Vendor-buyer ordering policy when demand is trapezoidal

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ABSTRACT

A joint vendor-buyer strategy is analyzed which is beneficial to both the players in the supply chain. The demand is assumed to be trapezoidal. It is established numerically that the joint venture decreases the total cost of the supply chain when compared with the independent decision of the buyer. To entice the buyer to order more units, a permissible credit period is offered by the vendor to the buyer. A negotiation factor is incorporated to share the cost savings.

Keywords: Vendor-buyer joint decision Credit period Negotiation factor Trapezoidal demand

1. Introduction

Silver and Meal (1969), Silver(1979), Xu and Wang (1991), Chung and Ting (1993, 1994), Bose et al. (1995), Hariga (1995), Giri and Chaudhari (1997), Lin et al. (2000), etc. discussed optimal ordering policy when demand is linearly changing with respect to time which is superficial in the market of fashion good, Air seats, Smart phones etc. Mehta and Shah (2003, 2004) assumed the demand to be exponentially time varying, which is again unrealistic for newly launched product. Shah et al. (2008) introduced the quadratic demand, which is again not observed in the market for indefinite period. In order to have an alternative demand pattern, the trapezoidal demand is considered. This type of demand increases for some time, then gets constant up to some time and afterwards decreases exponentially with time.

Most of the models available in the literature assumed that the buyer is dominant player to make the decision for procurement. This strategy may not be economical for the vendor. An integrated vendor-buyer policy should be analyzed which is beneficial to the players of the supply chain. Clark and Scarf(1960) proposed a mathematical model for vendor-buyer integration. Banerjee (1986) discussed an economic lot-size model when production is finite. Goyal (1988) extended Banerjee’s model by
relaxing the assumption of the lot-for-lot production. Shah et al. (2008) analyzed joint decision when demand is quadratic.

In this study, a joint vendor–buyer inventory system is analyzed when demand is trapezoidal. A negotiation factor is incorporated to share the savings. The credit period is offered to the buyer to attract the buyer for placing larger order.

2. Notations and Assumptions

The proposed study uses following notations and assumptions.

2.1 Notations

\[ K_v \] Buyer’s ordering cost per order
\[ A_v \] Vendor’s ordering cost per order
\[ C_b \] Buyer’s purchase cost per unit
\[ C_v \] Vendor’s purchase cost per unit
\[ I_b \] Inventory carrying charge fraction per unit per annum for the buyer
\[ I_v \] Inventory carrying charge fraction per unit per annum for the vendor
\[ I_b(t) \] Buyer’s inventory level at any instant of time \( t \)
\[ I_v(t) \] Vendor’s inventory level at any instant of time \( t \)
\[ n \] Number of orders during cycle time for the buyer (a decision variable)
\[ K_b \] Buyer’s total cost per unit time
\[ K_v \] Vendor’s total cost per unit time
\[ K_{ij} \] Total cost for vendor-buyer inventory System when they take independent decision
\[ K_f \] Total cost for vendor-buyer inventory System when they take joint decision
\[ T \] Vendor’s cycle time (a decision variable)
\[ T_b \] (= \( T/n \)), Buyer’s cycle time (a decision variable)
\[ M \] Credit period offered by the vendor to the buyer (a decision variable)
\[ r \] Continuous discounting rate

2.2 Assumptions

- A supply chain of single vendor and single buyer is considered.
- An inventory system deals with single item.
- The demand rate is trapezoidal. Its functional form is

\[ R(t) = \begin{cases} f(t), & 0 \leq t \leq u_1 \\ D_0, & u_1 \leq t \leq u_2 \\ g(t), & u_2 \leq t \leq T \end{cases} \]

where \( f(t) \) is linear in \( t \),
\(D_0 = f(u_i) = g(u_2)\), and \(g(t)\) is exponentially decreasing in \(t\)

\[
R(t) = \begin{cases} 
  a(1 + b_1t), & 0 \leq t \leq u_1 \\
  a(1 + b_2u_1), & u_1 \leq t \leq u_2 \\
  a(1 + b_2u_2)e^{-b_2(-u_2+t)}, & u_2 \leq t \leq T 
\end{cases}
\]

where \(a\) denotes scale demand, \(0 < b_1, b_2 < 1\) denotes rates of change of demand. (See Fig. 1)

- The lead time is zero and shortages are not allowed.
- The credit period is offered for settling the accounts due against purchases to attract the buyer to opt a joint decision policy.

3. Mathematical Model

Fig. 2 depicts the time-varying inventory status for the vendor and the buyer. The inventory changes due to trapezoidal demand for both vendor and buyer. The rate of change of inventory for both the players is governed by the differential equations:

\[
\frac{dI_v(t)}{dt} = -R(t), \quad 0 \leq t \leq T_v
\]

(1)

\[
\frac{dI_b(t)}{dt} = -R(t), \quad 0 \leq t \leq T
\]

(2)

with the boundary conditions \(I_v(T_v) = 0\), \(I_b(T) = 0\) and initial conditions \(I_v(0) = I_{vb}, I_b(0) = I_{nv}\). The solutions of the differential equations are
Using \( I_b(0) = I_{mb} \) , \( I_v(0) = I_{mv} \), the maximum procurement quantities for the buyer and the vendor are

\[
I_{mb} = a \left[ u_1 + \frac{b_1 u_1^2}{2} \right] + a(1+b_1 u_1)(u_2-u_1) + \frac{a(1+b_1 u_1)e^{b_2 u_2}}{b_2} \left[ e^{-b_2 u_2} - e^{-\frac{b_2}{b}T} \right], 0 \leq t \leq u_2
\]

and

\[
I_{mv} = a \left[ u_1 + \frac{b_1 u_1^2}{2} \right] + a(1+b_1 u_1)(u_2-u_1) + \frac{a(1+b_1 u_1)e^{b_2 u_2}}{b_2} \left[ e^{-b_2 u_2} - e^{-b_2 T} \right], u_1 \leq t \leq u_2
\]

respectively. During the cycle time \([0, T]\), the buyer’s

- Purchase Cost ; \( PC_b = nC_b I_{mb} \)
- Holding Cost ; \( HC_b = nC_b \int_0^T I_b(t) dt \)
- Ordering Cost ; \( OC_b = nA_b \)

Hence, the buyer’s total cost; \( K_b \) per unit time is

\[
K_b = \frac{1}{T} [PC_b + HC_b + OC_b]. \tag{5}
\]

The vendor’s inventory is the difference between the vendor-buyer combined inventory and the buyer’s inventory during n-orders. This is known as the joint two-echelon inventory model. The vendor’s

- Purchase Cost ; \( PC_v = nC_v I_{mv} \)
- Holding Cost ; \( HC_v = C_v \int_0^T I_v(t) dt - n \int_0^T I_b(t) dt \)
- Ordering Cost ; \( OC_v = A_v \)
Hence, the vendor’s total cost; $K_v$ per unit time is

$$K_v = \frac{1}{T} \left[ PC_v + HC_v + OC_v \right]$$  \hspace{1cm} (6)$$

The joint total cost $K$ is the sum of $K_b$ and $K_v$ where $T_b = \frac{T}{n}$. Thus $K$ is the function of discrete variable $n$ and continuous variable.

4. Computational Procedure

There are two cases to analyze $T$.

Case 1: When the vendor and the buyer take decision independently.

For given value of $n$, differentiate $K_b$ with respect to $T_b$ (equivalently, $T$) and solve $\frac{dK_b}{dT_b} = 0$. This $n$ and $T_b$ minimizes $K_v$ provided

$$K_v(n-1) \geq K_v(n) \leq K_v(n+1)$$  \hspace{1cm} (7)$$

satisfies. Here, the total cost per unit time with independent decision; $K_{nj}$ is given by

$$K_{nj} = \min_n [\min_n K_b + K_v]$$  \hspace{1cm} (8)$$

Case 2: When vendor and buyer make decision jointly.

The optimum value of $T$ and $n$ must satisfy the following conditions simultaneously:

$$\frac{\partial K}{\partial T} = 0 \text{ and } K(n-1) \geq K(n) \leq K(n+1)$$  \hspace{1cm} (9)$$

Thus, the total joint cost is

$$K_j = \min_n, T [K_b + K_v]$$  \hspace{1cm} (10)$$

It is obvious that $K_j \leq K_{nj}$. Hence, total cost savings $Sav_j$ is defined as $Sav_j = K_{nj} - K_j$. Now define buyer’s cost saving $Sav_b$ as $Sav_b = \alpha Sav_j$, where $0 \leq \alpha \leq 1$ is the negotiation factor. When negotiation factor equals to one, all saving goes to buyer; when it is equal to zero, all saving is in the vendor’s pocket. When negotiation factor is 0.5, the total cost savings is equally distributed between the vendor and the buyer. The present value of unit after a time interval $M$ is $e^{-rM}$, where $r$ is discounting rate. Solving the following equation

$$R(t)C_b (1 - e^{-rM}) = Sav_b$$  \hspace{1cm} (11)$$

the buyer’s credit period is given by

$$M = \frac{1}{r} \ln \left[ \frac{C_bR(t)}{C_bR(t) - Sav_b} \right]$$  \hspace{1cm} (12)$$
5. Numerical Example and Sensitivity Analysis

Consider following inventory parameters values in proper units:

\[
\begin{bmatrix}
a_1 & b_1 & A \_b & A \_c & C \_b & I \_b & I \_c & r
\end{bmatrix} = [40000, 0.04, 0.02, 600, 3000, 10, 6, 0.11, 0.10, 0.06]
\]

Let \( \alpha = 0.5 \). The optimal solution is listed in Table 1 for independent and joint decisions.

<table>
<thead>
<tr>
<th>Case</th>
<th>Case</th>
<th>( T_b )</th>
<th>( T )</th>
<th>( K_b )</th>
<th>( K_c )</th>
<th>( K )</th>
<th>( PJCR )</th>
<th>( M(\text{days}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Independent Decision)</td>
<td>3</td>
<td>0.179004</td>
<td>0.537012</td>
<td>407422</td>
<td>249129</td>
<td>656551</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 (Joint Decision)</td>
<td>2</td>
<td>0.288339</td>
<td>0.576678</td>
<td>408163</td>
<td>247815</td>
<td>655978</td>
<td>0.08735049</td>
<td>7.5864</td>
</tr>
</tbody>
</table>

The buyer’s cost and cycle time increases in joint decisions. The vendor gains $1314 and the buyer loses $741. This hinders the buyer to agree for joint decision. To entice the buyer to joint decision, the vendor offers the buyer a credit period of days with equal sharing of cost savings. This reduces the joint total cost \( PJCR \) by 0.08735049\% , where \( PJCR \) is defined as \( \frac{K_{NJ} - K_J}{K_J} \times 100 \). The convexity of total integrated cost and independent costs are shown in Fig. 3.

![Graph showing the convexity of total integrated cost and independent costs](image)

**Fig. 3.** Total cost for independent vs joint decision

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity Analysis of Demand Rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( a )</th>
<th>24000</th>
<th>32000</th>
<th>40000</th>
<th>48000</th>
<th>56000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{NJ} )</td>
<td>396567</td>
<td>526669</td>
<td>656551</td>
<td>786277</td>
<td>915884</td>
</tr>
<tr>
<td>( K_J )</td>
<td>396123</td>
<td>526156</td>
<td>655978</td>
<td>785649</td>
<td>915205</td>
</tr>
<tr>
<td>( PJCR )</td>
<td>0.112086397</td>
<td>0.097499601</td>
<td>0.08735049</td>
<td>0.079933915</td>
<td>0.074191028</td>
</tr>
<tr>
<td>( M(\text{days}) )</td>
<td>7.560363512</td>
<td>7.57972378</td>
<td>7.586401505</td>
<td>7.603471479</td>
<td>7.624091074</td>
</tr>
</tbody>
</table>

**Observations**

- Increase in fixed demand \( a \), decreases percentage of cost reduction and increases delay period. (See Fig. 4 and 5)
Table 3
Sensitivity Analysis of Linear Rate of Change ofDemand

<table>
<thead>
<tr>
<th>$b_1$</th>
<th>0.024</th>
<th>0.032</th>
<th>0.04</th>
<th>0.048</th>
<th>0.056</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{NJ}$</td>
<td>656278</td>
<td>664414</td>
<td>656551</td>
<td>656688</td>
<td>656825</td>
</tr>
<tr>
<td>$K_J$</td>
<td>655705</td>
<td>655842</td>
<td>655978</td>
<td>656115</td>
<td>656251</td>
</tr>
<tr>
<td>$PJCR$</td>
<td>0.08736858</td>
<td>0.087216128</td>
<td>0.08735049</td>
<td>0.08732251</td>
<td>0.087466533</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>7.582111052</td>
<td>7.570846658</td>
<td>7.586401505</td>
<td>7.588383352</td>
<td>7.603621229</td>
</tr>
</tbody>
</table>

Observations

- Increase in linear rate of change of demand $b_1$, may increase or decrease percentage of cost reduction and delay period. (See Fig. 4 and 5)

Table 4
Sensitivity Analysis of Exponential Rate of Change of Demand

<table>
<thead>
<tr>
<th>$b_2$</th>
<th>0.012</th>
<th>0.016</th>
<th>0.02</th>
<th>0.024</th>
<th>0.028</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{NJ}$</td>
<td>657145</td>
<td>656854</td>
<td>656551</td>
<td>656236</td>
<td>655909</td>
</tr>
<tr>
<td>$K_J$</td>
<td>656660</td>
<td>656327</td>
<td>655978</td>
<td>655611</td>
<td>655224</td>
</tr>
<tr>
<td>$PJCR$</td>
<td>0.073858618</td>
<td>0.080295341</td>
<td>0.08735049</td>
<td>0.09533092</td>
<td>0.1045444</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>6.848732823</td>
<td>7.214091565</td>
<td>7.586401505</td>
<td>7.981409057</td>
<td>8.410917618</td>
</tr>
</tbody>
</table>

Observations

- Increase in exponential rate of change of demand $b_2$, increases percentage of cost reduction and delay period. (See Fig. 4 and 5)

Table 5
Sensitivity Analysis of Buyer’s Ordering Cost

<table>
<thead>
<tr>
<th>$A_b$</th>
<th>360</th>
<th>480</th>
<th>600</th>
<th>720</th>
<th>840</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{NJ}$</td>
<td>656058</td>
<td>656208</td>
<td>656551</td>
<td>656979</td>
<td>657444</td>
</tr>
<tr>
<td>$K_J$</td>
<td>655120</td>
<td>655555</td>
<td>655978</td>
<td>656388</td>
<td>656787</td>
</tr>
<tr>
<td>$PJCR$</td>
<td>0.143179875</td>
<td>0.099610254</td>
<td>0.08735049</td>
<td>0.090038209</td>
<td>0.100032431</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>13.21695433</td>
<td>8.909292382</td>
<td>7.586401505</td>
<td>7.606669428</td>
<td>8.233058524</td>
</tr>
</tbody>
</table>

Observations

- Increase in Buyer’s Ordering Cost $A_b$, may increase or decrease percentage of cost reduction and delay period. (See Fig. 4 and Fig. 5)

Table 6
Sensitivity Analysis of Vendor’s Ordering Cost

<table>
<thead>
<tr>
<th>$A_v$</th>
<th>1800</th>
<th>2400</th>
<th>3000</th>
<th>3600</th>
<th>4200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{NJ}$</td>
<td>654317</td>
<td>655434</td>
<td>656551</td>
<td>657668</td>
<td>658786</td>
</tr>
<tr>
<td>$K_J$</td>
<td>653718</td>
<td>654897</td>
<td>655978</td>
<td>656983</td>
<td>657926</td>
</tr>
<tr>
<td>$PJCR$</td>
<td>0.091629724</td>
<td>0.081997627</td>
<td>0.08735049</td>
<td>0.104264494</td>
<td>0.130713788</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>9.417741873</td>
<td>7.690606444</td>
<td>7.586401505</td>
<td>8.474548917</td>
<td>10.02331229</td>
</tr>
</tbody>
</table>
Observations

- Increase in Vendor’s Ordering Cost $A$, may increase or decrease percentage of cost reduction and delay period (See Fig. 4 and 5)

Table 7
Sensitivity Analysis of Buyer’s Purchase Cost

<table>
<thead>
<tr>
<th>$C_b$</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{KJ}$</td>
<td>494342</td>
<td>575451</td>
<td>656551</td>
<td>737623</td>
<td>818661</td>
</tr>
<tr>
<td>$K_J$</td>
<td>493364</td>
<td>574720</td>
<td>655978</td>
<td>737158</td>
<td>818275</td>
</tr>
<tr>
<td>PJCR</td>
<td>0.198230921</td>
<td>0.12719237</td>
<td>0.08735049</td>
<td>0.0630801</td>
<td>0.047172405</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>18.172696</td>
<td>11.1807315</td>
<td>7.586401505</td>
<td>5.493492176</td>
<td>4.152027608</td>
</tr>
</tbody>
</table>

Observations

- Increase in Buyer’s purchase cost $C_b$, decreases percentage of cost reduction and delay period significantly (See Fig. 4 and 5)

Table 8
Sensitivity Analysis of Vendor’s Purchase Cost

<table>
<thead>
<tr>
<th>$C_v$</th>
<th>3.6</th>
<th>4.8</th>
<th>6</th>
<th>7.2</th>
<th>8.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{KJ}$</td>
<td>559134</td>
<td>607843</td>
<td>656551</td>
<td>705260</td>
<td>753969</td>
</tr>
<tr>
<td>$K_J$</td>
<td>558908</td>
<td>607449</td>
<td>655978</td>
<td>704495</td>
<td>753001</td>
</tr>
<tr>
<td>PJCR</td>
<td>0.040435993</td>
<td>0.064861412</td>
<td>0.08735049</td>
<td>0.108588421</td>
<td>0.128552286</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>2.815971583</td>
<td>5.064913311</td>
<td>7.586401505</td>
<td>10.41579493</td>
<td>13.53480563</td>
</tr>
</tbody>
</table>

Observations

- Increase in vendor’s purchase cost $C_v$, increases percentage of cost reduction and delay period significantly (See Fig. 4 and 5)

Table 9
Sensitivity Analysis of Inventory Carrying Charge Fraction of Buyer

<table>
<thead>
<tr>
<th>$I_b$</th>
<th>0.066</th>
<th>0.088</th>
<th>0.11</th>
<th>0.132</th>
<th>0.154</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{KJ}$</td>
<td>654297</td>
<td>655423</td>
<td>656551</td>
<td>657634</td>
<td>658665</td>
</tr>
<tr>
<td>$K_J$</td>
<td>653169</td>
<td>654649</td>
<td>655978</td>
<td>657193</td>
<td>658321</td>
</tr>
<tr>
<td>PJCR</td>
<td>0.1726965</td>
<td>0.118231296</td>
<td>0.08735049</td>
<td>0.067103575</td>
<td>0.052254143</td>
</tr>
<tr>
<td>$M$ (days)</td>
<td>12.01339335</td>
<td>9.300320019</td>
<td>7.586401505</td>
<td>6.331728601</td>
<td>5.295638308</td>
</tr>
</tbody>
</table>

Observations

- Increase in Buyer’s inventory carrying charge fraction $I_b$, decreases percentage of cost reduction and delay period significantly (See Fig. 4 and 5)
Table 10
Sensitivity Analysis of Inventory Carrying Charge Fraction of Vendor

<table>
<thead>
<tr>
<th>( I_v )</th>
<th>0.06</th>
<th>0.08</th>
<th>0.1</th>
<th>0.12</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{KJ} )</td>
<td>654845</td>
<td>655698</td>
<td>656551</td>
<td>657405</td>
<td>658258</td>
</tr>
<tr>
<td>( K_j )</td>
<td>654535</td>
<td>655275</td>
<td>655978</td>
<td>656648</td>
<td>657291</td>
</tr>
<tr>
<td>( PJCR )</td>
<td>0.047361868</td>
<td>0.06455305</td>
<td>0.08735049</td>
<td>0.115282465</td>
<td>0.147119008</td>
</tr>
<tr>
<td>( M(\text{days}) )</td>
<td>3.694344159</td>
<td>5.327546336</td>
<td>7.586401505</td>
<td>10.4889503</td>
<td>13.9710132</td>
</tr>
</tbody>
</table>

Observations

- Increase in Vendor’s inventory carrying charge fraction \( I_v \), increases percentage of cost reduction and delay period significantly. (See Fig. 4 and 5)

![Fig. 4. Total savings Vs. percentage of changes in affecting parameters](image1)

![Fig. 5. Delayed time in days Vs. percentage of changes in affecting parameters](image2)
6. Conclusions

In this paper, a mathematical model is developed to analyze an optimal ordering policy for a supply chain comprising of vendor-buyer inventory system when demand is trapezoidal. It is established that the joint decision lowers the total cost of an inventory system, even though the buyer’s cost increases significantly. To attract the buyer for a joint decision, a credit period has been offered by the vendor to the buyer to settle the account.

References


