

Notched plates in mixed mode loading (I+II): a review based on the local strain energy density and the cohesive zone model

F. Berto^{a*} and J. Gomez^b

^aDepartment of Industrial and Mechanical Engineering, NTNU, Trondheim, Norway

^bAdvanced Material Simulation, S.L. Asturias 3. Bilbao 48015, Spain

ARTICLE INFO

Article history:

Received 6 July, 2016

Accepted 3 November 2016

Available online

6 November 2016

Keywords:

Notched plates

Mixed mode loading

Local strain energy density

Cohesive zone model

ABSTRACT

Two procedures to evaluate fracture resistance of notched components are proposed in this contribution: the Strain Energy Density (SED) over a control volume and the Cohesive Zone Model (CZM). With the aim to simplify the application of the two fracture criteria, the concept of the 'equivalent local mode I' is presented. The control volume of the SED criterion and the cohesive crack of the CZM, have been rotated along the notch edge and centered with respect to the point where the elastic principal stress is maximum. Numerical predictions are compared with experimental results from U and V shaped notches under three point bending with notch root radius ranging from 0.2 to 4.0 mm. In parallel the loading conditions vary, from pure mode I to a prevailing mode II. All specimens were made of PMMA and tested at -60°C. The good agreement between theory and experimental results adds further confidence to the proposed fracture criteria.

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1. Introduction

In recent years, the mode I fracture of notched samples made of brittle or quasi-brittle materials has been studied by different authors (Carpinteri, 1987; Knésl, 1991; Nui et al., 1994; Seweryn, 1994; Gómez et al., 2000; Gomez & Elices, 2003; Strandberg, 2002; Atzori & Lazzarin, 2001; Gogotsi, 2003; Leguillon & Yosibash, 2003; Yosibash, 2004; Dini & Hills, 2004; Taylor, 2004; Lazzarin & Zambardi, 2001; Lazzarin & Berto, 2005). Under mixed mode (I+II) loading, the problem is more complex, even for brittle or quasi-brittle materials, and few failure criteria have been proposed for V-notches (Seweryn & Lucaszewicz, 2002; Yosibash et al., 2006) giving more or less accurate predictions.

The first task of this paper is to provide a new set of experimental results on fracture of U and V-notched samples, with different values of loading mixity, notch root radii and notch angles. The second

* Corresponding author.

E-mail addresses: filippo.berto@ntnu.no (F. Berto)

task is to investigate the possibility to use two fracture criteria in mixed mode fracture of notched components: the cohesive zone model and the strain energy density model, the latter applied to a finite size volume. After the description of the experimental program (all specimens made of PMMA and tested at -60°C), the paper presents the two different failure criteria as well as the numerical methods to use for fracture load assessments. The paper closes with a comparison between experimental data and expected values to failure.

2. Experimental results

Experiments were carried out by using specimens made of polymethyl-methacrylate (PMMA), a polymer that exhibits a non-linear behavior at room temperature and linear elastic up to fracture at -60°C (Gomez et al., 2005). The average mechanical properties of PMMA at -60°C appear in Table 1.

Table 1. Mechanical properties of PMMA at -60°C

Young's modulus	$E =$	5.05 ± 0.04 GPa
Tensile strength	$\sigma_u =$	128.4 ± 0.1 MPa
Fracture toughness	$K_{IC} =$	1.7 ± 0.1 MPa $\text{m}^{1/2}$
Poisson's ratio	$\nu =$	0.40 ± 0.01

The experimental program considers U and V-notched specimens, with different notch inclinations, notch opening angles, notch radii and boundary conditions. The geometry of the specimens is shown in Fig. 1. In all specimens the thickness was 14 mm, the height 28 mm, whereas the notch depth was 14 mm. Four different geometries were tested, as indicated in Fig. 1 namely: standard U-notched beams, standard V-notched specimen, beams with a tilted U-notch and beams with tilted V-notch. In standard vertical U-notched beams, seven values of the notch root radius, R , were tested; $R = 0.0, 0.2, 0.3, 0.5, 1.0, 2.0$ and 4.0 mm. The position of the loading point, b , was changed to obtain different mixed mode boundary conditions (i.e. $b = -3, 3, 9, 18, 27$ and 36 mm). A notch tip radius of 0.0 and 0.2 mm was analyzed only in combination with the distances $b = 9, 18, 27$ and 36 mm; each configuration was repeated three times so that a total number of 114 tests were performed for this group of notches.

For standard V-notched beams, three values of the notch opening angle α were analyzed: $\alpha = 30^{\circ}, 60^{\circ}$ and 90° . Mixed mode loading was introduced by changing the loading position b (Fig. 1). Two values were considered; $b = 1$ and 9 mm and 18 different samples were tested. To increase the mode II contribution, geometries with 45 degrees inclined notch have been considered (Fig 1). Five notch root radii, $R = 0.3, 0.5, 1.0, 2.0$ and 4.0 mm, and three support span, $m = 3, 9$ and 15 mm, were explored resulting in 15 different geometries and a total number of 45 tests.

For tilted V-notched beams, three notch angles were studied; $\alpha = 30^{\circ}, 60^{\circ}$ and 90° , and, two values of support span m were considered, as shown in Fig. 1; $m = 9$ and 18 mm. 18 different samples were tested. Before testing, the radius and the angle of the notch were measured with an image analysis system. The tests were performed on a servo controlled INSTRON 8803 testing machine. The load was measured with a 5/10 kN INSTRON load cell with ± 0.5 % error at full scale. An MTS strain gauge extensometer, with ± 1 mm nominal displacement and ± 0.15 % error at full scale was used for measuring the displacement of the point where the load was applied. The testing set-up was placed inside an INSTRON environmental chamber and cooled to -60°C . Cooling was achieved by pouring liquid nitrogen inside the chamber in a controlled and continuous flow. Temperature was measured using a K-type thermocouple placed on the sample surface, near the notch. Testing was done in three phases (Gomez et al. 2005); during the first the sample was cooled down to -60°C in about 30 min and subjected to a small pre-load under load control. Then, the sample temperature was stabilized, still under load control, for 45 minutes. And finally, the sample was tested under displacement control at rate of 0.03 mm/min at constant temperature. After each test the TPB experimental device has heated to avoid condensed water over it. All in all, 195 fracture tests were performed.

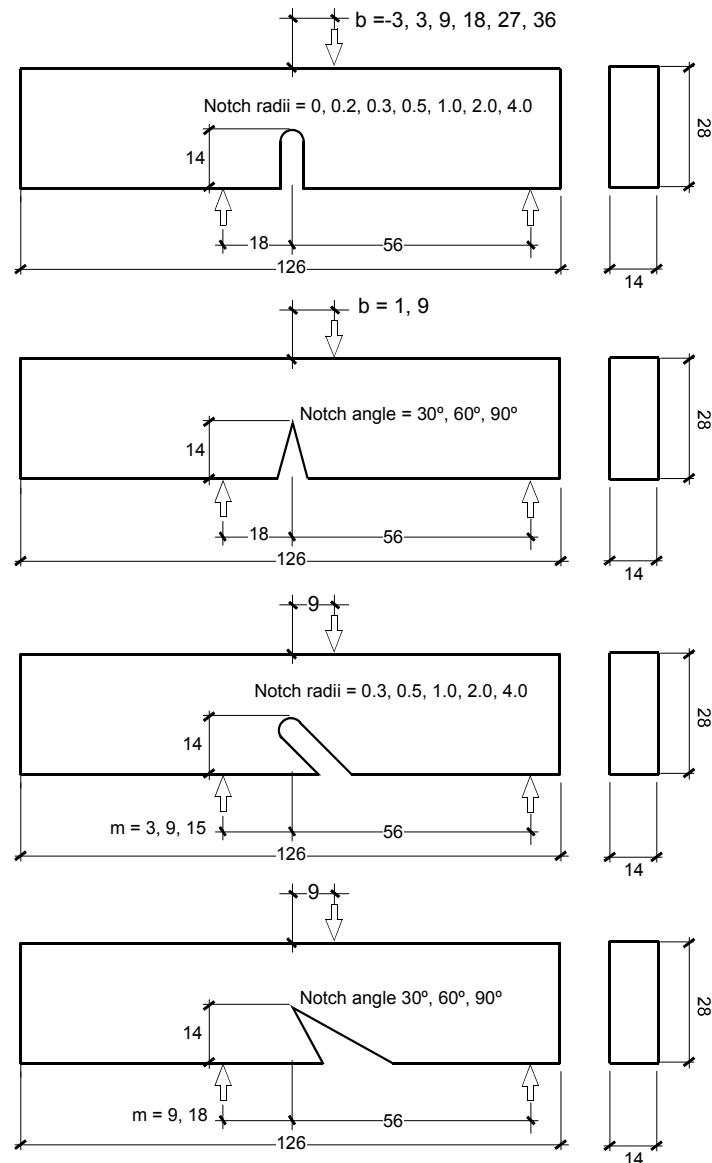


Fig. 1. Geometry and loading conditions. Data in mm

3. Failure criteria

3.1. The averaged strain energy density criterion

The averaged strain energy density criterion assumes that failure starts when the mean value of the strain energy density over a control volume is equal to a critical energy W_c (Yosibash et al., 2004; Lazzarin & Zambardi, 2001; Lazzarin & Berto, 2005). The control volume section is a circle in the case of cracks or sharp V-notches and a crescent shape in U or blunts V-notches (Lazzarin & Zambardi, 2001; Lazzarin & Berto, 2005). In both cases the size is constant, equal to a material parameter, R_c . In order to apply the criterion two independent parameters are needed: the critical value of the strain energy, W_c , and the critical length, R_c . For a linear elastic material these two parameters could be obtained from the ultimate tensile stress f_t and the fracture toughness K_{IC} .

Under mixed mode loading it is assumed the equivalent local mode I approach, that establishes the critical volume is no centered on the notch tip, rather on the point where the principal stress reaches its

maximum (Fig. 2). The crescent shape rotates rigidly, with no changes in shape and size (Gomez et al., 2007; Berto et al., 2007).

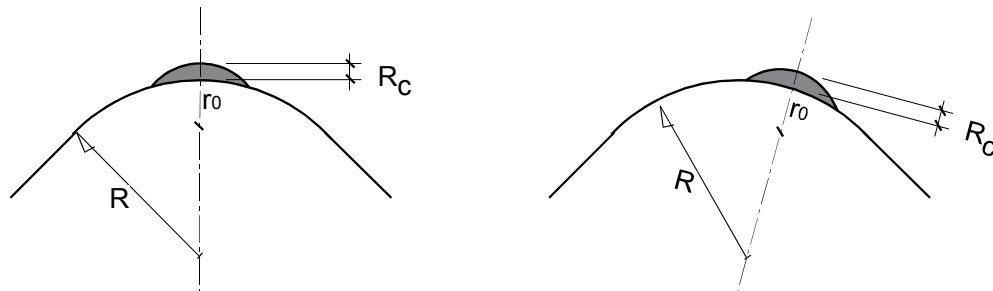


Fig. 2. Control volume under mixed mode loading

Following this criterion, the critical load is obtained when the mean strain energy density reaches the critical value W_c . The notched specimens have been numerically modelled using the finite element method with the commercial code ABAQUS v 6.3. Two calculations were performed for all geometries; the first one to determine the point of the notch edge where the principal stress reached its maximum value; the second one to compute the averaged strain energy density on the control volume (Berto et al., 2007).

3.2. The cohesive zone model criterion

This model establishes two different regions: the cohesive crack zone and the bulk of the body (Bazant & Planas, 1997; Elices et al., 2002). The behavior of the bulk material, PMMA at -60°C , can be approximated by an isotropic linear elastic material, and the behavior of the cohesive crack zone, simplified as a surface, is defined by the relationship between the stress transfer by the crack and the displacement of the lips, defined by the softening curve. In this research, the softening curve is assumed rectangular; this simple curve provides a reasonable estimation of the fracture behavior of PMMA at -60°C (Gomez et al. 2005). The rectangular softening curve depends only on two parameters: the cohesive stress f_t and the fracture energy G_F . The cohesive stress f_t is assumed equal to the tensile strength ($f_t = 128\text{MPa}$) and the fracture energy was obtained from the fracture toughness ($G_F = 480\text{ N/m}$). Under mixed mode loading the position of the cohesive crack zone is initially unknown. This problem could be overcome with the local mode I approach (Gómez et al., 2007), establishing that the cohesive cracks start at the point where the principal stress reaches its maximum value and propagates initially perpendicular to the notch edge. This criterion predicts the mechanical behavior of the notch geometry. Calculations were performed with the freeware finite element code COFE, developed in the Department of Materials Science at the Universidad Politécnica de Madrid (Planas & Sancho, 2007). Details of calculations could be found in (Berto et al., 2007).

4. Results

In order to check the applicability and the degree of accuracy of the strain energy density criterion (SED) and the cohesive zone model (CZM) when applied to fracture behavior assessments of PMMA specimens tested at -60°C under mixed mode loading, the numerical and the experimental values of the maximum load to failure were compared. Fig. 3 compares, for vertical standard U-notched beams, the experimental values of the critical load, with the numerical predictions based on the CZM model and the SED model. There are six figures corresponding to six different mixed load conditions, $b = -3, 3, 9, 18, 27, 36\text{ mm}$. As can be seen, the two models fit with a good accuracy all experimental results.

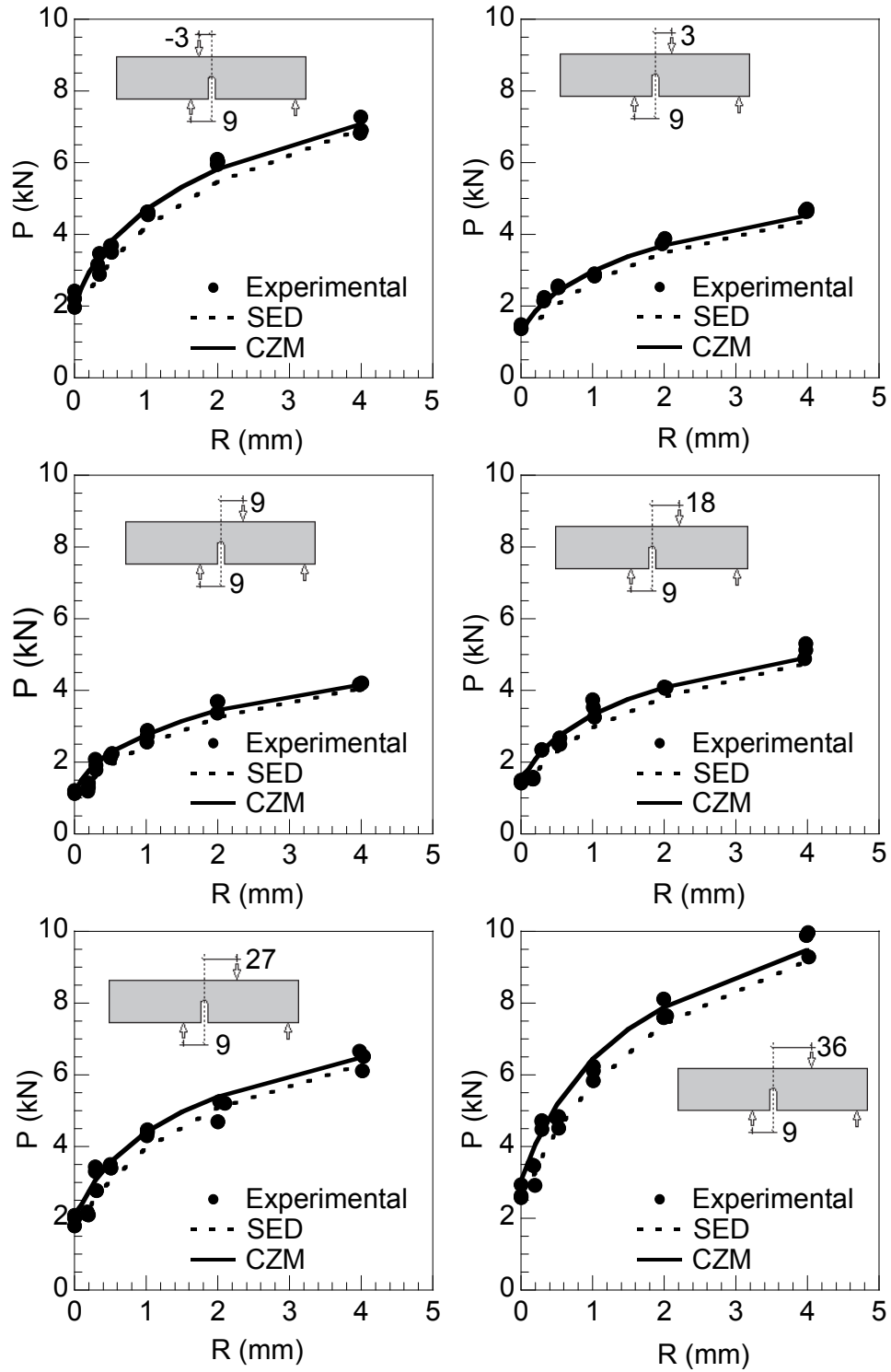


Fig. 3. Experimental and predicted values of the maximum load in standard U-notches

Fig. 4 compares for tilted U-notched beams, the critical loads experimental and numerical. The three figures correspond to three different mixity loading as obtained changing the support span; $m=15, 9, 3$ mm. The same comments, as for Fig. 3, can be drawn.

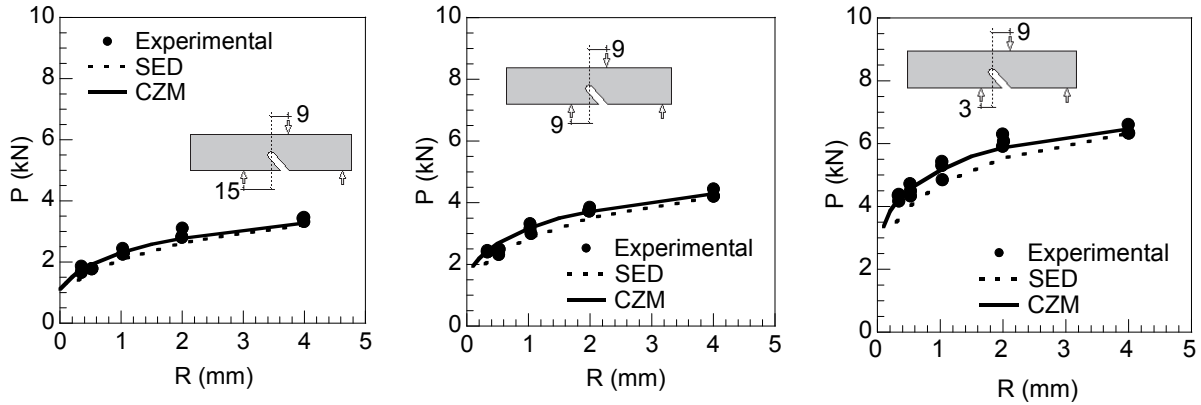


Fig. 4. Experimental and predicted values of the maximum load in tilted U-notches

Dealing with V-notches two limit values of the radius at the tip were introduced in the model; 0 and 0.1 mm. Fig. 5 compares the critical experimental loads and the numerical values for V-notched beams, the upper line corresponding to a radius of 0.1 mm and the lower one to a radius of 0 mm. As can be seen again, the two models fit all experimental results.

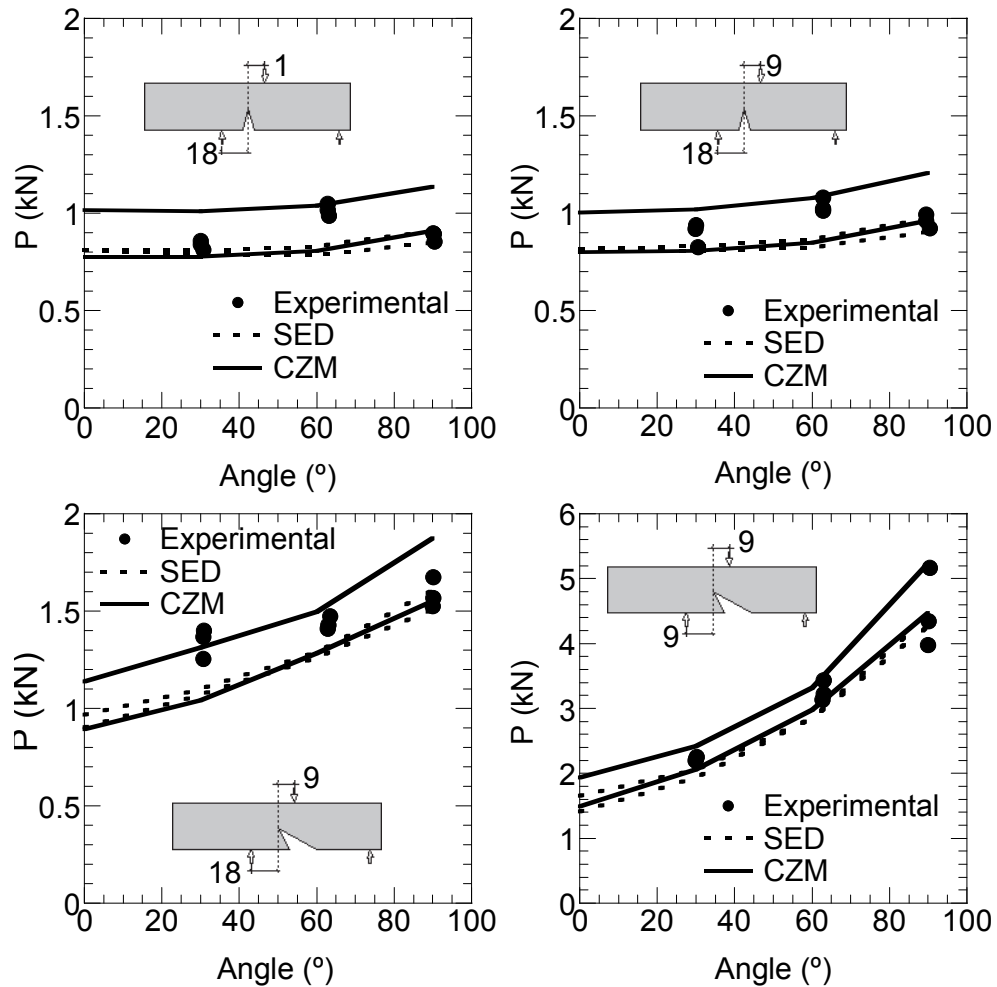


Fig. 5. Experimental and predicted values of the maximum load in V-notches

5. Conclusions

This research provides the results of a huge experimental program of fracture tests carried out on notched specimens under mixed mode (I+II) loading. A total number of 195 fracture tests has been performed by using a linear elastic material, PMMA at -60°C . Different notches, U and V, notch opening angles, notch root radii and loading conditions have been analyzed.

Two models are proposed: the cohesive zone model (CZM) and the strain energy density (SED) over a control volume. Both methods are able to predict the maximum load that a notched specimen can support under mixed mode. The agreement between the numerical predictions and the experimental results is good. This fact confirms the applicability of the two procedures to mixed mode loading conditions, the accuracy being comparable to that already shown under mode I, as documented in previous analyses carried out by the same authors.

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