A new supply chain management method with one-way time window: A hybrid PSO-SA approach

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\section*{ABSTRACT}

In this paper, we study a supply chain problem where a whole seller/producer distributes goods among different retailers. The proposed model of this paper is formulated as a more general and realistic form of traditional vehicle routing problem (VRP). The main advantages of the new proposed model are twofold. First, the time window does not consider any lower bound and second, it treats setup time as separate cost components. The resulted problem is solved using a hybrid of particle swarm optimization and simulated annealing (PSO-SA). The results are compared with other hybrid method, which is a combination of Ant colony and Tabu search. We use some well-known benchmark problems to compare the results of our proposed model with other method. The preliminary results indicate that the proposed model of this paper performs reasonably well.

\section*{1. Introduction}

Supply chain management is one of the most important parts of any production and service business and there are literally enormous numbers of publications devoted solely for better managing different costs involved. One of the issues in supply chain is to distribute independent goods from one supplier to different retailers and the primary concern is to build a mathematical model, which could consider realistic issues. The model is normally preferred to maintain a simple structure so that we could simply solve it using an available software packages. There are three primary issues influencing the complexity of the resulted problem formulation, which are as follows,

1. Normally, there is only a limited amount of storage for retailers and in some cases; they do not have any storage at all.
2. All retailers have their own customers and they need to supply goods in certain amount time called time window.
3. The number of vehicles for delivering goods is unknown and we must consider it as a variable.
There are various researches associated with supply chain problems specially the problem of one supplier and different retailers (Shapiro, 2001; Chen & Lee, 2004). Qu et al. (1999) proposed an integrated inventory–transportation system with a modified periodic-review inventory policy and a travelling-salesman component, which is a multi-item joint replenishment issue with simultaneous decisions made on inventory and transportation policies. They presented a heuristic technique to solve the resulted problem, minimizing the long-run total average costs including ordering, holding, backlogging, stopover and travel.

Korpela and Lehmusvaara (1999) presented a customer oriented method for evaluating and selecting alternative warehouse operators. They used analytic hierarchy process (AHP) to analyze the customer-specific requirements for logistics service and to evaluate the alternative warehouse operators. They used the priorities for a mixed integer linear programming (MILP) model to maximize the overall service performance of the warehouse network. Sabri and Beamon (2000) considered supply chain problem with different suppliers, producers and retailers for multi products with stochastic demand. Chan et al. (2005) proposed a genetic algorithm (GA) for multi-product supply chain problem with different routes.

Jayaraman and Pirkul (2001) proposed a multi commodity supply chain problems. They first provided an MILP formulation to the integrated model and then presented an efficient heuristic solution procedure for utilizing the solution generated from a Lagrangian relaxation of the problem. They proposed this heuristic procedure to measure the performance of the model with respect to solution quality and algorithm performance and implemented their problem for a real-world example to demonstrate the implications of the model. Dasci and Verter (2001) proposed a continuous model for production-distribution system design, where it allows considering the impact of problem parameters on facility design decisions and explained that discrete and continuous modeling approaches complement each other.

Nozick (2001) considered fixed charge facility location problem with coverage restrictions. In his model, a supplier could only serve a limited number of retailers in a circle. Wang et al. (2003) developed an approach of just-in-time distribution requirements planning for supply chain management. The primary objective of their proposed model was to establish an optimal distribution requirements planning model to minimize the total cost of manufacturing and transportation as well as the earliness/tardiness penalty in meeting retailer's requirements under limited warehouse capacity. They developed a simple mathematical model, which could be solved using direct implementation of linear programming techniques. Syarif et al. (2002) investigated the multi-stage logistic chain network using a spanning tree based GA approach. Their proposed model was a logistic chain network problem formulated by 0–1 mixed integer linear programming model. As a result, they need to develop metaheuristics to solve such problem and they used GA for this purpose. Zhou et al. (2002) developed a balanced allocation of customers to multiple distribution centers in the supply chain network and used GA to solve the resulted problem formulation.

Syam (2002) developed a model and methodologies for the location problem with logistical components. His modeling formulation was an integrated location–consolidation model, which provided two methodologies to solve the problem and the performance of the two methodologies was

Jayaraman and Ross (2003) presented a simulated annealing methodology for a problem of distribution network design and management, which consists of production, logistics, outbound and transportation (PLOT) design system. The system investigated a class of distribution network design problems, which is characterized by multiple product families, a central manufacturing plant site, multiple distribution center and cross-docking sites, and retail outlets (customer zones) which demand multiple units of several commodities. The system consists of two stages of the planning and operational stages. The PLOT system developed to implement the model provided for a high degree of user interaction in the generation of solutions. They proposed simulated annealing (SA) approach to solve the resulted problem formulation.

Miranda and Garrido (2004) incorporated inventory control decisions into a strategic distribution network design model with stochastic demand. They presented a non-linear-mixed-integer model and a heuristic solution technique, based on Lagrangian relaxation and the sub-gradient method and reported that the potential cost reduction, compared to the traditional approach, increases when the holding costs and/or the variability of demand are higher. Chan et al. (2001) proposed a multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands. They formulated a multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands defined as demands, which are generated upon completing site-specific service on their predecessors.

In this paper, we present an improved VRP modeling formulation by considering additional assumptions. The proposed model of this paper is solved using a new hybrid metaheuristics and the performance of the proposed model is compared with an alternative hybrid method with some benchmark problems.

2. Problem statement

2.1. Model assumptions

The following major assumptions hold for the problem formulation.

- There are some certain independent products, in which a major supplier has to distribute them among various retailers.
- The products are not perishable.
- There is severe penalty if supplier cannot meet demand in predefined time window.
- Time windows are also independent from each other, which is based on a soft one. In other word, in case there is a delay for delivering good, the retailers will accept it and there is no sale or discard.
- Time horizon is limited and it is considered only for one period. However, the proposed model of this paper can be expressed for rolling horizon planning.
- All different goods can be transported using a simple vehicle.
- All transportation facilities are unique.
- There are unlimited numbers of vehicles, which can be rented.
- It is not possible to use more than one vehicle to meet a demand for one single retailer.
- A particular vehicle can be used only for one single trip and we cannot use it for more than one route.
- No interruption on vehicles is permitted. In other word, they are supposed to work with good conditions.
- The loading and packaging are determined and route is also known.
- We assume all retailers are located in similar geographical locations and there is a unique difficulty for delivering goods for all of them.
- Transportation cost increases proportion to the distance between wholesaler and retailer.
- The primary objective is to minimize total costs.

2.2. Parameters

\( N \) Total number of loading and unloading centers, including the main depot,

\( i,j \) Loading and unloading goods \( i=1,..N, i=0 \) denotes the main depot,

\( k \) Main vehicle with \( k=1,...,K \),

\( E \) The number of goods,

\( e \) The type of delivery good,

\( tc_{ij} \) Total transportation cost between node \( i \) and node \( j \),

\( t_{ij} \) Total transportation time between node \( i \) and node \( j \),

\( f_e \) The completion time for production \( e \),

\( d_{i,e} \) Demand for product type \( e \) in location \( i \),

\( cap \) The capacity of each big vehicle,

\( w_e \) Weight of each product \( e \),

\( l_i \) The length of each time window,

\( s_i \) Time needed for downloading goods in location \( i \),

\( cc \) The penalty cost for delay delivery to retailer,

\( nc_i \) The number carrier for delivery to retailer.

2.3. Variables

\[
x_{ijk} = \begin{cases} 
1 & \text{If vehicle type } k \text{ works between node } i \text{ and } j \\
0 & \text{otherwise}
\end{cases}
\]

\[
y_{ke} = \begin{cases} 
1 & \text{If vehicle type } k \text{ delivers product type } e \\
0 & \text{otherwise}
\end{cases}
\]

\[
u_i = \begin{cases} 
1 & \text{If no vehicle can reach to node } i \text{ in time window} \\
0 & \text{otherwise}
\end{cases}
\]

\( t_i \) is the time a big vehicle reaches node \( i \).

\( tt_i \) is the amount of violated time for a big vehicle reaches node \( i \).
$t_s$ is the starting trip time a big vehicle.

$t_{ske}$ is the starting trip time a big vehicle for delivering product type $e$.

$K$ is the total number of big vehicles.

2.4 Problem statement

Based on the information given, the problem statement is formulated as follows,

$$\begin{align*}
\min & \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} tc_{i,j,k} x_{i,j,k} + ce \sum_{i=1}^{N} nc_i t_i \\
\text{subject to} & \\
\sum_{j=1}^{N} x_{0,j,k} &= 1 \quad \forall k \quad (2) \\
\sum_{i=1}^{N} x_{i,0,k} &= 1 \quad \forall k \quad (3) \\
\sum_{k=1}^{K} \sum_{j=0}^{N} x_{i,j,k} &= 1 \quad \forall i \quad (4) \\
\sum_{k=1}^{K} \sum_{i=0}^{N} x_{i,j,k} &= 1 \quad \forall j \quad (5) \\
\sum_{i=0}^{N} \sum_{e=1}^{E} d_{i,e} w_e \left( \sum_{j=0}^{N} x_{i,j,k} \right) &\leq cap \quad \forall k \quad (6) \\
t_j &= \sum_{k=1}^{K} \left( x_{0,j,k} \left( t_s + t_{0,j} \right) + (1 - x_{0,j,k}) \sum_{i=1}^{N} x_{i,j,k} \left( t_i + s_i + t_{i,j} \right) \right) \quad \forall j \quad (7) \\
\sum_{i=1}^{N} \sum_{j=0}^{N} d_{i,e} x_{i,j,k} &\leq 0 + My_{k,e} \quad \forall k, e \quad (8) \\
\sum_{i=1}^{N} \sum_{j=0}^{N} d_{i,e} x_{i,j,k} &> 0 - M (1 - y_{k,e}) \quad \forall k, e \quad (9) \\
t_{ske} &= y_{k,e} f_e \quad \forall k, e \quad (10) \\
t_s &\geq t_{ske} \quad \forall k, e \quad (11) \\
t_i - l_i &< 0 + Mu_i \quad \forall i \quad (12) \\
t_i - l_i &\geq 0 - M (1 - u_i) \quad \forall i \quad (13) \\
t_i &= u_i \left( t_i - t_j \right) \quad \forall i \quad (14)
\end{align*}$$
In this model, the objective function given in Eq. (1) is the minimization of total costs, which includes the cost of transportation and prevents violation from time window. Eq. (2) to Eq. (5) specify transportation characteristics. Eq. (6) shows the capacity of each big vehicle. Eq. (7) determines the time that a vehicle delivers a cargo at node $j$. Eq. (8) and Eq. (9) determines whether a vehicle type $k$ is responsible for delivering product type $e$. Eq. (12) and Eq. (13) check to see whether a vehicle can reach its time window for node $i$. Eq. (14) measures the amount of violation from window. Eq. (15) and Eq. (16) determines the type of variables. The proposed mathematical problem of this paper has two new contributions compared with the existing models in the literature. First, the time window is adjusted in one way, i.e. there is no lower limit and wholesaler has to pay penalty when the delivery violates the upper time. Second, the objective function considers a new type of cost for setup time. Note that this type of cost was already considered as part of other cost components in the literature.

The resulted problem formulation is a mixed integer nonlinear programming and we believe it is not possible to solve the resulted problem using regular software packages or any other direct implementation of optimization techniques. Based on the survey we have carried out, we believe the only reason we may solve this type of problems is to use either heuristic or metaheuristics approaches.

3. Solution procedure

The proposed model of this paper can be solved using different heuristic or metaheuristics. The proposed model of this paper uses a hybrid of particle swarm optimization (PSO) (Kennedy & Eberhart, 1995) and simulated annealing (SA) (Kirkpatrick et al., 1983). We present details of our implementation in next section.

3.1. Solution procedure for PSO

The following notations are used for the proposed hybrid PSO-SA method.

- $NOP$: The number of particles
- $n$: Index for particles, $n=1,\ldots,NOP$
- $Itermax$: Maximum number of iterations
- $j$: Index for the algorithm, $j=1,\ldots,Itermax$
- $c_1$: Weight of each particle in terms of decision making, $c_1 \in [0, 2]$
- $c_2$: Weight of each particle in terms of learning, $c_2 \in [0, 2]$
- $r_1, r_2$: Stochastic parameters with uniform distribution between zero and one
### 3.1.1 Initial solution

In the first step of the algorithm, we generate a set of random solutions by considering possible constraints. Fig 1 shows a sample of feasible solution.

![Fig. 1. A feasible solution between a wholesaler and retailers](image)

### 3.1.1 The procedure for the movement of each swarm

In order to present the pseudo code of the solution procedure we need to build a vector for physical position and a vector for the speed for each swarm. Let \((x_{jit}, P_j)\) be the position vector and \((V_n(x_{jit}), V_n(P_j))\) be the vector of speed. Fig. 2 shows the pseudo code of the proposed PSO method.
get \( \{w; c_1; c_2\} \)

for \( n=1 \) to \( NOP \)

\( r_1=\text{random}(0,1) \)
\( r_2=\text{random}(0,1) \)

for \( i=1 \) to \( n \)

for \( t=1 \) to \( T \)

\[
V_n(x_{i,t}^{t+1}) = r_1 c_1 (p_{best}(x_{i,t}^{t}) - x_{i,t}^{t}) + r_2 c_2 (g_{best}(x_{i,t}^{t}) - x_{i,t}^{t}) + w V_n(x_{i,t}^{t})
\]

\[
V_n(P_{i,t}^{t+1}) = r_1 c_1 (p_{best}(P_{i,t}^{t}) - P_{i,t}^{t}) + r_2 c_2 (g_{best}(P_{i,t}^{t}) - P_{i,t}^{t}) + w V_n(P_{i,t}^{t})
\]

\[
x_{i,t}^{t+1} = x_{i,t}^{t} + V_n(x_{i,t}^{t+1})
\]

\[
P_{i,t}^{t+1} = P_{i,t}^{t} + V_n(P_{i,t}^{t+1})
\]

end for

end for

end for

Fig. 2. Pseudo code for proposed PSO (Kennedy & Eberhart, 1995)

3.2. Solution procedure for SA

The proposed metaheuristics approach uses SA for local search needed in each iteration of PSO code and details of our implementation are shown in Fig 3.
Fig. 3. The pseudo code of SA method (Kirkpatrick et al., 1983)

Note that we use $\exp(-|z|)$ as a fitness function for the local search since the proposed model is of minimization type and standard PSO is in maximization type.

3.3. Termination criteria

There are literally various termination criteria, which could be used and the proposed model of this paper terminates PSO algorithm whenever the difference between $f_{\text{average}}$ and $f_{\text{best}}$ reaches a small value, say $\varepsilon$, or the number of iterations reaches the maximum value, $\text{Iter}_{\text{max}}$. The important point is to set a suitable value for $\varepsilon$ and one alternative to set a proper value for $\varepsilon$ is as follows,

$$\varepsilon \leq \alpha \cdot (\text{fitness}_{\text{max}} - \text{fitness}_{\text{min}}),$$

(17)

$$\varepsilon \leq \alpha \left( \frac{Z_{\text{NIS}} - Z_{\text{PIS}}}{(1 + Z_{\text{PIS}})(1 + Z_{\text{NIS}})} \right),$$

(18)

where $\alpha$ is an arbitrary value, $\text{fitness}_{\text{min}}$ and $\text{fitness}_{\text{max}}$ are minimum and maximum values of fitness function, respectively. $Z_{\text{PIS}}$ and $Z_{\text{NIS}}$ are minimum and maximum values of the objective function after repeating algorithm several times such that $Z_{\text{PIS}} \leq Z \leq Z_{\text{NIS}}$. In this paper, we use $0.05 \leq \alpha \leq 0.1$.

3.3. Parameter setting

One of the important issues in metaheuristics is to set appropriate values for input parameter settings. The parameter setting plays an important role on the convergence of the proposed model. Table 1 shows the parameter setting for the proposed model of this paper. Note that the values of the parameters have been adopted from the work by Jamili et al. (2011).

Table 1

<table>
<thead>
<tr>
<th>PSO</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of particles</strong></td>
<td><strong>Number of temperature change</strong></td>
</tr>
<tr>
<td>45</td>
<td>480</td>
</tr>
<tr>
<td><strong>Number of iterations</strong></td>
<td>$\alpha$</td>
</tr>
<tr>
<td>600</td>
<td>0.98</td>
</tr>
<tr>
<td><strong>$c_1$</strong></td>
<td><strong>Initial temperature $T_0$</strong></td>
</tr>
<tr>
<td>1.49</td>
<td>4800</td>
</tr>
<tr>
<td><strong>$c_2$</strong></td>
<td><strong>Final temperature $T_f$</strong></td>
</tr>
<tr>
<td>1.49</td>
<td>100</td>
</tr>
<tr>
<td><strong>$w$</strong></td>
<td></td>
</tr>
<tr>
<td>0.639</td>
<td></td>
</tr>
</tbody>
</table>

3.4. Test problems

Another important factor for validating the performance of a model is to use benchmark problems. In this case, we may compare the performance of our proposed model with alternative ones from the literature. The proposed model of this paper uses standard test problems introduced by Solomon (1987). This set of benchmark problems consist of 56 standard problems including $RC1$, $RC2$, $C1$, $C2$, $R1$ and $R2$. The problems are divided in three classes and each class is divided into two parts. In class $R1$, $R2$, customers' position and their demands are stochastic. In class $C1$, $C2$ customers are divided based on their demand and in the last class, $RC1$ and $RC2$ are combinations of stochastic and classifications. For all problems, we use Euclidian norm to measure the performance of algorithms.
3.6. Results

In order to measure the performance of the proposed model, we have coded the proposed model and compared the results with alternative method, which is a hybrid of Ant colony and Tabu search (Iibri, 2010). We ran both codes for three pre-specified times of 45, 75 and 105 seconds. Each time, we repeat the run 30 times. Table 2 and Table 3 show the results for the proposed model and alternative method.

**Table 2**
The performance of the proposed PSO-SA

<table>
<thead>
<tr>
<th>Problem</th>
<th>CPU time limit (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>max</td>
</tr>
<tr>
<td>C101</td>
<td>100.00</td>
</tr>
<tr>
<td>C102</td>
<td>94.89</td>
</tr>
<tr>
<td>R101</td>
<td>57.47</td>
</tr>
<tr>
<td>R102</td>
<td>32.28</td>
</tr>
<tr>
<td>RC101</td>
<td>82.81</td>
</tr>
<tr>
<td>RC102</td>
<td>81.45</td>
</tr>
</tbody>
</table>

**Table 3**
The performance of Ant-Tabu

<table>
<thead>
<tr>
<th>Problem</th>
<th>CPU time limit (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>max</td>
</tr>
<tr>
<td>C101</td>
<td>4.64</td>
</tr>
<tr>
<td>C102</td>
<td>30.97</td>
</tr>
<tr>
<td>R101</td>
<td>48.91</td>
</tr>
<tr>
<td>R102</td>
<td>51.80</td>
</tr>
<tr>
<td>RC101</td>
<td>42.29</td>
</tr>
<tr>
<td>RC102</td>
<td>43.21</td>
</tr>
</tbody>
</table>

In order to measure the performance of the proposed model against the alternative one, we use the following criteria,

\[ R_{G_{i,j}} = 1 - \frac{BR_{i,j} - RA}{RA} \]  \hspace{1cm} (19)

where \( R_{G_{i,j}} \) is the relative closeness of algorithm \( i \) and \( j \) and \( RA \) is the best solution after 30 independent run under the same amount of time and \( BR_{i,j} \) is the quality of the solution \( j \) by algorithm \( I \) in each 30 run. According to this criteria a higher value of algorithm \( i \) means a better value.

Table 4 and Table 5 show the results of the performance of the proposed model against alternative methods. Based on the results of Table 4-5, the proposed PSO-SA seems to perform better than Ant-Tabu in terms of CPU time. However, as the CPU time increases we cannot make any conclusion about the performance of these two methods.
Table 4
RG values for the proposed PSO-SA

<table>
<thead>
<tr>
<th>Problem</th>
<th>45</th>
<th>75</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
<td>max</td>
</tr>
<tr>
<td>C101</td>
<td>73.12</td>
<td>43.08</td>
<td>100.00</td>
</tr>
<tr>
<td>C102</td>
<td>27.17</td>
<td>50.73</td>
<td>100.00</td>
</tr>
<tr>
<td>R101</td>
<td>3.38</td>
<td>27.23</td>
<td>100.00</td>
</tr>
<tr>
<td>R102</td>
<td>7.58</td>
<td>12.66</td>
<td>100.00</td>
</tr>
<tr>
<td>RC101</td>
<td>54.91</td>
<td>71.48</td>
<td>100.00</td>
</tr>
<tr>
<td>RC102</td>
<td>26.90</td>
<td>44.74</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Table 5
RG value for the proposed Ant-Tabu

<table>
<thead>
<tr>
<th>Problem</th>
<th>45</th>
<th>75</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
<td>max</td>
</tr>
<tr>
<td>C102</td>
<td>9.79</td>
<td>13.46</td>
<td>28.20</td>
</tr>
<tr>
<td>R101</td>
<td>3.51</td>
<td>8.11</td>
<td>18.74</td>
</tr>
<tr>
<td>R102</td>
<td>25.99</td>
<td>27.65</td>
<td>47.15</td>
</tr>
<tr>
<td>RC101</td>
<td>21.68</td>
<td>31.26</td>
<td>34.41</td>
</tr>
<tr>
<td>RC102</td>
<td>19.23</td>
<td>24.57</td>
<td>26.07</td>
</tr>
</tbody>
</table>

4. Conclusion

Managing supply chain is one of the most important cost items and any attempt to reduce this cost item could help increase competition in today's sluggish market. The present study introduced a more realistic form of VRP by considering setup cost and a one-way time window. The problem formulation was considered an NP-Hard problem and we believe there is no direct solution to find optimal solution for real-world problems. The proposed model of this paper proposed a new meta-heuristic approach, PSO-SA, and compared the results of the proposed model with an alternative hybrid method, Ant-Tabu. The performance of the proposed model was examined using some benchmark problems and they are compared with an alternative one.

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