Coordination of a three-level supply chain under disruption using profit sharing and return policy contracts

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Abstract

This paper studies a three-level supply chain with one retailer, one distributor and one manufacturer under disruption. First, we show that profit sharing and return policy contracts established between members can coordinate this supply chain. Then, we investigate the effect of demand disruption in supply chain and we illustrate the possibility of breaking the initial desirable coordination. We consider two possible scenarios for the demand disruption separately; afterwards we define the revised coordinator parameters to respond to their effect in supply chain. Numerical examples for validation and evaluation of our model are also elaborated.

1. Introduction

Coordination between independent firms in a supply chain relationship has been an important research issue, recently. Without coordination, distribution channel members determine their own decision variables independently in order to maximize one’s own profits. It is well documented in marketing and economics literature that non-coordinated decisions lead to “double marginalization”. It can decrease the competition of supply chain in comparison with other supply chains. Most of the researches focus on coordination schemes on supply chain by contractual approach. Cachon (2003) illustrated extensive researches on supply chain coordination with contracts approach. Tsay et al. (1998) provided a comprehensive review of supply chain contracts models. Mirzaee et al. (2012) illustrated the coordination of supply chain with combined contracts. A revenue sharing contract as a coordination mechanism has played an important role in coordination literature. Cachon and Larivier (2005) studied strengths and limitations of the revenue sharing contract in supply chain with one manufacturer and
multiple retailers. Dana and Spier (2001) considered the use of revenue sharing contract in a
decentralized channel with a perfect competitive downstream market and stochastic demand. They
explained that a revenue sharing contract could encourage downstream firms to choose the optimal
values of supply chain. Krishnan and Winter (2011) evaluated the role of revenue sharing contract on
supply chain. A return policy or in some articles buy back contract as a coordination mechanism tries to
decrease the risk of ordering quantity for the retailer and will cause an increase in profit of supply
chain. Some articles have used this contract in supply chain. Chen (2011) evaluated the effect of this
contract and quantity discount contract in newsvendor model. Yao et al. (2008) analyzed the impact of
changing of price in coordinating supply chain under return policy. They considered a model with one
manufacturer and one retailer and showed that this contract would improve supply chain performance.
Ding and Chen (2008) considered a coordination of a three-level supply chain with flexible return
policy. This paper used a return policy and profit sharing contracts to coordinate a three-level supply
chain without disruption firstly.

After an optimal coordination scheme has been established, the environment is often disrupted by some
unexpected haphazard events, such as: terrorism, earthquake, financial crisis, raw material shortage,
and so on. Disruption can significantly affect the performance of the supply chain members and
planned coordination schemes. Therefore, it is meaningful to know how the supply chain can still be
coordinated under different levels of disruptions. Disruption has evaluated in some of articles. Disruption
could be happened in two kinds: disruption in supply or demand. Khedlekar (2012) and
Azarmehr et al. (2012) considered a supply chain under supply disruption. Xiao et al. (2007)
investigated the coordination mechanism for a two-level supply chain with retail competition when the
demands are disrupted. Xiao and Qi (2008) assessed a coordination of two-level supply chain with two
competing retailers. They considered an all-unit quantity discount and an incremental quantity discount
in it. Qi et al. (2004), Chen and Xiao (2009), Huang et al. (2012) and Lei et al. (2012) also considered a
two-level supply chain with demand disruption in different situations. Cao et al. (2013) considered the
demand and cost disruptions in supply chain simultaneously. They also evaluated effect of disruption
on revenue sharing contract and the optimal strategies with disruption.

Three-level supply chain coordination has been considered rarely because of complexity. Arshinder et
al. (2009) introduced a theoretic model for coordination of three-level supply chains by using contracts.
Munson and Rosenblatt (2001) considered a coordinating of three-level supply chain by quantity
discount contract. Giannoccaro and Pontrandolfo (2004) showed a three-level supply chain could be
coordinated by revenue sharing contracts, but they did not consider the effect of disruption in supply
chain that would cause the coordination mechanism to be ruined. Some of the other articles such as
Seifert et al. (2012) and Ding and Chen (2008) have considered coordination of three-level supply
chains with different situations and disparate contracts. Neither of them considers the effect of
disruption on coordination of supply chain.

This paper considers a three-level supply chain with one manufacturer, one distributor and one retailer.
First, we define necessary conditions that the supply chain could be coordinated without disruption
using a return policy contract between the manufacturer and the distributor, and also a profit sharing
contract between the distributor and the retailer. A profit sharing contract has the same structure like
revenue sharing contract, but only the profit of sales will share. Then we will evaluate the effect of
demand disruption on our model and how it can affect the optimal parameters. The revised parameters
for coordination under disruption will be defined. Disruption is considered in two scenarios: when the
market demand increase and when the market demand decrease. Assumptions and parameters of model
will be defined in part 2. Supply chain coordination without demand disruption in part 3 is considered.
In part 4, we investigated on demand disruption in supply chain and all situations that can be happened
and the coordination that may be broken off. Simulations and numerical examples for validation of our
model are evaluated in part 5. At last, Conclusion is provided in part 6.
2. Assumptions and Notations

We consider a three-level supply chain consisting of one retailer, one distributor and one manufacturer when one kind of product in supply chain would be produced. First, the retailer determines the order quantity to the distributor based on the prediction of stochastic demand and wholesale price that the distributor determines, afterwards the distributor and the manufacturer set their plans based on the order quantity. The manufacturer produces goods and then the products will be delivered to distributor and after that to the retailer before the beginning of the selling season. For establishing the model, the following notations are utilized:

- $c_r$: The retailer’s marginal cost per unit,
- $c_d$: The distributor’s marginal cost per unit,
- $c_m$: The manufacturer’s marginal cost per unit,
- $q$: The order quantity that the retailer request,
- $p$: The retailer’s price,
- $v$: The return price per unit of leftover inventories at the end of selling season that the distributor pays the retailer,
- $D$: The demand during the selling season,
- $w_{dr}$: The wholesale price that is determined by distributor for the retailer per unit,
- $w_{md}$: The wholesale price that is determined by manufacturer for the distributer per unit,
- $g_r$: The shortage cost of retailer for not satisfied demand,
- $g_d$: The shortage cost of distributor for not satisfied demand,
- $g_m$: The shortage cost of manufacturer for not satisfied demand,
- $\Pi_r$: The expected profit function of the retailer,
- $\Pi_d$: The expected profit function of the distributer,
- $\Pi_m$: The expected profit function of the manufacturer,
- $\Pi$: The expected profit function of the total supply chain.

Also let $g = g_r + g_d + g_m$, $c = c_r + c_d + c_m$ and let $f$ be the probability density function and $F$ be the distribution function of the demand. We assume $F$ is differentiable and strictly non-decreasing and $F(0) = 0$. $\mu$ is an expected value of demand, $S(q) = q - \int_0^q F(y)dy$ is the expected sale, $L(q) = (q - D)^+ = q - S(q)$ is the expected surplus inventory and $L(q) = (D - q)^+ = \mu - S(q)$ is the expected lost sale where $(x)^+ = \max\{0, x\}$. In basic model, we assume $w_{md} > c_m$, $w_{dr} > c_d + w_{md}$ and $v < w_{dr}$. Other needful parameters will be introduced during this paper.

3. Supply chain without Disruption

In this section, a three-level supply chain without disruption will be considered. First, for the coordination of supply chain, we need to earn the optimal parameters of centralized supply chain and after that, we can use them for establishing the coordination between members in decentralized model.

3.1. Centralized supply chain

In centralized supply chain, it is supposed that all members are trying to integrate the channel and to increase the total profit of the supply chain. Thus the objective function of the centralized supply chain to maximize the total profit of the supply chain which consists of the summation of all the manufacturer, the distributor and the retailer expected profits that can be calculated as follows:

$$\Pi = \Pi_r + \Pi_d + \Pi_m$$  \hspace{1cm} (1)

That it equals to:

$$\Pi = ps(q) - cq - gL(q) = (p + g)s(q) - cq - g\mu$$  \hspace{1cm} (2)
The following first order condition is necessary and not sufficient for coordination
\[ \frac{\partial n}{\partial q} = (p + g)(1 - F(q)) - c = 0. \] (3)

Let \( q^* \) be an order quantity obtained from Eq. (3). So we would have
\[ F(q^*) = \frac{(p - c + g)}{(p + g)} \] (4)

Because \( \frac{\partial^2 n}{\partial q^2} = -(p + g)f(q) < 0 \), the expected profit function of total supply chain is concave. So the optimal order quantity of supply chain is equal to \( q^* \) and the coordination of supply chain will be achieved when the retailer selects \( q^* \). Therefore, we need to introduce a mechanism to encourage the retailer for selecting the optimal quantity of total supply chain.

3.2. Decentralized supply chain without coordination

In a decentralized supply chain in non-coordinated state, all the manufacturer, the distributor and the retailer decide separately to improve their profit share. First of all, the manufacturer determines its wholesale price for the distributor and the distributor determines its wholesale price for the retailer next. Finally, the retailer determines the order quantity based on the distributor’s wholesale price. Consequently, the profits of members are as below:

For the retailer:
\[ \Pi_r = ps(q) - (w_{dr} + c_r)q + v l(q) - g_r L(q) \] (5)

For the distributor:
\[ \Pi_d = (w_{dr} - w_{md})q - c_d q - g_d L(q) - v l(q) \] (6)

For the manufacturer:
\[ \Pi_m = (w_{md} - c_m)q - g_m L(q) \] (7)

So the retailer makes a decision to maximize its own profit, which means \( \frac{\partial \Pi_r}{\partial q} = 0 \), therefore the retailer will determine \( q_r^* \) as follows,
\[ F(q_r^*) = \frac{(p - w_{dr} - c_r + g_r)}{(p - v + g_r)} \] (8)

In order to \( w_{dr} \geq c_r + c_d + c_m = c \) and because \( F \) is non-decreasing and also based on Eq. (4) and Eq. (8), it is obvious that we would have \( q_r^* < q^* \). Therefore, the coordination of supply chain will not be achieved. As a result, we need to use an appropriate contract or contracts for establishing supply chain coordination that is used in next part.

3.3. Decentralized supply chain with coordination

In this section, for establishing supply chain coordination, a profit sharing contract between the retailer and distributor and also a return policy between the distributor and manufacturer are introduced. Under the profit sharing contract between the distributor and the retailer, the retailer pays the distributor the wholesale price \( w_{dr} \) for each unit purchased, plus a percentage of the sale profit that the retailer generates \( \varphi_{dr}(0 \leq \varphi_{dr} \leq 1) \). \( w_{dr} \) and \( \varphi_{dr} \) are the parameters of this contract, which depend on each other; this means that it is necessary to define one of them and the other would be calculated from the equations. We use a return policy contract between the distributor and the manufacturer, the distributor pays the manufacturer the wholesale price for each unit purchased \( w_{md} \) but the manufacturer pays the distributor \( b (v < b < w_{md}) \) as a return price per unit of leftover inventories at the end of selling
season, therefore \( w_{md} \) and \( b \) are the parameters of this contract that dependent on each other. We need to define one of them and the other would be calculated from the equations. The leftover inventories have been given from the retailer to the distributer and instead, the distributer pays \( v \) for them where it is permanent amount and defined before the selling season and this leftover inventories will be given to the manufacturer with return price of \( b \), which is defined based on the contract and it can be changed in order to \( w_{md} \). According to the coordinator contracts, we can calculate the expected profit for each member as below:

\[
\begin{align*}
\Pi_r &= q_{dr}pF(q) - (w_{dr} + c_r)q - g_rL(q) + vl(q) \\
\Pi_d &= (1 - q_{dr})pF(q) + (w_{dr} - w_{md})q - c_d q + (b - v)l(q) - g_d L(q) \\
\Pi_m &= (w_{md} - c_m)q - bl(q) - g_m L(q)
\end{align*}
\]

We assume the parameters \( q_{dr} \), \( b \) are given, we have two variables that they must be calculated, then for coordinating of the supply chain we will let the optimal order quantity of the centralized supply chain \( q^* \) in Eq. (12) and Eq. (14).

Based on the expected profit function of the retailer, \( \frac{\partial \Pi_r}{\partial q} \) must be zero, so we would have:

\[
(1 - F(q))q_{dr}p - w_{dr} - c_r + g_r + vF(q) = 0 \tag{12}
\]

By rewriting of above equation and substituting of \( q^* \), we can calculate \( w_{dr} \) from below that we will call that as \( w_{dr}^* \).

\[
w_{dr} = (1 - F(q^*)) (g_r + q_{dr}p) + vF(q^*) - c_r \tag{13}
\]

Based on the expected profit function of the distributer, \( \frac{\partial \Pi_d}{\partial q} \) must be zero, so we would have

\[
(1 - F(q))((1 - q_{dr})p + g_d) + w_{dr} - w_{md} - c_d + (b - v)F(q) = 0 \tag{14}
\]

By rewriting of above equation and substituting of \( q^* \), we can calculate \( w_{md} \) from below that we will name that as \( w_{md}^* \).

\[
w_{md} = (1 - F(q^*))((1 - q_{dr})p + g_d) + w_{dr} - c_d + (b - v)F(q^*) \tag{15}
\]

We do not have to consider the expected profit function of the manufacturer because the parameters \( w_{dr}^* \), \( w_{md}^* \) will be calculated from Eq. (13) and Eq. (15). If the distributer and the manufacturer select \( w_{dr}^* \), \( w_{md}^* \), respectively, the retailer will select the optimal quantity of supply chain, that means the coordination of supply chain will be achieved.

4. Supply chain under Disruption

In the above model, the static case of supply chain coordination was studied, but many factors will affect the demand, practically. When a disruption occurs, the coordination scheme of supply chain may become invalid and the coordination mechanisms can be ruined. Therefore, we consider the effect of demand disruption on supply chain coordination and also will introduce revised plans to the coordination. Disruption makes the market demand change from \( f \) to \( h \) and also the cumulative distribution function and the expected value will be \( H \) and \( \mu_h \), respectively, we have \( S_h(q) = q - \int_0^q H(y)dy \) as an expected sale, \( l_h(q) = (q - D)^+ = q - S_h(q) \) as an expected surplus inventory and \( L_h(q) = (D - q)^+ = \mu_h - S_h(q) \) as an expected lost sale. After disruption, the new quantity of supply chain is not the optimal quantity \( q^* \) that will cause the extra production costs for the retailer, the distributer and the manufacturer. We would have:
\[\begin{align*}
\bar{\Pi}_r &= \varphi_{dr}pS_h(q) - (\tilde{w}_{dr} + c_r)q - g_rL_h(q) + vI_h(q) \\
\bar{\Pi}_d &= (1 - \varphi_{dr})pS_h(q) + (\tilde{w}_{dr} - \tilde{w}_{md})q - c_dq + (b - v)I_h(q) - g_dL_g(q) - \gamma_{1d}(q - q^*)^- - \gamma_{2d}(q^* - q)^+ \\
\bar{\Pi}_m &= (\tilde{w}_{md} - c_m)q - bl_h(q) - g_mL_h(q) - \gamma_{1m}(q - q^*)^- - \gamma_{2m}(q^* - q)^+
\end{align*}\]

where \(\bar{\Pi}_r, \bar{\Pi}_d, \bar{\Pi}_m\) are the expected profit of the retailer, the distributor and the manufacturer with disruption, respectively. The wholesale price with disruption will change, the wholesale price that the distributor determines for the retailer is \(\tilde{w}_{dr}\) and the wholesale price that the manufacturer determines for the distributor is \(\tilde{w}_{md}\). \(\gamma_{1d} > 0\) is the unit extra production costs for additional products of the distributor and \(\gamma_{2d} > 0\) is the unit costs the supply chain decreases the production quantity for the distributor. For the manufacturer, \(\gamma_{1m} > 0\) is the unit extra production costs for additional products and \(\gamma_{2m} > 0\) is the unit costs the supply chain decreases the production quantity.

The total expected profit of supply chain would be:

\[\bar{\Pi} = \bar{\Pi}_r + \bar{\Pi}_d + \bar{\Pi}_m,\]

and Eq. (19) can be rewritten as below:

\[\bar{\Pi} = pS_h(q) - cq - gL_h(q) - (\gamma_{1d} + \gamma_{1m})(q - q^*)^- - (\gamma_{2d} + \gamma_{2m})(q^* - q)^+ .\]

Disruption can be happened as two scenarios; when the market demand increases and when the market demand decreases. We will consider both cases separately and explain conditions of coordination for this kind of supply chain. Obviously, when the market demand does not change; the parameters are still valid without disruption. We let \(\bar{q}^* = \text{argmax} \bar{\Pi} \text{for } q \geq 0\).

4.1. When the market demand increase

In this part, we consider the disruption as the market demand increases and define the optimal parameters for coordination of supply chain. The optimal parameters of coordinator contracts are not valid and must change.

**Theorem 1:** If disruption makes market size increasing [which means for any \(\geq 0\), \(H(q) \leq F(q)\)] so we will have \(\bar{q}^* \geq q^*\); and \(\bar{q}^*\) will calculate from the below equation:

\[H(q) = \frac{p - c + g - \gamma_{1d} - \gamma_{1m}}{p + g} .\]

and \(\bar{q}^*_1\) is an optimal quantity calculated from above equation.

**Proof:** If disruption makes market size increasing, we suppose the optimal quantity is less than \(q^*\) in this situation, then for any \(q < q^*\) the expected profit is equal to:

\[\bar{\Pi} = pS_h(q) - cq - gL_h(q) - (\gamma_{2d} + \gamma_{2m})(q^* - q)^+ .\]

Since we have for any \(q \geq 0\), \(H(q) \leq F(q)\) and we know \(H(\cdot)\) is strict non-decreasing function of \(q\), and from the assumption of \(q < q^*\), we know \(H(q) \leq H(q^*)\) then we will have \(H(\bar{q}^*) \leq H(q^*) \leq F(q^*)\), which means \(\frac{p - c + g}{(p + g)} > \frac{p - c + g + \gamma_{2d} + \gamma_{2m}}{(p + g)}\) that it is impossible, so when disruption makes market demand increasing, the optimal production quantity of total supply chain should be more or equal to the optimal production quantity \(q^*\). Finally, Eq. (18) will be reduced to:
\( \Pi = pS_h(q) - cq - gL_h(q) - (Y_{1d} + Y_{1m})(q - q^*)^+ \),

while the second order condition of function \( \frac{\partial^2 \Pi}{\partial q^2} = -(p + g)h(q) < 0 \) for \( q > q^* \) is less than zero and so the expected profit function is strictly concave; Therefore, we solve \( \frac{\partial \Pi}{\partial q} = 0 \) for solving \( \Pi \), then we will have:

\[
H(q) = \frac{p - c + g - Y_{1d} - Y_{1m}}{(p + g)}.
\]

(24)

And we let \( q_1^* \) be the optimal quantity of Eq. (24), then \( q_1^* \) is a maximum point of the function \( \Pi \). ■

We need to introduce new parameters for coordination, based on Eq. (24) the optimal quantity of centralized supply chain under disruption is \( \tilde{q}_1^* \), so we have

\[
H(q_1^*) = \frac{p - c + g - Y_{1d} - Y_{1m}}{(p + g)}
\]

(25)

So for coordination, \( q_1^* \) must satisfy \( \frac{\partial \Pi_r}{\partial q} = 0 \), \( \frac{\partial \Pi_d}{\partial q} = 0 \), \( \frac{\partial \Pi_m}{\partial q} = 0 \), therefore, from the \( \frac{\partial \Pi_r}{\partial q} = 0 \), we have:

\[
(1 - H(q_1^*)) (g_r + \varphi_{dr} p) + \nu H(q_1^*) - \tilde{w}_{dr} - c_r
\]

(26)

Then, from the \( \frac{\partial \Pi_d}{\partial q} = 0 \), we have

\[
(1 - H(q_1^*)) ((1 - \varphi_{dr}) p + g_d) + \tilde{w}_{dr} - \tilde{w}_{md} - c_d + (b - \nu) H(q_1^*) - Y_{1d}
\]

(27)

If two above conditions are established, \( \frac{\partial \Pi_m}{\partial q} \) would be zero and it does not have to consider that. So we can find \( \tilde{w}_{dr} \) and \( \tilde{w}_{md} \) from Eq. (26) and Eq. (27) that we name them as \( \tilde{w}_{dr}^*, \tilde{w}_{md}^* \), respectively. If the manufacturer determines \( \tilde{w}_{md}^* \) and the distributor determines \( \tilde{w}_{dr}^* \), the coordination of supply chain will be achieved while the market demand increases.

4.2. When the market demand decreases

In this part, we study the disruption as the market demand decreases and define the optimal parameters for coordination of supply chain.

**Theorem 2:** If disruption makes market size decreasing [which means for \( q \geq 0 \), \( H(q) \geq F(q) \)], so we will have \( \tilde{q}^* \leq q^* \); and \( \tilde{q}^* \) will calculate from the following equation

\[
H(q) = \frac{p - c + g + Y_{2d} + Y_{2m}}{(p + g)}
\]

(28)

Let \( \tilde{q}_2^* \) be an optimal quantity calculated from the above equation.

**Proof:** It is almost similar to theorem 1 then we do not have to prove it. ■

For establishing the coordination as the market demand decreases, based on Eq. (28) the optimal quantity of centralized supply chain is \( \tilde{q}_2^* \), so we have \( q_2^* \)

\[
H(q_2^*) = \frac{p - c + g + Y_{2d} + Y_{2m}}{(p + g)}
\]

(29)

So for coordination, \( q_2^* \) must satisfy \( \frac{\partial \Pi_r}{\partial q} = 0 \), \( \frac{\partial \Pi_d}{\partial q} = 0 \), \( \frac{\partial \Pi_m}{\partial q} = 0 \), therefore, from the \( \frac{\partial \Pi_r}{\partial q} = 0 \), we have:
\[(1 - H(q_r^*) - \varphi_{dr} p + v) - \bar{w}_{dr} - c_r = 0 \quad (30)\]

Then, from the \(\frac{\partial \Pi_m}{\partial q} = 0\), we have:

\[(1 - H(q_r^*))(1 - \varphi_{dr} p + g_d) + \bar{w}_{dr} - \bar{w}_{md} - c_d + (b - v)H(q_r^*) + \gamma_{2d} = 0 \quad (31)\]

If two above conditions would be established, \(\frac{\partial \Pi_m}{\partial q}\) would be zero and it does not have to be considered. Then we can find \(\bar{w}_{dr}\) and \(\bar{w}_{md}\) from Eq. (30) and Eq. (31) that we name them \(\bar{w}_{dr}^*, \bar{w}_{md}^*\). If the manufacturer determines \(\bar{w}_{md}^*\) and the distributor determines \(\bar{w}_{dr}^*\), the coordination of supply chain will be achieved when the market demand decreases.

5. Simulation and analysis

In this section, we present the simulations and numerical examples to describe how the model will coordinate with or without disruption. Moreover, the effect of combined contracts on supply chain will be considered and also the risk of uncertainty from disruption in model will be evaluated. We assume \(c_r = 10, c_d = 10, c_m = 30, g_m = g_d = g_d = 5, v = 20, p = 200\) and the demand is uniformly distributed, \(D \sim (a_1, a_2)\). In this part, the supply chain coordination will be evaluated by simulation in MATLAB software with and without disruption. Based on our model that has been discussed before, disruption will be considered as two separate parts.

5.1. Supply chain without disruption

In this part, we consider supply chain without disruption. We assume \(a_1 = 200, a_2 = 300, \varphi_{dr} = 0.5\) and \(b = 30\) and then the optimal parameters for supply chain will calculate as: \(q^* = 276.74, w_{dr}^* = 29.77, w_{md}^* = 51.86, \Pi_r = 33724, \Pi_d = 35848, \Pi_m = 6050, \Pi = 75652\). Therefore, under the given parameters, if the manufacturer and the distributor select their wholesale prices while 51.86 and 29.77; respectively, the retailer will order the optimal quantity that coordinate the supply chain. Now, we are going to evaluate the effect of coordinator parameters on supply chain that will be shown in Table 1 and Table 2.

5.1.1. The effect of profit sharing parameter

Table 1 shows the optimal parameters when \(b\) is constant with different amount of \(\varphi_{dr}\). In this situation, we want to consider the effect of profit sharing parameter on the supply chain when the parameter of the return policy contract is constant. Therefore, \(w_{md}\) and the expected profit of the manufacturer will not be changed. We have \((w_{dr}, \varphi_{dr})\) as the parameters of the profit sharing contract. It is reasonable, when the share of the retailer \(\varphi_{dr}\) increases based on the contract, the wholesale price \((w_{dr})\) of the distributor will increase because of dependency on \(\varphi_{dr}\), but this compensation will not be enough. Then, the expected profit will increase for the retailer and the distributor will decrease. This change will not affect the expected profit of total supply chain and it will be still 75652 and the optimal order quantity will be 276.74. This change just affect the share of profit between the retailer and the distributor.

<table>
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<tr>
<th>(\varphi_{dr})</th>
<th>(b)</th>
<th>(w_{dr}^*)</th>
<th>(w_{md}^*)</th>
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<th>(\Pi_d)</th>
<th>(\Pi_m)</th>
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</table>
5.1.2. The effect of the return policy parameter

Table 2 shows the optimal parameters when $q_{dr}$ is constant with different amount of $b$. In this situation, we want to consider the effect of the return policy parameter on the supply chain when the parameter of the profit sharing contract is constant. Therefore, $w_{dr}$ and the expected profit of the retailer will not be changed. We have $(w_{md}, b)$ as the parameters of the return policy contract. When $b$ increases, $w_{md}$ will increase and that is reasonable because of dependency on $b$. The increase of $b$ will be appropriate for the distributor, but it is not enough in comparison with the increase in $w_{md}$. So, the expected profit for the manufacturer increases and for the distributor decreases. This change will not affect the expected profit of total supply chain and the optimal order quantity and will be still 75652 and 276.74, respectively. This change is the exchange of the profit sharing between the manufacturer and the distributor.

Table 2
The effect of the return policy parameter

<table>
<thead>
<tr>
<th>$q_{dr}$</th>
<th>$b$</th>
<th>$w_{dr}^*$</th>
<th>$w_{md}^*$</th>
<th>$\Pi_r$</th>
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<td>55.70</td>
<td>33724</td>
<td>34786</td>
<td>7112</td>
</tr>
<tr>
<td>0.5</td>
<td>40</td>
<td>29.77</td>
<td>59.53</td>
<td>33724</td>
<td>33724</td>
<td>8174</td>
</tr>
</tbody>
</table>

5.2. Supply chain under disruption

Disruption in demand in our model has been verified as two scenarios. When the market demand increases and when the market demand decreases. In this part, we analyze the effect of disruption in two situations. We suppose $q_{dr} = 0.5, b = 30$ as before and for this parameters, we will evaluate the effect of changes market on supply chain. For the sake of simplicity, we have supposed the distribution of the demand as same as last part, but the market demand changes.

5.2.1. Increase in market demand

In this part, we will investigate the effect of enhancement in market demand on supply chain and then, we assume $\gamma_{1d} = \gamma_{1m} = 2$. Table 3 shows the increase in market demand for different size of it. The wholesale prices will increase for both the manufacturer and the distributor in comparison without disruption based on part 5.1 and it will not be adverse for the members of supply chain and also for the total supply chain. Since, the demand enhanced, the optimal order quantity, the profit of the members and the total supply chain will enhance. It is clear when compare data based on part 5.1 with data from Table 3.

Table 3
The optimal parameters with coordination when the market demand increases

<table>
<thead>
<tr>
<th>$\gamma_{1d} = \gamma_{1m}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$q_{1}^*$</th>
<th>$\bar{w}_{dr}^*$</th>
<th>$w_{md}^*$</th>
<th>$\bar{\Pi}_r$</th>
<th>$\bar{\Pi}_d$</th>
<th>$\bar{\Pi}_m$</th>
<th>$\bar{\Pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>210</td>
<td>320</td>
<td>292.37</td>
<td>31.35</td>
<td>53.21</td>
<td>34109</td>
<td>36852</td>
<td>6755</td>
<td>77716</td>
</tr>
<tr>
<td></td>
<td>220</td>
<td>340</td>
<td>309.86</td>
<td>31.35</td>
<td>53.21</td>
<td>34976</td>
<td>37850</td>
<td>7125</td>
<td>79951</td>
</tr>
<tr>
<td></td>
<td>230</td>
<td>360</td>
<td>327.35</td>
<td>31.35</td>
<td>53.21</td>
<td>35901</td>
<td>38905</td>
<td>7496</td>
<td>82302</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>380</td>
<td>344.84</td>
<td>31.35</td>
<td>53.21</td>
<td>36871</td>
<td>40007</td>
<td>7867</td>
<td>84745</td>
</tr>
</tbody>
</table>

5.2.2. The effect of $\gamma_{1d}, \gamma_{1m}$ on supply chain

Obviously, as the market demand increases, the optimal order quantity will be more than the optimal order quantity without disruption, that was shown before. So, with regard to the effect of unit extra costs for both the manufacturer and the distributor, we only need to consider supply chain under increasing demand. Table 4 shows the optimal parameters when the unit extra costs have changed. For
evaluating this parameters, we suppose the market demand increases and after that we will keep it constant. For the simplicity, we assume \( y_{1d} = y_{1m} \) and define \( a_1 = 220 \) and \( a_2 = 340 \). According to Table 4, as the unit extra costs enhance, that means the risk of surplus inventory for the manufacturer and the distributor will increase and they will increase their wholesale prices to compensate a part of this risk, finally the retailer will decline the order quantity. Therefore, the expected profit of the retailer will decrease and the expected profit of the manufacturer and the distributor will increase. These are reasonable from the equations in part 4.1 and calculated data as below. The expected profit of total supply chain will also decrease, because the risk of supply chain grows up and the costs of quantity add more than the optimal quantity in situation without disruption.

### Table 4

The effect of \( y_{1d}, y_{1m} \) on supply chain

<table>
<thead>
<tr>
<th>( y_{1d} = y_{1m} )</th>
<th>( \bar{w}_{dr}^* )</th>
<th>( w_{md}^* )</th>
<th>( q_1^* )</th>
<th>( \bar{\Pi}_r )</th>
<th>( \bar{\Pi}_d )</th>
<th>( \bar{\Pi}_m )</th>
<th>( \bar{\Pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>31.35</td>
<td>53.21</td>
<td>309.86</td>
<td>34976</td>
<td>37850</td>
<td>7125</td>
<td>79951</td>
</tr>
<tr>
<td>3</td>
<td>32.14</td>
<td>53.88</td>
<td>308.74</td>
<td>34749</td>
<td>37863</td>
<td>7278</td>
<td>79890</td>
</tr>
<tr>
<td>4</td>
<td>32.93</td>
<td>54.56</td>
<td>307.63</td>
<td>34523</td>
<td>37877</td>
<td>7431</td>
<td>79832</td>
</tr>
<tr>
<td>5</td>
<td>33.72</td>
<td>55.23</td>
<td>306.51</td>
<td>34298</td>
<td>37892</td>
<td>7585</td>
<td>79776</td>
</tr>
<tr>
<td>Increase</td>
<td>increase</td>
<td>increase</td>
<td>decrease</td>
<td>decrease</td>
<td>increase</td>
<td>increase</td>
<td>decrease</td>
</tr>
</tbody>
</table>

5.2.3. Decrease in market demand

In this part, we pursue the effect of reduction in market demand on supply chain and then, we assume \( y_{2d} = y_{2m} = 2 \). Table 5 shows the decrease in market demand for different size of it. The wholesale prices for both the manufacturer and the distributor will decrease in comparison without disruption based on part 5.1. Based on our model and in practice, it is reasonable that the optimal order quantity, the profit of the members and the total supply chain will decline. That is obvious when we compare data based on part 5.1 with data from Table 5.

### Table 5

The optimal parameters with coordination when the market demand decreases

<table>
<thead>
<tr>
<th>( y_{2d} = y_{2m} )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( q_2 )</th>
<th>( \bar{w}_{dr} )</th>
<th>( w_{md}^* )</th>
<th>( \bar{\Pi}_r )</th>
<th>( \bar{\Pi}_d )</th>
<th>( \bar{\Pi}_m )</th>
<th>( \bar{\Pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>190</td>
<td>280</td>
<td>260.74</td>
<td>28.19</td>
<td>34.79</td>
<td>33393</td>
<td>38988</td>
<td>1217</td>
<td>73598</td>
</tr>
<tr>
<td>180</td>
<td>260</td>
<td>242.88</td>
<td>28.19</td>
<td>34.79</td>
<td>32792</td>
<td>37966</td>
<td>1096</td>
<td>71854</td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>240</td>
<td>225.02</td>
<td>28.19</td>
<td>34.79</td>
<td>32390</td>
<td>37143</td>
<td>975</td>
<td>70508</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>220</td>
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<td>34.79</td>
<td>32285</td>
<td>36617</td>
<td>853</td>
<td>69755</td>
<td></td>
</tr>
</tbody>
</table>

5.2.4. The effect of \( y_{2d}, y_{2m} \) on supply chain

Once the market demand decreases, the optimal order quantity will be less than the optimal order quantity without disruption, that was shown before. Then, regarding to the effect of \( y_{2d}, y_{2m} \) for both the manufacturer and the distributor respectively, we only need to consider the supply chain under decreasing demand. Table 6 shows the optimal parameters when the \( y_{2d}, y_{2m} \) have changed. For evaluation of this parameters, we suppose the market demand decline one time, and after that we will keep it constant. We assume \( a_1 = 180 \) and \( a_2 = 260 \).

According to Table 6, when \( y_{2d} \) and \( y_{2m} \) increase, the risk of inventory is reduced for the manufacturer and the distributor. Then, they will decrease their wholesale prices to compensate a part of this risk so that the retailer would order more quantity to decrease their disadvantages. Consequently, the expected profit of the retailer will increase because of excess in quantity and reduction in wholesale
price and the expected profit of the manufacturer and the distributor will decrease. These are suitable from the equations in part 4.2 and calculated data as Table 6. The total profit of supply chain will also diminish, because the risk of supply chain increase and the costs of quantity less than the optimal quantity without any enhancement in disruption.

<table>
<thead>
<tr>
<th>( \gamma_2d = \gamma_2m )</th>
<th>( \tilde{w}_{dr}^* )</th>
<th>( w_{md}^* )</th>
<th>( q_2^* )</th>
<th>( \tilde{\Pi}_r )</th>
<th>( \tilde{\Pi}_d )</th>
<th>( \tilde{\Pi}_m )</th>
<th>( \tilde{\Pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>28.19</td>
<td>34.79</td>
<td>242.88</td>
<td>32792</td>
<td>37966</td>
<td>1096</td>
<td>71854</td>
</tr>
<tr>
<td>3</td>
<td>27.4</td>
<td>33.93</td>
<td>243.63</td>
<td>32972</td>
<td>37955</td>
<td>858</td>
<td>71785</td>
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<tr>
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<td>26.6</td>
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<td>37944</td>
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<td>71717</td>
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<td>25.81</td>
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<td>245.12</td>
<td>33333</td>
<td>37935</td>
<td>383</td>
<td>71617</td>
</tr>
</tbody>
</table>

Increase ➔ decrease decrease increase increase decrease decrease decrease

6. Conclusion

This paper investigated the coordination of a three-level supply chain with one retailer, one distributor and one manufacturer that encountered with the demand disruption. One kind of product produced by the manufacturer and it is delivered to the distributor and then to the retailer before the beginning of the selling season. First, it has been shown the supply chain coordination with combined contracts consist of profit sharing contract between the retailer and the distributor and also return policy contract between the distributor and the manufacturer was achieved. Next, we displayed that the initial contracts could not attain the coordination under disruption. Therefore, with the changes in market demand, the conditions of coordination have to change. We have considered two possible scenarios for the disruption: At first, as the market demand increases and then, when the market demand decreases. In both situations, the optimal parameters for the coordination of the supply chain have been defined. We have shown, disruption can ruined the plans of coordination but with redefinition of parameters of combined contracts, the coordination can be achieved again. We have analyzed performance of our model by simulation and different examples. The effect of main parameters and increase in cost of disruption on supply chain have been presented and evaluated. When the costs of uncertainty in both scenarios increase, the total supply chain will be harmed.

References


