

A Simulated Annealing method to solve a generalized maximal covering location problem**Mohammad Saeed Jabalameli^a, Behzad Bankian Tabrizi^{a*}, Mohammad Moshref Javadi^a**^aDepartment of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran**ARTICLE INFO****ABSTRACT***Article history:*

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The maximal covering location problem (MCLP) seeks to locate a predefined number of facilities in order to maximize the number of covered demand points. In a classical sense, MCLP has three main implicit assumptions: all or nothing coverage, individual coverage, and fixed coverage radius. By relaxing these assumptions, three classes of modelling formulations are extended: the gradual cover models, the cooperative cover models, and the variable radius models. In this paper, we develop a special form of MCLP which combines the characteristics of gradual cover models, cooperative cover models, and variable radius models. The proposed problem has many applications such as locating cell phone towers. The model is formulated as a mixed integer non-linear programming (MINLP). In addition, a simulated annealing algorithm is used to solve the resulted problem and the performance of the proposed method is evaluated with a set of randomly generated problems.

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1. Introduction

The concept of covering problems was originally proposed by Toregas et al. (1971) and it was extended by Berlin and Liebman (1974) afterwards. In a typical covering problem, a customer is called *covered* if it is within a predefined distance, covering radius, from one of the closest facilities. There are two major categories of covering problems in the literature. Location set covering problem (LSCP), originally introduced by Church and Reville (1974), with the objective of covering all the customers to minimize the number of located facilities and maximal covering location problem (MCLP), originally defined by Reville et al. (1976), with the objective of maximizing total weighted covered customers according to specified number of facilities. According to Berman et al. (2010) there are three main assumptions in coverage problem as follows,

- 1- *All or nothing coverage:* based on this assumption, a customer is *covered* if it is located within the coverage radius of a facility, while it is *uncovered* if it is outside the coverage radius of a facility.

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- 2- *Individual coverage*: according to this assumption, the coverage of each customer depends on the closest established facility to the customer and the next closest facility does not affect the coverage of the customer.
- 3- *Fixed coverage radius*: the coverage radius of each facility is a fixed parameter and is not a decision variable.

There are, however, many real-world applications in which these assumptions are relaxed (Berman et al., 2010). The following summarizes the new classification of these problems:

- 1- *Gradual covering models*: in this type of models, instead of all or nothing coverage, a coverage function is used. This function determines the proportion of demands covered in a specified distance from a facility. Church and Robert (1984) presented the first model in this class and applied a step-coverage function for discrete version. Berman and Krass (2002) proposed the network version of this problem with a step coverage function. Fig. 1 shows a stepwise coverage function with break points R_1 , R_2 and R_3 . Berman et al. (2003) studied the gradual model using general linear non-increasing coverage function in discrete and network version. This function is shown in Fig. 2. Eiselt and Marianov (2009) extended the gradual covering problem in the framework of set covering location problem. Berman et al. (2009) proposed the ordered gradual cover location problem (OGCLP) which combines the characteristics of gradual cover and ordered median models. Finally, Drezner et al. (2010) incorporated uncertainty in coverage radius in gradual cover models.
- 2- *Cooperative cover models*: when an individual coverage assumption is relaxed, all of the established facilities may have an effect on coverage of a demand point. In other words, consider a case where each facility emits a signal. The amount of signal received by each customer from each facility is determined based on the distance between the facility and the customer. Thus, the total amount of signals received by each customer is the sum of signals received from all of the facilities. A customer is considered covered if the total amount of received signals is greater than a predefined threshold. Berman et al. (2010) recently proposed this concept for both maximal covering location problems and location set covering problems in plane for the Euclidean distance case.
- 3- *Variable radius models*: In these models, the coverage radius of a particular facility is considered as a function of establishing cost of the facility, which means the more the establishing cost of the facility, the greater the coverage radius of the facility. Berman et al. (2009) are the first who introduced this idea. The goal of their proposed model was to determine the locations, the number and the coverage radius of each facility to cover all of the demand points with the minimum locating cost. They investigated this covering model with variable radius for both discrete and plane cases.

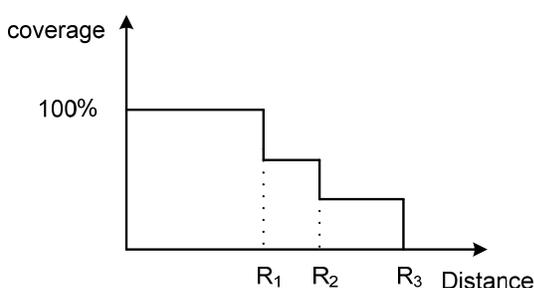


Fig. 1. The stepwise coverage function

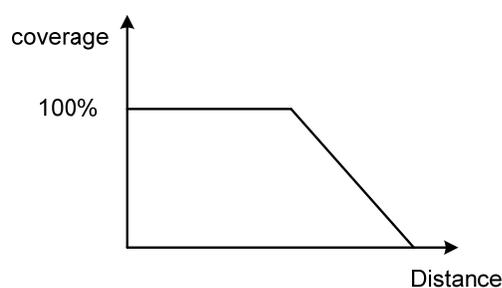


Fig. 2. The linear coverage function

There are also other real-world cases where we need to relax all assumptions of all or nothing coverage, individual coverage, and fixed coverage radius, simultaneously. For instance, consider a problem of locating cell phone towers. The following circumstances exist in this problem:

- 1- The strength of received signals for each mobile phone user is inversely correlated with the square of the distance between the user and the tower. In other words, as the distance between the user

and the cell phone tower increases, the probability of the connection with the tower decreases (gradual coverage).

- 2- The coverage radius of the cell phone tower increases as the signal strength of the tower is amplified (variable radius).
- 3- Each mobile phone user receives signals from different cell phone towers with various strength levels. If a cell phone user fails to connect to the closest tower, he/she may connect to the next closest towers. In other words, in addition to the closest tower, other towers, with less probability, may become available to cover cell phone users (cooperative coverage).

In this paper, we study a special form of the maximal covering location problem, which considers gradual cover, variable radius, and cooperative cover assumptions. The proposed model of this paper is formulated as a mixed integer non-linear problem (MINLP) and the resulted problem is solved using some meta-heuristic method.

The rest of the paper is organized as follows. In section 2, the problem is described and formulated. Section 3 describes the proposed solution method which is based on simulated annealing algorithm. The SA approach is applied to solve the generated test problems in Section 4. Finally, Section 5 summarizes the contribution of the paper.

2. Problem description and formulation

This section presents the mathematical formulation of the problem, which considers gradual coverage, variable radius and cooperative coverage, simultaneously. Suppose that J is the set of candidate locations to locate the facilities. Also, assume that the demand points and candidate locations to locate the facilities are the nodes of network G . Given an available budget, the goal of the model is to determine the numbers, the locations and the size and the coverage radius of each facility to maximize the total weighted covered demand points. The following notations are used to formulate the problems:

indices

- i set of demand points
 j set of candidate locations

Parameters

- F_j fixed cost of locating a facility at location j
 w_i weight of demand point i
 d_{ij} distance between customer i and candidate location j
 B amount of available budget
 l_j the strength of signal emitted by facility j
 T_i the threshold for coverage of customer i
 a_j a percentage of coverage radius of facility j in which all of the emitted signal by facility j is received
 $f_i(d)$ coverage function which determines the amount of signal received by customer i located in distance d from the facility
 $\varphi_j(r)$ variable establishing cost of facility j with coverage radius r
 M sufficiently large number

Decision Variables

- $p_i = \begin{cases} 1 & \text{if customer } i \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$

$$x_j = \begin{cases} 1 & \text{if a facility is located at location } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to facility } j \\ 0 & \text{otherwise} \end{cases}$$

r_j coverage radius of facility j

$$z_{ij} = \begin{cases} 1 & \text{if the distance between customer } i \text{ and facility } j \text{ is less than the initial coverage radius of the facility} \\ 0 & \text{otherwise} \end{cases}$$

$$z'_{ij} = \begin{cases} 1 & \text{if the distance between customer } i \text{ and facility } j \text{ is between the initial and secondary coverage radius of the facility} \\ 0 & \text{otherwise} \end{cases}$$

$$z''_{ij} = \begin{cases} 1 & \text{if the distance between customer } i \text{ and facility } j \text{ is greater than the secondary coverage radius of the facility} \\ 0 & \text{otherwise} \end{cases}$$

In the proposed model, each facility must cover all assigned customers; thus, the coverage radius of the established facility is equal to the farthest demand point assigned to the facility:

$$r_j = \max_{i \in N} \{y_{ij} d_i(x_j)\}$$

There are two types of fixed, F_j , and variable costs, $\varphi_j(r)$ associated with the establishment of a facility j with coverage radius r . Berman et al. (2009) have proposed different types of variable cost functions. Thus, total cost to located facilities is as follows,

$$\sum_{j \in N} F_j X_j + \sum_{j \in N} \varphi_j(r_j).$$

It is assumed that each facility emits a signal with strength l_j . All customers within the initial coverage radius of the facility receive the signal, completely. Moreover, customers between the initial and the secondary coverage radius of the facility receive partial signals and finally, customers who are located outside the secondary coverage radius of the facility receive no signal. A customer is considered covered if the total amount of received signals from all located facilities is greater than a specified threshold (T_i). The secondary coverage radius (r_j) of each facility is defined according to the distance of farthest allocated customer to the facility. If a_j is a percentage of coverage radius of facility j in which all of the customers completely receive signals emitted by facility j , $a_j r_j$ would be the initial coverage radius of facility j . Furthermore, customers who are between the initial and secondary radius of facility j partially receive signals from the facility. The amount of signals received by each customer is a function of the distance between the customer and the facility. This function is called the coverage function. Different types of coverage functions can be found in the literature. One of the simplest kinds of these functions is the linear one, which is defined as follows,

$$f_i(d) = \frac{r_j - d_{ij}}{r_j - a_j r_j}, \quad d_{ij} \in [a_j r_j, r_j]$$

Therefore, we can formulate the covering facility location problem with gradual coverage, variable radius and cooperative coverage as follows,

$$\max \sum_{i \in N} w_i p_i \quad (1)$$

$$s.t. \sum_{j \in N} F_j x_j + \sum_{j \in N} \varphi_j(r_j) \leq B, \quad (2)$$

$$r_j \geq d_{ij} y_{ij} \quad i, j \in N, \quad (3)$$

$$y_{ij} \leq x_j \quad i, j \in N, \quad (4)$$

$$\sum_{j \in N} l_j [z_{ij} + f_i(d_{ij}) z'_{ij}] \leq T_i + MP_i, \quad i \in N, \quad (5)$$

$$\sum_{j \in N} l_j [z_{ij} + f_i(d_{ij}) z'_{ij}] \geq T_i - M(1 - P_i), \quad i \in N, \quad (6)$$

$$d_{ij} \geq a_j r_j z'_{ij} \quad i, j \in N, \quad (7)$$

$$d_{ij} \leq a_j r_j + (1 - z_{ij}) M \quad i, j \in N, \quad (8)$$

$$d_{ij} \geq r_j z''_{ij} \quad i, j \in N, \quad (9)$$

$$d_{ij} \leq r_j + (1 - z'_{ij}) M \quad i, j \in N, \quad (10)$$

$$z_{ij} + z'_{ij} + z''_{ij} = 1, \quad i, j \in N, \quad (11)$$

$$y_{ij}, x_j, p_i, z_{ij}, z'_{ij}, z''_{ij} \in \{0, 1\}, \quad (12)$$

$$r_j \geq 0, \quad (13)$$

The objective function (1) maximizes the weighted sum of covered demand points. Constraint (2) gives the available budget constraint. In other words, total establishing cost of facilities, which includes fixed and variable costs must be equal or less than the available budget. Constraint (3) ensures that the coverage radius of facility j must be equal or greater than the farthest allocated customer to the facility. Constraint (4) states that if demand point i is allocated to facility j , the facility in location j must be established. Constraint (5) and (6) ensure that if the total amount of received signals by the customer i from the located facilities is greater than the threshold value, then the customer is considered covered ($p_i = 1$) and otherwise the customer is uncovered ($p_i = 0$). Note that when $Z_{ij} = 1$, demand point i completely receives signal emitted by facility j while when $Z'_{ij} = 1$, only a portion of the signal from facility j is received. Moreover, when $Z''_{ij} = 1$, customer i receives no signal from facility j . Constraint (7)-(10) impose that if the distance between demand point i and facility j is less than the initial coverage radius of facility j ($d_{ij} \leq a_j r_j$), then $Z_{ij} = 1, Z'_{ij} = 0, Z''_{ij} = 0$. Also, if the d_{ij} is between the initial coverage radius and the secondary coverage radius of facility j ($a_j r_j \leq d_{ij} \leq r_j$), then $Z_{ij} = 0, Z'_{ij} = 1, Z''_{ij} = 0$, and finally, if the d_{ij} is greater than the secondary coverage radius of facility j ($d_{ij} \geq r_j$), then $Z_{ij} = 0, Z'_{ij} = 0, Z''_{ij} = 1$. Constraint (11) shows that only one of variables Z_{ij}, Z'_{ij} , and Z''_{ij} equals to 1 for each i and j . Finally, constraint (12) and (13) determine the type and range of the variables. Since the maximal covering location problem is NP-hard and the proposed model of this paper is an extension of MCLP where three covering assumptions are relaxed, we can conclude that the proposed model is NP-hard. Therefore, exact algorithms can reach the solution in reasonable time only for small size problems. Therefore, heuristic and meta-heuristic methods are normally implemented to deal with medium and large-scale problems. In this paper, we present a new simulated annealing (SA) algorithm to solve the proposed problem and the results are compared with some upper bounds in order to measure the efficiency of the resulted problem.

3. Simulated annealing algorithm

Simulated annealing (SA) is one of the well known algorithms which is inspired from physical annealing of solids. It was first introduced by Kirkpatrick (1983) to solve large combinatorial

optimization problems. SA attempts to escape from the local optima by probabilistically choosing non-improving solutions. The following summarizes the characteristics of SA algorithm.

3.1. Solution representation

The first step to implement a meta-heuristic algorithm is to encode solutions of the problem such that the operators of the algorithm can be performed on. A two dimensional array with 2 rows and n columns is used to represent the solutions. The first row consists of binary numbers where 1 represents established facility. The second row is filled by integer numbers from 1 to n and represents the last customer which is allocated to an established facility. This number determines the covering radius of the facility. Fig. 3 shows a solution of a six-node problem. For instance, the value in the second row of the solution for facility 3 is 5 which means the coverage radius of facility 3 is d_{35} .

1	2	3	4	5	6
0	0	1	0	1	0
-	-	5	-	2	-

Fig. 3. Solution representation for a six-node problem

3.2. Initial solution

The quality of solutions using meta-heuristic algorithms highly depends on the quality of the initial solution. In the proposed algorithm, the initial solution is generated as follows: first, the alleles in the first row of the solution are set to 1 with the probability p ; otherwise, 0. The value of parameter p depends on the number of nodes of the network, the cost of established facility and the available budget. In the second step, the second row of the solution is filled with random numbers from 1 to n where the corresponding allele in the first row is 1.

3.3. Neighborhood search strategy

New solution is generated by altering the elements of current solution. A two-step procedure is used to obtain a new neighbourhood solution.

- a) *Altering the alleles of the first row:* two facilities are randomly selected and if they are both located facilities (the alleles are 1), both are moved to the closed facilities (1 change to 0). If both alleles are 0, then they are changed to 1, and finally, if one allele is 1 and the other is 0, then the values are exchanged. For instance, if facilities 2 and 3 are selected, after the first step we have:

1	2	3	4	5	6
0	0	1	0	1	0

→

1	2	3	4	5	6
0	1	0	0	1	0

- b) *Altering the alleles of the second row:* to complete the process of neighborhood solution generation, the alleles in the second row of the current solution where the corresponding alleles in the first row are 1 (located facilities) are changed k times. This change is performed by filling the second row with randomly generated numbers from 1 to n . As a result, k new solutions with different coverage radius are obtained for the solution yielded from the previous step. At last, the solution with the best objective function value is selected as the new solution. Note that k is a parameter which is determined according to the size of the problem.

3.4. Fitness function evaluation

The following fitness function is used based on the objective function of the proposed model,

1- Calculate the value of $b_{ij}, \forall i, j \in N$, using the following formula,

$$b_{ij} = \begin{cases} l_j & \text{if } d_{ij} \leq a_j r_j \\ l_j f_i(d_{ij}) & \text{if } a_j r_j \leq d_{ij} \leq r_j \\ 0 & \text{if } d_{ij} \geq r_j \end{cases}$$

2- Assume that N_i is the set of covered customers ($\sum_{j \in N} b_{ij} \geq T_i$). Thus, the fitness function value of solution P is calculated as $F_i(p) = \sum_{i \in N_i} w_i$.

3.5. Feasibility of solutions

Available budget constraint may impose infeasibility to the solutions of the algorithm; in other words, total establishing cost of facilities exceed available budget. The following procedure assures the feasibility of the solutions. In the current solution, without any changes to the first row of the solution, the second row is filled with random numbers from 1 to n where the corresponding allele in the first row is 1. This process is performed M times where M depends on the dimensions of the problem and for each altered solution, the established cost is calculated to examine the feasibility of the solution ($\sum_{j \in N} F_j x_j + \sum_{j \in N} \varphi_j(r_j) \leq B$). When a feasible solution is generated, the solution is accepted.

Otherwise, it can be concluded that there is no feasible solution with the current number of established facilities. Thus, one of 1s in the first row is changed to 0, randomly. This process continues until one feasible solution is obtained.

3.6. SA algorithm

Set the temperature $T = T_0$. Generate an initial solution and calculate its objective function value $F(p)$

Set the best found solution to $F(p)$

1- Repeat the following steps until $T \geq T_{\min}$

(a) Generate a new solution p' and evaluate $F(p')$, Let $\Delta F = F(p') - F(p)$

(b) If $\Delta F \geq 0$ perform the move to p' and go to step 2d,

(c) If $\Delta F \leq 0$ perform the move to p' with probability $e^{-\frac{\Delta F}{T}}$ and go to step 2e, Otherwise retain p and go to step 2e

(d) If $F(p')$ is better than the best found solution, update the best found solution

(e) Multiply $T = \alpha T$

2- The best found solution is the result of the algorithm.

4. Computational results

4.1 Data set generation

To compare the results of the proposed algorithm, a set of random problems are generated. The coordination of each node of the network is selected randomly using U (5,30) distribution. Furthermore, the distance between nodes is calculated using the Euclidean distance type. Also, the weight of each node (w_i) is generated randomly between 10 and 100. The cost of establishing each facility is a random number between 200 and 400 for fixed cost and 10 and 20 for variable cost,

respectively. Finally, variable establishing cost function of facilities is assumed to be $c_j r$ ($\varphi_j(r) = c_j r$). Other parameters' values are: $a_j = 0.5$, $B = 1000$, $l_j = 7$, $T_i = 5$.

4.2 Parameter setting

The performance of SA algorithm highly depends on the values of parameters of the algorithm. Table 1 summarizes the initial values for initial temperature (T_0), cooling rate (α), and final temperature (T_{min}) to investigate the best value for each parameter. A 50-node problem was tested against various values and the results are given from Tables 2 to 4. The best values are $T_0 = 10000$, $\alpha = 0.99$, $T_{min} = 0.005$.

Table 1
SA parameters' levels

parameters	levels		
T_0	1000	5000	10000
α	0.9	0.95	0.99
T_{min}	0.01	0.005	0.001

Table 2
Parameter setting results for initial temperature

problem No.	1000		5000		10000	
	Objective Function	Time	Objective Function	Time	Objective function	Time
1	2135	21	2160	23	2348	16
2	2294	16	2356	18	2521	20
3	2137	17	2395	21	2359	21
4	2335	13	2470	15	2150	17
5	2228	13	2228	21	2461	25
Average	2225.8	16	2321.8	19.6	2367.8	19.8

Table 3
Parameter setting results for α

problem No.	0.9		0.95		0.99	
	Objective function	Time	Objective function	Time	Objective function	Time
1	2348	16	2336	41	2485	206
2	2521	20	2371	42	2435	204
3	2359	21	2359	38	2591	202
4	2150	17	2583	36	2583	189
5	2461	25	2628	32	2628	192
Average	2367.8	19.8	2455.4	37.8	2544.4	198.6

Table 4
Parameter setting results for T_{min}

problem No.	0.01		0.005		0.001	
	Objective function	Time	Objective function	Time	Objective fu nction	Time
1	2485	206	2628	195	2481	185
2	2435	204	2628	194	2583	241
3	2591	202	2453	205	2530	243
4	2583	189	2481	212	2504	245
5	2628	192	2639	215	2592	267
Average	2544.4	198.6	2565.8	204.2	2538	236.2

Fig. 4 shows the improvement of solutions after adjusting each parameter. In this Figure, initial value is where the parameters are in the lowest levels. By setting each parameter, the algorithm achieves better solutions with respect to their objective function values.

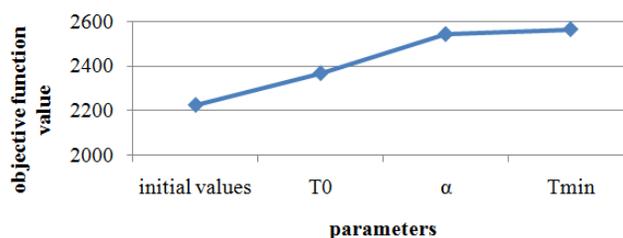


Fig. 4. Improvement of performance of the algorithm after parameter setting

4.3. Numerical results

The proposed SA algorithm was used to solve 6 generated test problems consisting of 10, 20, 30, 50, 70, and 90 nodes. Because of long CPU time to obtain the optimized solution, the comparison is performed with upper bound values of solutions which are achieved using linear programming relaxation; As a result, binary variables of the model are assumed to be continuous variables between 0 and 1 and then the converted model was solved using BARON solver of GAMS. Each test problem was solved 5 times and the percentage deviation of the objective value from the upper bound solution is calculated using the following equation,

$$\%Gap = \frac{\text{upper bound} - \text{SA solution}}{\text{upper bound}}$$

Objective function value, computation time, upper bound and %Gap are shown in Table 5. Fig. 5 compares the SA average objective function values and upper bound values. It can be concluded from Table 5 and Fig. 5 that SA solutions slightly deviate from the upper bound values and the results are acceptable. Fig. 6 shows the average percentage deviation of the solutions using SA from the upper bound values. As shown in this figure, this percentage increases for larger problem sizes.

Table 5

Objective function value, CPU time, upper bound, and %Gap for the test problems

problem No.	n	Objective function	Time (sec.)	upper bound	%GAP	n	Objective function	Time (sec.)	upper bound	%GAP
1	10	498	8	505	0.0138	20	1084	18	1125	0.0364
2	10	477	8	505	0.0554	20	1107	21	1125	0.0160
3	10	477	7	505	0.0554	20	1107	20	1125	0.0160
4	10	498	8	505	0.0138	20	1107	20	1125	0.0160
5	10	498	8	505	0.0138	20	1107	23	1125	0.0160
ave.		489.6	7.8	505	0.0304		1102.4	20.4	1125	0.0200
1	30	1612	66	1625	0.0080	50	2628	195	2692	0.0237
2	30	1612	65	1625	0.0080	50	2628	194	2692	0.0237
3	30	1605	62	1625	0.0123	50	2453	205	2692	0.0887
4	30	1457	68	1625	0.1033	50	2481	212	2692	0.0783
5	30	1511	62	1625	0.0701	50	2639	215	2692	0.0196
ave.		1559.4	64.6	1625	0.0403		2565.8	204.2	2692	0.0468
1	70	3792	288	3994	0.0505	90	4607	641	4853	0.0506
2	70	3764	248	3994	0.0575	90	4669	625	4853	0.0379
3	70	3811	263	3994	0.0458	90	4573	649	4853	0.0576
4	70	3655	247	3994	0.0848	90	4493	619	4853	0.0741
5	70	3792	258	3994	0.0505	90	4463	654	4853	0.0803
ave.		3762.8	260.8	3994	0.0578		4561	637.6	4853	0.0601

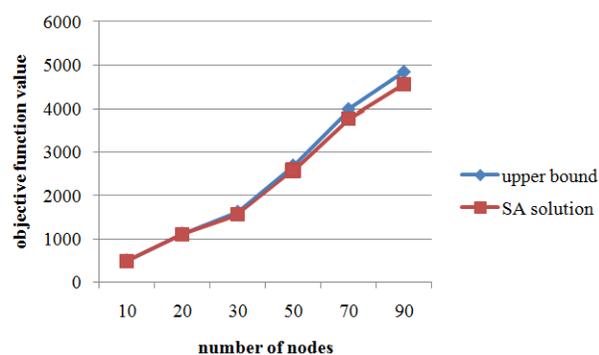


Fig. 5. SA versus upper bound values

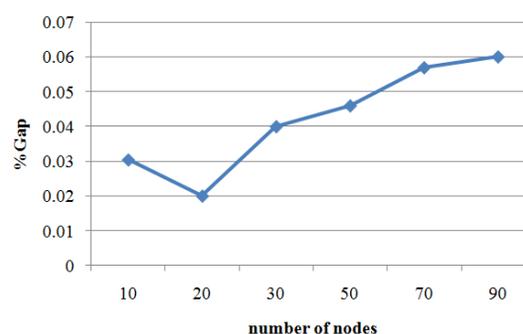


Fig. 6. %Gap for different problem sizes

5. Conclusion and future research

In this paper, we developed the maximal covering location problem which combines the characteristics of the gradual cover, cooperative cover, and variable radius models. This is one of the most important problems in location analysis because of its widespread real-world applications. The problem investigated in this paper was formulated as a MINLP. Since the complexity of the presented problem is NP-Hard, a meta-heuristic method based on simulated annealing algorithm was used to solve the proposed problem. The results show that the proposed method reaches near optimal solutions in reasonable amount of time. For further research, one can use other meta-heuristic methods to solve the problem and compare the results with the results of this paper.

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