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Marketplace channel encroachment under private brand introduction of online platform

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A B S T R A C T
This paper studies the marketplace channel introduction of contract manufacturers and the response of the platform with an option to introduce a private brand. We develop a game-theoretical model to examine a three-tier e-commerce supply chain including a contract manufacturer (CM), an original equipment manufacturer (OEM) and a platform and derive the equilibrium results. We find that the marketplace channel introduction of the CM and the platform's private brand introduction influence each other. More specifically, marketplace channel encroachment may discourage the platform from introducing a private brand, and this preference is reinforced as the referral fee increases. Interestingly, the introduction of the platform's private brand increases the likelihood of contract manufacturer encroachment, which is mediated by the difference between the two private brands of the CM and platformas the difference increases, the CM prefers to enter the marketplace channel. Furthermore, only contract manufacturer encroachment (or private brand introduction for the platform and the CM perform their strategies simultaneously. In the extension section, in addition to demonstrating the validity of our main results when the CM and the OEM act as a single entity, we also find that the first-mover advantage of the platform may reduce the possibility of the contract manufacturer encroachment.

1. Introduction

With the rapid development of the Internet and logistics, increasing numbers of online resellers (also called online platforms) who are built as pure resellers for selling national brands of original equipment manufacturers (OEMs), are embracing the marketplace mode (Abhishek, Jerath & Zhang, 2016), whereby they allow upstream firms such as contract manufacturers (CMs) direct access to customers through the platform websites for a commission rate (also called referral fee). In practice, faced with these opportunities provided by online platforms, numerous contract manufacturers could encroach to compete with the national brands of OEMs by establishing their private brands through the marketplace channel (Chen, Shum & Xiao 2012; Chen, Liang & Yao, 2019; Kaya, 2011; Cui, 2019; Shi, 2019; Niu, Wang & Guo, 2015). For example, on JD.com, Septwoleves brand's shirts manufactured by Esquel Group are sold in JD.com's self-owned stores (i.e., reselling mode). Meanwhile, Esquel Group has established the Determinant brand, one of its private brands, to compete directly with Septwoleves brand through the marketplace channel. In China, well-known e-retailers such as Taobao and Pinduoduo have launched several programs to support upstream contract manufacturers to enter the marketplace, which may accelerate the private brand introduction for these contract manufacturers (Chen, Liang & Yao, 2019). Besides providing marketplace channels, a growing number of platforms have begun to develop their own-label brands to squeeze the market. For example,

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the supermarket giant Walmart sells national brand products of Procter & Gamble as well as its private brand of Great Value (Zhang, Song & Zhu, 2021). Recent years have witnessed the rapid development of private brands of online platforms such as JD.com and Amazon. The sales of Kogan.com's private brands accounted for nearly half of its total gross profit in 2019 (Statista, 2020). The sales of JD.com's private brands were close to 3 billion yuan (\$464 million) in 2020, up 200 percent from the previous year (Journal, 2020).

The examples mentioned above involve the marketplace channel encroachment of CMs and the private brand introduction of online platforms, and thus inevitably leads to the market competition among CMs, OEMs and platforms regarding different products and channels. From the perspective of competition, the marketplace introduction of CMs may cause a response of the platform with an option to introduce a private brand. Although (Zhang, Song & Zhu, 2021; Li, Leng & Zhu, 2018) have considered the influence of the online direct channel encroachment on the private brand introduction, how the marketplace channel introduction affects the private brand introduction remains an open question. Note that in the marketplace channel, CMs can obtain the pricing power without investing in some infrastructure such as websites, and have direct contact with consumers but need to pay a referral fee to the platform. Therefore, our work focuses on the impact of the referral fee and the platforms, which has been ignored by the existing literature. Motivated by these considerations, we aim to investigate the following questions: Given the contract manufacturer's marketplace channel introduction strategy, should the platform introduce a private brand introduction? How do these strategies affect the profits of the OEM and the supply chain?

To conduct this study, we consider an e-commerce supply chain with an OEM that outsources its production services to a CM selling national brand products to a platform that then resells them to consumers. The contract manufacturer has the option to establish a private brand PB1 to encroach through the marketplace channel, and in response, the platform decides whether to introduce its private brand PB2. To study the interactions between the marketplace channel introduction of the contract manufacturer and the platform's private brand introduction, four scenarios are established. Furthermore, we also analyze the impact of the game sequence and integrated strategy on equilibrium results in the extension section. This study contributes to the extant literature as follows. First, the platform's private brand introduction and the contract manufacturer's marketplace encroachment could complicate the relationship among supply chain members. Therefore, in an e-commerce supply chain, it is important to consider the interplay between the marketplace channel introduction and the private brand introduction, which, to the best of our knowledge, has not been studied before. Second, our work not only complements the existing literature on channel encroachment and private brand introduction but also provides some new explanations for supply chain channel selection. Lastly, we derive several interesting findings by characterizing the equilibrium results. For example, when the difference between the two private brands of the contract manufacturer and the platform is small, the marketplace encroachment of the contract manufacturer may prevent the platform from introducing the private brand because the fierce competition between them reduces the agency fee and the profit from reselling the national brand, and therefore decreases the overall profit of the platform. We also find that the private brand introduction of the platform encourages the contract manufacturer to enter the marketplace channel, this is because the private brand introduction of the platform reduces the profit of the contract manufacturer, and in response, the contract manufacturer may introduce the marketplace channel to reduce its profit loss. Furthermore, the whole supply chain becomes better off when either of the two strategies (i.e., the contract manufacturer encroachment and the platform's private brand introduction) is implemented, but performing simultaneously the two strategies may hurt the supply chain. In addition, we find that the first-mover advantage of the platform may decrease the possibility of marketplace channel encroachment.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 shows the model framework. Section 4 derives equilibrium results. Section 5 states the analysis and comparison of equilibrium results. Section 6 extends the main model. The conclusion is presented in Section 7.

2. Literature review

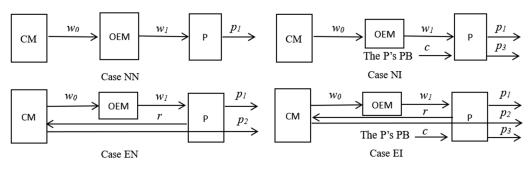
The first stream of literature related to our work focuses on supplier encroachment. By investing in infrastructure such as websites to establish the online direct channel, suppliers could have direct contact with consumers, which is called supplier encroachment and has been considered by many scholars (Chen & Yao, 2019; Huang, Guan & Chen, 2018; Ha, Long & Nasiry, 2016; Chen & Wang, 2015; Arya, Mittendorf & Sappington, 2007; Arya & Mittendorf, 2013). The majority of the literature regarding supplier encroachment mainly investigates the competition effect of encroachment (Liu & Zhang, 2006; Balasubramanian, 1998) and the coordination role of the supplier's direct channel (Tsay & Agrawal, 2004; Chiang, Chhajed & Hess, 2003; Cai, 2010; Wei, Wang & Lu 2021). Recently, online platforms taking a portion of revenues could also provide plenty of opportunities for upstream suppliers to encroach through the marketplace channel (Zhao & Hou, 2021; Zhang & Ma, 2022; Wei & Dong, 2022). For example, Reference (Zhang & Ma, 2022) explores whether the supplier and the platform should introduce the marketplace channel by incorporating the logistics service strategy, and find that the level of logistics service has a significant impact on the channel choice. Our work differs from the existing literature in two distinct ways. First, instead of assuming that upstream suppliers establish their own websites to operate direct channels and can obtain the whole

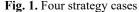
profit from direct channels, we assume that upstream suppliers sell products through the marketplace channel of the platforms and need to share a portion of revenues with the platform. Second, as far as we know, this paper is among the first to consider the interplay between the platform's private brand introduction strategy and the marketplace channel encroachment of upstream contract manufacturers.

Our research also relates closely to the vast literature on private brand introduction. Based on physical retailers, there is vast literature on whether offline retailers should introduce their private brands (Wang, Chen & Song, 2021; Scott Morton & Zettelmeyer, 2004; Ryan, Sun & Zhao, 2012; Ru, Shi & Zhang, 2015; Raju, Sethuraman & Dhar, 1995; Nenycz-Thiel, Sharp & Romaniuk, 2010; Jin , Wu & Hu, 2017; Hökelekli, Lamey & Verboven, 2017; Cui, Chiu & Li, 2016; Choi & Coughlan, 2006). For example, Reference (Ru & Zhang, 2015) studies the impact of private brand introduction on the chain members and find that the private brand introduction of a power retailer may benefit both the manufacturer and the retailer. With the development of online platforms, some scholars have begun to pay attention to the introduction of private brands for these platforms (Zhang & Hou, 2022; Huang & Liu, 2022). Different from offline stores, online platforms can provide a traditional reselling mode as well as a marketplace mode which allows the upstream firm to encroach by sharing a portion of revenues with the platforms (Ha, Tong & Wang 2022; Wang & Chaolu 2022; Liu, Yang & Liu 2022). Although the private brand introduction of offline retailers is influenced by many aspects: Brand loyalty (Gabrielsen & Sørgard, 2007), channel leadership (Chun & Lee, 2018), advertising (Karray & Martín-Herrán, 2019), quality differentiation (Li & Chen, 2022) and online channel introduction (Zhang & Zhu, 2021; Li & Zhu, 2018), the understanding of the impact of online platforms' private brands on the marketplace mode is limited. Our research complements the extant literature by considering the investigation of private brand introduction of online platforms and the response of upstream CMs with an option to encroach through the marketplace channel.

3. The model

This paper considers an e-commerce supply chain including a contract manufacturer (denoted by CM, he), an OEM, and an online platform (denoted by P, she). Besides providing the production services for the national brand (NB) of the OEM who sells its NB product through the platform using the conventional reselling mode, the contract manufacturer has the option to introduce a private brand named PB1 and sell them through the marketplace channel by paying a referral fee r to the platform. Meanwhile, in response, the platform can decide whether to build a private brand named PB2. Note that we follow (Ha, Tong & Wang 2022; Wang & Chaolu 2022) to assume that the referral fee r (0 < r < 1) is exogenous and available on the platform's website. To analyze the interaction among the contract manufacturer, OEM and platform, we examine four different cases based on whether the marketplace channel and the platform's private brand are introduced, which is illustrated in Fig. 1. First, the marketplace channel is not entered by the contract manufacturer, and the platform does not introduce a private (denoted by scenario NN). Second, the contract manufacturer establishes a private brand to enter the marketplace channel and a private brand is not introduced by the platform (denoted by scenario NI). Finally, the contract manufacturer enters the marketplace channel and a private brand is introduced by scenario EI).





We assume that the perceived value of NB is the highest among the three types of products (i.e., NB, PB1 and PB2) and is normalized to one (Li & Zhu, 2018). The parameter θ ($0 < \theta < 1$) represents the perceived value of PB1 through the marketplace channel, and thus captures the combined effect of the marketplace channel and the contract manufacturer's private brand. We use b (0 < b < 1) to denote the perceived value of PB2 sold by the platform. Therefore, NB product sold through the reselling channel has the highest value perceived by consumers, and the perceived value of PB1 sold by the contract manufacturer through the marketplace channel is lower or higher than that of PB2 sold by the platform. All consumers buy at most one unit of PB or NB products, and make their purchasing decisions by maximizing their utilities (Gabrielsen & Sørgard, 2007). We assume that v, the consumer's reservation price of buying the NB product, is uniformly distributed over the interval [0,1]. Thus, a consumer's utility is $v - p_1$ when buying one unit of NB, $\theta v - p_2$ when buying one unit of PB1 sold by the contract manufacturer through the marketplace channel and $bv - p_3$ when buying one unit of PB2 sold by the 406

In the cases of NN, EN and NI, the demand functions can be straightforwardly derived depending on the consumer's utilities. In the case wherein the contract manufacturer enters the marketplace channel and the platform introduces PB2, three types of products give one consumer four options regarding making a decision of purchasing one product: a NB, or PB1, or PB2, or nothing. To obtain the demand in these four segments, we consider the two situations wherein $0 < b < \theta < 1$ and $0 < \theta < b < 1$, which implies that the perceived value of PB1 through the marketplace channel is higher or lower than that of PB2. In the case where $0 < b < \theta < 1$, the indifference point between purchasing NB and PB1 is $\frac{p_1 - p_2}{1 - \theta}$, the point between buying PB1 and PB2 is $\frac{p_2 - p_3}{\theta - b}$ and the point between buying PB2 and nothing is $\frac{p_3}{b}$. The demand functions of NB, PB1 and PB2 are $d_1 = 1 - \frac{p_1 - p_2}{1 - \theta}, d_2 = \frac{p_1 - p_2}{1 - \theta} - \frac{p_2 - p_3}{\theta - b}$ and $d_3 = \frac{p_2 - p_3}{\theta - b} - \frac{p_3}{b}$, respectively. Following the same logic as that in the case where $0 < b < \theta < 1$, we can obtain the demand functions in the case where $0 < \theta < b < 1$ as follows: $d_1 = 1 - \frac{p_1 - p_3}{1 - b}, d_2 = \frac{p_1 - p_2}{1 - b} - \frac{p_2 - p_3}{b - b}$.

Furthermore, the game among the contract manufacturer, OEM and the platform includes two stages. The first is the strategy decision. In this stage, the contract manufacturer first determines whether to enter the marketplace channel by establishing a private brand, and then the platform decides whether to introduce a private brand PB2. In section 6, we consider another case wherein the platform first set her strategy decision and then the contract manufacturer determines his encroachment strategy. The second stage is the pricing decision stage. In this stage, following (Wu, Zhang & Zhou 2022; Liu, Yang & Liu 2022), the CM first sets the wholesale price w_0 and/or p_2 , and then the OEM decides w_1 for the NB, and finally the platform decides the retail prices for NB and/or PB3 (we also examine an alternative scenario where the CM first sets w_0 , and then the OEM decides w_1 , and finally the contract manufacturer and the platform simultaneously determine the retail prices for the three types of products, and find that our results remain qualitatively unchanged). Without loss of generality, we follow (Choi & Fredj 2013) to normalize the selling costs of NB and PB products to zero. Normalizing the contract manufacturer's production cost for the platform's PB2 is c ($c \ge 0$) (Liu, Yang & Liu 2022; Balasubramanian & Maruthasalam 2021).

Note that $c < min\left\{\frac{3(1-b)b}{4-3b}, \frac{2b(1-r)(b-\theta)}{3b+br-4\theta}\right\}$ if $b < \theta$; $c < min\left\{\frac{3(1-b)b}{4-3b}, \frac{(b-1)(b-\theta)(6b-\theta-5r\theta)}{6b^2-8b+6\theta-3b\theta+2r\theta-br\theta-\theta^2-r\theta^2}\right\}$ if $b < \theta$, which can ensure that the demand for all products is positive. To better explore the strategic interactions between the CM and the platform, we further assume that the production cost of PB2 is zero in the benchmark model. Moreover, we also consider the case where the cost of PB2 is greater than zero and find that our main conclusions are still valid when *c* is small (we exclude the trivial case wherein *c* is large, which can ensure that the demand for PB2 is positive).

4. Equilibrium solutions of pricing subgames

4.1 Case NN

Under scenario NN, the contract manufacturer wholesales the NB product at a wholesale price w_0 to the OEM who then resells them to the platform at a price w_1 , and finally, the product is sold at p_1 by the platform to consumers. According to Section 3, the demand function is $d_1 = 1 - p_1$, and the profit functions of the firms are as follows:

$\pi_{CM}^{NN} = d_1 w_0$	(1)
$\pi_{OEM}^{NN} = d_1(w_1 - w_0)$	(2)
$\pi_P^{NN} = d_1(p_1 - w_1)$	(3)

By using backward induction, all the equilibrium outcomes are summarized in Tab. 1.

Optimal butcomes under unterent scenarios						
Equilibriu m	N N	EN	NI	EI if $b < \theta$	EI if $b < \theta$	
p_1^*	7	$7-3\theta$	7 - 3b	$7b + 7br - 14\theta + b\theta - 7br\theta + 6\theta^2$	$\frac{14b-6b^2-7\theta-3b\theta-7r\theta+5br\theta+2\theta^2+2r\theta^2}{14b-6b^2-7\theta-3b\theta-7r\theta+5br\theta+2\theta^2+2r\theta^2}$	
Ρ1	8	8	8	$8(b+br-2\theta)$	$8(2b - \theta - r\theta)$	
p_2^*	/	$\frac{\theta}{2}$	/	$(b-\theta)\theta$	$(b-\theta)\theta$	
		2		$b + br - 2\theta$	$2(2b-\theta-r\theta)$	
p_3^*	/	/	<u>b</u>	$\frac{b(1+r)(b-\theta)}{2}$	$\frac{(b-\theta)(4b-\theta-r\theta)}{4}$	
P 3	,	,	2	$2(b+br-2\theta)$	$4(2b-\theta-r\theta)$	
*	1	$1 - r\theta$	1 - b	$b + br - 2\theta + b\theta - 3br\theta + 2r\theta^2$	1 - b	
w_0^*	2	2	2	$2(b+br-2\theta)$	2	
*	3	$3 - \theta - 2r\theta$	3(1-b)	$3b + 3br - 6\theta + b\theta - 7br\theta + 2\theta^2 + 4r\theta^2$	3(1-b)	
w_1^*	4	4	4	$4(b+br-2\theta)$	4	

Table 1

4.2 Case NI

Under scenario NI, the contract manufacturer sells the NB product through the OEM channel, and the platform introduces a private brand, which is shown in Fig. 1. According to Section 3, the demand functions are $d_1 = 1 - \frac{p_1 - p_3}{1 - b}$ and $d_3 = \frac{p_1 - p_3}{1 - b} - \frac{p_3}{1 - b}$, and the profit functions of the firms are as follows:

$$\pi_{CM}^{NI} = d_1 w_0 \tag{4}$$

4.3 Case EN

Under scenario EN, the contract manufacturer not only wholesales the NB product through the OEM channel, but also builds a private brand to enter the marketplace channel, which is shown in Fig. 1. According to Section 3, the demand functions are $d_1 = 1 - \frac{p_1 - p_2}{1 - \theta}$ and $d_2 = \frac{p_1 - p_2}{1 - \theta} - \frac{p_2}{\theta}$, and the profit functions of the firms are as follows:

$$\pi_{CM}^{EN} = d_1 w_0 + d_2 p_2 (1 - r) \tag{7}$$

$$\pi_{CM}^{EN} = d_1 (w_1 - w_0) \tag{8}$$

$$\pi_{P}^{EN} = d_1(p_1 - w_1) + d_2 p_2 r$$
(9)

4.4 Case EI

Under scenario EI, the contract manufacturer sells the NB product through the OEM channel, and the platform introduces a private brand, which is shown in Fig. 1. Referring to the demand functions in the case where $0 < 0 < b < \theta < 1$ and $0 < \theta < b < 1$ in Section 3, we can derive the profit functions of the firms as follows:

$$\pi_{CM}^{EI} = d_1 w_0 + d_2 p_2 (1 - r) \tag{10}$$

$$E_{OEM}^{EI} = d_1(w_1 - w_0) \tag{11}$$

$$\pi_P^{EI} = d_1(p_1 - w_1) + d_2 p_2 r + d_3 p_3 \tag{12}$$

5. Comparison and analysis of equilibrium outcomes

Depending on the equilibrium solutions under four different cases given in section 4, this section studies firms' equilibrium decisions regarding marketplace channel encroachment and private brand introduction. For mathematical tractability, we refer to $t = \frac{\theta}{b} > 0$ as the ratio of the PB1's perceived value to the value of PB2, and thus this ratio could denote the degree of differentiation between PB1 and PB2. When t approaches one, the difference is small; otherwise, the difference is large. Before we proceed, we obtain the relationship between the equilibrium prices under different cases, which is shown in the lemma below.

Lemma 1. (a)
$$p_1^{NN*} > p_1^{NI*}$$
, $p_1^{EN*} > p_1^{EI*}$, $p_2^{EN*} > p_2^{EI*}$, $p_3^{NI*} > p_3^{EI*}$; (b) $w_0^{NN*} > w_0^{NI*}$, $w_0^{EN*} > w_0^{EI*}$, $w_1^{NN*} > w_1^{NI*}$, $w_1^{EN*} > w_1^{EI*}$.

Lemma 1 shows the relationship among equilibrium prices. Regardless of whether the contract manufacturer encroaches or not, the wholesale and retail prices for the national brand are higher when the platform does not introduce her private brand than when the private brand is established by the platform due to product competition. In the same way, the private brand retail price of the contract manufacturer (platform) becomes lower after the platform (contract manufacturer) introduces the private brand. These relationships are not affected by the referral fee charged by the platform and the differentiation between the two private brands.

5.1 Platform's private brand introduction when CM does not encroach

Lemma 2. Suppose the contract manufacturer enters the marketplace channel to encroach. We have $\pi_p^{N*} > \pi_p^{N*}$ otherwise.

Lemma 2 shows that when the contract manufacturer does not build a marketplace channel to encroach, the e-commerce platform always prefers to build its private brand. This conclusion appears in numerous papers (Zhang, Song & Zhu, 2021; Li, Cai & Chen 2022) and can be explained by the competition and cannibalization effects of the private brand introduction, which benefits the platform but hurts the upstream enterprises.

5.2 Platform's private brand introduction when CM encroaches

Proposition 1. Suppose the contract manufacturer enters the marketplace channel to encroach. We have $\pi_P^{EI*} < \pi_P^{EN*}$ if $t_1(r) < t < 1 + r$; $\pi_P^{EI*} > \pi_P^{EN*}$ otherwise.

Proposition 1 illustrates the private brand introduction strategy of the platform when the contract manufacturer encroaches. It is intuitive that in most cases, the platform has the incentive to introduce her private brand because of the increased total sales due to competition. However, one counterintuitive result shows that when the contract manufacturer encroaches through the marketplace channel, the platform may not introduce her private brand if the differentiation between the platform's private brand and the contract manufacturer's private brand is small. This is because when the contract manufacturer set up a marketplace channel on the platform, which implies that the platform's revenue comes from the PB1's agency fee and the national brand's profit. In this case, if the platform introduces a private brand that has a similar perceived value of PB1, there will be intensified competition between the two brands, which increases total sales but reduces the national brand's profit and PB1's agency fees. Therefore, it reduces the overall profit of the platform.

Corollary 1. The market channel introduction of the contract manufacturer may discourage the platform from introducing a private brand, and this tendency is more obvious as the referral fee r increases (i.e., $\frac{\partial(1+r-t_1(r))}{\partial r} > 0$).

The platform may have three sources of revenue: the profits from selling PB2 and reselling NB, and the agency fee of PB1 sold by the contract manufacturer. Compared with the case where the platform does not introduce PB2, as disclosed in Proposition 1, when the difference between PB1 and PB2 is small, the competition becomes intense and may lead to the reduction of the national brand profit and the PB1's agency fee, and finally makes the platform worse off. Meanwhile, this effect becomes greater as the referral fee charged by the platform increases. Thus, Corollary 1 is straightforward. This finding emphasizes the influence of marketplace channel encroachment on private brand introduction of e-commerce platforms. In other words, when the contract manufacturer has encroached to sell his private brand through the marketplace channel, the platform could make herself better off by introducing her private brand products that are significantly different from PB1.

5.3 Marketplace channel encroachment decision

5.3.1 Impact of platform's private brand introduction on marketplace channel encroachment

Lemma 3. (a) If the platform does not introduce a private brand, $\pi_{CM}^{EN*} > \pi_{CM}^{NN*}$ when $r < \frac{3}{4}$; $\pi_{CM}^{EN*} < \pi_{CM}^{NN*}$ otherwise. (b) If the platform introduces a private brand, $\pi_{CM}^{EI*} > \pi_{CM}^{NI*}$ when (i) $r < \frac{6t+1}{8t-1}$ and t > 1 or (ii) t < 1; $\pi_{CM}^{EI*} < \pi_{CM}^{NI*}$ otherwise.

Lemma 3 illustrates marketplace channel encroachment of the contract manufacturer in the case where the platform operates different private brand introduction strategies. One might imagine that when the referral fee charged by the platform is relatively high, the contract manufacturer will choose not to encroach through the marketplace channel, while Lemma 3(b) implies that, regardless of the referral fee's value, when the perceived value of PB1 is lower than that of PB2, he will prefer to set up a marketplace channel with the platform. The reason behind this is as follows. When the referral fee is low, the contract manufacturer can always maximize his profit because of the right to set the retail price of PB1. When the referral fee is high, if the value of PB1 is higher than PB2 but lower than the NB, the increased retail price of PB1 reduces the retail and wholesale prices of NB due to competition, and thus makes him worse off. However, when the value of PB1 is lower than the other brands, the increased retail price of PB1 does not affect the contract manufacturer's profit from selling NB but increases the revenue of the marketplace channel, and finally makes the contract manufacturer better off.

Proposition 2. The private brand introduction of the platform encourages the contract manufacturer to encroach by establishing his own brand. This preference is affected by the ratio t--as the ratio increases, the contract manufacturer prefers more to encroach.

Proposition 2 shows that regardless of the value of t, when the platform introduces her own brand, the probability of the contract manufacturer encroaching through the marketplace of the platform increases. This probability is affected by t--as t decreases (i.e., the perceived value of PB1 decreases or PB2's value increases), there is a higher probability of contract manufacturer encroachment. On one hand, the introduction of platform private brands could weaken the competition between PB1 and NB, which may make the contract manufacturer more likely to enter a marketplace channel. On the other hand, when t is large, the PB1 sold by the contract manufacturer through the marketplace competes fiercely with NB, which may reduce the contract manufacturer's profit from selling NB and thus hurt him. As t decreases (i.e., the perceived value of PB1 decreases or PB2's value increases), the introduction of PB1 reduces the profit loss from selling NB but increases the income of the marketplace channel, and therefore makes the contract manufacturer better off.

5.3.2 Equilibrium outcomes of the whole game

We have discussed the marketplace channel encroachment of the contract manufacturer and the platform's private-brand introduction strategy. Next, we will examine the impact of these strategies on the profits of the OEM and the whole supply chain, as well as the equilibrium results of the whole game. These results are illustrated in the following propositions.

Proposition 3. The whole supply chain becomes better off when either the contract manufacturer encroaches by selling PB1 through the marketplace channel or the platform introduces her private brand PB2, but may get worse off when two firms introduce their private brands simultaneously. Moreover, the OEM becomes worse off when the contract manufacturer encroaches and/or the platform introduces a private brand.

In general, when the contract manufacturer or the platform encroaches by introducing private brands, the profit of the supply chain increases because of the increased total sales because of competition between the private and national brands. However, one interesting result shows that when the two firms introduce private brands at the same time, the fierce competition among the three products (i.e., PB1, PB2, and NB) in the market may lead to price wars, which hurts the profit of the whole supply chain. Furthermore, once there is a private brand competing with OEM's national brand, the OEM will be hurt.

Proposition 4. When $t_1(r) < t < 1 + r$ and $0 < r < \frac{3t+1}{4t}$, the equilibrium outcome is case EN; when (i) $\frac{6t+1}{8t-1} < r < 1$ and t > 1 + r or (ii) $\frac{3t+1}{4t} < r < 1$ and t < 1 + r, scenario NI is the equilibrium outcome; otherwise, the equilibrium outcome is case EI.

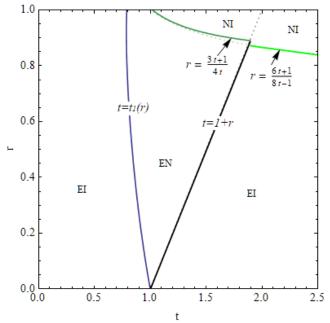


Fig. 2. The equilibrium outcome of the whole game

Fig. 2 illustrates the equilibrium solution for the entire game. When the difference between the two private brands is not large (i.e., parameter t approaches 1), the contract manufacturer chooses to encroach by selling PB1 through the marketplace channel, while the platform does not introduce her private brand. When the perceived value of PB1 is significantly less than that of PB2 or when the value of PB1 is significantly higher than that of PB2 and the referral fee is not high, both the contract manufacturer and platform will introduce their private brands, which may hurt the both firms' profits because fierce price wars suffer when the two private brands close. Interestingly, when the value of PB1 is significantly higher than that of PB2 and the referral fee is high, the manufacturer will not encroach but the platform will introduce her brand. This is because, at this time, the introduction of PB1 reduces the contract manufacturer's profit from selling NB, and thus decreases the profit of the whole contract manufacturer. Finally, we note that scenario NN does not appear at equilibrium, which implies that the upstream contract manufacturer and/or the downstream platform have the motivation to introduce their private brands to compete with OEM's national brands. Consider the equilibrium result. From the perspective of the contract manufacturer, in most cases, the contract manufacturer chooses to encroach by selling his private brand through the marketplace channel. However, when the referral fee is sufficiently high, if $b > \theta$, the contract manufacturer should choose to develop a private brand; otherwise, he may choose not to build a private brand. From the platform's perspective, in order to increase the profit of the platform, she can establish her private brand by setting up a private brand that is quite different from PB1. Meanwhile, the degree of differentiation between PB1 and PB2 should increase with the increase of the referral fee. Otherwise, the private brand introduction of the platform will hurt herself.

6 The extended models

6.1 Model with a different strategy sequence

In the base model, we assume that the contract manufacturer moves first by setting his strategy and then the platform determines whether to introduce her private brand. Here, we investigate an extension of the base model by assuming that the

platform moves first and then the contract manufacturer decides his encroachment strategy. Therefore, we can obtain the equilibrium result of the whole game.

Proposition 5. When $\frac{6t+1}{8t-1} < r < 1$, the equilibrium outcome is case NI; when $t_1(r) < t < 1 + r$ and $0 < r < \frac{3}{4}$, scenario EN is the equilibrium outcome; when $t_2(r) < t < t_3(r)$ and $\frac{3}{4} < r < 1$, the equilibrium outcome is scenario NN; the equilibrium outcome is case EI otherwise.

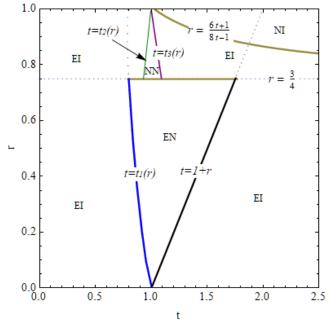


Fig. 3. The equilibrium outcome of the whole game

Fig. 3 shows that in the case where the platform first sets whether to introduce a private brand and the contract manufacturer then moves by determining whether to encroach, the equilibrium outcome of the whole game is similar to the above. However, Figure 3 also illustrates that scenario NN emerges in equilibrium when the difference between the two private brands is small and the referral fee is sufficiently high. This is because in region NN of Fig. 3, when the platform first decides its strategy, if the platform introduces her private brand, the contract manufacturer will be encouraged to encroach (see Proposition 2), which leads to fierce competition and damages the profit of the platform. In this case, the platform chooses not to introduce her private brand and the contract manufacturer will also choose not to encroach due to the high referral fee charged by the platform.

6.2 Integrated upstream firms

This subsection considers an extension of the base model by assuming that the contract manufacturer and the OEM integrate. By using the standard method in the baseline model, we can get the equilibrium result and find the proposition below.

Proposition 6. The main results are robust when the contract manufacturer and the OEM integrate as a union.

Proposition 6 states that when the contract manufacturer and the OEM cooperate as a centre, our main results remain qualitatively unchanged. Specifically, the encroachment of the integrated firm's marketplace channel may prevent the platform from introducing a private brand, while the private brand introduction of the platform increases the likelihood of the marketplace channel introduction of the contract manufacturer. These findings provide useful insights for the platform in making decisions regarding private brand introduction and managing upstream firm encroachment through the marketplace channel.

7. Conclusion

In the e-commerce supply chain, the introduction of the marketplace channel and the introduction of the online platform private brand are common business practices. However, the existing literature separately examines the marketplace channel encroachment and the private brand introduction. By considering both issues simultaneously, this article studies the interplay between the contract manufacturer choosing to enter the marketplace to sell a private brand PB1 and the platform with an option to introduce a private brand PB2. Based on the contract manufacturer's channel strategy and the platform's brand strategy, we examine four different scenarios, and derive equilibrium outcomes. Our results first show that the marketplace channel encroachment of the contract manufacturer and the platform's private brand introduction influence each other. On one hand, the encroachment of contract manufacturers may prevent the platform from introducing her private brand. Specifically,

the e-commerce platform always introduces her private brand when the contract manufacturer does not enter a marketplace channel, but may not introduce her private brand when the contract manufacturer entering the marketplace sells his PB1 is similar to that of the platform's PB2. On the other hand, the platform's private brand introduction increases the probability of the contract manufacturer encroachment. This probability increases in the perceived value of PB1 but decreases in the perceived value of PB1. Therefore, when the platform (CM) sets its private brand introduction strategy (marketplace channel encroachment), the CM's marketplace channel encroachment (the platform's private brand introduction) should be carefully considered due to the interactions between them. In addition, when the contract manufacturer and/or the platform enter the market by introducing private brands, the OEM always becomes worse off and the profit of the whole supply chain may decrease.

Second, we derive the equilibrium results of the whole game and find that the equilibrium outcomes are influenced by the differentiation between the two private brands and the referral fee charged by the platform. Specifically, when the differentiation is small, the contract manufacturer prefers to encroach and the platform chooses not to introduce her private brand; when the perceived value of PB1 sold by the contract manufacturer is much higher than that of PB2, and the referral fee is high, the contract manufacturer will not enter the marketplace and the platform will introduce her private brand. Interestingly, when the differentiation is small and the referral fee is low, both the manufacturer and the platform could introduce private brands, which may hurt both the contract manufacturer and the platform. These conclusions imply that the firms should carefully consider the impacts of the referral fee and the difference between the two private brands when deciding their strategies.

Finally, we extend the benchmark model to two other cases, and find that when the platform first decides whether to introduce the private brand and then the contract manufacturer decides whether to encroach, scenario NN (i.e., the two firms do not introduce their private brands) may emerge in equilibrium. In other words, in the region of scenario NN in Fig. 3, the platform has the first-mover advantage to make her better off. Therefore, when platform managers decide whether to develop a private brand, the timing of private brand introduction also should be considered.

Our research provides managers with several useful insights to understand and make decisions on marketplace channel encroachment and private brand introduction in e-commerce supply chains. For future research, first, we assume that the costs of marketplace channel encroachment and private brand introduction are normalized to zero in our base model, and one can extend our model to examine the impacts of these costs on the equilibrium results. Second, in practice, as long as the contract manufacturer meets the entry conditions of the online platform such as JD.com, the contract manufacturer can successfully open a marketplace channel on these platforms, which has also been studied in the extant literature (Chen, Zhao & Li 2020; Huang, Huang & Liu 2022; Zhang & Hou 2022). However, some other platforms may prevent contract manufacturers from encroaching, which may be considered in the future. Finally, in the three-tier supply chain, the OEM may take some actions to prevent other members from introducing private brands, and these reactions could also serve as a direction for our future research.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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Appendix

Proof of Case NN. Given $\frac{\partial^2 \pi_P^{NN}}{\partial p_1^2} = -2 < 0$, we can get $p_1 = \frac{1+w_1}{2}$ from the equation $\frac{\partial \pi_P^{NN}}{\partial p_1} = 0$. Plugging $p_1 = \frac{1+w_1}{2}$ into the equation (2), we have $\frac{\partial^2 \pi_{OEM}^{NN}}{\partial w_1^2} = -1 < 0$, which indicates that π_{OEM}^{NN} is concave in w_1 . From $\frac{\partial \pi_{OEM}^{NN}}{\partial w_1} = 0$, we get $w_1 = \frac{1+w_0}{2}$. Plugging $w_1 = \frac{1+w_0}{2}$ into the equation (1), we have $\frac{\partial^2 \pi_{CM}^{NN}}{\partial w_0^2} = -0.5 < 0$, which indicates that π_{CM}^{NN} is concave in w_0 . From $\frac{\partial \pi_{CM}^{NN}}{\partial w_0} = 0$, we get $w_0 = \frac{1}{2}$. Substituting w_0 into w_1 and p_1 , we can get the optimal outcomes of case NN.

Proof of Case NI. Using the equation (6), we have $\frac{\partial^2 \pi_P^{NI}}{\partial p_1^2} = \frac{2}{b-1}$, $\frac{\partial^2 \pi_P^{NI}}{\partial p_1 \partial p_3} = \frac{2}{1-b}$, $\frac{\partial^2 \pi_P^{NI}}{\partial p_3^2} = \frac{2}{b(b-1)}$, and $\frac{\partial^2 \pi_P^{NI}}{\partial p_3 \partial p_1} = \frac{2}{1-b}$. The Hessian

matrix
$$H$$
 of π_P^{NI} to p_1 and p_3 is $H = \begin{pmatrix} b-1 & 1-b \\ \frac{2}{1-b} & \frac{2}{b(b-1)} \end{pmatrix}$. We can get that $|H| = \frac{4}{b(1-b)} > 0$ and $\frac{\partial^2 \pi_P^{NI}}{\partial p_1^2} < 0$, which means that π_P^{NI}

is strictly concave in p_1 and p_3 . Then from $\frac{\partial \pi_P^{NI}}{\partial p_1} = 0$ and $\frac{\partial \pi_P^{NI}}{\partial p_3} = 0$, we obtain $p_1 = \frac{1+w_1}{2}$ and $p_3 = \frac{b}{2}$.

Plugging $p_1 = \frac{1+w_1}{2}$ and $p_3 = \frac{b}{2}$ into the equation (5), we have $\frac{\partial^2 \pi_{OEM}^{OE}}{\partial w_1^2} = \frac{1}{b-1} < 0$, which indicates that π_{OEM}^{NI} is concave in w_1 . From $\frac{\partial \pi_{OEM}^{OE}}{\partial w_1} = 0$, we get $w_1 = \frac{1-b+w_0}{2}$. Plugging $w_1 = \frac{1-b+w_0}{2}$ into the equation (4), we have $\frac{\partial^2 \pi_{OEM}^{NI}}{\partial w_0^2} = \frac{1}{2(b-1)} < 0$, which indicates that π_{CM}^{NI} is concave in w_0 . From $\frac{\partial \pi_{CM}^{NI}}{\partial w_0} = 0$, we get $w_0 = \frac{1-b}{2}$. Substituting w_0 into w_1 , p_1 and p_3 , we can get the optimal outcomes of case NI.

Proof of Cases EN and EI. The proof of Cases EN and EI is similar to that of Case NI, and is omitted for brevity.

Proof of Lemma 1. We can easily obtain Lemma 1 by comparing the wholesale and retail prices, and therefore we omit it here.

Proof of Lemma 2. By comparison, we have: $\pi_P^{NI*} - \pi_P^{NN*} = \frac{15b}{16} > 0$.

Proof of Proposition 1. When $\theta > b$ (i.e., t > 1), $\pi_P^{EI*} - \pi_P^{EN*} = \frac{b(1-r)^2 t(t-1-r)}{4(2t-1-r)^2}$, it is shown that if 1 < t < 1 + r, $\pi_P^{EI*} < \pi_P^{EN*}$; otherwise, $\pi_P^{EI*} > \pi_P^{EN*}$. When $\theta < b$ (i.e. t < 1), $\pi_P^{EI*} - \pi_P^{EN*} = \frac{b(60-88t-108rt+31t^2+114rt^2+67r^2t^2-3t^3-22rt^3-35r^2t^3-16r^3t^3)}{64(rt-2+t+rt)^2}$, we can show that if $t_1(r) < t < 1$, $\pi_P^{EI*} < \pi_P^{EN*}$; otherwise, $\pi_P^{EI*} > \pi_P^{EN*} = \frac{b(1-r)^2 t(t-1-r)^2}{64(rt-2+t+rt)^2}$, we can show that if $t_1(r) < t < 1$, $\pi_P^{EI*} < \pi_P^{EN*}$; otherwise, $\pi_P^{EI*} > \pi_P^{EN*} = \frac{1}{64(rt-2+t+rt)^2}$, we can show that if $t_1(r) < t < 1$, $\pi_P^{EI*} < \pi_P^{EN*}$; otherwise, $\pi_P^{EI*} > \pi_P^{EN*} = \frac{1}{64(rt-2+t+rt)^2}$, and its expression is omitted due to complexity. Therefore, we can prove Proposition 1

Proof of Lemma 3. The proof of Lemma 3 is similar to that of Lemma 2, and is omitted for brevity.

Proof of Proposition 2. By comparing Lemma 3(a) and Lemma 3(b), Proposition 2 is straightforward.

Proof of Proposition 3. The proof of Proposition 3 is similar to that of Lemma 2, and is omitted for brevity.

Proof of Proposition 4. When t > 1 + r or $t < t_1(r)$, we show that regardless of the CM's strategies, the platform always introduces a private brand. In this case, if $r > \frac{6t+1}{8t-1}$ and t > 1 + r, $\pi_{CM}^{EI*} < \pi_{CM}^{NI*}$; otherwise, $\pi_{CM}^{EI*} > \pi_{CM}^{NI*}$ (see Lemma 3). In other words, when $\frac{6t+1}{8t-1} < r < 1$ and t > 1 + r, scenario NI is the equilibrium outcome; when (i) $\frac{6t+1}{8t-1} > r$ and t > 1 + r or (ii) $t_1(r) > t$, the equilibrium outcome is case EI. Similarly, in the area $t_1(r) < t < 1 + r$, we can find the equilibrium outcomes. Therefore, we prove Proposition 4.

Proof of Propositions 5 and 6. Following a similar logic in proving Proposition 4, we obtain Propositions 5 and 6. Note that

 $t_2(r)$ is the only meaningful root of the equation $-60 + 92t + 44rt - 35t^2 - 54rt^2 - 3r^2t^2 + 4t^3 + 8rt^3 + 4r^2t^3 = 0$ for 0 < t < 1, and its expression is omitted due to complexity. $t_3(r) = \frac{-5+23r+12r^2+2\sqrt{2(1-r)^2(1+11r+18r^2)}}{2(16r-1)}$.



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