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The consumer web-rooming on different sales models and retailing channel expansion

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A B S T R A C T

In the context of multichannel retailing, the phenomenon of web-rooming, where consumers research online and purchase offline, has become widespread. Therefore, we consider supply chain consisting of a manufacturer, an e-commerce platform, and an offline retailer, we study the impact of consumer web-rooming effect on the three sales modes of online channels, which are directly operated by the manufacturer, resold by the e-commerce platform, or co-exist with direct operation and resale, and comparatively analyze the pricing, demand, and profit of each channel under the three modes. The conditions for optimal pricing decisions are further explored through numerical simulation. It is found that profit always increases whether the online channel is opened by the manufacturer or the e-commerce company. The coexistence of direct sales and resale does not always increase profit for offline retailers, which must be discussed in the context of the sales model before channel expansion. The existence of web-rooming not only affects the decision-making of manufacturers and e-commerce platforms, but also always harms the interests of e-commerce platforms, and as the intensity of web-rooming deepens, the revenue of e-commerce platforms becomes smaller. Offline retailers benefit from web-rooming and experience a slowdown in profit growth as the intensity of the phenomenon increases. For manufacturers, the impact of changes in the intensity of the web-rooming is analyzed in relation to platform commission rates and online retail prices.

1. Introduction

The increasing availability of multiple shopping channels has expanded the choices for consumers. Consumers can choose to shop entirely online, shop entirely offline, combine offline browsing with online purchases, and combine online browsing with offline purchases. Consumer shopping habits can be divided into two categories, including show-rooming and web-rooming. Show-rooming involves browsing in physical stores but making purchases online, while web-rooming involves browsing online but making purchases in physical stores (Saeed et al., 2022). E-commerce platforms (e.g., Amazon, Jingdong, and Tmall) utilize consumers' online browsing history and purchase records to create user profiles and analyze their shopping preferences by using data analysis technologies. They employ big data marketing strategies to provide consumers with real-time product recommendations that align with their consumption patterns and characteristics. This allows consumers to access digital information about products anytime and anywhere(Chen et al., 2020; Liu et al., 2020). The consumer will be able to access digital information about the product at any time and from any location. Reselling is a business practice in which an e-commerce platform acquires products from manufacturers at wholesale rates and subsequently sells them directly to consumers through its own distribution channels at retail prices. Additionally, manufacturers can leverage e-commerce platforms to establish direct-to-consumer stores, with the e-commerce company earning a commission as a means of generating revenue(Chen et al., 2018). The e-commerce company receives a commission to generate profit. In terms of the overall market, expanding network channels increases the distribution intensity of a firm and improves customer accessibility to the firm's products. This, in turn, leads to an increase in the firm's market share (Fu & Guo, 2021). Considering these circumstances, it is imperative to examine the potential implications of manufacturers or e-commerce platforms expanding

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their online sales channels. Specifically, it is important to understand how the growth of the market and the power dynamics within the channel will influence decision-making processes for the relevant parties and the overall supply chain system. Addressing these inquiries is of utmost importance for informed decision-making within the supply chain community, as it can aid members who are operating in the online market in mitigating any adverse effects stemming from the web-rooming.
This paper aims to develop three supply chain models that incorporate manufacturers, retailers, and multiple sales channels. These models consider the presence of platform commissions and web-rooming effects. The primary objective is to examine and elucidate two specific concerns:

(1) The present study investigates the effects of manufacturers expanding their online direct channels or e-commerce companies expanding their retail channels on the decision-making, profitability, and performance of various supply chain members. The research examines and analyzes the equilibrium decisions made by supply chain members in three distinct scenarios, including the manufacturer's online direct channel, the e-commerce platform's resale retail channel, and the coexistence of direct resale and offline retail channels. Through a comparison of supply chain members' decisions and an analysis of changes in profit performance across these three scenarios, the study aims to explore the conditions, advantages, and disadvantages associated with the expansion of online channels by manufacturers and e-commerce platforms.
(2) In the context where direct selling and online reselling coexist as distribution channels, the study undertook a comprehensive examination of the effects of commission coefficients, market scale expansion, and varying channel power structures on the profitability of supply chain participants. Furthermore, the objective is to elucidate the underlying mechanisms that account for the differential influences of these two factors on the performance of distinct channels.
Compared to existing studies, this paper offers following improvements and innovations:
(1) Some scholars solely concentrate on the “show-rooming” or “web-rooming” and attempt to mitigate the adverse impact of the "web-rooming effect" on the willingness and effort of the subject of free-riding by utilizing contractual or coordinating mechanisms that leverage the manufacturer's dominance. This paper examines the detrimental effects of the "web-rooming effect" on the promotional willingness and effort level of the free-riding subject. Additionally, this paper explores the negative impact of the "web-rooming" effect following channel expansion. This paper focuses on the scenario of online coexistence after channel expansion. It aims to analyze the possibility of expanding online channels, the scope of application, and the advantages and disadvantages from the perspective of the entire supply chain. This analysis will provide a theoretical foundation for e-commerce platforms to seek ways of improving their performance.
(2) Scholars have primarily concentrated on comparing two sales modes in dual-channels, and they neglected the examination of the coexistence of two sales modes in multi-channels. This research paper aims to investigate the hitchhiking behaviors of consumers when utilizing online channels provided by manufacturers or e-commerce platforms. Additionally, it seeks to elucidate the underlying mechanisms that contribute to the diverse influences of commission coefficients, market scale expansion, and distinct channel power structures on the performance of various channels. By clarifying these mechanisms, the paper offers theoretical recommendations to enhance informed decision-making for participants within multichannel supply chains.

2. Literature Review

2.1 Show-rooming and web-rooming phenomenon

The show-rooming phenomenon and the web-rooming phenomenon can be seen as special forms of a combination of free-riding behavior and cross-channel purchasing behaviour(Fernández et al., 2018). Many scholars have focused on the negative impact of the showroom phenomenon on the revenue of supply chain members in offline channels and put forward some countermeasures. For instance, Zhang et al. (2021) considered the learning behavior of consumers in the product information search stage, and found that although the showroom phenomenon increases competition between online and offline merchants, the offline retailers can adopt differentiated product strategies to mitigate the competition and increase revenues. Li et al. (2021) found that manufacturers choose non-showroom strategies when the amount of information describing the product in the showroom is at a moderate level, while online retailers choose non-showroom strategies. Li et al. (2020) studied the impact of showroom placement on retailers, and they found that when the setup costs are relatively low and the proportion of local consumers is high, the more likely retailers with physical showrooms are to increase their profits. Ma and Hu (2022) considered the reference quality effect in combination with the dynamic operation of O2O supply chain, and found that physical shops are most affected by showrooms and they should maintain high pricing and service to enhance brand premium to maintain revenue. Li et al. (2023) focused on whether retailers should expand online channels and the “in-showroom effect” and “out-showroom effect” after channel expansion, and found that manufacturers' profits may decrease with the increase of the out-showroom effect.

Although the showroom phenomenon has gained widespread scholarly attention, most studies have recognized the Internet as the preferred source of information and the offline merchants as the primary channel of purchase (Yadav & Pavlou, 2014). For instance, the empirical analysis from Flavin et al. (2020) found that counter-showrooms are more likely to lead to consumers' personal attributions than showrooms, which implies that consumers perceive themselves as reliable and in control of the purchase outcome. However, existing research paid less attention to the impact of online channel revenues on the web-rooming phenomenon and the strategies that online supply chain members should adopt to deal with it. Van and Dach (2005) found that more than 20 per cent of consumers are hitchhikers from online shops to traditional retail shops and from traditional retail shops to online shops, with offline retailers retaining far fewer customers in both the positive and negative directions. Meanwhile, He et al. (2022) distinguished the search-based and experiential products and adopted an advertising strategy for the web-rooming phenomenon. The study found that for search-based products, consumers would experience the web-rooming phenomenon when the cost of privacy is high, and targeted advertising is more likely to increase the revenue of experiential products. Ma et al. (2021) compared the situations with and without the web-rooming phenomenon by combining resale and proxy sales in a platform-based supply chain, and found that the profit of offline retailers increases and then decreases with the enhancement of the web-rooming effect. It can be found that few scholars have considered the impact of free-riding diversion from different channels within the same online platform and the influence of the web-rooming effect on platform commissions.

2.2 Dual-channel supply chain pricing and multichannel retailing

Zhang et al. (2020) identified the optimal sales pricing for direct and retail channels in forward supply chains and the optimal collection pricing for retail channels and third parties in backward supply chains for the general case of centralized and decentralized models. Other scholars have also considered supply chain members and consumer traits in their research. For instance, Wang et al. (2022) used the Stackelberg game approach to explore the pricing strategy of a dual-channel supply chain in which retailers are risk-averse and consumers have channel preferences, and the study shows that both partners and consumers can benefit from retailers' risk control in dual-channel supply chains. Moderating pricing strategies not only helps members to cope with shifts in channels, modes, etc., but also adapts to different supply chain environments.

The existing body of scholarly literature on multichannel retailing focuses on three key inquiries. Firstly, does the adoption of multichannel retailing result in increased sales for retailers? Secondly, what is the significance of expanding channels in this context? Lastly, should prices be maintained consistently across all channels? These questions have yet to be definitively answered. Some scholars have posited that the implementation of a multichannel strategy may have adverse effects on the profitability of supply chain participants. Cao et al. (2016) studied the expansion of offline channels by retailers who have multiple distribution channels for selling products to customers. It was found that while these new channels can help retailers develop new customer segments and increase demand for their products, they may also cannibalize existing channels and increase operating costs. And these will harm retailers' profits.

Other scholars have a different view. Multi-channel sales can increase consumer satisfaction and loyalty, which in turn can lead to better economics for retailers than single-channel sales, Kim and Chun(Kim & Chun, 2018) found that manufacturers are more likely to use online or omnichannel channels as the level of competition increases. If consumers do not prefer different online channels, manufacturers will be more likely to adopt an omnichannel strategy. Jeffers and Nault(Jeffers & Nault, 2011) found that a multichannel strategy may lead to higher retail prices and increased profits for the industry as a whole.

3.Models

3.1 Description of the model

This paper assumes the presence of a dominant player as a manufacturer, with an offline retailer and an e-commerce platform as followers in the market. The online direct sales channel refers to the manufacturer selling its products directly to consumers through the e-commerce platform. Additionally, the manufacturer also sells its products through offline retailers. These two channels compete with each other, and consumers can choose to purchase from e-commerce platforms or brick-and-mortar retailers. The free-riding consumer will learn about the product on the e-commerce platform and then make the purchase through the offline channel. Manufacturers are now faced with the choice between direct sales through e-commerce platforms or reselling through e-commerce platforms. They also must decide whether to develop their own channels if they choose to resell through e-commerce platforms or not. This paper examines the equilibrium decisions of three models in the presence of the web-rooming effect by using the parameters and symbols described below:

Table 1
Description of parameters and symbols

Parameters	Description
s	Level of promotional service effort
λ	Commission factor for e-commerce platforms ($0<\lambda<1$)
β	Dual-channel cross-price elasticity coefficient ($0<\beta<1$)
γ	Multi-channel cross-price elasticity coefficient ($0<\gamma<1$)
c	Manufacturer's unit cost of production
τ	Proportion of free-riding consumers in new market entries ($0<\tau<1$)
θ	Percentage of market size diverted by online channels ($0<\theta<1$)
δ	Channel Expansion Diversion Ratio of Online Channels after Online Market Scale Expansion ($\theta<\delta<1$)

η	Cost-of-effort factor for services ($\eta>0$)
α	Underlying market size
k	Percentage diversion of online base market size by manufacturer's direct channel
m	Proportion of new online consumers diverted by manufacturers' direct channels
w	Product wholesale price
P_r	Offline retail channel prices
P_{ei}	$I = m, d$ Price of online direct channel/price of online retail channel
D_r	Demand for offline retail channels
D_{ei}	$I = m, d$, Online Direct Channel Demand/Online Retail Channel Demand
π_j	$j = R,E,M,$ Offline retailer profit / e-commerce platform profit / manufacturer profit

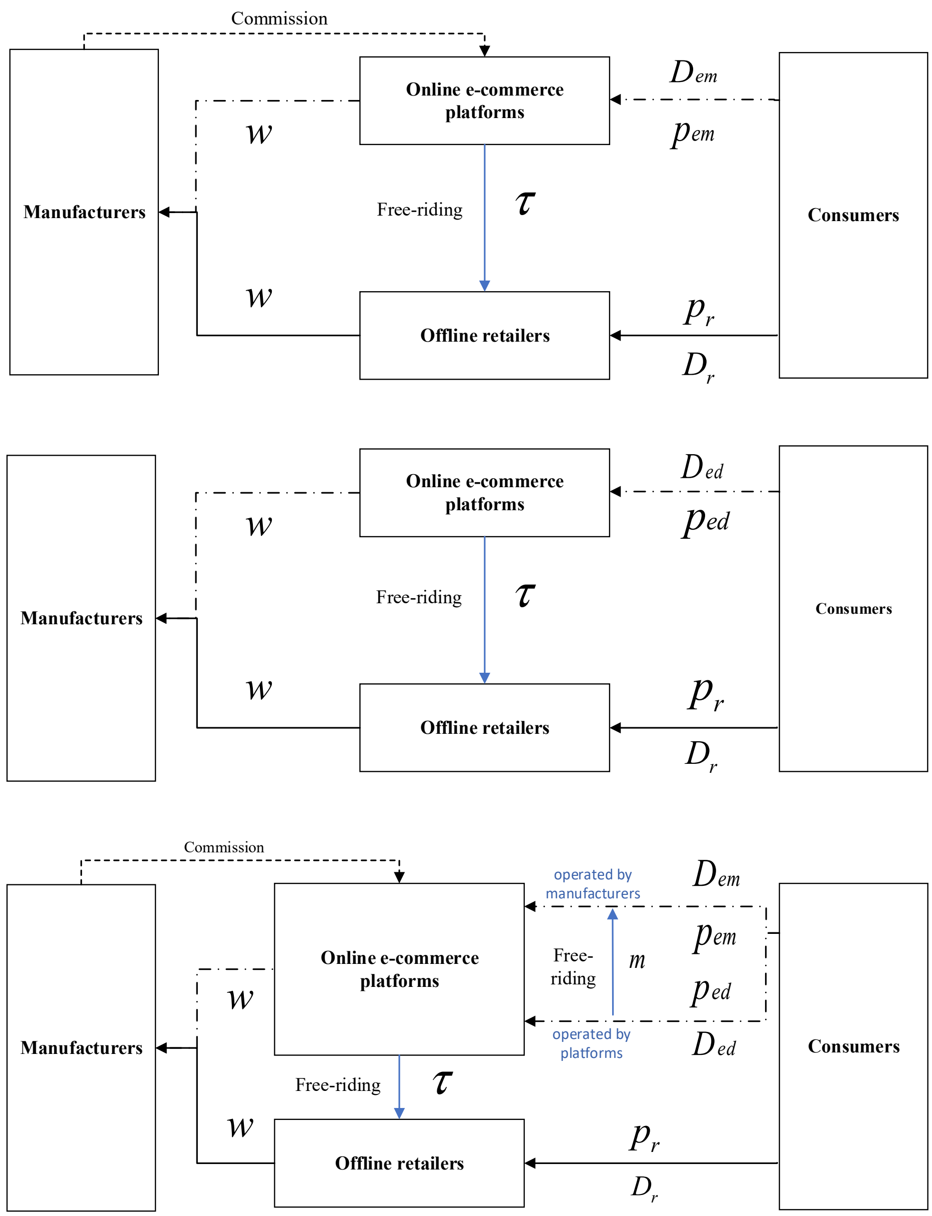

Fig. 1. Structure of the three secondary supply chain systems

3.2 Model Solution and Discussion

In this paper, it is assumed that a decision maker in the online channel exerts a service effort level of S. The market attracts an increased group of consumers S, and some of the consumers in this group have free-riding behavior. Therefore, the number of consumers lost to the decision maker after the service effort is τS and τ is the proportion of free-riding consumers. The cost of the decision maker's service effort is $\frac{\eta s^2}{2}$, where η is the cost-of-service effort factor with $\eta > 0$. The inventory surplus is not considered.

3.2.1 Manufacturer Direct Model

In this model, the third-party e-commerce company plays the role of an online marketplace platform. The manufacturer sells its products to offline retailers at a wholesale price of w and also establishes a direct flagship shop through the third-party e-commerce platform to sell its products. Additionally, the manufacturer pays the third-party e-commerce platform a commission based on the transaction amount in proportion to the amount of λ. It is important to note that the commission does not include the extra cost of promotional services, which is to be borne separately by the manufacturer. The retailer's retail price of the product is P_r and the manufacturer's online direct price is P_{em}. However, the system has a free-riding effect due to the service effort s made by the manufacturer. This leads to the demand function for the offline retail channel and the manufacturer direct channel in the manufacturer direct model:

$$D_r = (1-\theta)\times\alpha - p_r + \beta\times p_{em} + \tau\times s \tag{1}$$

$$D_{em} = \theta\times\alpha - p_{em} + \beta\times p_r + (1-\tau)\times s \tag{2}$$

where α represents the fundamental market demand, the price elasticity coefficient of demand across channels is 1, and the cross-price elasticity coefficient is β ($0<\beta<1$). For the sake of calculation, the operation cost of the e-commerce platform is not considered. Therefore, the profit functions of retailers, e-commerce platforms and manufacturers are:

$$\pi_R = (p_r - w)\times D_r \tag{3}$$

$$\pi_E = \lambda\times p_{em}\times D_{em} \tag{4}$$

$$\pi_M = (w-c)\times D_r + (p_{em}-c)\times D_{em} - \lambda p_{em} D_{em} - \frac{\eta s^2}{2} \tag{5}$$

Equilibrium solutions exist both Hessian matrix negative definite conditions are: $0 < \beta < 2\sqrt{\frac{2-2\lambda}{8+(-8+\lambda)\lambda}}$.

Lemma 1: Obtaining equilibrium solutions for supply chain members by using backward induction:

$$w_1^* = \frac{c(-1+\beta)\left(4-4\lambda+\beta(4+(-2+\beta)\lambda)\right)+(-1+\lambda)\left(4\alpha(1+(-1+\beta)\theta)-2s\beta(-2+\lambda)+\alpha\beta(-\beta+(-2+\beta)\theta)\lambda+s\left(4-\beta(4+(-2+\beta)\lambda)\right)\tau\right)}{8(-1+\lambda)+\beta^2\left(8+(-8+\lambda)\lambda\right)}$$

$$p_{em1}^* = \frac{-c(-1+\beta)\left(-4+\beta(-4+\lambda)\right)+4s(-1+\lambda)+\alpha\left(4\theta(-1+\lambda)-\beta(-1+\theta)(-4+3\lambda)\right)+s\left(4-4\lambda+\beta(-4+3\lambda)\right)\tau}{8(-1+\lambda)+\beta^2\left(8+(-8+\lambda)\lambda\right)}$$

$$p_{r1}^* = \frac{\left(c(-1+\beta)\left(2(1+\beta)^2-(2+\beta)\lambda\right)+(-1+\lambda)\left(\alpha\left(6+2\beta^2(-1+\theta)-6\theta-\beta\theta(-4+\lambda)\right)+s\left(\beta(-4+\lambda)(-1+\tau)+6\tau-2\beta^2\tau\right)\right)\right)}{8(-1+\lambda)+\beta^2\left(8+(-8+\lambda)\lambda\right)}$$

To ensure that wholesale prices are lower than channel prices,

$$\theta > \frac{1-\beta-2\lambda}{2+\beta(-2+\lambda)-3\lambda}$$

$$c < \frac{1}{(-1+\beta)\left(-4+(-1+\beta)\beta\right)\lambda}\begin{pmatrix} 2s\left(2+\beta(-2+\lambda)\right)(-1+\lambda)+s\left(8-8\lambda+\beta\left(-8+\left(9+\beta(-1+\lambda)-2\lambda\right)\lambda\right)\right)\tau \\ +\alpha\left(4(-1+2\theta)(-1+\lambda)+\beta^2\lambda(-1+\theta+\lambda-\theta\lambda)+\beta\left(-4+3\lambda+\theta\left(8+\lambda(-9+2\lambda)\right)\right)\right) \end{pmatrix}$$

In a two-channel supply chain, the existence of an equilibrium solution presupposes θ > 0.5. This is because when the market share of the online channel is small, the profits generated by the online channel for the manufacturer are not sufficient to cover the incurred costs. Only when the scale of demand for the online channel reaches a certain level will the manufacturer adopt the dual-channel sales model.

Proposition 1: The retailer's product price p_r, the manufacturer's direct channel price p_{em} and the wholesale price w all increase with the manufacturer's level of service effort. The relationship between $\frac{\partial p_{r1}^*}{\partial s} > \frac{\partial w_1^*}{\partial s}$ and $\frac{\partial p_{r1}^*}{\partial s} > \frac{\partial p_{em1}^*}{\partial s}$ implies that the level of promotional effort increases retailer product prices more than wholesale and direct channel prices. However, the relationship between the level of promotional effort and the level of wholesale and direct channel prices varies depending on the thresholds formed by the proportion of free-riding consumers, the cross-price elasticity coefficient β, and the proportion of e-commerce platform commissions λ.

Proposition 1 suggests that when a manufacturer takes on a service effort independently, it typically offsets the higher cost of improving its service by increasing prices in the direct channel. Then, retailers leverage this to raise retail prices and generate more revenue. Simultaneously, the manufacturer raises its wholesale prices to boost profitability through taking advantage of the growing number of consumers in the system.

Proposition 2: $\frac{\partial p_{r1}^{*}}{\partial \tau} > \frac{\partial w_{1}^{*}}{\partial \tau} > 0 > \frac{\partial p_{em1}^{*}}{\partial \tau}$. Both the retailer's product price and the manufacturer's wholesale price increase as the proportion of new consumers entering the market who are free-riding increases, with the retailer's product price increasing more than the manufacturer's wholesale price. In the online free-riding channel, the price of the manufacturer's online direct channel decreases as the proportion of free-riders increases. For manufacturers, the extent of change in the selling price p_{em} through direct channels and wholesale price w is influenced by the cross-price elasticity coefficient β and the e-commerce platform's commission rate λ. The increase in wholesale prices is smaller than the decrease in direct channel selling prices when $\frac{-1+2\lambda}{-1+\lambda} < \beta < 2\sqrt{2}\sqrt{-\frac{-1+\lambda}{8-8\lambda+\lambda^2}}$ is used, and the opposite is true when $0 < \beta < \frac{-1+2\lambda}{-1+\lambda}$ is used.
Proposition 2 states that when a manufacturer incurs service effort costs, the manufacturer will choose to reduce the selling price in the direct channel to attract more consumers. On the other hand, the wholesaler will choose to increase the wholesale price in order to maximize profit through taking advantage of the increased number of consumers in the offline retail channel. The extent to which the manufacturer adjusts the selling price p_{em} in the direct channel and the wholesale price w will depend on the relationship between the percentage of commissions charged by e-commerce platforms and the percentage of consumers who switch to other channels due to retail price increases.

3.2.2 E-commerce retail model

Under this model, the third-party e-commerce company sells the products through its own online platform channel, and the manufacturer is not involved in retail sales. After the manufacturer determines the wholesale price for selling products to third-party e-commerce platforms and offline retailers, the e-commerce and offline retailers then determine the retail price for their respective channels based on the wholesale price. The e-commerce platform alone is responsible for providing the service effort. It is assumed that the manufacturer gives the same wholesale price to both the third-party e-commerce platform and the retailer. This leads to the development of the demand function for the offline retail channel and the e-commerce online retail channel in the model:

$$D_r = (1-\theta)\times\alpha - p_r + \beta^* + p_{ed} + \tau\times s \tag{6}$$

$$D_{ed} = \theta\times\alpha - p_{ed} + \beta^* + p_r + (1-\tau)s \tag{7}$$

The profit functions of the retailer, the e-commerce platform and the manufacturer are:

$$\pi_R = (p_r - w)\times D_r \tag{8}$$

$$\pi_E = (p_{ed} - w)\times D_{ed} - \frac{\eta s^2}{2} \tag{9}$$

$$\pi_M = (w-c)\times(D_r + D_{ed}) \tag{10}$$

Lemma 2: Obtaining equilibrium solutions for supply chain members by using backward induction:

$$w_2^{\ *} = \frac{4(s+\alpha)+(-1+\beta)\left(c\left(-8+(-1+\beta)\beta(4+\beta)\right)-\beta\left(\alpha(2+\beta-\beta\theta)+s(2+\beta\tau)\right)\right)}{2(-1+\beta)\left(-8+(-1+\beta)\beta(4+\beta)\right)}$$

$$p_{ed2}^{\ *} = \frac{1}{4(-1+\beta)\left(-2+\beta^2\right)\left(-8+(-1+\beta)\beta(4+\beta)\right)}\begin{pmatrix} -8\alpha+c(-2+\beta)(-1+\beta)(1+\beta)\left(-8+(-1+\beta)\beta(4+\beta)\right) \\ +\alpha(-1+\beta)\left(32\theta+\beta\left(24+\beta\left(12-22\theta+\beta\left(-7+3\beta(-1+\theta)+\theta\right)\right)\right)\right) \\ +s\left(8(-5+4\tau)+\beta\left(8-32\tau+\beta\left(34-22\tau+\beta\left(-4+23\tau+\beta\left(-6+(2-3\beta)\tau\right)\right)\right)\right)\right) \end{pmatrix}$$

$$p_{r2}^{\ *} = \frac{1}{8(-1+\beta)\left(-2+\beta^2\right)\left(-8+(-1+\beta)\beta(4+\beta)\right)}\begin{pmatrix} c(-1+\beta)\left(-8+(-1+\beta)\beta(4+\beta)\right)\left(-4+\beta\left(-2+\beta+\beta^2\right)\right) \\ -2s\left(8+(-1+\beta)\beta\left(-24+\beta\left(-12+\beta(7+3\beta)\right)\right)\right)+s(-1+\beta)\left(64+\beta^2\left(-52+\beta\left(-2+\beta(9+\beta)\right)\right)\right)\tau \\ +\alpha\left(-80+64\theta+\beta\left(16-64\theta+\beta\left(76-52\theta+\beta\left(-12+50\theta+\beta\left(-19+11\theta+\beta\left(2+\beta-(8+\beta)\theta\right)\right)\right)\right)\right)\right) \end{pmatrix}$$

Proposition 3: Wholesale prices w, offline retailer product prices P_r, and e-commerce online retail channel prices P_{ed} increase with the level of service effort by the manufacturer where $\frac{\partial w_2^*}{\partial s} > 0, \frac{\partial p_{ed2}^*}{\partial s} > 0, \frac{\partial p_{r2}^*}{\partial s} > 0$. Unlike the manufacturer-direct model, the relationship between the magnitude of the increase in the three prices with the level of promotional service varies with the coefficient of cross-price elasticity β and the proportion of consumers who are free-riders τ. Nevertheless, the increase in the price of the offline retailer's product with the level of service $\frac{\partial p_{r2}^{*}}{\partial s}$ always increases with τ, and the increase in the price of the e-commerce online retail channel $\frac{\partial p_{ed2}^{*}}{\partial s}$ decreases with τ.

Proposition 3 suggests that when an e-commerce firm increases its service effort, it typically offsets the higher cost by raising prices in the online retail channel. Similarly, retailers raise their retail prices to generate more revenue, and manufacturers increase their wholesale prices to boost profits. This finding aligns with the conclusion of Proposition 1. The relationship between the magnitude of the three boosts varies depending on the cross-price elasticity coefficient β and the proportion of consumers who free-ride τ .

Proposition 4: $\frac{\partial p_{r2}^{*}}{\partial s} > \frac{\partial w_{2}^{*}}{\partial s} > 0 > \frac{\partial p_{ed2}^{*}}{\partial s}$. It can be seen that both the retailer's product price and the manufacturer's wholesale price increase as the proportion of new consumers entering the market who are free riders increases, with the retailer's product price increasing more than the manufacturer's wholesale price. Among the free-riding online channels, e-commerce online retail channel prices decrease as the proportion of free-riding consumers increases and there is a $\left|\frac{\partial p_{ed2}^{*}}{\partial s}\right| > \left|\frac{\partial p_{r2}^{*}}{\partial s}\right|$ and the difference in magnitude increases as the level of service effort s and the cross-price elasticity coefficient β increase.

Proposition 4 shows that when the e-commerce platform bears the cost of the service effort alone, the e-commerce platform will choose to reduce the selling price of the online channel in an attempt to reduce the flow of consumers to the offline channel in order to maintain revenues. As the service cost leads to an increase in the number of consumers in the system, the wholesaler will choose to increase the wholesale price in an attempt to make more profit.

3.2.3 Multi-Channel Coexistence Models

In this model, the manufacturer opens a flagship shop directly on the e-commerce platform and sells products through offline retailers and third-party e-commerce platforms. The manufacturer determines the wholesale price w_3 and selling price p_{em3} for the direct channel, and then sells the products to both the e-commerce platform and retailers. The third-party e-commerce platforms and offline retailers determine the retail price p_{ed3} for their respective channels based on the wholesale price. The manufacturer pays a commission to the third-party e-commerce platform on λ percent of the transactions made through the direct channel. The e-commerce platform undertakes its own promotional service efforts and incurs the associated costs. The demand functions for the retailer's offline channel, the manufacturer's direct channel, and the e-commerce retail channel in a multi-channel coexistence scenario are as follows:

$$D_r = (1-\theta)\times\alpha - p_r + \gamma(p_{em} + p_{ed}) + \tau\times s \tag{11}$$

$$D_{em} = k\times\delta\times\alpha - p_{em} + \gamma(p_r + p_{ed}) + m\times(1-\tau)\times s \tag{12}$$

$$D_{ed} = (1-k)\times\delta\times\alpha - p_{ed} + \gamma(p_r + p_{em}) + (1-m)\times(1-\tau)\times s \tag{13}$$

where m denotes the proportion of diversion of new consumers by manufacturer direct channels ($0<m<1$). k denotes the proportion of diversion of the manufacturer direct channel to online market size($0<k<1$).

The profit functions of the retailer, the e-commerce platform and the manufacturer are:

$$\pi_R = (p_r - w)\times D_r \tag{14}$$

$$\pi_E = (p_{ed} - w)\times D_{ed} + \lambda\times p_{em}\times D_{em} - \frac{\eta s^2}{2} \tag{15}$$

$$\pi_M = (w-c)\times(D_r - D_{ed}) + (p_{em} - c)\times D_{em} - \lambda\times p_{em}\times D_{em} \tag{16}$$

Lemma 3: Equilibrium solutions for each decision variable in a multichannel supply chain:

$$\begin{aligned} w_3^{*} = {} & \frac{1}{4A(1+\gamma)}(c\left(-1+\gamma+2\gamma^2\right)\left(-2F\left(-8+(-1+\gamma)\gamma(4+\gamma)\right)+\left(32+\gamma\left(8+\gamma(2+\gamma)\left(-18+\gamma(3+2\gamma)\right)\right)\right)\lambda-\gamma^2(2+\gamma)^2\lambda^2\right) \\ & -s(-1+\lambda)(16+2m\left((-2-\gamma)(-1+2\gamma)F^2+\gamma\left(-1+\gamma+\gamma^2\right)\left(-12+\gamma\left(-6+\gamma(5+2\gamma)\right)\right)\lambda\right)(-1+\tau)+\gamma(1+\gamma)\left(4+\gamma\left(-8+\gamma(-2+4\gamma+\lambda)\right)\right)\tau))) \\ & +\frac{\left[\begin{aligned}&(\alpha(-1+\lambda)(-2F\left(4(-1+(-1+k)\delta+\theta)+\gamma\left(-2(1+\delta+3k\delta-\theta)+\gamma\left(1+(2-6k)\delta-\theta+\gamma\left(1+\gamma+3k\delta-\gamma\delta+2k\gamma\delta-(1+\gamma)\theta\right)\right)\right)\right)\\&+\gamma(24k\delta+\gamma(-4(-5-5\delta+3k\delta+5\theta)+\gamma\left(22+22\delta-46k\delta-22\theta+\gamma\left(-3-2\delta-6k\delta+3\theta+\gamma\left(-9-8\delta+14k\delta+9\theta+2\gamma\left(-1+(-1+2k)\delta+\theta\right)\right)\right)\right)))\lambda))\end{aligned}\right]}{4F(1-2\gamma)(1+\gamma)\left(-8+(-1+\gamma)\gamma(4+\gamma)\right)\lambda+4(1+\gamma)\left(-16+\gamma\left(24+\gamma\left(30+\lambda^2+\gamma\left(-22+\gamma\left(-15+\gamma(5+2\gamma)\right)+\lambda^2\right)\right)\right)\right)} \end{aligned}$$

$$p_{em3}^{*} = \frac{1}{2B(-1+\lambda)}\left(\begin{aligned}&\left(w_3^{*}\right)\left(2\gamma\left(8+\gamma\left(4+\gamma(3+\gamma)(-1+\lambda)-2\lambda\right)-6\lambda\right)\right)-c(2+\gamma)(-1+2\gamma)\left(4+\gamma\left(2-\gamma+(2+\gamma)\lambda\right)\right)\\&-(-1+\lambda)\left(\begin{aligned}&\alpha\left(8k\delta+\gamma\left(-4\left(-1+(-1+k)\delta+\theta\right)+\gamma\left(2-\gamma+2\delta-6k\delta+(-2+\gamma)\theta\right)\right)\right)\\&+2ms\left(-4+\gamma(2+3\gamma)\right)(-1+\tau)+s\gamma\left(4+\gamma(2-\gamma\tau)\right)\end{aligned}\right)\end{aligned}\right)$$

$$\begin{aligned}
p_{ed3}{}^{*} = {} & \frac{1}{4BF(-1+\lambda)}((w_3{}^{*})\Big(4\big(B(-2+\gamma)(1+\gamma)(-1+\lambda)-\gamma^2(2+\gamma)\big(8+\gamma(4+\gamma(3+\gamma)(-1+\lambda)-2\lambda)-6\lambda\big)(1+\lambda)\big)^2\Big) \\
& +2B\big(2(-1+m)s+2(-1+k)\alpha\delta+\alpha\gamma(-1+\theta)\big)(-1+\lambda)+c\gamma(2+\gamma)^2(-1+2\gamma)(1+\lambda)\big(4+\gamma(2-\gamma+(2+\gamma)\lambda)\big) \\
& +(-1+\lambda)(\alpha\gamma(2+\gamma)\big(8k\delta+\gamma\big(\gamma(2+(2-6k)\delta+\gamma(-1+\theta)-2\theta)-4(-1+(-1+k)\delta+\theta)\big)\big)(1+\lambda) \\
& +2ms\big(16\tau+\gamma(2+\gamma)\big(-(-4+\gamma(2+3\gamma))(1+\lambda)+(-4(1+\lambda)+(-2+\gamma)\gamma(5+\lambda))\tau\big)\big) \\
& +s\big(-32\tau+\gamma\big(16\tau+\gamma(2+\gamma)\big(2(2+\gamma)(1+\lambda)+8(3+\lambda)\tau+\gamma(-16+\gamma-3\gamma\lambda)\tau\big)\big)\big)))
\end{aligned}$$

$$\begin{aligned}
p_{r3}{}^{*} = {} & \frac{1}{8BF(-1+\lambda)}((w_3{}^{*})\big(2B\big(-4+\gamma(-2+\gamma+\gamma^2)\big)(-1+\lambda)-2\gamma^2\big(8+\gamma(4+\gamma(3+\gamma)(-1+\lambda)-2\lambda)-6\lambda\big)\big(4+\gamma(2-\gamma+(2+\gamma)\lambda)\big)\big) \\
& +(-1+\lambda)(\alpha\gamma\big(8k\delta+\gamma\big(\gamma(2+(2-6k)\delta+\gamma(-1+\theta)-2\theta)-4(-1+(-1+k)\delta+\theta)\big)\big)\big(4+\gamma(2-\gamma+(2+\gamma)\lambda)\big) \\
& +2ms\gamma\big(16+\gamma(2+\gamma)\big(\gamma^2(-1+\lambda)(-3+\tau)-4\lambda(-1+\tau)-2\gamma(5+\lambda+\tau+\lambda\tau)\big)\big) \\
& +s\left(64\tau+\gamma\left(-32\tau+\gamma(2+\gamma)\left(\begin{array}{l}8+\gamma^3(-1+\lambda)\tau-8(7+2\lambda)\tau \\ +4\gamma(1+\lambda+9\tau)+2\gamma^2(-1+\lambda+2\tau+4\lambda\tau)\end{array}\right)\right)\right)) \\
& -2B\big(-2(-1+m)s\gamma+\alpha\big(4-2(-1+k)\gamma\delta+\gamma^2(-1+\theta)-4\theta\big)\big)(-1+\lambda)+c\gamma(2+\gamma)(-1+2\gamma)\big(4+\gamma(2-\gamma+(2+\gamma)\lambda)\big)^2)
\end{aligned}$$

where

$$A=-16(-1+\lambda)+\gamma\left(24(-1+\lambda)+\gamma\left(\begin{array}{l}-30+22\gamma-15\gamma^2(-1+\lambda)+5\gamma^3(-1+\lambda) \\ +2\gamma^4(-1+\lambda)-(-30+\lambda)\lambda-\gamma\lambda(22+\lambda)\end{array}\right)\right)$$

$$B=\big(-8+\gamma^2(2+\gamma)(6-\gamma+(2+\gamma)\lambda)\big)$$

$$F=\big(-2+\gamma^2\big)$$

Proposition 5: $\frac{\partial w_3^*}{\partial s}>0, \frac{\partial p_{em3}^*}{\partial s}>0, \frac{\partial w_{ed3}^*}{\partial s}>0, \frac{\partial w_{r3}^*}{\partial s}>0$. Both selling price and wholesale price across channels increase with the level of promotional services, which is similar to the conclusion of Propositions 1 and 3. And the relationship between the four decision variables changes with the values of τ, m, γ.

Proposition 5.1: $\frac{\partial w_3^*}{\partial s\partial\tau}>0, \frac{\partial p_{r3}^*}{\partial s\partial\tau}>0$. As the level of service effort on the e-commerce platform increases, the increase in wholesale prices and retailer prices in the offline channel continues to rise as the proportion of free-riding consumers increases. Interestingly, the trend of the two online channels with respect to the proportion of free-riding consumers as the level of service effort increases on the e-commerce platforms $\frac{\partial p_{ed3}^*}{\partial s\partial\tau}>0$ and $\frac{\partial p_{em3}^*}{\partial s\partial\tau}>0$ varies with the value of m for the proportion of online consumers choosing the manufacturer's direct channel. The calculation shows that $\frac{\partial p_{ed3}^*}{\partial s\partial\tau}>0$ and $\frac{\partial p_{em3}^*}{\partial s\partial\tau}>0$ do not hold at the same time. For example, as the level of service effort of e-commerce platforms and the proportion of free-riding consumers increase, the price increase in both channels will not increase at the same time. Nevertheless, it may slow down at the same time, irrespective of the proportion of online consumers choosing the directly-managed channel. The price increases in both online channels will not increase at the same time, but it may slow at the same time.

Proposition 5.2: $\frac{\partial w_3^*}{\partial s\partial m}<0, \frac{\partial p_{ed3}^*}{\partial s\partial m}<0, \frac{\partial p_{r3}^*}{\partial s\partial m}<0$. As the manufacturer's direct channel attracts more users from the e-commerce retail channel, the rise in the manufacturer's wholesale price, the e-commerce channel retail price. And the retailer's offline channel price slows down, while the change in the manufacturer's direct channel price $\frac{\partial p_{ed3}^*}{\partial s\partial m}$ accelerates with the rise in the coefficient of cross-price elasticity less than a certain threshold. Proposition 5.2 suggests that as the manufacturer's direct channel is increasingly diverted to the online channel, e-merchants and retailers will opt to decrease the rate of price growth in the higher retail channel. This is done to prevent more consumers from shifting to the direct channel. Additionally, the manufacturer will reduce the increase in wholesale prices at this stage to sustain demand and profitability in the non-direct channel. However, the rate of price growth in the manufacturer's direct channel will either increase or decrease depending on the magnitude of the cross-price elasticity coefficient. This means that if a significant number of customers switch to other channels due to a price increase, the rate of price growth in the direct channel will decline.

4.Numerical Simulation Comparison

Since the expressions for price, product demand, and profit of each decision maker in the multichannel supply chain system are complex, numerical simulation is employed to compare the effects of different sales modes of online channels and the coexistence of multiple channels on the decision variables, product sales of each channel, and the profit of each decision maker in each model. The values of the parameters are from Li et al.(Li et al, 2023). The initial parameter values are shown in Table 2:

Table 2
The initial values of the parameters

Parameters	α	θ	δ	c	β	η	k	λ	m	s	γ
Values	300	0.5	0.9	30	0.4	5	0.4	0.15	0.4	7	0.2

4.1 Comparative analysis of wholesale prices

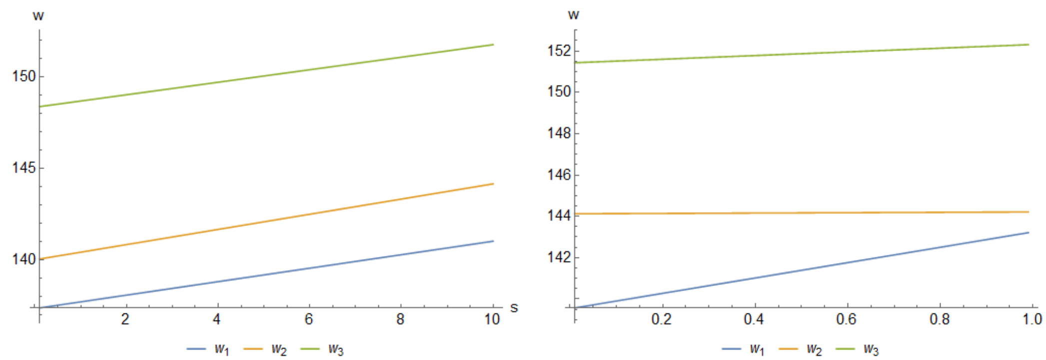

Fig. 2. Comparison of Manufacturers' Wholesale Prices under Growth in Promotional Service Levels (Left) and Hitchhiking Coefficients (Right)

The manufacturer's wholesale prices are shown for each of the three models. As the service level increases, it can be observed that w_1 is always smaller than w_2 and the gap does not narrow significantly. This may be since in the direct channel, the manufacturer can directly contact and transact with the end-consumers without relying on other distributors or retailers. In other words, it not only benefits from selling the products wholesale. The direct channel only needs to pay commissions to the e-commerce platform on sales and cover the costs associated with service efforts. This reduces other intermediary channel costs and allows for selling products at relatively low wholesale prices. Whereas, when there is only an e-commerce retail channel, the manufacturer does not have to pay platform commissions and service effort costs. However, as the manufacturer can only benefit from selling the product wholesale, the wholesale price is raised to maintain revenue. As the number of free-riding consumers increases, wholesale prices are higher when multiple channels coexist compared to when there is only a single online channel. Additionally, the increase in prices is slower when there is only a direct manufacturing channel. In the absence of a direct sales channel, the manufacturer only generates revenue from wholesale and is therefore minimally affected by free-riding consumers. In cases where the manufacturer has a direct channel, sales to offline retailers tend to grow faster due to the inverse showroom effect. As a result, wholesale revenues from retailers surpass the profits from sales in the direct channel. In order to generate more revenue, the manufacturer raises the wholesale price, despite the negative impact it has on the retail channel on the e-commerce platform.

4.2 Comparative analysis of channel selling prices

As depicted in in Figs. (3-5), when multiple channels coexist, the prices in both online channels are lower compared to that when there is only a single channel. Furthermore, the prices in the manufacturer's direct channel exhibit more significant changes before and after the expansion of e-commerce online retail channels. For retailers, the influence of e-commerce's expansion into online retail channels is smaller, while the influence of manufacturers' expansion into direct channels on offline retail prices is more significant.

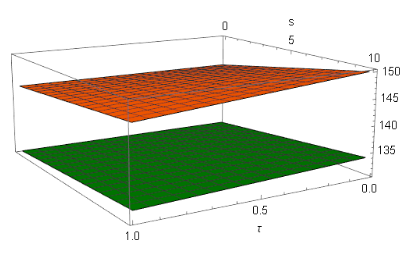

Fig. 3. Comparison of Prices in Manufacturers' Direct Channels

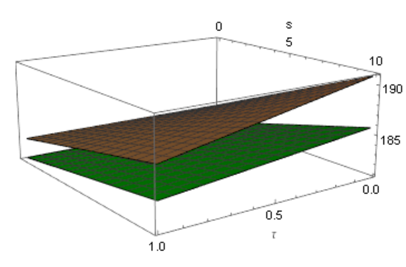

Fig. 4. Price comparison of e-commerce online retail channels

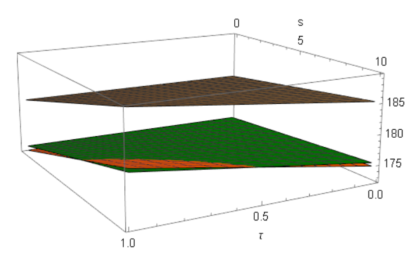

Fig. 5. Retailer offline retail channel price comparison

As the number of free-riding consumers increases, prices decrease in both online channels. However, prices in the manufacturer's direct channel are less affected. As the level of promotional services increases, prices increase in all three models, with the most significant increase occurring in the online retail channel. Offline retail prices in a multi-channel coexistence are significantly lower than e-commerce retail channels and only marginally higher than that of the manufacturer-direct model. In fact, they may even be lower than the manufacturer-direct model as the level of promotional service and the proportion of free-riding consumers increases.

4.3 Comparative analysis of channel demand

As shown in Figs. (5-8), when multiple channels coexist, the demand in the manufacturer's direct channel decreases more significantly and is less influenced by the level of promotional service and the proportion of free-riding consumers.
Demand in the e-tail channel rises slightly, but it is overtaken by the e-tail model when the proportion of free-riding consumers is low, and the level of promotional services is high. This channel is also significantly affected by the proportion of free-riding consumers and the level of promotional services.

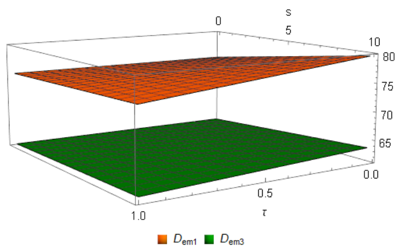

Fig. 6. Comparison of Manufacturer Direct Channel Demand

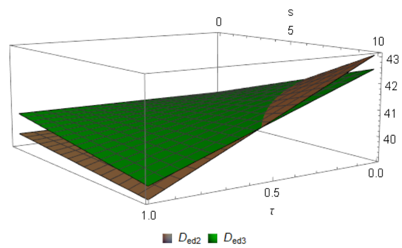

Fig. 7. Comparison of e-commerce online retail channel demand

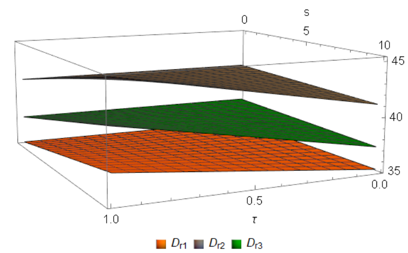

Fig. 8. Comparison of retailers' offline retail channel demand

For retailers, demand during the period of multichannel co-existence falls between the other two models. It is higher than that in the case of the manufacturer-direct model but lower than that in the case of e-commerce retailing. Demand is affected by the proportion of free-riding consumers to the same extent, but it is most significantly influenced by the level of promotional services.

4.4 Comparative analysis of the profits of each supply chain members

As shown in Figs. (9-11), when multiple channels coexist, the combination of selling price and demand from the previous two sections reveals that although the selling price and demand in the manufacturer's direct channel have declined significantly, the increase in market size allows the manufacturer to sell more products through the wholesale channel. Additionally, the manufacturer can increase the wholesale price to expand its profits without relying on the sales in the direct channel. The manufacturer's profit is the highest among all decision makers.

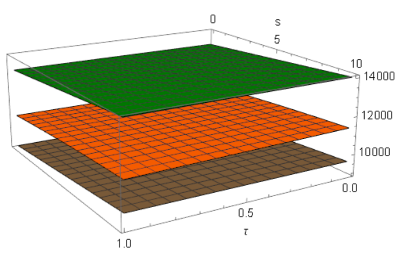

Fig. 9. Comparison of Manufacturers' Profits

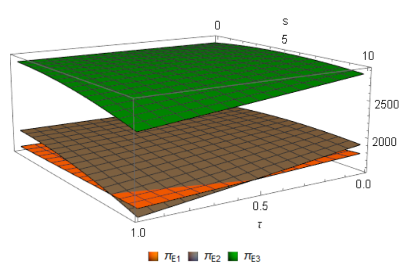

Fig. 10. Comparison of e-commerce profits

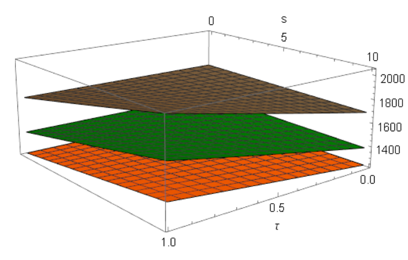

Fig. 11. Retailer Drainage Profit Comparison

For e-commerce platforms, profits are highest when multiple channels coexist, and there is a significant increase in profits when channels are expanded. Profits in the e-commerce retail model are so close to those in the manufacturer-direct model that the manufacturer-direct model has a higher return on the e-commerce platform's commission when the level of promotional service effort is high, and the proportion of free-riding consumers is high. This may be since the higher cost of service does not result in sufficient consumer growth because of the inverse showroom effect. The e-commerce platform, as the sole entity responsible for bearing the cost of promotional service efforts, experiences a decrease in profit as the level of promotional service effort increases. The rate of decline accelerates with the increase in service level, indicating diminishing marginal returns for promotional services.

For retailers, multi-channel coexistence is more profitable than the manufacturer-direct model, but it is less profitable than the e-commerce retail model. Combined with Figures 5 and 8, the influence of increased market size resulting from online channel expansion on retailers' offline channel demand and prices is uncertain. This influence needs to be discussed within the context of the online channel sales model before channel expansion.

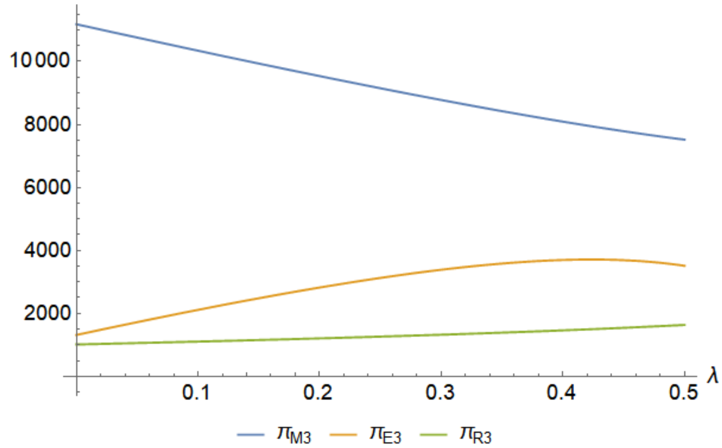

Fig. 12. Comparison of the impact of commission factors under multi-channel co-existence

As depicted in Fig. 12, manufacturers' profits consistently decrease as commission draws increase in the scenario of multichannel coexistence. However, the rate of decline slows down over time. On the one hand, the profits of e-commerce platforms initially increase with the index and eventually exhibit a more significant decline. The profits of offline retailers, on the other hand, will rise. This is because the increase in commissions will compel manufacturers to raise the prices of their products to maintain profits. As a result, consumers will turn away from these channels, which causes a decline in the number of commissions and ultimately leads to lower e-commerce profits.

4.5 Comparative analysis of commissions

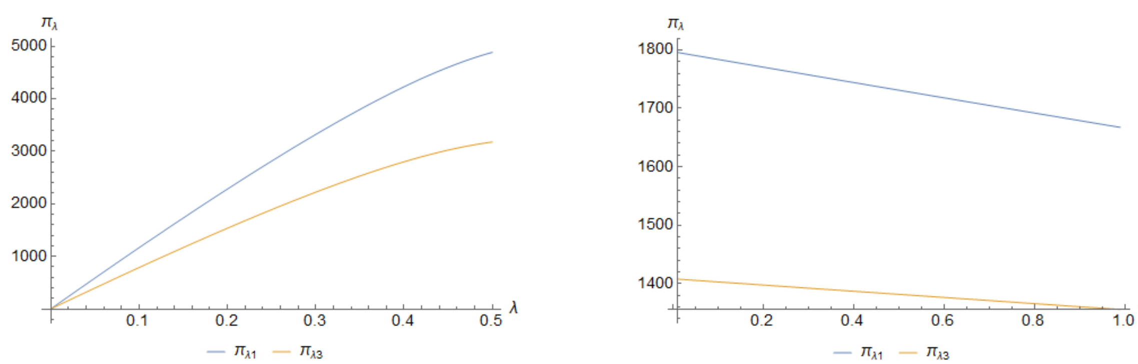

Fig. 13. Comparison of the number of commissions with different commission draw rates (left) and free-riding rates (right)

When there is a manufacturer's direct sales channel, the e-commerce platform earns commissions by taking a cut of the direct sales channel's sales. In the figure, $\pi_{\lambda1}$ represents the commission in the manufacturer-owned model, and $\pi_{\lambda3}$ represents the commission in the multi-channel co-existence model. The number of commissions under the multichannel coexistence model is generally smaller than the number of commissions in the manufacturer-direct model. This is because in the multichannel coexistence scenario, the e-commerce retail channel competes for consumers in the online channel, which reduces the number of consumers in the manufacturer-direct channel. As a result, the number of commissions is lower. As the commission draw percentage increases, it can be observed that the rate of increase decreases in both models. Commissions under the multichannel coexistence model rise more slowly than under the manufacturer direct model. The number of commissions under both models is gradually increasing, and the number of commissions decreases sharply when the draw ratio exceeds a certain threshold. However, this threshold has already surpassed the upper limit of the draw ratio commonly set by e-commerce platforms, so it will not be discussed. As the number of free-riding consumers increases, commissions decline in both models. This is because the inverse show-rooming effect drives consumers to offline retail channels, which leads to fewer consumers using online channels and ultimately results in lower sales. Commissions in the manufacturer-owned model $\pi_{\lambda1}$ are more significantly affected than that in the multichannel co-existence model $\pi_{\lambda3}$. This is since there is only the manufacturer-owned online channel in the manufacturer-owned model, whereas the online retail channel shares the loss of consumers due to the inverse showroom effect in the multichannel coexistence model.

4.6 impact of power structure and degree of market expansion on profits

This section assumes that the larger the share of the underlying market, the greater the power of a decision maker. As the degree of market expansion increases, both manufacturers and e-commerce platforms experience faster profit growth, with e-commerce platforms experiencing slower profit growth. Offline merchants will experience a marginal decrease in profits. This is due to the expansion of market scale and manufacturers took the opportunity to increase the wholesale price of products, which results in e-commerce and retailers of procurement costs increased significantly. However, as market size continues to increase, sales growth and retailers raising their own retail prices will slow the decline in profits or even increase them. Thus, the expansion of market size is not entirely favorable for retailers. The expansion of online channels can lead to a decline in sales of offline channels. This, combined with the fact that manufacturers may take the opportunity to raise wholesale prices, which leads to higher input costs for retailers and a decline in retailer profits to some extent and over time. When multiple channels coexist, it can be observed that as the size of the online market captured by the manufacturer's direct channel increases, the profits of both the manufacturer and the e-commerce platform tend to fall and then rise, with the manufacturer's profits reaching the low value earlier and rising above the initial profits. For e-commerce, the increase in market size leads to an increase in sales in the manufacturer's direct channel, which in turn leads to an increase in profits for e-commerce in the form of commissions. If market size increases less and e-commerce power is weaker (lower market share), the e-commerce profits may be lower than those of offline retailers.

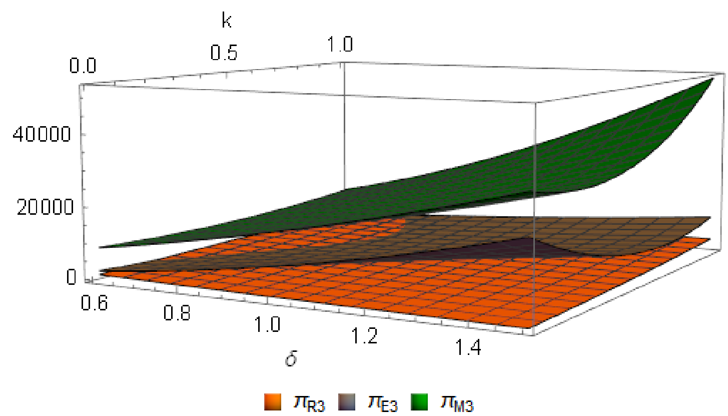

Fig. 14. Comparison of the impact of online channel power structure and market size expansion on profits when multiple channels co-exist

5. Discussion and Conclusion

With the continuous growth of the market size of online sales channels, the "web-rooming phenomenon" has become a common phenomenon. The web-rooming effect will have a negative impact on the willingness, motivation and performance of decision makers with online sales channels. Therefore, this paper takes the expansion of online channels by manufacturers and e-commerce platforms in the supply chain as a way to mitigate or eliminate the negative impact of the web-rooming effect, and explores the three models of opening direct-managed channels by manufacturers, opening online retail channels by e-commerce companies, and coexistence of multiple channels on the basis of a comprehensive consideration of the role of intra-platform diversion and the level of effort of promotional services. The three models of manufacturer-opened direct channels, e-commerce-opened online retail channels, and multichannel coexistence are examined in terms of their influence on channel selling price, product sales (demand), and profit for each subject in the supply chain. In addition, the impact of elements such as the degree of expansion of the online market size and the power structure of the online channel on the supply chain system is also tentatively explored. The management lessons learned are as follows:

First, expanding online channels to increase profits is a proven strategy for manufacturers or e-commerce companies given the increased market size that online channel expansion will bring. It is generally accepted that supply chain members who develop channels will compete with existing channels, which leads to a decrease in supply chain profits. In this paper, it was found that the profits of manufacturers and e-commerce companies increase significantly regardless of who develops the online channel and the degree of market expansion brought about by the development of the channel. Manufacturers and e-commerce companies can earn more profits from multiple channels than from dual channels due to lower consumer purchasing costs and marketing efforts.

Second, even after considering the web-rooming effect, the addition of online channels may not necessarily increase demand for the original channels of offline retailers, nor may it increase the profits of offline retailers. The expansion of online channels may lead to a decline in sales of products in retailers' offline channels. In addition, manufacturers may take the opportunity to raise wholesale prices in pursuit of high profits, and retailers may experience a downward trend in profits over time if they do not actively take countermeasures. This suggests that the increase in market size may not be entirely beneficial to retailers, and the increase in market size may reduce the benefits to retailers due to the “web-rooming effect”. For example, retailers should take steps to differentiate their promotional efforts across channels. This requires a specific discussion of the influence of service levels, free riders, and other elements in the context of the online channel sales model in the absence of channel expansion.

Thirdly, in the coexistence of dual and multichannel, as the proportion of 'free-riding' consumers increases, the profitability of the supply chain members on the free-riding side always decreases. As the level of promotional services increases, the profits of the supply chain member that does not have to pay the cost of promotional services increases.

Fourth, prices on all channels increase with the level of promotion by supply chain members. However, when the online retail channel is opened up by an e-commerce company in a manufacturer-direct model, the price increase for offline retailers is small. Channels interact and are interdependent. Supply chain members should make comprehensive judgements and careful decisions on pricing through taking into account factors such as channel share, market size expansion and power structure.
In summary, to diminish the negative impact of the web-rooming, decision-makers affected by free-riding can incorporate promotional service levels into their decision making, rather than just making price decisions. Second, they can choose differentiated wholesale prices or differentiated promotional service levels for different channels. Third, they can consider cost-sharing or benefit-sharing agreements among supply chain members to coordinate the distribution relationship among them. Finally, new technologies such as block-chain can be considered in conjunction with actual products to reduce constraints.

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Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author(s) used [DeepL and Wordvice AI] to [translate and proofread]. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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Appendices

Proof of Lemma1

To achieve the Stackelberg equilibrium, the best response of the follower in the second stage should be determined first. The leader's decision problem is solved based on the follower's response. We can get H to be a Hessian of π_M

$$H=\begin{pmatrix} -1 & \frac{\beta}{2}-\frac{1}{2}\beta(-1+\lambda) \\ \frac{\beta}{2}-\frac{1}{2}\beta(-1+\lambda) & -2\left(-1+\frac{\beta^2}{2}\right)(-1+\lambda) \end{pmatrix}$$

H is a negative definite when $0<\beta<2\sqrt{\frac{2-2\lambda}{8+(-8+\lambda)\lambda}}$ .Therefore, is concave in p_{em} and w_1 . Let $\frac{\partial \pi_M}{\partial p_{em}}=0$ and $\frac{\partial \pi_M}{\partial p_{w1}}=0$,then we get $w_1^*, p_{em1}^*, p_{r1}^*$.

Proof of Proposition 1

In order to obtain the range of first-order derivatives of the optimal solution with respect to a parameter, the research used the Reduce function in Mathematica to obtain the results. Combined consideration of parameter value ranges and threshold limits.

$$\frac{\partial w_1^*}{\partial s}=\frac{(-1+\lambda)\left(2\beta\lambda(-1+\tau)-4\tau-\beta^2(-2+\lambda)\tau\right)}{8-8\lambda+\beta^2\left(-4+\lambda(4+\lambda)\right)}>0 \qquad \frac{\partial p_{em1}{}^*}{\partial s}=\frac{\beta(2-3\lambda)\tau+4\lambda\tau-4(-1+\lambda+\tau)}{8-8\lambda+\beta^2\left(-4+\lambda(4+\lambda)\right)}>0$$

$$\frac{\partial p_{r1}{}^*}{\partial s}=-\frac{(-1+\lambda)\left(\beta(2+\lambda)-\left(-6+\beta(2+2\beta+\lambda)\right)\tau\right)}{8-8\lambda+\beta^2\left(-4+\lambda(4+\lambda)\right)}>0$$

$$\frac{\partial p_{r1}{}^*}{\partial s}-\frac{\partial w_1{}^*}{\partial s}=\frac{(-1+\lambda)\left(-\beta(-2+\lambda)(-1+\tau)-2\tau+\beta^2\lambda\tau\right)}{8-8\lambda+\beta^2\left(-4+4\lambda+\lambda^2\right)}>0$$

$$\frac{\partial p_{r1}{}^*}{\partial s}-\frac{\partial p_{em1}{}^*}{\partial s}=\frac{2\beta^2(-1+\lambda)\tau-2(-1+\lambda)(-2+5\tau)+\beta\left(2+\lambda^2(-1+\tau)-4\tau+\lambda(-1+4\tau)\right)}{8-8\lambda+\beta^2\left(-4+4\lambda+\lambda^2\right)}>0$$

$$\frac{\partial w_1^*}{\partial s}-\frac{\partial p_{em1}^*}{\partial s}=\frac{-\beta\lambda^2\left(2+(-2+\beta)\tau\right)-2\left(2+\left(-4+\beta+\beta^2\right)\tau\right)+\lambda\left(4-8\tau+3\beta^2\tau+\beta(2+\tau)\right)}{8-8\lambda+\beta^2\left(-4+4\lambda+\lambda^2\right)}$$

$$\begin{cases} >0, \dfrac{-2(-1+\lambda)(-2+\beta\lambda)}{8(-1+\lambda)+\beta^2\left(2-3\lambda+\lambda^2\right)-\beta\left(-2+\lambda+2\lambda^2\right)}<\tau<1 \\ \leq 0, 0<\tau\leq\dfrac{-2(-1+\lambda)(-2+\beta\lambda)}{8(-1+\lambda)+\beta^2\left(2-3\lambda+\lambda^2\right)-\beta\left(-2+\lambda+2\lambda^2\right)} \end{cases}$$

The rationale and process for calculating the results in Propositions 2, 3.2.2 and 3.2.3 are similar to those shown above.