

Volume flexible multi items inventory system with imprecise environment

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ABSTRACT

This paper addresses a multi items volume flexible system for time dependent decaying items with the concept of machine breakdown and imprecise environment. In this study, partially backlogged shortages have been discussed. All the costs are fuzzified with signed distance method. Numerical examples are given to illustrate the theoretical results and sensitivity analysis is given to validate the results for various parameters.

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1. Introduction

Most of the studies have taken into consideration only the crisp aspect of inventory models. In crisp studies, all the parameters of the model have definite values and there is no uncertainty associated with them. However, in reality, there are some uncertainties also about the values of different cost parameters involved in the system. For any particular problem in the crisp scenario, the aim is to maximize or minimize the objective function under the given constraints. However, in many practical situations, the decision maker may not be in a position to specify the objective or the constraints, precisely, but rather may specify them in the “fuzzy sense”. In such a case, the decision maker has to depend on the fuzzy methods of solving the problem. Fuzzification grants authenticity to the model in the sense that it allows vagueness in the whole setup which brings it closer to reality.

In a perfect competitive situation, the holding cost, shortages cost, lost sale cost and the set-up cost may fluctuate a little from its actual value in the different replenishment cycles. These are some

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uncertainties in inventory optimization tasks that cannot be treated appropriately by usual probabilistic models. Therefore, it is realistic to consider the idea of fuzzy.

In 1965, the first publication in fuzzy set theory by Zadeh (1965) showed the intention to accommodate uncertainty in the non-stochastic sense. Zimmermann (1985) developed a tolerance approach to transform a fuzzy decision making problem to a regular crisp optimization problem and showed that it could be solved to obtain a unique exact optimal solution with highest membership degree using classical optimization algorithms. Padmanabhan and Vrat (1990) formulated an inventory problem of deteriorating items with two objectives of minimization of total average cost and wastage cost in crisp environment. Goyal (1991) discussed a procedure for determining the economic manufacturing policy for a product and the economic ordering policy for the raw materials required by the product. The demand for the product was constant over time and shortages were not permitted. Tse and Makis (1994) considered preventive replacement with two types of machine failure – major and minor. When a major failure occurs, the failed unit is replaced by a new one and the interrupted lot was aborted. They have used the concept of crisp. Srinivasan and Lee (1996) developed a manufacturing model when the demand rate was taken as variable, processing time follows an arbitrary probability distribution. Roy and Maiti (1998) formulated an inventory problem of deteriorating items with two objectives, namely, maximizing total average profit and minimizing total cost in fuzzy model. Roy and Maiti (2000), Mahapatra and Maiti (2005) have solved the classical order level inventory models in fuzzy environment. Yao and Wu (1999) and Dey et al. (2005) have considered the production model with fuzzy environment. Goyal and Giri (2003) developed the production–inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. Giri and Dohi (2005) presented an exact formulation of EMQ model under a general framework in which the time to machine failure, corrective and preventive repair times all are assumed to follow arbitrary probability distributions. Wee and Chung (2005) considered an optimal production model for deteriorating items with constant demand. The objective was to determine the inventory models to maximize the net benefit of the production system. In their study, deterioration rate was constant and shortages were not allowed. Singh et al. (2008) developed an inventory model for decaying items with variable demand and variable holding cost in crisp environment. Maity and Maiti (2008) developed the optimal production policy for an inventory control system of deteriorating items in fuzzy environment. Here, the deterioration of the items and depreciation of sales were at a constant rate. Singh and Singh (2008) considered the fuzzy inventory model for finite rate of replenishment using signed distance method. Chakraborty et al. (2008) presented a generalized economic manufacturing quantity model for an unreliable production system in which the production facility may shift from an ‘in-control’ state to an ‘out-of-control’ state at any random time and may ultimately break down afterwards. Chakraborty et al. (2009) developed a production inventory system for deteriorating items in which the production facility may not only shift from an ‘in-control’ state to an ‘out-of-control’ state. Xu and Zhai (2010) discussed the coordination problem for single-period supply chain problems with fuzzy demand. Mahata and Mahata (2011) considered the EOQ model for a retailer to reflect the inventory system situation in the fuzzy sense. Wang et al. (2012) developed continuous review inventory models in which a fraction of demand is backordered and the remaining fraction is lost during the stock out period under fuzzy demands. Singh et al. (2013) discussed a centralized reverse channel structure with flexible manufacturing under the stock out situation. Shukla et al. (2013) considered EOQ model for deteriorating items with exponential demand rate and shortages. Hsu (2012) discussed an EOQ for items with imperfect quality and inspection errors.

This paper investigated an Economic Manufacturing Quantity model for time dependent decaying items and selling price demand with volume flexible and fuzzy environment. We assumed that different machines ‘ A_i ’ (1,2,...n) are dedicated to the production of different items ‘ i ’ with different production rates ‘ P_i ’. The management of production in machine A_i is vested with the management unit ‘ B_i ’. It is assumed that a machine may become out of order during its working time. As a result, there is a mean time for every machine between its failure/breakdowns. During a breakdown of a machine, there is

demand although there is no production. In such a situation, the demand is met until the inventory level falls below the quantity demanded. When inventory level becomes less than the demand, the concerned management unit B_i is rendered fully idle. This situation occurs when the customer is a wholesaler having the demand of a big lot size and the concerned management unit can't meet this demand because the stock size is less than the quantity demanded. Therefore, we considered the idle time of each management unit; this idle time leads to an additional cost for the last man hours. We have considered the capital available for manufacturing the items is limited. Shortages are allowed with partial backlogging. After that a numerical example is provided and sensitivity analysis is carried out to demonstrate the significance of considering profit maximization technique.

2. Assumptions and Notations

The mathematical models in this study are developed on the basis of the following assumptions and notations:

Assumptions

The following assumptions for the proposed model, which are as follows:

1. The model is developed for multiple items.
2. The demand for the item is a downward slopping function of the price.
3. Deterioration rate is time dependent.
4. Production rate per unit time is considered as a decision variable.
5. Invested capital for production is limited.
6. Machine breakdown is considered during the production period.
7. Idle time to the management unit is considered.
8. Unit production cost for the i^{th} item ($i=1,2,3,\dots, n$) is a function of the production rate.
9. Holding cost is a linear increasing function of time.
10. Shortages are allowed with partial backlogging during the idle time.
11. Backlogging rate is taken as constant.
12. Time horizon is infinite.

Notations

The following notations are assumed in our study:

- $Q_i(t)$ The on-hand inventory of the i^{th} item at time t
 P_i The production rate per unit time for the i^{th} item
 $\theta_i t$ Time dependent deterioration rate $0 < \theta_i < 1$
 μ_i The mean time between successive breakdowns of the machines ($A_i, i=1,2,\dots,n$)
 $\psi_i(t_i)$ Probability density function of t_i
 C Unit purchasing cost with $C < S_p^i$
 m_i The mean time of repair of i^{th} machine
 τ_i The mean duration of a breakdown of machines ($A_i, i=1,2,\dots, n$)
 $\phi_i(\tau_i)$ Probability density function of τ_i
 $C_h^i + \gamma t$ The cost of carrying one unit of i^{th} item in inventory per unit time
 C_s^i The cost of shortages per unit time of i^{th} item
 C_{LS}^i The cost of lost sale per unit time of i^{th} item

- $\eta_i(P_i)$ The cost for production of a unit time of the i^{th} item ($i=1,2,\dots, n$) and
 $\eta_i(P_i) = (r_i + \frac{g_i}{P_i} + \alpha_i P_i)$, where r_i is material cost, α_i is tool or die cost and g_i
 is energy and labor cost
 S_p^i The selling price per unit of i^{th} item
 W_i The cost per unit of idle time of the management unit B_i
 CAP The total capital available for production of all the items
 D_i The annual demand as a decreasing function of price; $D_i(S_p^i) = \alpha_i(S_p^i)^{-\beta_i}$,
 Where $\alpha_i > 0$ and $\beta_i > 1$

3. Formulation of the Model

In this model a multi-items inventory system with machine breakdown and volume flexibility has been considered. The production cycle begins with zero stock. Production starts at time $t=0$ and the stock reaches a level $Q_i(t_i) \{i=1,2,\dots,n\}$ at times $t=t_i$ after adjusting demand rates D_i . At times $t=t_i$ machines A_i , becomes out of order. Then repairing of machines A_i starts and takes times τ_i to comeback into working state. Here, t_i and τ_i are random variables which follow probability distribution functions ψ_i and Φ_i , respectively. The governing equations for the inventory system are as follows:

$$Q_i'(t) + \theta_i t Q_i(t) = P_i - \alpha_i (S_p^i)^{-\beta_i} \quad 0 \leq t \leq t_i \quad (1)$$

$$Q_i'(t) + \theta_i t Q_i(t) = -\alpha_i (S_p^i)^{-\beta_i} \quad t_i \leq t \leq x \quad (2)$$

With the boundary condition $Q_i(0) = 0$ and $Q_i(x) = 0$, where $i=1,2,\dots,n$, solution of the Eq. (1) and Eq. (2) are:

$$Q_i(t) = (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left(t + \frac{\theta_i t^3}{6} \right) e^{-\frac{\theta_i t^2}{2}} \quad 0 \leq t \leq t_i \quad (3)$$

$$Q_i(t) = \alpha_i (S_p^i)^{-\beta_i} \left[(x-t) + \frac{\theta_i}{6} (x^3 - t^3) \right] e^{-\frac{\theta_i t^2}{2}} \quad t_i \leq t \leq x \quad (4)$$

The idle times of the management units $\{B_i, i=1,2,\dots,n\}$ due to a breakdown of the machines $\{A_i, i=1,2,\dots,n\}$ are

$$\begin{cases} \text{if } \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \geq \tau_i & u_i = 0 \\ \text{if } \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} < \tau_i & u_i = \tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \end{cases}$$

The expected cost per breakdown of the machine $\{A_i, i=1,2,\dots,n\}$ during the idle time is given by:

$$\begin{aligned} E_{ic}^i &= W_i \int_0^\infty \left[\int_{\frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}}}^\infty \left(\tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \right) \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i \\ &= \frac{W_i}{\mu_i} \left(\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1 \right) \left[\mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left(\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} \right) \right] \end{aligned} \quad (5)$$

The expected shortage cost for i^{th} item during the idle time is

$$E_{Sc}^i = C_S^i \alpha_i (S_p^i)^{-\beta_i} \int_0^\infty \left[\int_{\frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}}}^\infty \left(\tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \right) \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i \quad (6)$$

$$= C_S^i [\alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3]$$

The expected lost sale cost for i^{th} item during the idle time is given by:

$$\begin{aligned} E_{Sc}^i &= C_{LS}^i \int_0^\infty \left[\int_0^\infty \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} (1-\delta) \alpha_i (S_p^i)^{-\beta_i} \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i \\ &= \frac{C_{LS}^i (1-\delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \left[\mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} (\mu_i^2 + \frac{\theta_i \mu_i^3}{3}) \right] \end{aligned} \quad (7)$$

Now, the total inventory of i^{th} item is as follows:

$Inv_i(t_i)$ = Inventory during $[0, t_i]$ + Inventory during $[t_i, x]$

$$\begin{aligned} Inv_i(t_i) &= \left[\int_0^{t_i} (C_h^i + \gamma t) Q_i(t_i) dt_i + \int_{t_i}^x (C_h^i + \gamma t) Q_i(t_i) dt_i \right] \\ &= (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left[C_h^i \left(\frac{t_i^2}{2} - \frac{\theta_i t_i^4}{12} \right) + \gamma \left(\frac{t_i^3}{3} - \frac{\theta_i t_i^5}{15} \right) \right] + \alpha_i (S_p^i)^{-\beta_i} \left[C_h^i \left(\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 t_i^2}{2 \alpha_i (S_p^i)^{-2\beta_i}} \right. \right. \\ &\quad - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) t_i^2}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{t_i^2}{2} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i t_i^4}{6 \alpha_i (S_p^i)^{-3\beta_i}} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i t_i^4}{12 \alpha_i (S_p^i)^{-4\beta_i}} + \frac{\theta_i t_i^4}{12} \\ &\quad + \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i t_i^4}{6 \alpha_i (S_p^i)^{-\beta_i}} \left. \right) + \gamma \left(\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 t_i^3}{3 \alpha_i (S_p^i)^{-3\beta_i}} + \frac{t_i^3}{6} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) t_i^4}{2 \alpha_i (S_p^i)^{-\beta_i}} + \frac{\theta_i t_i^5}{15} \right. \\ &\quad \left. + \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i t_i^5}{40 \alpha_i (S_p^i)^{-5\beta_i}} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i t_i^5}{6 \alpha_i (S_p^i)^{-3\beta_i}} + \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i t_i^5}{8 \alpha_i (S_p^i)^{-\beta_i}} \right) \end{aligned} \quad (8)$$

As a result, the expected inventory cost for i^{th} item is as follows:

$$\begin{aligned} E_{inc}^i &= \int_0^\infty Inv_i(t_i) \psi_i(t_i) dt_i \\ &= \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \left[C_h^i (\mu_i^3 - 2\theta_i \mu_i^4) + \gamma (2\mu_i^3 - 8\mu_i^5) \right] + \alpha_i (S_p^i)^{-\beta_i} \left[C_h^i \left(\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} \right. \right. \\ &\quad - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 \\ &\quad - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} \left. \right) + \gamma \left(\frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} + \mu_i^4 - \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-5\beta_i}} \right. \\ &\quad \left. + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-3\beta_i}} + 15(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^5 \right) \end{aligned} \quad (9)$$

The, production cost per unit of i^{th} item is given by:

$$\eta_i(P_i) = r_i + \frac{g_i}{P_i} + \alpha_i P_i$$

For that reason, the expected production cost for i^{th} item is given by:

$$E_{prc}^i = \sum_{i=1}^n \int_0^\infty \eta_i(P_i) P_i t_i \psi_i(t_i) dt_i = \sum_{i=1}^n (r_i + \frac{g_i}{P_i} + \alpha_i P_i) P_i \mu_i \quad (10)$$

Then the expected set up cost for i^{th} item, which is as follows:

$$SE_{UP}^i = \sum_{i=1}^n \int_0^\infty S_{UP}^i t_i \psi_i(t_i) dt_i = \sum_{i=1}^n S_{UP}^i \quad (11)$$

The reliability of spare parts of a machine follows exponential probability distribution function. The probability density functions of exponential distribution is as follows

$$\psi_i(t_i) = \frac{1}{\mu_i} e^{-t_i/\mu_i}$$

$$\phi_i(\tau_i) = \frac{1}{m_i} e^{-\tau_i/m_i}$$

The expected total profit per breakdown is given by:

$ETP(P_1, P_2, \dots, P_n)$ = Expected revenue from selling items - Expected production cost - Expected holding cost - Expected cost for idle time - Expected shortages cost - Expected lost sale cost - Expected set up cost

$$\begin{aligned} &= \sum_{i=1}^n [(S_p^i - (r_i + \frac{g_i}{P_i} + H_i P_i)) P_i \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \{C_h^i (\mu_i^3 - 2\theta_i \mu_i^4) + \gamma (2\mu_i^3 - 8\mu_i^5)\} \\ &- \alpha_i (S_p^i)^{-\beta_i} \{C_h^i (\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &- \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}}) + \gamma (\frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i-\beta})^3 \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &+ \mu_i^4 - \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-5\beta_i}} + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &+ 15(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^5\} - \frac{W_i}{\mu_i} (\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1) \{\mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) (\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} \\ &- \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}})\} - C_S^i \{\alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3\} \\ &- \frac{C_{LS}^i (1 - \delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \{\mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} (\mu_i^2 + \frac{\theta_i \mu_i^3}{3})\} - S_{UP}^i] \end{aligned} \quad (12)$$

Consequently, one has to maximize the profit function:

$$ETP(P_1, P_2, \dots, P_n)$$

Subject to

$$\sum_{i=1}^n (r_i + \frac{g_i}{P_i} + \alpha_i P_i) P_i \mu_i \leq CAP$$

$$P_1 \geq D_1, P_2 \geq D_2, \dots, P_n \geq D_n$$

3.1 Imprecise Model

On applying the fuzzy parameters i.e. fuzzified holding cost, set up cost, shortages cost, lost sale cost. The expected total profit per breakdown is as follows,

$$\begin{aligned} ETP(P_1, P_2, \dots, P_n) &= \sum_{i=1}^n [(S_p^i - (r_i + \frac{g_i}{P_i} + H_i P_i)) P_i \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \{\tilde{C}_h^i (\mu_i^3 - 2\theta_i \mu_i^4) + \gamma (2\mu_i^3 - 8\mu_i^5)\} \\ &- \alpha_i (S_p^i)^{-\beta_i} \{\tilde{C}_h^i (\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &- \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}}) + \gamma (\frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i-\beta})^3 \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &+ \mu_i^4 - \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-5\beta_i}} + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-3\beta_i}} \\ &+ 15(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^5\} - \frac{W_i}{\mu_i} (\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1) \{\mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) (\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} \\ &- \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}})\} - C_S^i \{\alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3\} \\ &- \frac{C_{LS}^i (1 - \delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \{\mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} (\mu_i^2 + \frac{\theta_i \mu_i^3}{3})\} - S_{UP}^i] \end{aligned} \quad (13)$$

$$\begin{aligned}
& -\frac{2(P_i - \alpha_i(S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i(S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i(S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i(S_p^i)^{-\beta_i}} + \gamma \left(\frac{2(P_i - \alpha_i(S_p^i)^{-\beta_i})^3 \mu_i^4}{\alpha_i(S_p^i)^{-3\beta_i}} + \mu_i^4 \right. \\
& - \frac{3(P_i - \alpha_i(S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i(S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i(S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i(S_p^i)^{-5\beta_i}} + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i(S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i(S_p^i)^{-3\beta_i}} \\
& \left. + 15(P_i - \alpha_i(S_p^i)^{-\beta_i}) \theta_i \mu_i^5 \right) - \frac{W_i}{\mu_i} \left(\frac{P_i}{\alpha_i(S_p^i)^{-\beta_i}} - 1 \right) \{ \mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i(S_p^i)^{-\beta_i}) \\
& \left(\frac{2\mu_i^3}{m_i \alpha_i(S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i(S_p^i)^{-\beta_i}} \right) \} - \tilde{C}_s^i \{ \alpha_i(S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i(S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i(S_p^i)^{-\beta_i}) \theta_i \mu_i^3 \} \\
& - \frac{\tilde{C}_{LS}^i (1 - \delta) \alpha_i(S_p^i)^{-\beta_i}}{\mu_i} \left\{ \mu_i - \frac{(P_i - \alpha_i(S_p^i)^{-\beta_i})}{m_i \alpha_i(S_p^i)^{-\beta_i}} \left(\mu_i^2 + \frac{\theta_i \mu_i^3}{3} \right) \right\} - \tilde{S}_{UP}^i
\end{aligned}$$

where (\sim) denotes the fuzzification of the parameters. In practical it is not easy to decide the holding cost for long period. Therefore, it becomes reasonable to locate holding cost in an interval $[C_h^i - \Delta_1, C_h^i + \Delta_2]$, where $0 < \Delta_1 < C_h^i$ and $\Delta_1, \Delta_2 > 0$. The error in the value of holding cost will be larger if the deviation of it is farthest from C_h^i obviously error will attain its maximum at $C_h^i - \Delta_1$ and $C_h^i + \Delta_2$. In fuzzy system error is changed to the confidence level such as: if error is zero then the confidence level to be 1. Maximum error is attained at $C_h^i - \Delta_1$ and $C_h^i + \Delta_2$ and then the confidence level is zero. If the value lies in the interval then the confidence level is any real value between zero and one. The above conditions are represented by the fuzzy triangular numbers.

$$\tilde{C}_h^i = [C_h^i - \Delta_1, C_h^i, C_h^i + \Delta_2], \quad (14)$$

where, $0 < \Delta_1 < C_h^i$, $\Delta_1, \Delta_2 > 0$ and the signed distance of C_h^i is given by the relation

$$d(\tilde{C}_h^i, \tilde{0}) = C_h^i + \frac{1}{4}(\Delta_2 - \Delta_1) \quad (15)$$

$$d(\tilde{C}_h^i, \tilde{0}) > 0 \text{ and} \quad (16)$$

$$d(\tilde{C}_h^i, \tilde{0}) \in [C_h^i - \Delta_1, C_h^i, C_h^i + \Delta_2]$$

$d(\tilde{C}_h^i, \tilde{0})$ is the estimated total holding cost during the planning period $[0, x]$ based on the signed distance. In a perfect competitive situation, the shortages cost, lost sale cost and set up cost of a unit per unit time in a planning period x may also fluctuate from its actual value. Let us suppose that it lies in the interval $[C_S^i - \Delta_3, C_S^i + \Delta_4]$, $[C_{LS}^i - \Delta_5, C_{LS}^i + \Delta_6]$ $[S_{UP}^i - \Delta_7, S_{UP}^i + \Delta_8]$ as discussed above, we considered a fuzzy triangular number to represent the shortages cost, lost sale cost and set up cost as below:

$$\tilde{C}_S^i = [C_S^i - \Delta_3, C_S^i, C_S^i + \Delta_4], \quad (17)$$

$$\tilde{C}_{LS}^i = [C_{LS}^i - \Delta_5, C_{LS}^i, C_{LS}^i + \Delta_6], \quad (18)$$

$$\tilde{S}_{UP}^i = [S_{UP}^i - \Delta_7, S_{UP}^i, S_{UP}^i + \Delta_8], \quad (19)$$

where, $0 < \Delta_3 < C_S^i$, $0 < \Delta_5 < C_{LS}^i$, $0 < \Delta_7 < S_{UP}^i$ and $\Delta_3, \Delta_4 > 0$, $\Delta_5, \Delta_6 > 0$ and $\Delta_7, \Delta_8 > 0$

and the signed distance of C_S^i , C_{LS}^i , C_{UP}^i is given by the formula

$$d(\tilde{C}_S^i, \tilde{0}) = C_S^i + \frac{1}{4}(\Delta_4 - \Delta_3), \quad (20)$$

$$d(C_S^i, \tilde{0}) > 0 \text{ and } d(C_S^i, \tilde{0}) \in [C_S^i - \Delta_3, C_S^i, C_S^i + \Delta_4], \quad (21)$$

$$\text{and } d(\widetilde{C}_{LS}^i, \tilde{0}) = C_{LS}^i + \frac{1}{4}(\Delta_6 - \Delta_5), \quad (22)$$

$$d(\widetilde{C}_{LS}^i, \tilde{0}) > 0 \text{ and } d(\widetilde{C}_{LS}^i, \tilde{0}) \in [C_{LS}^i - \Delta_5, C_{LS}^i, C_{LS}^i + \Delta_6], \quad (23)$$

$$\text{and } d(\widetilde{S}_{UP}^i, \tilde{0}) = S_{UP}^i + \frac{1}{4}(\Delta_8 - \Delta_7), \quad (24)$$

$$d(\widetilde{S}_{UP}^i, \tilde{0}) > 0 \text{ and } d(\widetilde{S}_{UP}^i, \tilde{0}) \in [S_{UP}^i - \Delta_7, S_{UP}^i, S_{UP}^i + \Delta_8] \quad (25)$$

From Eq. (13), we have

$$F(\widetilde{C}_h^i, \widetilde{C}_S^i, \widetilde{C}_{LS}^i, \widetilde{S}_{UP}^i) = ETP(P_1, P_2, \dots, P_n) \quad (26)$$

Now by the fuzzy triangular rule, we have

$$F(\widetilde{C}_h^i, \widetilde{C}_S^i, \widetilde{C}_{LS}^i, \widetilde{S}_{UP}^i) = (F_1, F_2, \dots, F_n) \quad (27)$$

$F(\widetilde{C}_h^i, \widetilde{C}_S^i, \widetilde{C}_{LS}^i, \widetilde{S}_{UP}^i)$ is the estimated fuzzified total cost and F_1, F_2, F_3 are obtained as:

$$\begin{aligned} F_1 = & \sum_{i=1}^n [(S_p^i - (r_i + \frac{g_i}{P_i} + H_i P_i)) P_i \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \{ (C_h^i - \Delta_1) (\mu_i^3 - 2\theta_i \mu_i^4) + \gamma (2\mu_i^3 - 8\mu_i^5) \}] \\ & - \alpha_i (S_p^i)^{-\beta_i} \{ (C_h^i - \Delta_1) (\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ & - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}}) + \gamma (\frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} + \mu_i^4 \\ & - \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-3\beta_i}} \\ & + 15(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^5 \} - \frac{W_i}{\mu_i} (\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1) \{ \mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \\ & (\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}}) \} - (C_S^i - \Delta_3) \{ \alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i \\ & + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3 \} - \frac{(C_{LS}^i - \Delta_5)(1 - \delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \{ \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} \\ & (\mu_i^2 + \frac{\theta_i \mu_i^3}{3}) \} - (S_{UP}^i - \Delta_7)] \end{aligned} \quad (28)$$

$$F_2 = F_{(P_1, P_2, \dots, P_n)}(C_h^i, C_S^i, C_{LS}^i, S_{UP}^i) \quad (29)$$

$$\begin{aligned} F_3 = & \sum_{i=1}^n [(S_p^i - (r_i + \frac{g_i}{P_i} + H_i P_i)) P_i \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \{ (C_h^i + \Delta_2) (\mu_i^3 - 2\theta_i \mu_i^4) + \gamma (2\mu_i^3 - 8\mu_i^5) \}] \\ & - \alpha_i (S_p^i)^{-\beta_i} \{ (C_h^i + \Delta_2) (\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} \\ & - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}}) + \gamma (\frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} + \mu_i^4 \\ & - \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i (S_p^i)^{-\beta_i})^5 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{\theta_i \mu_i^5}{15} - \frac{20(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^5}{\alpha_i (S_p^i)^{-3\beta_i}} \\ & + 15(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^5 \} - \frac{W_i}{\mu_i} (\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1) \{ \mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \\ & (\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}}) \} - (C_S^i - \Delta_3) \{ \alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i \\ & + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3 \} - \frac{(C_{LS}^i - \Delta_5)(1 - \delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \{ \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} \\ & (\mu_i^2 + \frac{\theta_i \mu_i^3}{3}) \} - (S_{UP}^i - \Delta_7)] \end{aligned} \quad (30)$$

$$\begin{aligned}
 & -\frac{3(P_i - \alpha_i(S_p^i)^{-\beta_i})\mu_i^4}{\alpha_i(S_p^i)^{-\beta_i}} + \frac{3(P_i - \alpha_i(S_p^i)^{-\beta_i})^5\theta_i\mu_i^5}{\alpha_i(S_p^i)^{-\beta_i}} + \frac{\theta_i\mu_i^5}{15} - \frac{20(P_i - \alpha_i(S_p^i)^{-\beta_i})^3\theta_i\mu_i^5}{\alpha_i(S_p^i)^{-3\beta_i}} \\
 & + 15(P_i - \alpha_i(S_p^i)^{-\beta_i})\theta_i\mu_i^5 \left\} - \frac{W_i}{\mu_i} \left(\frac{P_i}{\alpha_i(S_p^i)^{-\beta_i}} - 1 \right) \{ \mu_i^2 - 30\theta_i\mu_i^3 - (P_i - \alpha_i(S_p^i)^{-\beta_i}) \right. \\
 & \left. \left(\frac{2\mu_i^3}{m_i\alpha_i(S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i\alpha_i(S_p^i)^{-\beta_i}} \right) \right\} - (C_S^i + \Delta_4) \{ \alpha_i(S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i(S_p^i)^{-\beta_i}) \mu_i \\
 & + 2(P_i - \alpha_i(S_p^i)^{-\beta_i})\theta_i\mu_i^3 \} - \frac{(C_{LS}^i + \Delta_6)(1 - \delta)\alpha_i(S_p^i)^{-\beta_i}}{\mu_i} \left\{ \mu_i - \frac{(P_i - \alpha_i(S_p^i)^{-\beta_i})}{m_i\alpha_i(S_p^i)^{-\beta_i}} \right. \\
 & \left. \left(\mu_i^2 + \frac{\theta_i\mu_i^3}{3} \right) \right\} - (S_{UP}^i + \Delta_8)]
 \end{aligned}$$

Defuzzification of fuzzy number $F_{(P_1, P_2, \dots, P_n)}(\tilde{C}_h^i, \tilde{C}_S^i, \tilde{C}_{LS}^i, \tilde{S}_{UP}^i)$ using signed distance method is given by:

$$\begin{aligned}
 F_{(P_1, P_2, \dots, P_n)}(\tilde{C}_h^i, \tilde{C}_S^i, \tilde{C}_{LS}^i, \tilde{S}_{UP}^i, \tilde{0}) &= F_{(P_1, P_2, \dots, P_n)}(C_h^i, C_S^i, C_{LS}^i, S_{UP}^i) + \frac{1}{4} \sum_{i=1}^n \left[-\frac{(P_i - \alpha_i(S_p^i)^{-\beta_i})}{\mu_i} \right. \\
 & (\Delta_2 + \Delta_1)(\mu_i^3 - 2\theta_i\mu_i^4) - \alpha_i(S_p^i)^{-\beta_i} (\Delta_2 + \Delta_1) \left(\frac{(P_i - \alpha_i(S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i(S_p^i)^{-2\beta_i}} \right. \\
 & \left. - \frac{2(P_i - \alpha_i(S_p^i)^{-\beta_i})\mu_i^3}{\alpha_i(S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i(S_p^i)^{-\beta_i})^3\theta_i\mu_i^4}{\alpha_i(S_p^i)^{-3\beta_i}} - \frac{2(P_i - \alpha_i(S_p^i)^{-\beta_i})^4\theta_i\mu_i^4}{\alpha_i(S_p^i)^{-4\beta_i}} \right. \\
 & \left. + 2\theta_i\mu_i^4 - \frac{4(P_i - \alpha_i(S_p^i)^{-\beta_i})\theta_i\mu_i^4}{\alpha_i(S_p^i)^{-\beta_i}} \right) - (\Delta_4 + \Delta_3) \{ \alpha_i(S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i(S_p^i)^{-\beta_i}) \mu_i \\
 & + 2(P_i - \alpha_i(S_p^i)^{-\beta_i})\theta_i\mu_i^3 \} - (\Delta_6 + \Delta_5) \frac{(1 - \delta)\alpha_i(S_p^i)^{-\beta_i}}{\mu_i} \left\{ \mu_i - \frac{(P_i - \alpha_i(S_p^i)^{-\beta_i})}{m_i\alpha_i(S_p^i)^{-\beta_i}} \right. \\
 & \left. \left(\mu_i^2 + \frac{\theta_i\mu_i^3}{3} \right) \right\} - (\Delta_8 + \Delta_7)]
 \end{aligned} \tag{31}$$

Eq. (31) gives an estimate of profit function. For maximization of profit function, differentiate the Eq. (31) with respect to P_1, P_2, \dots, P_n . The results are illustrated numerically with the help of the software *Mathematica5.2*.

4. Numerical Illustrations

The model has been explored numerically as well. There are following data which can be used to explain the model, based on the studies in proper unit. We have studied for two items. The following numerical study has been used to find the optimal solution of the multi items production model.

The common input parameters are: $\gamma=0.6, \delta=0.4,$

Item 'i'	μ_i	r_i	g_i	H_i	α_i	β_i	C_{LS}^i
1	5	250	2500	0.02	8500	2.0	26
2	6	200	2400	0.03	8000	2.2	28
Item 'i'	θ_i	W_i	m_i	S_p^i	C_S^i	C_h^i	
1	0.07	40	0.65	60	14	15	
2	0.06	45	0.75	50	17	15	
Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8
2	3	4	3.5	2.5	3	4.5	5

The optimal values of P_1 and P_2 are 7.51556 and 5.91907 respectively. The optimal net profit is 136989.

5. Sensitivity Analysis

The sensitivity analysis is presented due to the changes in the fuzzy parameters for the production and profit.

Table 1
Sensitivity Analysis of Optimal Solution w.r.t. System Parameters

Parameter	% Change	-30%	-20%	-10%	10%	20%	30%
Δ_1	P_1	+1.1861	+0.7929	+0.3976	-0.3998	-0.8022	-1.2071
	P_2	+0.4651	+0.3100	+0.1551	-0.1551	-0.3102	-0.4653
	Profit	+4.6055	+3.055	+1.5198	-1.5052	-2.9951	-4.4697
Δ_2	P_1	-1.8192	-1.2071	-0.6008	+0.5956	+1.1861	+1.7703
	P_2	-0.6981	-0.4653	-0.2326	+0.2325	+0.4651	+0.6974
	Profit	-6.6538	-4.4697	-2.2520	+2.2856	+4.6055	+6.9604
Δ_3	P_1	-0.0049	-0.0033	-0.0016	+0.0017	+0.0033	+0.0051
	P_2	-0.0007	-0.0005	-0.0003	+0.0002	+0.0005	+0.0007
	Profit	-0.0343	-0.0234	-0.0117	+0.0109	+0.0226	+0.0336
Δ_4	P_1	+0.0044	+0.0029	+0.0015	-0.0015	-0.0028	-0.0043
	P_2	+0.0007	+0.0003	+0.0002	-0.0002	-0.0005	-0.0007
	Profit	+0.0292	+0.0197	+0.0095	-0.0102	-0.0204	-0.0299
Δ_5	P_1	+0.0013	+0.0009	+0.0005	-0.0004	-0.0008	-0.0012
	P_2	+0.0002	0.0	0.0	0.0	-0.0002	-0.0002
	Profit	+0.0066	+0.0044	+0.0022	-0.0022	-0.0044	-0.0066
Δ_6	P_1	-0.0015	-0.0009	-0.0005	+0.0005	+0.0011	+0.0016
	P_2	-0.0002	-0.0002	0.0	0.0	0.0	+0.0002
	Profit	-0.0080	-0.0058	-0.0029	+0.0022	+0.0051	+0.0073

6. Observations

From the sensitivity analysis, it has been observed that

1. Δ_1 as increases, P_1 , P_2 and the profit are decreases and Δ_1 as decreases, P_1 , P_2 and the profit are increases. The graphical representation of P_1 , P_2 and the profit w.r.t. Δ_1 is illustrated by figure 1.
2. As Δ_2 increases, P_1 , P_2 and the profit are increases and as Δ_2 decreases, P_1 , P_2 and the profit are decreases. The variation of P_1 , P_2 and the profit w.r.t. Δ_2 is shown by figure 2.
3. On increasing the Δ_3 , P_1 , P_2 and the profit are increases and on decreasing the Δ_3 , P_1 , P_2 and the profit are decreases. Graphically, it is shown by figure 3.
4. If Δ_4 is increases then P_1 , P_2 and the profit are decreases and if Δ_4 is decreases then P_1 , P_2 and the profit are increases. Figure 4 represented the variation of P_1 , P_2 and the profit w.r.t. ' Δ_4 '.
5. As Δ_5 increases, P_1 and the profit are decreases and as Δ_5 decreases, P_1 and the profit are increases and P_2 is almost constant to change in Δ_5 . It is shown by the figure 5.
6. If Δ_6 is increases then P_1 and the profit are increases and if Δ_6 is decreases, P_1 and the profit are decreases and P_2 is almost stable to change in Δ_6 . Figure 6 represented the variation of P_1 , P_2 and the profit w.r.t. ' Δ_6 '.

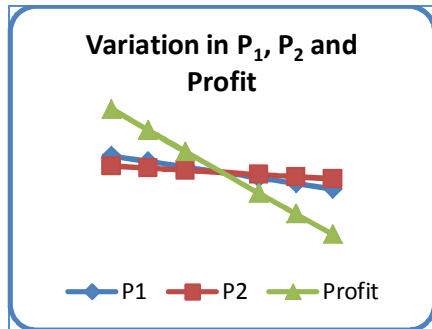


Fig. 1. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_1

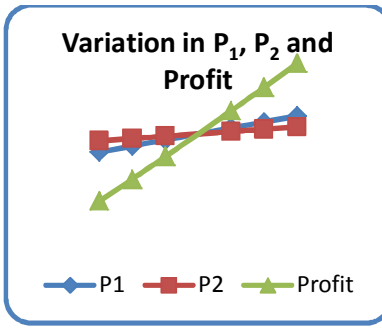


Fig. 2. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_2

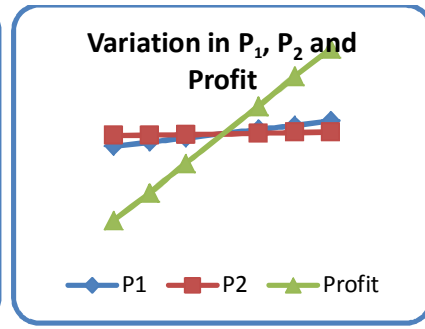


Fig. 3. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_3

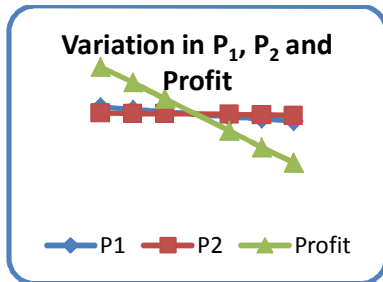


Fig. 4. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_4

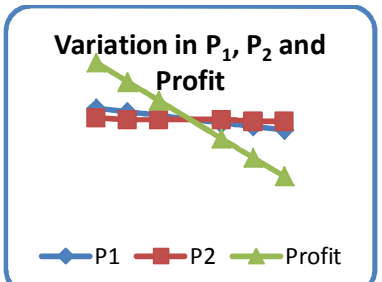


Fig. 5. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_5

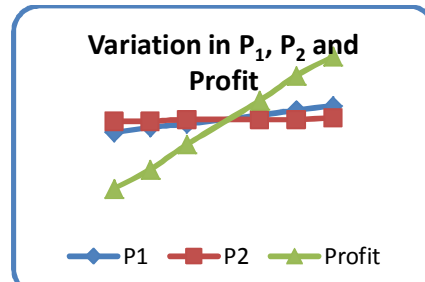


Fig. 6. Graphical representation of sensitivity of the P_1 , P_2 and Profit w.r.t. Δ_6

7. Conclusions

In this paper, an EMQ multi items model for unreliable production systems with the process of deterioration and machine breakdown has been discussed. In a competitive market, price of goods plays an important factor. Generally, a reduced price encourages a customer to buy more. For fitting in with realistic circumstances, the environment of the whole study has been taken as fuzzified. Model has been solved numerically with sensitivity analysis. All these facts have been shown graphically.

We have many real life situations in which multi items inventories are required. For instance: a pharmacist keeps a number of medicines of different brands, readymade clothes shop keeps dresses of different things in different colors and in different size, shoe store stocks shoes of various models and sizes. The presented model is much more realistic and practical.

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