+\theta)}\left(\frac{1}{b}(e^{bT_1}-1)+\frac{1}{\theta}(e^{-\theta T_1}-1)\right).$$

For small value of θT_1 and using Taylor series approximation, we have

$$I_{t1}=\frac{\alpha(\lambda-1)a}{2}T_1^2. \tag{5}$$

Since $I_2(T_2)=0$, and using similar steps, the total inventory in a non – production period can be formulated as:

$$I_2(t_2)=\frac{a}{(b+\theta)}(e^{(\theta+b)T_2}e^{-\theta t_2}-e^{bt_2}), \tag{6}$$

$$I_{t2}=\frac{a(\theta-b)}{2(\theta+b)}T_2^2. \tag{7}$$

Since $I_1=I_2$, when $t_1=T_1$ and $t_2=0$, then:

$$\frac{\alpha(\lambda-1)a}{(b+\theta)}(e^{bt_1}-e^{-\theta t_1})=\frac{a}{(b+\theta)}(e^{(\theta+b)T_2}e^{-\theta t_2}-e^{bt_2}), \tag{8}$$

$$T_2\approx\alpha(\lambda-1)(T_1+\frac{(b-\theta)}{2}T_1^2). \tag{9}$$

The total serviceable inventory in a rework production period, the total serviceable inventory in a rework non–production period and their work non–production time are derived as follows:

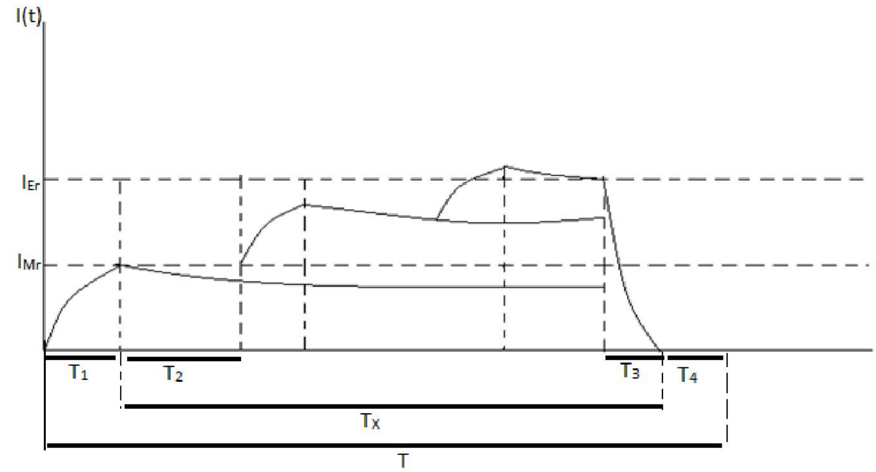

Fig. 3. Recoverable inventory level of 3 production setups and 1 rework setup

$$I_{t3} = \frac{(p_r - a)}{2} T_3^2, \tag{10}$$

$$I_{t4} = \frac{a(\theta - b)}{2(\theta + b)} T_4^2. \tag{11}$$

Since $I_3 = I_4$, when $t_3 = T_3$ and , then:

$$T_4 \approx \frac{p_r(T_3 - \frac{\theta}{2}T_3^2) - a(T_3 - \frac{(b-\theta)}{2}T_3^2)}{a}. \tag{12}$$

The inventory level of recoverable items is illustrated in Fig. 3. The inventory level of recoverable items in a production period can be modelled as:

$$\frac{dI_{r1}(t_{r1})}{dt_{r1}} + \theta I_{r1}(t_{r1}) = (1-\alpha)p. \qquad 0 \le t_{r1} \le T_1 \tag{13}$$

Since $I_{r1}(0) = 0$, the inventory level of recoverable items in a production period is:

$$I_{r1}(t_{r1}) = \frac{(1-\alpha)\lambda a}{(\theta + b)}(e^{btr_1} - e^{-\theta t_{r1}}), \tag{14}$$

The total recoverable inventory in a production up time in one setup is:

$$TTI = \frac{(1-\alpha)\lambda a}{2} T_1^2. \tag{15}$$

Since there are m production setups in one cycle, the total inventory for recoverable items in one cycle is:

$$I_{v1} = \sum_1^m \frac{(1-\alpha)\lambda a}{2} T_1^2 = \frac{(1-\alpha)m\lambda a}{2} T_1^2. \tag{16}$$

The initial recoverable inventory level in each production setup is equal to I_{Mr} and it can be modeled as:

$$I_{Mr} = \frac{(1-\alpha)\lambda a}{(\theta+b)}(e^{bT_1} - e^{-\theta T_1}). \tag{17}$$

Using Taylor series approximation, I_{Mr} can be written as:

$$I_{Mr} = (1-\alpha)\lambda a(T_1 + \frac{(b-\theta)}{2}T_1^2). \tag{18}$$

The inventory level of recoverable items in a non- production period is:

$$\frac{dI_{r2}(t_{r2})}{dt_{r2}} + \theta I_{r2}(t_{r2}) = 0. \qquad 0 \le t_{r2} \le (m-1)T_1 + mT_2 \tag{19}$$

When $t_{r2} = 0$, inventory level is equal to I_{Mr}, so the inventory level of recoverable items in a non-production time for each production setup can be modeled as:

$$I_{r2}(t_{r2}) = I_{Mr}e^{-\theta t_{r2}}. \qquad 0 \le t_{r2} \le (m-1)T_1 + mT_2 \tag{20}$$

The total inventory of recoverable items in *m* non production periods can be formulated as:

$$I_{v2} = \sum_{k=1}^{m} \int_{t_{r2}=0}^{(k-1)T_1+kT_2} I_{Mr}e^{-\theta t_{r2}} dt_{r2},$$

$$I_{v2} = \sum_{k=1}^{m} I_{Mr}\left(((k-1)T_1 + kT_2) - \frac{\theta((k-1)T_1 + kT_2)^2}{2} \right). \tag{21}$$

Inventory level of recoverable item in the end of production cycle is equal to maximum inventory level of recoverable items in a production setup reduced by deteriorating rate during production up time and down time. The inventory level can be formulated as follows:

$$I_{Er} = \sum_{k=1}^{m} I_{Mr}\left(1 - \theta((k-1)T_1 + kT_2) - \frac{(\theta((k-1)T_1 + kT_2))^2}{2} \right). \tag{22}$$

Substitute I_{Mr} from Eq. (18), we have

$$I_{Er} = \sum_{k=1}^{m} (1-\alpha)\lambda a(T_1 + \frac{(b-\theta)}{2}T_1^2)\left(1 - \theta((k-1)T_1 + kT_2) - \frac{(\theta((k-1)T_1 + kT_2))^2}{2} \right). \tag{23}$$

The inventory level of recoverable item in a rework period can be formulated as:

$$\frac{dI_{r3}(t_{r3})}{dt_{r2}} + \theta I_{r3}(t_{r3}) = -p_r, \qquad 0 \le t_{r3} \le T_3 \tag{24}$$

By solving Eq. (24), the inventory level of recoverable item in a rework period can be modelled as:

$$I_{r3}(t_{r3}) = \frac{p_r}{\theta}(e^{\theta(T_3 - t_{r3})} - 1). \tag{25}$$

The total inventory of recoverable items in a rework period can be modelled as:

$$I_{v3}(t_{r3}) = \int_{t_{r3}=0}^{T_3} \frac{p_r}{\theta}(e^{\theta(T_3-t_{r3})}-1)dt_{r3} = \frac{p_r T_3^2}{2}. \tag{26}$$

When $t_{r3}=0$, the number of recoverable inventory is equal to I_{Er}, then we have

$$I_{Er} = \frac{p_r}{\theta}(e^{\theta T_3}-1), \tag{27}$$

$$T_3 = \frac{I_{Er}}{p_r}. \tag{28}$$

Substitute I_{Er} from Eq. (23), we have

$$T_3 = \frac{1}{p_r}\sum_{k=1}^{m} I_{Mr}\left(1-\theta((k-1)T_1+kT_2)-\frac{(\theta((k-1)T_1+kT_2))^2}{2}\right). \tag{29}$$

The total recoverable inventory can be formulated as:

$$TRI = I_{v1}+I_{v2}+I_{v3},$$

$$TRI = \frac{(1-\alpha)m\lambda a}{2}T_1^2 + I_{v2} = \sum_{k=1}^{m} I_{Mr}\left(((k-1)T_1+kT_2)-\frac{\theta((k-1)T_1+kT_2)^2}{2}\right)+\frac{p_r T_3^2}{2}. \tag{30}$$

The number of deteriorating item is equal to the number of items produced minus the number of total demands. The total deteriorating units can be modeled as:

$$D_i = (m\alpha p T_1 + p_r T_3) - d(m(T_1+T_2)+T_3+T_4). \tag{31}$$

The total inventory cost consists of production setup cost, rework setup cost, serviceable inventory cost, recoverable inventory cost and deteriorating cost. The total inventory cost per unit time can be modelled as follows:

$$TCT(m,T_1) = \frac{mK_s + K_r + h_s(m(I_{t1}+I_{t2})+I_{t3}+I_{t4}) + h_r TRI + D_c D_i}{m(T_1+T_2)+T_3+T_4}, \tag{32}$$

The optimal solution must satisfy the following condition:

$$\frac{\partial TCT(m,T_1)}{\partial T_1} = 0. \tag{33}$$

3. Numerical Example

From the previous data, we consider

K_s = \$30 per production setup, K_r = \$5 per rework setup, $\lambda = 2$, $a = 100$, $b = 0.2$, $p_r = 1.5$ units per unit time, h_s = \$ 15 per unit per unit time, h_r = \$ 2 per unit per unit time, $\alpha = 4$, D_c = \$12 per unit and θ = 1.2. in appropriate units. With the help of the software, the optimal cost is equal to the \$3764.47 when $T_1 = 0.130862$ and m =3. Fig. 1 shows that the number of production setup is sensitive to the changes in parameters. The optimal production period for varying parameters is shown in Fig. 4. The optimal total cost per unit time for varying parameters is shown in Table 2. Fig. 5 shows that the total cost per unit time for varying parameters. The total cost per unit time for varying T_1 and m is shown in Fig. 6. Fig. 6. shows that the total cost per unit time is convex for small values of T_1.

Table 1
Sensitivity analysis of m and T_1

Parameter	-40%		-20%		20%		40%	
	m	T_1	m	T_1	m	T_1	m	T_1
K_s	3	0.0882288	3	0.0882999	3	0.0883469	3	0.0882402
K_r	3	0.0882959	3	0.0882483	3	0.0882165	3	0.08828
h_s	3	0.0958018	3	0.907646	3	0.881324	3	0.911737
h_r	3	0.144336	1	0.108916	3	0.0852951	1	0.0848731
α	3	0.133799	3	0.106444	3	0.0753443	3	0.116642
a	2	0.213107	3	0.0852956	3	0.0987392	3	0.116897
b	3	0.0847809	3	0.0865957	3	0.0899751	3	0.16081
P_r	2	0.25024	2	0.142196	3	0.155509	4	0.0635245
D_c	3	0.076721	3	0.0821448	3	0.0960248	3	0.113166

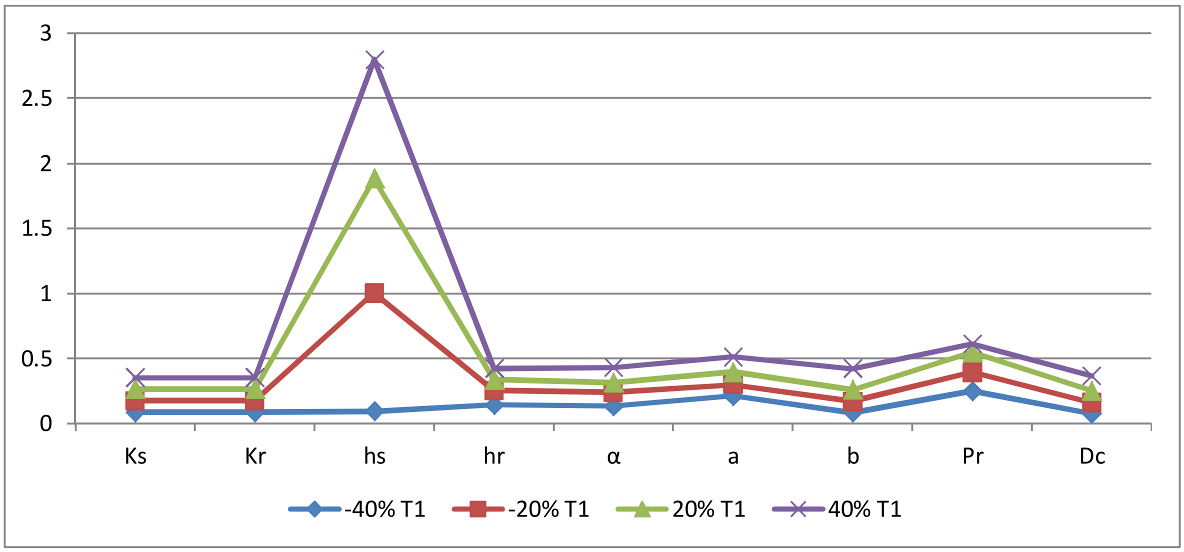

Fig. 4. T_1 Sensitivity Analysis

The sensitivity analysis is performed by changing each of the parameters by -40%, -20%, 20% and 40%. One parameter is taken at a time and the remaining parameters are kept unchanged. The *m* and T_1 values for different values of parameters are shown in Table1. The optimal production time (T_1^*) decreases with the increasing K_r, α, a, p_r, h_s, and h_r values, and it increases when the value of parameters b, D_c and K_s increase.

Table 2
Sensitivity analysis the total cost per unit time ($)

Parameter	-40%	-20%	20%	40%
K_s	7038.76	6844.12	6780.88	6973.12
K_r	6903.75	6910.66	6915.11	6905.95
h_s	4335.79	5896.86	9937.43	5063.7
h_r	4074.73	7414.7	7419.59	6966.2
α	7556.91	7677.18	7794.04	7260.47
a	5275.72	5759.6	9238.58	8858.8
b	14095.3	10105.7	6219.22	1436.61
P_r	3812.01	6458.92	7015.09	4515.25
D_c	12850.6	8126.93	7547.43	7382.36

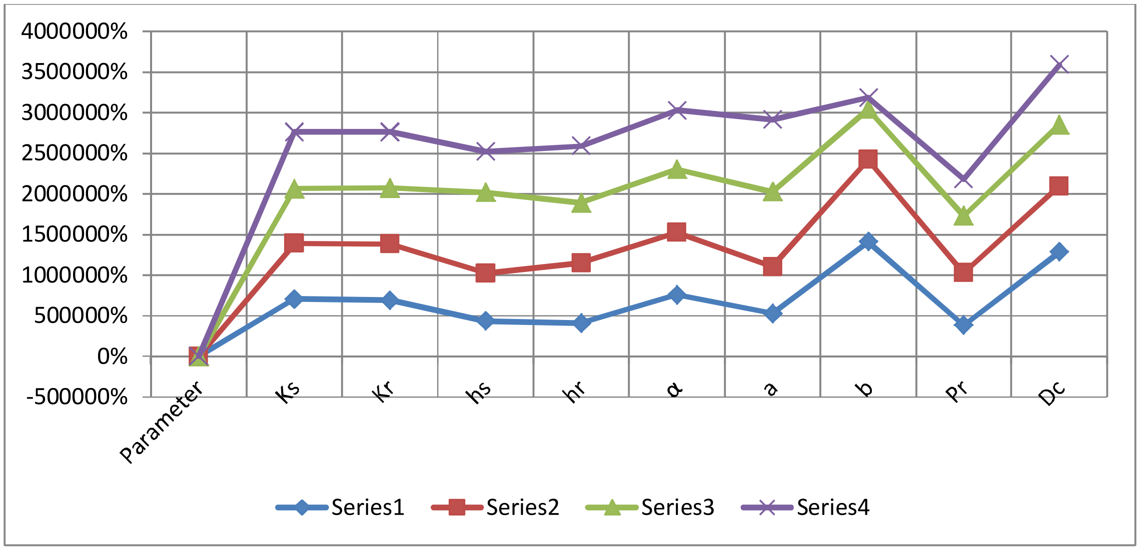

Fig. 5. Total cost per unit time sensitivity analysis

4. Conclusion

In this paper, we developed an economic production model for time dependent demand with multiple productions and rework setups. Production facility produced items in *m* production setups and one rework setup (*m*, 1) policy. The major reason of reverse logistic and green supply chain is rework, so it reduced the cost of production and other ecological problems. Most of the researchers developed a rework model without deteriorating items. A numerical example and sensitivity analysis has been shown to describe the model. An extension to this paper can be done to consider different production, demand, and deterioration scheme in each cycle. Further research also can be accomplished with different rates.

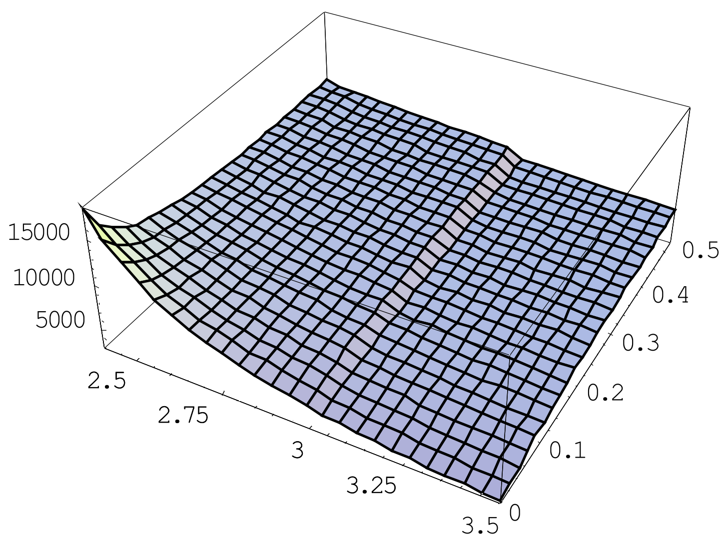

Fig. 6. Total cost per unit time in varies of T_1

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