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Horizontal oscillations of the wood sawing support during the cutting process in a wood sawing line using a vertical bandsaw

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A B S T R A C T

The wooden trolley (in the sawing line using a vertical bandsaw) during work generally moves by iron wheels on two rails placed on the concrete floor. Due to the effect of noticeable cutting force applied by the saw blade and the vehicle's uneven weight on the wheels, it causes the vehicle to vibrate during the horizontal movement, affecting the stability of the wood sawing circuit. By modeling the problem of vibration of beams located on Winkler elastic foundations, subjected to mobile concentrated loads, the article has built a system of differential equations of vibration of two rails, giving an expression to calculate the vibration amplitude of the swan plane. The influence of the flexural rigidity EI of the rails and the elastic stiffness k of foundation on the vibration amplitude of the cutting plane is studied and analyzed. The use of the Dirac Delta function and the employed analytical solution allow the designers to evaluate the accuracy and reliability of the calculated results.

1. Introduction

The wooden trolley (in the sawing line using a vertical bandsaw) during work will move on six wheels and on two rails placed on a concrete floor (Fig. 1). Although the wooden trolley moves at a low speed, due to the cutting load and force of the saw blade and the uneven weight of the cart on the wheels, it causes the cart to vibrate while moving. This can affect the stability of the wood sawing circuit and consequently the recovery rate and surface smoothness are reduced. To limit this vibration, in addition to increasing the stiffness of the wooden trolley frame, it is also important to investigate the relationship between bending stiffness *EI* of the rail and the foundation coefficient *k*. Thus, the construction and installation of the machine should be made in such a way that during sawing the wood, the oscillation amplitude of the saw circuit is always within allowable limits.

Research on the vibration of a wooden trolley during the sawing process can be modeled as the problem of vibration of rectangular shape beams resting on elastic foundations (known as Winkler), subjected to mobile concentrated loads. The problem of vibration of a single beam located on the elastic foundation may have several industrial and engineering applications. Depending on the technical requirements, it will lead to problem models of beams located on different foundations. For instance, Coskun & Engin (1999) and Coskun (2000) analyzed the non-linear vibrations of beams located on Winkler elastic foundations. Motaghian et al. (2011), studied the vibration of beams under free-free boundary conditions and resting on the locally elastic-foundation under mixed boundary conditions. Kukla (1991) investigated the free-vibration response of beam shape samples supported by stepped elastic-foundation with different boundary conditions. De Rosa &

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Maurizi (1998) studied the effects of concentrated masses and the Pasternak type foundation model on the free-vibration of the type Euler beams. The free vibrations of horizontally curved beams under flexural and torsional modes were studied by Lee & Jeong (2016). They utilized the Pasternak foundations for their investigations and analyses. Khademi-Kouhi et al. (2023, 2024) studied the vibration response of fiber metal laminate (FML) thin cylindrical shells supported on Pasternak type elastic foundation having different boundary conditions.

Along with the study of beams on foundation models, several numerical, analytical and even experimental efforts have been done till now. For example, Chen et al. (2004) suggested a mixed method (i.e. combined differential quadrature method and state space method) for analyzing free-vibration and bending of thick beams supported on Pasternak type foundation. Yokoyama (1991,1996) presented numerical method based on finite element (FE) simulation for obtaining the vibration behavior and response of a uniform Timoshenko beam-column. The investigated problem in Yokoyama's work was considered on the two-parameter foundation with elastic behavior. Adair et al. (2018) also used the variational iteration method to calculate the natural frequency of the Timoshenko beams supported on the 2-parameter elastic foundations. Gülkan & Alemdar (1999) provided solution for the shape functions of a beam segment resting on a generalized type of the 2-parameter elastic-foundation. The exact shape functions can be used to derive exact analytical expressions for the coefficients of stiffness matrix for an element, the working equivalent nodal forces for arbitrary lateral loads, and the consistent mass-coefficient and geometric stiffness matrix. Amiri & Onyango (2010) obtained the behavior and response of a single beam supported on an elastic foundation subjected to repeated rolling concentrated loads using Fourier transform.

There are many technical problems about the dynamics of beams on elastic type foundations in which their models are based on the Winkler hypothesis. Zhou (1993) and Eisenberger (1994) worked on a general solution for determining the vibrations of beams resting on a variable Winkler elastic foundation. Kacar et al. (2011) examined the free-vibrational response of beams resting on several Winkler type elastic foundations by using the differential-transform-technique (DTT). Al-Hosani et al. (1999) fully introduced the basic solution and boundary integral equation for thick Reissner plates placed on Winkler type elastic foundations, taking into account the influence of horizontal normal stress generated from ground reactions on the surface of the plate. The basic solution is obtained and extracted from the Hankel integral and Fourier transformations in terms of the complex Bessel functions. Beams supported by foundations with elastic behavior have received great research attention due to their wide practical and industrial applications. After pioneer presentation of an analytical solution for beam elements resting on elastic foundations using the differential equation method by Timoshenko (1941), many other researchers used such beam models and theories with several load and boundary conditions (e.g. Lamprea-Pineda et al., 2022; Hadji, & Bernard, 2020; Akhazhanov et al., 2023; Zhang et al., 2023; Samaei et al., 2015; Abdelrahman et al., 2023; Dinev, 2012). The soil-like medium as the foundation is often modeled by the simple Winkler type model that considers a number of mutually independent spring elements. Accordingly, the Winkler model is often adopted for analyzing the behavior of beam shape elements and parts. In this method the vertical or normal displacement of nodes is assumed to be proportional to the contact pressure at any point. On the other hand, several works can be found in the literature for investigating the cutting force and vibration of bandsaw machines and blades (see for example, Moradpour et al., 2013,2016; Gendraud et al., 2003; Balighate & Dhanal, 2018; Yang & Mote Jr, 1990; Okai, 2009; Ajayeoba et al., 2020; Porankiewicz et al., 2011; Marchal et al., 2009).

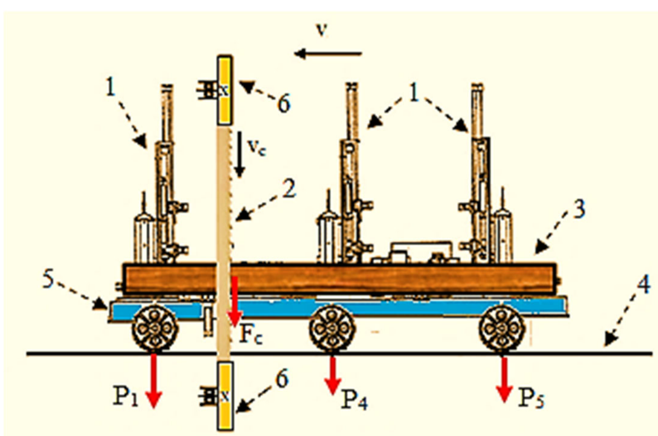

Fig. 1. Vertical cross-sectional diagram of the wooden trolley (in the sawing line using a vertical bandsaw)
1 -*Wooden clamps,* 2 - *Vertical bandsaw,* 3 - *Sawed logs,* 4 - *Rails,* 5-*Stroller frame,* 6- *Saw wheel*

In this paper, using the vibrating beam model placed on the Winkler type elastic-foundation, the analytical solution of the vibrating differential equation of the beam that is under the influence of mobile concentrated loads is solved and derived by the method known as "separation of variables". Along with the use of the Delta Dirac function and its properties, the expression for calculating the amplitude of horizontal vibration of the wood cutting groove plane is determined. Based on this result, the pair of EI and k values are established to match the horizontal vibration amplitude of the wood cutting groove plane to reach the maximum allowed.

2. Calculation of vibration of the wood sawing slot plane during the wood cutting process

2.1 Calculation of the force exerted by the weight of the wooden trolley

First, the force exerted by the weight of trolley (in the sawing line using a vertical bandsaw) is calculated. This machine is placed on the two rails at the contact points between rail and wheel when the cart is moving with a constant speed $v(m/s)$. Assuming that the trolley frame is an absolutely rigid structure, the axles of the six wheels are rigidly mounted to the frame at points A_i with the distances shown in Fig. 2. At A_i points the loads $P_i(N)$ are applied and at point M there is a downward wood shear force F_c.

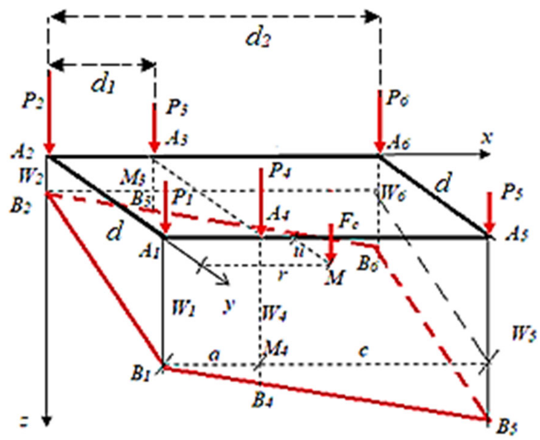

Fig. 2. Calculation model of the load acting on the rails at the point of contact of the trolley wheel.

Assume that the wheels of the trolley, with the forces acting as shown in Fig. 2, are placed on elastic springs with a stiffness coefficient $\bar{k}$. If F_i is called the force of the trolley acting on the rail at points A_i (the point of contact of the wheel and rail) and W_i is the length of the compressed spring (Fig. 2), then $F_i = \bar{k}\, W_i$ .Pay attention to the condition that points $B_1B_2B_3B_4B_5B_6$ are coplanar, so they have the following relations:

$$\frac{W_3 - W_2}{W_6 - W_2} = \frac{d_1}{d_2} = \frac{W_4 - W_1}{W_5 - W_1} \tag{1}$$

$$W_6 - W_2 = W_5 - W_1 \tag{2}$$

From the condition of equilibrium moment forces for sides A_1A_2, A_2A_6, A_1A_5 together with Eqs. (1-2) and pay attention to $r = vt$, leading to:

$$\begin{cases} W_3 + W_4 + \dfrac{d_2}{d_1}W_5 + \dfrac{d_2}{d_1}W_6 & = \dfrac{1}{\bar{k}}V_1 + \dfrac{1}{\bar{k}}\dfrac{vt}{d_1}F_c \\ W_1 + W_4 + W_5 & = \dfrac{1}{\bar{k}}V_2 \\ W_2 + W_3 + W_6 & = \dfrac{1}{\bar{k}}V_3 \\ -\dfrac{d_2 - d_1}{d_1}W_2 + \dfrac{d_2}{d_1}W_3 - W_6 & = 0 \\ -\dfrac{d_2 - d_1}{d_1}W_1 + \dfrac{d_2}{d_1}W_4 - W_5 & = 0 \\ W_1 - W_2 - W_5 + W_6 & = 0 \end{cases} \tag{3}$$

in which

$$
\begin{aligned}
&V_1 = \left(P_3 + P_4\right) + \frac{d_2}{d_1}\left(P_5 + P_6\right)\\
&V_2 = P_1 + P_4 + P_5 + \frac{d+u}{d}F_c\\
&V_3 = P_2 + P_3 + P_6 + \frac{u}{d}F_c
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&V = \left(V_1 \;\; V_2 \;\; V_3 \;\; 0 \;\; 0 \;\; 0\right)^T\\
&E = \left(\frac{v}{a}F_c \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)^T\\
&F = \left(F_1 \;\; F_2 \;\; F_3 \;\; F_4 \;\; F_5 \;\; F_6\right)^T
\end{aligned}
\tag{5}
$$

where

$$
A = \begin{bmatrix}
0 & 0 & 1 & 1 & \frac{d_2}{d_1} & \frac{d_2}{d_1}\\
1 & 0 & 0 & 1 & 1 & 0\\
0 & 1 & 1 & 0 & 0 & 1\\
0 & -\frac{d_2-d_1}{d_1} & \frac{d_2}{d_1} & 0 & 0 & -1\\
-\frac{d_2-d_1}{d_1} & 0 & 0 & \frac{d_2}{d_1} & -1 & 0\\
1 & -1 & 0 & 0 & -1 & 1
\end{bmatrix}
\tag{6}
$$

and pay attention to $F_i = \overline{k}\,W_i$ we get that F is the solution of the linear algebraic equation:

$$
A.F = V + Et \tag{7}
$$

values of F_i all having the form below:

$$
F_i = a_i + b_i t \tag{8}
$$

in which $\alpha = \left(a_1 \;\; a_2 \;\; a_3 \;\; a_4 \;\; a_5 \;\; a_6\right)^T$ is the solution of the equation $A.\alpha = V$,
and $\beta = \left(b_1 \;\; b_2 \;\; b_3 \;\; b_4 \;\; b_5 \;\; b_6\right)^T$ is the solution of the equation $A.\beta = E$. Thus, the values of a_i, b_i only depend on the values of the parameters d, d_1, d_2, h, P_i, F_c, v.

2.2. Calculation of beam placed on Winkler type elastic foundation

2.2.1 Differential equations of vibrations of beams resting on the Winkler elastic foundation

Consider a beam of length *L*, with two ends hinged, the Young modulus *E* and the moment of inertia of the beam *I*, placed on an Winkler foundation with elastic coefficient *k*. Select the Oxz coordinate system with the Ox axis along the beam axis, the Oz axis pointing downwards. The beam is subjected to a load distributed along the beam with a strength $q(x,t)(\mathrm{N}/m)$. It is assumed that during the working process the beam and the foundation do not separate from each other, that is, the settlement of the foundation and the deflection of the beam are always equal at any cross-section with any x coordinate. For a typical Euler-Bernoulli type beam, the differential equation of motion can be explained as below relation:

$$
\frac{\partial^2 W(x,t)}{\partial t^2} + \frac{EI}{\rho}\frac{\partial^4 W(x,t)}{\partial x^4} + \frac{k}{\rho}W(x,t) = 0\,,\;\; 0 \le x \le L \tag{9}
$$

in which $W(x,t)$ is the displacement and $\rho(x)$ (kg/ m) is the mass density of the beam in units of length. Equation (9) is the equation of free oscillation of a uniform beam with constant cross-section lying on an elastic foundation. In the case, the beam is loaded in the distribution $q(x,t)$ (N/m), we get the differential-equation of the beam's vibration:

$$\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{EI}{\rho}\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{k}{\rho}.W(x,t)=\frac{1}{\rho}q(x,t) \qquad (10)$$

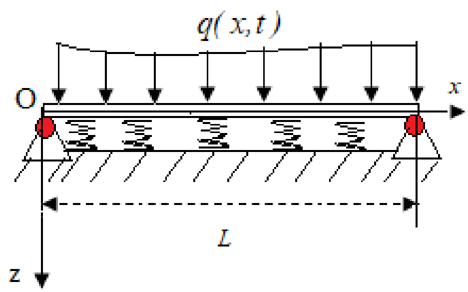

Fig.3. Description of the beam lying on a Winkler type foundation and loaded by a distributed force of $q(x,t)$.

The general solution of the differential Eq. (9) is found by the method of separation of variables:

$$W(x,t)=\sum_{n=1}^{\infty}\left[A_n\cos\omega_n t+B_n\sin\omega_n t\right]\left[C_n\cos\frac{n\pi}{L}x+D_n\sin\frac{n\pi}{L}x\right] \qquad (11)$$

in there:

A_n, B_n - The coefficients are determined from the initial condition,
C_n, D_n - The coefficients are determined from the boundary condition at $x=0$ and $x=L$.
Substitute Eq. (11) into Eq. (9) and since the expression is true for all x and t, we get:

$$\omega_n^2=\frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4+\frac{k}{\rho} \quad , \quad n=1,2,... \qquad (12)$$

2.2.2 Differential equation for the vibration of a beam on Winkler foundation, both ends supporting joints, under variable load and moving with velocity along the beam

Consider the beam resting on the elastic Winkler type foundation subjected to a variable load defined by $q_0=a+bt$ and moving with a constant velocity *v* (Fig. 4)

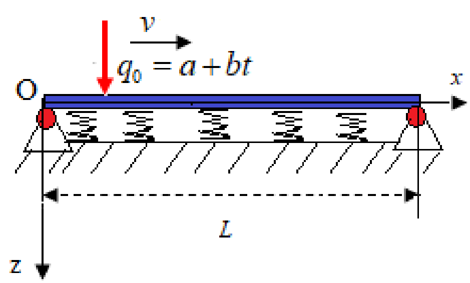

Fig. 4. The model for calculating vibration of beam resting on elastic type Winkler foundation, subjected to load $q_0=a+bt$ moving with a constant velocity *v*.

Using the Delta Dirac function, the load distributed on the beam in this case has the form $\mathrm{q(x,t)}=(a+bt)\delta(x-vt)$, leading to Eq. (10) of the form:

$$\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{EI}{\rho}\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{k}{\rho}.W(x,t)=\frac{(a+bt)}{\rho}\delta(x-vt) \quad (13)$$

$$W(x,t)=\sum_{i=1}^{\infty}T_i(t).X_i(x) \quad (14)$$

Find the solution for relation (13) in the form:

Because the boundary conditions need to be satisfied:

$$W(0)=W(L)=0\ ;\ W^{(2)}(0)=W^{(2)}(L)=0 \quad (15)$$

$$W(x,t)=\sum_{i=1}^{\infty}T_i(t)\sin\frac{i\pi}{L}x \quad (16)$$

lead to
substitute Eq. (16) into Eq. (13), lead to:

$$\sum_{i=1}^{\infty}\left[\ddot{T}_i+\left(\frac{EI}{\rho}\left(\frac{i\pi}{L}\right)^4+\frac{k}{\rho}\right)T_i\right].\sin\frac{i\pi}{L}x=\frac{(a+bt)}{\rho}\delta(x-vt) \quad (17)$$

Multiply both sides of Eq. (17) by $sin\frac{n\pi}{L}x$ then integrate over $[0,L]$ and pay attention to the properties of the resulting Delta Dirac function receive

$$\ddot{T}_n+\left(\frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4+\frac{k}{\rho}\right)T_n=\frac{2(a+bt)}{\rho L}\int_0^L\delta(x-vt)\sin\frac{n\pi x}{L}dx=\frac{2(a+bt)}{\rho L}\sin\frac{n\pi}{L}vt \quad (18)$$

$$\ddot{T}_n+\omega_n^2T_n=(A+Bt)\,sin\,\Omega_n t \quad (19)$$

Eq. (18) has the form:

in which $\omega_n^2=\frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4+\frac{k}{\rho}$ is natural vibration frequency of the beams; $\Omega_n=\frac{n\pi v}{L}$ is forced oscillation frequency and

$$A=\frac{2a}{\rho L}\ ;\ B=\frac{2b}{\rho L} \quad (20)$$

The general solution of Eq. (19) will be:

$$T_n=A_n\,cos\,\omega_n t+B_n\,sin\,\omega_n t+\frac{1}{\left(\omega_n^2-\Omega_n^2\right)}\left[(A+Bt)\,sin\,\Omega_n t-\frac{2\Omega_n B}{\left(\omega_n^2-\Omega_n^2\right)}cos\,\Omega_n t\right] \quad (21)$$

Leading to the solution of Eq. (13) will be:

$$W(x,t)=\sum_{n=1}^{\infty}\left\{A_n\cos\omega_n t+B_n\sin\omega_n t+(K_n+M_n t)\sin\Omega_n t+N_n\cos\Omega_n t\right\}.\sin\frac{n\pi}{L}x \quad (22)$$

$$K_n=\frac{2a}{\rho L\left(\omega_n^2-\Omega_n^2\right)};M_n=\frac{2b}{\rho L\left(\omega_n^2-\Omega_n^2\right)}\ ;N_n=-\frac{4\Omega_n b}{\rho L\left(\omega_n^2-\Omega_n^2\right)^2} \quad (23)$$

in there:

2.2.3 Vibration differential equation of the beam, subjected to 3 concentrated loads that are moving with a constant velocity along the beam

Consider a beam subjected to 3 loads $q_1=a_1+b_1t$, $q_2=a_2+b_2t$, $q_3=a_3+b_3t$; the loads are spaced apart as shown in Fig. 5 and they move along the beam at a constant speed v .

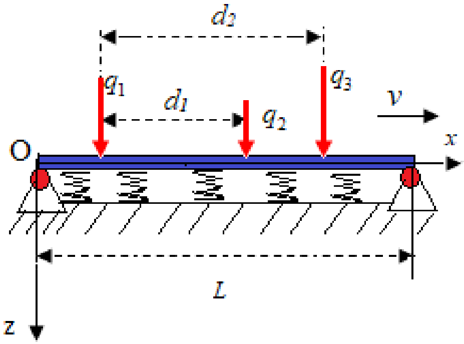

Fig. 5. The model for calculating the vibration of a beam placed on Winkler elastic foundation and subjected to three concentrated loads q_1, q_2, q_3 in different positions, moving at a constant speed *v*.

Using the Delta Dirac function for the loads distributed on the beam, the differential equation of vibration (i.e. Eq. (13)) of the beam in this case takes the form:

$$\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{EI}{\rho}\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{k}{\rho}W(x,t)=\frac{1}{\rho}\left[q_1\delta(x-vt)+q_2\delta(x+d_1-vt)+q_3\delta(x+d_2-vt)\right] \tag{24}$$

To satisfy the boundary condition of Eq. (15), we find solution of Eq. (24) in the form:

$$W(x,t)=\sum_{n=1}^{\infty}T_n(t)\sin\frac{n\pi}{L}x \tag{25}$$

then T_n is the solution of the equation:

$$\ddot{T}_n+\omega_n^2 T_n=\frac{2}{\rho L}\left[q_1\sin\frac{n\pi}{L}vt+q_2\sin\frac{n\pi}{L}(vt-d_1)+q_3\sin\frac{n\pi}{L}(vt-d_2)\right] \tag{26}$$

$$\omega_n^2=\frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4+\frac{k}{\rho} \tag{27}$$

in which
setting

$$\bar{a}_i=\frac{2a_i}{\rho L}\ ;\ \bar{b}_i=\frac{2b_i}{\rho L}\ ;\ i=1,2,3$$
$$\bar{c}_1=0\ ;\ \bar{c}_2=\frac{n\pi}{L}d_1\ ;\ \bar{c}_3=\frac{n\pi}{L}d_2\ ;\ \ \Omega_n=\frac{n\pi v}{L} \tag{28}$$

Eq. (26) becomes:

$$\ddot{T}_n+\omega_n^2 T_n=\sum_{i=1}^{3}\left(\bar{a}_i+\bar{b}_i t\right)sin\left(\Omega_n t-\bar{c}_i\right) \tag{29}$$

The general solution of Eq. (29) has the following form:

$$T_n(t)=A_n\cos\omega_n t+B_n\sin\omega_n t+\frac{1}{\left(\omega_n^2-\Omega_n^2\right)}\sum_{i=1}^{3}\left[\left(\bar{a}_i+\bar{b}_i t\right)\sin\left(\Omega_n t-\bar{c}_i\right)-\frac{2\Omega_n\bar{b}_i}{\left(\omega_n^2-\Omega_n^2\right)}\cos\left(\Omega_n t-\bar{c}_i\right)\right] \tag{30}$$

leading to the general solution of Eq. (24) having the form:

$$W(x,t)=\sum_{n=1}^{\infty}\left\{A_n\cos\omega_n t+B_n\sin\omega_n t+\frac{1}{\left(\omega_n^2-\Omega_n^2\right)}\sum_{i=1}^{3}\left[\left(\bar{a}_i+\bar{b}_i t\right)\sin\left(\Omega_n t-\bar{c}_i\right)-\frac{2\Omega_n\bar{b}_i}{\left(\omega_n^2-\Omega_n^2\right)}\cos\left(\Omega_n t-\bar{c}_i\right)\right]\right\}\sin\frac{n\pi}{L}x \tag{31}$$

Let Eq. (31) to satisfy the initial condition $W\big|_{t=0}=0$; $\left.\frac{\partial W}{\partial t}\right|_{t=0}$, get:

$$A_n = \frac{1}{\left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\bar{a}_i \, sin\bar{c}_i + \frac{2\Omega_n \bar{b}_i}{\left(\omega_n^2 - \Omega_n^2\right)} cos\bar{c}_i \right] \tag{32}$$

$$B_n = \frac{1}{\omega_n \left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\bar{b}_i \, sin\bar{c}_i - \bar{a}_i \Omega_n \, cos\bar{c}_i + \frac{2\Omega_n \bar{b}_i}{\left(\omega_n^2 - \Omega_n^2\right)} sin\bar{c}_i \right]$$

2.3. Equation of horizontal vibration of the wood sawing slot plane during the wood cutting process

In addition to the normal direction, the wood sawing machine can be subjected to horizontal vibration during exerting the cutting force. The horizontal vibration response of the plane is studied here.

2.3.1. The amplitude of horizontal vibration of the saw circuit at point with height h above the plane of the two rails

The wooden trolley (in the sawing line using a vertical bandsaw) is arranged to run on two rails with distance *d* with loads P_i at points $A_i \ (i = 1...6)$ on the wheel axles with distances from each other as shown in the Fig. 6. The rails have length $L(m)$ and constant stiffness *EI*. The rails are located on a concrete foundation with elastic coefficient of $k(N/m^2)$.

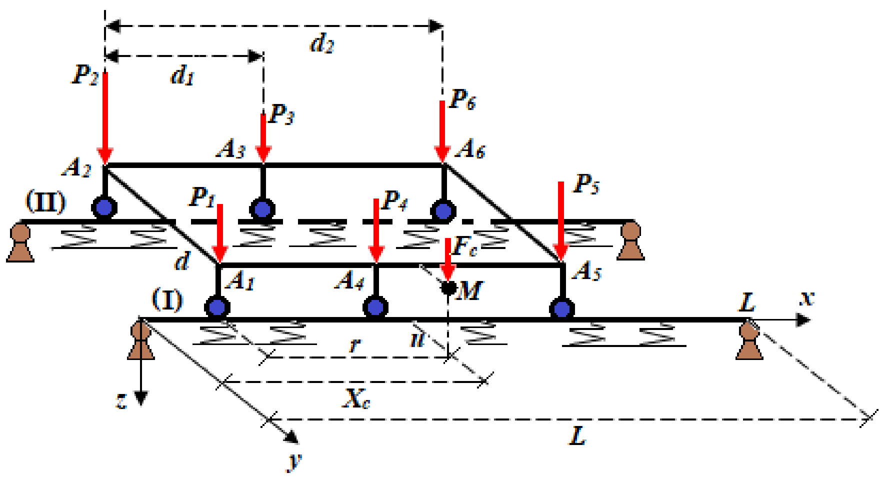

Fig. 6. Model for calculating vibrations of the wooden trolley during wood sawing process.

Choose a coordinate system with the *x*- axis pointing in the rail direction and the *z*- axis pointing down. The saw blade position is at *M*, with coordinates $\left(X_c, u, h\right)$ (height *h* compared to the plane of the two rails). The wooden trolley moves at a constant speed $v(m/s)$. According to the calculation method as in section (2.1), the forces acting on the rails at the contact points with the wheel are of the form $F_i = a_i + b_i.t$.

Applying calculation results according to formula (31) for rail beam (I) and rail beam (II) yields:

$$W^{(I)}(x,t) = \sum_{n=1}^{\infty} \left\{ A_n^{(1)} \cos\omega_n t + B_n^{(1)} \sin\omega_n t + \frac{1}{\left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\left(\bar{a}_i^{(1)} + \bar{b}_i^{(1)} t\right) \sin\left(\Omega_n t - \bar{c}_i\right) - \frac{2\Omega_n \bar{b}_i^{(1)}}{\left(\omega_n^2 - \Omega_n^2\right)} \cos\left(\Omega_n t - \bar{c}_i\right) \right] \right\} \sin\frac{n\pi}{L} x \tag{33}$$

$$W^{(II)}(x,t) = \sum_{n=1}^{\infty} \left\{ A_n^{(2)} \cos\omega_n t + B_n^{(2)} \sin\omega_n t + \frac{1}{\left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\left(\bar{a}_i^{(2)} + \bar{b}_i^{(2)} t\right) \sin\left(\Omega_n t - \bar{c}_i\right) - \frac{2\Omega_n \bar{b}_i^{(2)}}{\left(\omega_n^2 - \Omega_n^2\right)} \cos\left(\Omega_n t - \bar{c}_i\right) \right] \right\} \sin\frac{n\pi}{L} x \tag{34}$$

in there: $\bar{c}_1 = 0$; $\bar{c}_2 = \frac{n\pi}{L} d_1$; $\bar{c}_3 = \frac{n\pi}{L} d_2$

$$\bar{a}_1^{(1)} = \frac{2a_1}{\rho L}; \bar{b}_1^{(1)} = \frac{2b_1}{\rho L}; \ \bar{a}_2^{(1)} = \frac{2a_4}{\rho L}; \ \bar{b}_2^{(1)} = \frac{2b_4}{\rho L}; \bar{a}_3^{(1)} = \frac{2a_5}{\rho L}; \ \bar{b}_3^{(1)} = \frac{2b_5}{\rho L}; \tag{35}$$

$$\bar{a}_1^{(2)} = \frac{2a_2}{\rho L}; \ \bar{b}_1^{(2)} = \frac{2b_2}{\rho L}; \ \bar{a}_2^{(2)} = \frac{2a_3}{\rho L}; \ \bar{b}_2^{(2)} = \frac{2b_3}{\rho L}; \ \bar{a}_3^{(2)} = \frac{2a_6}{\rho L}; \ \bar{b}_3^{(2)} = \frac{2b_6}{\rho L};$$

$$A_n^{(m)} = \frac{1}{\left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\bar{a}_i^{(m)} \sin\bar{c}_i + \frac{2\Omega_n \bar{b}_i^{(m)}}{\left(\omega_n^2 - \Omega_n^2\right)} \cos\bar{c}_i \right] \tag{36}$$

$$B_n^{(m)} = \frac{1}{\omega_n \left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3} \left[\bar{b}_i^{(m)} \sin\bar{c}_i - \bar{a}_i^{(m)} \Omega_n \cos\bar{c}_i + \frac{2\Omega_n \bar{b}_i^{(m)}}{\left(\omega_n^2 - \Omega_n^2\right)} \sin\bar{c}_i \right]; \quad m = 1, 2$$

$$\omega_n^2 = \frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4 + \frac{k}{\rho}; \qquad \Omega_n = \frac{n\pi v}{L}$$

Let φ be the rotation angle about the x axis of the wood sawing slot plane, lead to:

$$\tan\varphi = \frac{W^{(\mathrm{I})}(X_c, t) - W^{(\mathrm{II})}(X_c, t)}{d} \tag{37}$$

If the fluctuation is small there will be $\tan\varphi \approx \varphi$, therefore:

$$\varphi(t) = \frac{1}{d}\sum_{n=1}^{\infty}\Big\{ \left(A_n^{(1)} - A_n^{(2)}\right)\cos\omega_n t + \left(B_n^{(1)} - B_n^{(2)}\right)\sin\omega_n t + \\ + \frac{1}{\left(\omega_n^2 - \Omega_n^2\right)} \sum_{i=1}^{3}\left[\left(\bar{a}_i^{(1)} - \bar{a}_i^{(2)} + \left(\bar{b}_i^{(1)} - \bar{b}_i^{(2)}\right)t\right)\sin\left(\Omega_n t - \bar{c}_i\right) - \frac{2\Omega_n\left(\bar{b}_i^{(1)} - \bar{b}_i^{(2)}\right)}{\left(\omega_n^2 - \Omega_n^2\right)}\cos\left(\Omega_n t - \bar{c}_i\right)\right]\Big\} \sin\frac{n\pi}{L}X_c \tag{38}$$

The amplitude of horizontal vibration of the saw circuit at point M - with height h above the surface of the rail will be:

$$A = h.\varphi(t) \tag{39}$$

In expression (38) the right side is the function chain. If $\Omega_n \neq \omega_n$ (when resonance does not occur), then the coefficients are infinitesimals on the same order as $\frac{1}{n^2}$ when $n \to \infty$, so the series converges uniformly for all x and t values. Therefore, when calculating, we will take the sum of a finite number of terms and be able to evaluate this error.

2.3.2. Check for resonance

Eq. (39) represents the horizontal oscillation amplitude of the saw plane at a point with height h - resonance (or beat phenomenon) will occur if the natural oscillation frequency ω_n coincides (or is nearly equal) with forced oscillation frequency Ω_n. It is necessary to calculate when these phenomena will occur. In order for $\omega_n = \Omega_n$, the velocity v needs to be satisfied:

$$v^2 = \frac{\left[\frac{EI}{\rho}\left(\frac{n\pi}{L}\right)^4 + \frac{k}{\rho}\right]}{\left(\frac{n\pi}{L}\right)^2} \tag{40}$$

Calculating with the parameters $\mathrm{EI} = 30\times10^6\,\mathrm{Nm}^2$; $\rho = 40\ \mathrm{kg/m}$; $\mathrm{L} = 15\ \mathrm{m}$; $\mathrm{k} = 6\times10^6\ \mathrm{N/m}^2$, we will get the v values corresponding to the mods $n$for resonance to occur, recorded in **Table 1.**

Table 1. The speed of the wooden trolley for resonance to occur the ten vibrational modes.

mode (n)	$\omega_n^2 = \Omega_n^2$	$v(m/s)$	mode (n)	$\omega_n^2 = \Omega_n^2$	$v(m/s)$
1	151443.1	1858.1	6	2020255	1131.1
2	173089.6	993.2	7	3614877	1296.9
3	266890.9	822.2	8	6060928	1469.3
4	519433	860.3	9	9618164	1645.3
5	1051936	979.4	10	14580976	1823.2

With the calculation results in Table 1, it was found that, a resonance phenomenon occurs at the smallest speed of 822.2 m/s (with mode $n=3$). Therefore, with a speed of the wooden trolley of only $0.05\ m/s$, resonance cannot occur.

2.4. Calculation applications

Some applications of the presented vibrational analyses are illustrated in the following.

2.4.1. Calculation for the amplitude of horizontal vibration of the saw circuit

Calculations this section are done using the parameters illustrated in Table 2 and the payloads are placed at points A_i:

$P_1 = 8kN;\ P_2 = 6kN;\ P_3 = 6.5\ kN;\ P_4 = 7.2\,kN;\ P_5 = 8.5\ kN;\ P_6 = 8.2\ kN;$ number of terms taken in infinite sum $N = 30$. The results of calculation for the horizontal vibration amplitude at point M on the saw circuit - with height $h = 1.2\,m$ are shown on the graphs in Fig. 7 for two cases (a) $\mathrm{EI} = 30\times10^6\,\mathrm{Nm}^2;\ \mathrm{k} = 12\times10^6\ \mathrm{N/m}^2$ and (b) $\mathrm{EI} = 80\times10^6\,\mathrm{Nm}^2;\ \mathrm{k} = 17\times10^6\ \mathrm{N/m}^2$.

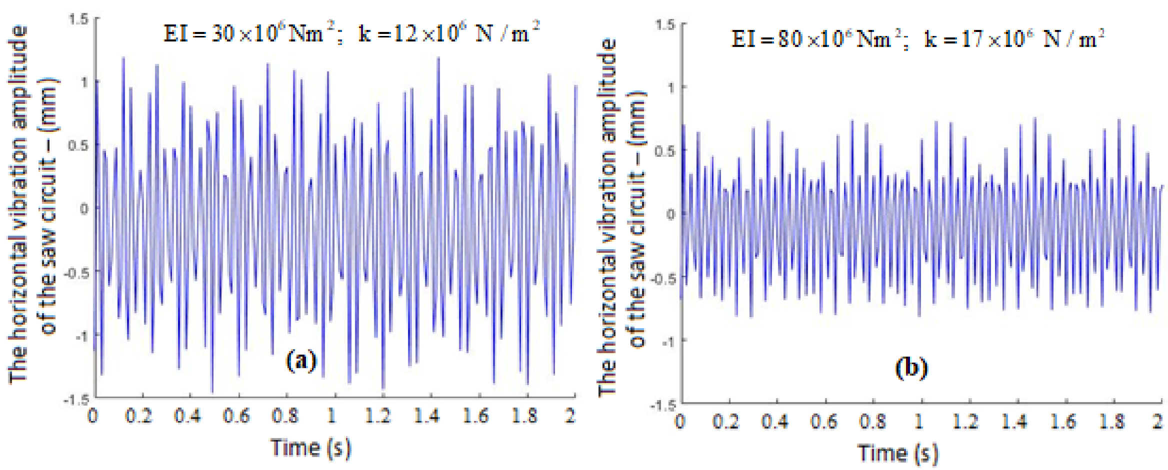

Fig. 7. Graphs of the horizontal vibration amplitude of the wood cutting circuit for two different cases (a) $\mathrm{EI}=30\times10^6\mathrm{Nm}^2;\ \mathrm{k}=12\times10^6\ \mathrm{N/m}^2$ and (b) $\mathrm{EI}=80\times10^6\mathrm{Nm}^2;\ \mathrm{k}=17\times10^6\ \mathrm{N/m}^2$.

Table 2. The calculation parameters for analyzing horizontal vibration.

Symbol	Content	Value
d_1	Distance between middle and top wheel axles	$2.5[m]$
d_2	Distance between the last and first wheel axles	$6.5[m]$
d	Distance between two rails	$1.5[m]$
v	Speed of the wooden trolley	$0.05[m/s]$
L	Length of rail beam	$15[m]$
b	Width of rail beam	$0.1[m]$
b_c	Width of tooth set of vertical bandsaw	$2.5[mm]$
n_c	Teeth per Inch (TPI)	*4*
L_{cut}	Wood cutting circuit length	20 [inch]
n_{cr}	Number of saw blade teeth in the cutting circuit	$n_{cr} = n_c L_{cut}$
ω_c	Revolving speed of saw wheel	700[rpm]
D	Diameter of saw wheel	$38[inch]$
K	Specific cutting resistance	$18[N/mm^2]$
h_c	Wood cutting tooth depth	$h_c = \dfrac{6.10^4.v}{\pi.D.n_c\omega_c}\ [mm]$
F_c	Cutting force for wood	$F_c = Kh_c b_c n_{cr}\ [N]$
ρ	The mass per unit length of rails	$40[\mathrm{kg}/m]$
X_c	x coordinate of vertical bandsaw	$3[m]$
u	y coordinate of vertical bandsaw	$0.5[m]$
h	The height of the point calculates the amplitude of horizontal vibration of the wood cutting circuit.	$1.2[m]$

Calculation results in a time period of 120*s* (the time it takes to saw a 6*m* log) show that the amplitude of horizontal vibrations at the point on the saw plane (1.2 *m* high) reaches the maximum value of 1.5 *mm* in case (a) and 0.92 *mm* in case (b) as can be seen from Fig.7.

2.4.2. Determine reasonable values for the EI and k parameter pair

In the wood sawing technique, the horizontal vibration amplitude of the wood sawing plane is only allowed within a very small limit typically for (<1.5 *mm*). Therefore, in the design and construction and installation of the wood sawing line, it is necessary to choose the appropriate parameters, so that when cutting the wood parts, the above technical requirements for sawing are guaranteed. Notice that, from formulas (38) and (39), when the parameters of the wooden trolley are fixed, then the amplitude of horizontal vibration of the wood cutting plane depends on the parameter *EI* (i.e., the flexural rigidity of the rails) and k (that is the elastic stiffness of the foundation). For each pair of *EI* and *k* values, the maximum value of the horizontal vibration amplitude of the sawing plane is obtained. These results are recorded and illustrated in **Table 3**. From here, it is possible to select suitable pairs of *EI* and *k* values so that the horizontal vibration amplitude of the wood sawing plane is within the allowable limits during the wood sawing process.

Table 3. The maximum value of the horizontal vibration amplitude $A(mm)$ of the sawing plane for different pairs of EI and k values.

$\mathrm{EI}.10^{-6}(Nm^2)$	$k.10^{-5}(N/m^2)$						
	12	**13**	**14**	**15**	**16**	**17**	**18**
30	1.501	1.428	1.335	1.237	1.194	1.146	1.081
35	1.449	1.37	1.286	1.213	1.152	1.086	1.046
40	1.414	1.31	1.252	1.182	1.126	1.076	1.023
45	1.358	1.292	1.225	1.152	1.09	1.043	1.001
50	1.339	1.248	1.208	1.136	1.084	1.022	0.972
55	1.298	1.25	1.161	1.105	1.06	0.999	0.956
60	1.284	1.2	1.131	1.075	1.036	0.983	0.939
65	1.247	1.18	1.111	1.063	1.004	0.963	0.925
70	1.226	1.155	1.094	1.038	0.994	0.939	0.905
75	1.212	1.129	1.079	1.015	0.98	0.925	0.9
80	1.183	1.127	1.066	0.994	0.948	0.92	0.888
85	1.175	1.109	1.052	0.991	0.945	0.908	0.839

3. Conclusion

By using a beam model located on a Winkler type foundation and subjected to different mobile concentrated loads, the study has solved the problem of oscillation of a wooden trolley in the sawing line using a vertical bandsaw, leading to determining the expression for calculating the amplitude of horizontal vibration of the wood cutting groove plane. The use of the Delta Dirac function and the solution found by method of separation of variables allows to evaluate the high accuracy as well as the reliability of the obtained results. Based on this result, the pair of bending stiffness values EI of the rail and the foundation coefficient k are set to match the maximum allowable value of the horizontal vibration amplitude of the wood cutting groove plane.

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