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Fracture mechanics of cellular structures: past, present, and future directions

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ABSTRACT

This review article aims to provide a greater understanding of on-going research in fracture behavior of additively manufacturable microcellular structures. Despite growing recent investigations in the mechanics of microcellular structures, predominantly on their constitutive behavior and structural optimization, understanding the fracture behavior is still in its infancy, particularly for functionally patterned microcellular structures. While presenting a comprehensive review of the past and current research activities, this paper discusses potential future directions that are necessary to fully cover unexplored areas in this field.

1. Introduction

Wood, cork, and turtle shells are a few of many examples of cellular materials in nature (Ritchie et al., 2009; Novitskaya et al., 2011; Wegst et al., 2015; Naleway et al., 2015; Habibi et al., 2015; Bührig-Polaczek et al., 2016; Liu et al., 2017). The name of cellular material is self-explanatory as they consist of void and solid material (i.e., the matrix) where each void surrounded by the solid structure is called a cell. Fig. 1 depicts some of the natural and artificial (man-made) cellular solids. The simplest structure for such materials is a two-dimensional (2D) array of polygons that together form a plane area (Gibson and Ashby, 1997). The variety of cell patterns is high and some of them are square cells, triangular (such as equilateral or isosceles triangle) cells, honeycombs, etc. and each pattern affects the mechanical properties of the material and may or may not create anisotropy. For instance, when all the sides of a square cell have the same thickness, then the cell is considered isotropic because it shows the same mechanical property both in the x and y directions. However, if the vertical sides of it acquire a thickness more or less than the horizontal sides, then it exhibits different mechanical behavior in the x and y directions and hence, the cell would be anisotropic (cf. Fig. 1 and Fig. 4).

Fig. 1. Some examples of natural and man-made cellular structures: a) wing of a dragonfly, b) honeycomb, c) titanium implants

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Some patterns exhibit a unique behavior in which the Poisson's ratio is negative meaning they considerably experience an increase in volume when stretched and vice versa. These materials are known as auxetic cellular materials (Evans et al., 1991; Novak et al., 2016) and they naturally can be found in pyrolytic graphite and skin (Voigt, 1928; Lees et al., 1991; Yeganeh-Haeri et al., 1992). They have a wide variety of applications such as sports helmets (Sanami et al., 2014), textiles (Alderson and Alderson, 2005; Wang & Hu, 2014), biomedicine (Evans & Anderson, 2000; Scarpa, 2008), etc. (Chan & Evans, 1998; Scarpa et al., 2004; Yang et al., 2004; Yang et al., 2013; Krödel et al., 2014;; Imbalzano et al., 2017; Imbalzano et al., 2018) and the first man-made auxetic structures were used as moderator core of a magnox nuclear reactor (Muto et al,. 1963). Some of the 2D cell patterns that make the material act as such are re-entrant structures (Robert, 1985), structures with rotating units (Grima et al., 2005; Grima et al., 2007), missing rib structure (Smith et al., 2000), chiral structures (Prall & Lakes, 1997), and so on (for instance see (Larsen et al., 1997; Theocaris et al., 1997; Alderson et al., 2010; Cho et al., 2014; Körner and Liebold-Ribeiro, 2014 Shan et al., 2015). Auxetic materials exhibit some enhanced properties like high shear and indentation resistance (Ju & Summers, 2011; Coenen & Alderson 2011). There are two issues that are worth discussing. Firstly, all the mentioned 2D structures can be extended in the normal direction to get a 3D unidirectional auxetic structure, and secondly, composite structures with rotating units and molecular auxetic materials can show similar behavior, so having a porous media to generate auxetic action is not always required (Milton et al., 1992; Grima & Evans, 2000).

In a more realistic pattern, the cells are polyhedral and packed in three dimensions to fill space. The three-dimensional (3D) cellular materials are divided into two different subcategories including closed-celled and open-celled. If the cell edges and faces are sealed off so that the cells are contained to themselves and there is no connection between the voids, the material is said to be a closed-cell. However, if the cell has solid edges and its void is connected to the neighboring cell void through the open face, then it is called open-celled. Obviously, there are some partly open and partly closed cellular materials too (Gibson & Ashby, 1997; Bhate et al., 2019).

The random or periodic repetition of these cells creates a porous medium with extraordinary mechanical properties. Efficient sound and heat isolation, having a high stiffness-to-weight ratio, and good energy absorption are a small fraction of them (Exerowa and Kruglyakov, 1997; Férey, 2008; Burschka et al. 2013; Prabhu et al., 2015; Schaedler & Carter, 2016; Huang et al., 2018; Huang et al., 2020; Liu et al., 2021).

These characteristics have made such materials be used in aircraft, automobiles, energy-efficient buildings, railway industry, tissue engineering, and shipbuilding to name but a few (Fuganti et al., 2000; Banhart, 2001;Freyman et al., 2001; Salimon et al., 2005; Srivastava & Sahoo, 2007; Murr et al., 2010; Smith et al., 2012; Raja & Prabhu, 2013; Heo et al., 2013; Najmon et al., 2018; Hang et al., 2021; Benedetti et al., 2021). One of the most important characteristics of cellular materials is that one can tailor their mechanical property to the desired purpose.

A spatially graded cellular structure is a porous medium with variable density along specific directions due to the uneven distribution of the void and the solid part throughout the material. The existence of such patterns is abundant in nature (Gandy et al., 1999; Michielsen & Stavenga, 2008; Schaedler & Carter, 2016; Benedetti et al., 2021). Theoretically, such patterns have been created in nature during millions of years of evolution so that the total weight of the structure remains low while some of its important mechanical properties of it are enhanced (e.g., strength or energy obsession of the material) (Li et al., 2019). For instance, Fig. 1a shows the wing of a dragonfly and in this figure, the cells are reasonably scattered in a way that performance becomes optimized. These naturally optimized patterns have inspired researchers to design and fabricate some lightweight cellular structures with favorable mechanical properties (Ashby et al., 2000; He et al. 2014; Rahman et al., 2022). Due to the recent development of manufacturing methods such as the emergence of additive manufacturing (AM), the production of desired cellular material with preferred mechanical properties has become drastically more convenient and subsequently, the usage of spatially graded cellular structures has been increased (Li et al., 2018; Wang et al., 2018). Basically, the material properties of cellular solids are mainly reliant on three factors including relative density (porosity), the material of the solid (matrix), and the cell structure (Gibson & Ashby, 1977; Jop et al., 2006; Steinmetz et al., 2013; Cisse et al., 2016). In addition to that, the printing orientation may cause material anisotropies with respect to the printing direction. The layer-by-layer fabrication of additively manufactured components can cause delamination at the interface of layers that may result in failure of the components. It is well-known that cracks may initiate and propagate at the interface of such layers (Ayatollahi and Mirsayar, 2011; Mirsayar, 2013; Arabi et al., 2014; Mirsayar, 2014; Mirsayar et al., 2014; Mirsayar & Park, 2015; Mirsayar & Park, 2016a; Mirsayar, 2019; Mirsayar & Hartl, 2019; Vashisth & Mirsayar, 2020; Mirsayar & Hartl, 2020; Mirsayar, 2021a; Mirsayar, 2021b, Ameri et al. 2020, 2021, 2022a,b, Kafshgar et al. 2021) adversely influencing the material properties of the printed structures.

Since the application of porous materials in the industry is abundant, the fracture study for such materials seems to be of utmost importance (for instance see Torabi et al., 2021b). As is well known, the principles of fracture mechanics are based on continuum mechanics so they cannot be applied to cellular materials as they are because of their discrete nature for sometimes the void can exceed 90% of the material. One strategy to construct a mechanical model with a continuous mass for a cellular medium is using a homogenization method. The goal of homogenization is to construct a homogeneous fictitious material whose global (macroscopic) characteristics are a good approximation of the original material which is microscopically heterogeneous (Cioranescu & Donato, 1999).

Analyzing cellular materials and studying their behavior under different loading conditions require high computational effort. One way to circumvent this arduous task is utilizing the homogenization method where the heterogeneous structure can be replaced by a homogeneous equivalent medium (HEM) in the modeling (Iltchev et al., 2015; Somnic & Jo, 2022). The idea is, that the properties of a heterogeneous material can be determined by analyzing a small portion of it called representative volume element (RVE) (Evans et al., 1998). There are several homogenization methods (Somnic & Jo, 2022) including beam theory approach (Gibson et al., 1982; Masters and Evans, 1996), strain energy equivalence (Hohe & Becker, 1999; Staszak et al., 2021), micropolar theory (Askar & Cakmak, 1968; Chen et al., 2014), asymptotic homogenization approach (AH) (Hassani and Hinton, 1998), Bloch's theorem and Cauchy-Born hypothesis (Phani et al., 2006), multi-Scale homogenization method (Vigliotti, 2014), machine learning approach (Arbabi et al., 2020). Several more homogenization methods for cellular materials have been proposed and in what follows, some of them are mentioned.

Duster et al. (2012) suggested a numerical homogenization method for computing the effective material properties of the cellular materials. By providing various numerical examples, the efficiency of the proposed procedure was examined and verified even for complex microstructure materials. Ma (2016) developed a new homogenization method based on the principle of virtual displacement and the principle of virtual forces. The new method incorporates an engineering insight into the problem instead of being merely a mathematical approach. It was shown that the suggested method is suitable for multiscale analysis problems and more easily can handle design problems and mechanical simulations. Gad et al. (2021) used an extended version of Hill's Lemma based on the micropolar elasticity theory and suggested a new strain energy-based method for homogenizing 2D and 3D cellular materials. Furthermore, closed-form expressions were achieved for calculating effective classical and micropolar stiffness tensors, and their accuracy was examined. Later, they (Gad & Gao, 2021) provided another strain energy-based homogenization method that was capable of being applied to all types of 2D and 3D cellular structures with no geometrical constraints.

Sigmund (1994) proposed a novel procedure, called inverse homogenization, for establishing materials with arbitrary prescribed positive semi-definite constitutive tensors. This method is formulated as a topology optimization problem and the given parameters are homogenized coefficients. The objective is to find the interior topology of a base cell with the lowest weight possible that incorporates the prescribed constitutive parameters. The inverse problem can be solved by a multiple constraint optimality criteria method, in which the constraints are defined by prescribed constitutive parameters and the cost function is the weight of the unit cell to be minimized. This approach has been used in various studies as the base method for the determination of the desired mechanical property (for instance see (Coelho et al., 2016; Cherkaev et al., 2018; Ferro et al., 2020). Gibson and co-workers were the first researchers who conducted an extensive study to understand the mechanical properties of cellular materials (Gibson et al., 1982). They modeled the plastic yield, elastic buckling, and brittle fracture of cellular structures under multiaxial stresses and developed some approximate equations to describe their failure (Gibson et al., 1989) and compared their model to experimental data (Triantafillou et al., 1989). Also, they investigated the response of the cellular materials to multiaxial stress states and presented the constitutive modeling of such materials (for instance see (Triantafillou & Gibson, 1990)). Fracture studies on notched or cracked components are under various loading conditions are abundant in the literature (Torabi & Shahbazian, 2020a,b; Torabi et al., 2021a,b; Shahbazian et al., 2022), However, regarding cellular materials, these investigations have mostly been done under pure mode I and the effect of anisotropy is almost always neglected. Some of the cellular materials fracture studies reported in the literature are presented below.

Huang and Lin (1996) proposed an expression for in-plane brittle fracture toughness in cellular materials and compared the results with the existing data in PVC foams. Their work indicated that the cell geometry, relative density, and the modulus of rupture of solid cell-wall material are the most important factors that the fracture toughness depends on. Chen et al. (1998) developed a 2D generalized continuum model for cellular materials based on a strain gradient model. The proposed analytical method showed that mode I, mode II, and the mixed-mode I/II fracture toughness is inverse proportional to the square root of the cell size and proportional to the thickness of cell walls. Fleck and Qiu (2007) studied several 2D elastic-brittle lattices with different cell patterns by the means of the finite element method and the dependence of mode I and mode II fracture toughness upon relative density was determined for each lattice. Andrews and Gibson (2001) used 2D finite element simulations and studied the influence of cracks, notches, holes, and cell size on the tensile strength of the cellular solid. They reported that in contrast to specimens weakened by a hole or a notch, the results obtained from the cracked specimens were sensitive to the cell size. Mirsayar (2022a) investigated the mixed-mode I/II fracture behavior of 2D lattices with various cell geometries and considered the material anisotropies corresponding to the build orientation. In this study, a new fracture criterion was constructed, and the onset of fracture and the angle of crack propagation were theoretically estimated. Then, for the verification, the results were compared to those obtained from finite element analyses.

Due to the various applications of 3D lattices in the industry such as the core of sandwich panels (Finnegan et al., 2007; Moongkhamklang et al., 2008; Liu et al., 2015), understanding their fracture behavior has been a popular subject of research. For instance, Qian et al. (2011) discussed a 3D lattice fracture model to study the fracture process in a wide range of multiphase materials. The results of the model were the load-displacement diagram showing the constitutive relationship, and the cracks patterns and propagation. Gu et al. (2019) used a brittle fracture criterion and finite element analysis to calculate the material property of the octet-truss lattice fabricated by an isotropic base material. They used various cracked models and concluded that Young's modulus and the fracture toughness of the studied lattice were nearly isotropic, but the strength greatly depended on the lattice orientation. The mechanical responses of the 3D Kagome lattice structure made of titanium alloy by selective laser melting method were studied by Wei et al. (2018). Various material properties were theoretically and experimentally obtained, and the results were discussed and compared to each other. Xu et al. (2019) utilized fused deposition modeling to 3D print lattice materials with different printing directions. Then, their mechanical properties were used as simulation input to foreknow the deformation and fracture behavior under uniaxial tension. Finally, the results were compared to those acquired by experimental tests, and the influence of printing directions was discussed.

Structural efficiency is an important factor in designing and fabricating cellular structures and it can be convincingly achieved by designing hierarchical cellular materials. Recently, topology optimization has become an efficient procedure to accomplish this task. The purpose of utilizing topology optimization in cellular structures is to find an optimal cell distribution within a design domain to gain the desired mechanical property. The following paragraphs briefly address some of the research that optimization is one of the main objectives of the investigation. Zang and Sun (2006) introduced the concept of design element and studied the scale effect of the microstructure on the optimal topology of 2D cellular materials with periodic cell patterns and finally suggested a method to decrease the computational cost. They concluded that the topology design results are greatly affected by the number and the aspect ratio of RVE. One of the natural cellular materials is bone. Khanoki and Pasini (2012) used the homogenization method to obtain the mechanical property of a hip implant both on macro and micro scales. Then, the multi-objective optimization method was utilized to achieve optimum material cell distribution that simultaneously minimizes bone resorption and bone-implant interface stresses. Then, the results obtained from the structure with the optimum pattern were compared to those acquired from a dense titanium implant and the efficiency of the proposed design was validated. Radman et al. (2013a) studied periodic isotropic cellular materials and after using homogenization to calculate the elasticity matrix of the material, they used the bidirectional evolutionary structural optimization technique to determine the optimal cell distribution that maximizes the shear modulus of the material. They examined the efficiency of their proposed methodology by presenting some 2D and 3D examples. Functionally graded materials (FGM) are the subject of many investigations because of their unique mechanical behavior. The objective of creating such materials is to combine the best of different materials to achieve their good performance (Li & Han, 2018). Radman et al. (2013b) proposed a new computationally efficient approach by utilizing topology optimization to design functionally graded materials with maximum shear modulus. To maximize the stiffness of cellular materials with periodic microstructures, Huang et al. (2013) introduced a topology optimization algorithm and the effective properties of the heterogeneous material were acquired by the means of homogenization theory. Liu and Tovar (2013) used homogenization theory to propose a design procedure for attaining 2D optimal macro-scale structures and the corresponding optimal meso-scale periodic material designs with maximum output displacement. They validated the suggested methodology through numerical results.

The nonlinear behavior of cellular materials can be caused by two factors of geometric and material nonlinearity. Studies have shown that topology optimization is greatly dependent on geometric nonlinearity and it can considerably change the design solution (Buhl et al., 2000). Carstensen and Guest (2016) considered cellular materials with nonlinear behavior and maximized their absorbed energy capacity by utilizing topology optimization design and they compared their results to those obtained from numerical analyses. Chen et al. (2018) determined the optimal periodic cell distribution over a cellular structure to achieve extreme material properties by presenting a new topology optimization approach based on the moving iso-surface threshold method. For this purpose, a penalty function was suggested to generate the objective function for designing the material under isotropic symmetric conditions. Also, the macroscopic material property was obtained by utilizing a finite element-based homogenization method. Chen et al. (2018) used a strain energy-based homogenization method to extract the macroscopic nonlinear stress-strain relationship from the microstructure of cellular materials and optimized the microstructure of elastoplastic cellular materials under various macroscopic strain loadings. The objective of this investigation was to maximize the strain energy density of the base cell of the material. By considering a lightweight cellular structure with uniform cell patterns, Qiao et al. (2019), proposed a hybrid algorithm for the topology optimization of such materials. The suggested procedure combined solid isotropic material with penalization and bi-directional evolutionary structural optimization. They validated their results with two numerical examples. Lu and Tong (2021) formulated a method for two-scale topology optimization of cellular structure and its anisotropic material to determine the macro and micro-structural topologies and the orientations of microstructures. The proposed algorithm is able to analytically determine the optimum orientation of the microstructure made of an anisotropic material and at the same time, it can solve both compliant mechanisms and minimum compliance problems.

2. Potential Future Research Directions

It can be deduced from the literature review presented above that the fracture behavior in cellular materials is still not fully understood and most of the fracture investigations are restricted to the pure mode I loading conditions whereas, in reality, such materials are subjected to different and more complex boundary conditions. In addition to that, the material anisotropy and the printing orientation have a huge impact on the fracture behavior of the material, and it causes mixed-mode loading conditions. To the best knowledge of the authors, the investigation on fracture analysis in cellular materials under mixed-mode I/II loading conditions and considering the anisotropic of the material is at its infancy and the related studies are indeed limited in the literature.

Thus, evaluation of the mixed-mode I/II fracture of cellular materials by considering the effects of cell patterns and the printing conditions seems to be a very important and potential direction of future research. Also, developing an optimized cell distribution and thickness is important and may be performed so that the material acquires the highest possible fracture toughness near the stress concentrators. By studying the effect of cell thickness and distribution, an optimized cell scattering may be obtained in which the material shows the highest possible resistance to fracture and at the same time has the same relative density as the un-optimized cellular medium. Extensive experimental and theoretical works need to be performed to study the effects of different geometrical and material factors on the fracture behavior of cellular structures. Herein, four main areas of potential future research are explained separately.

i) Mixed-mode fracture experiments

Having known that most available test data on fracture behavior of cellular structures are for pure mode I only, future research can be focused on mixed-mode experimentations. Different fracture specimens with different cell patterns, cell distributions, and printing orientations can be created by additive manufacturing. Experimental fracture tests can be performed on different specimen types and the effect of mentioned factors on the fracture behavior will be discussed. The results can be used to develop/compare/or verify the theoretical fracture solutions.

ii) Numerical homogenization of cellular patterns

Homogenization methods can be utilized to model virtual homogeneous fracture specimens equivalent to the mechanical properties of the original cellular structure. databases of homogeneous equivalent specimens can be created, and they can be linked to the corresponding manufactured samples. The mechanical properties of the modeled specimens can be acquired by the means of finite element analysis (FEA). This approach, if works properly, can significantly reduce computational costs of the developed fracture models.

iii) Developing mixed-mode fracture criteria and validation

The direction of crack propagation and the fracture toughness of the modeled samples can be calculated theoretically. To this end, suitable mixed-mode fracture criteria can be proposed. Since the recent studies have demonstrated that the processing zone might be highly affected by the mode mixity (Mirsayar, 2021c; Mirsayar and Shahbazian, 2022), the generated fracture criteria can cover this issue. The data obtained from the proposed criteria can be compared to those acquired from experimental tests.

iv) Optimizing the cell patterns for enhanced fracture toughness

As the last suggested, future research, the distribution, and the thickness of cells throughout the structure can be optimized with the goal of achieving the highest fracture toughness possible. The cells in the vicinity of the stress concentrators (i.e., notches, etc.) play the most important role here. Accordingly, the spatially graded pattern is required to obtain the optimized specimen in which the number of the cells and/or their thickness in the structure may gradually change over the surface of the sample, and subsequently the local properties of the material change with it as well. However, the general mechanical behavior will stay the same. In the end, the acquired optimized model together with the corresponding regular specimen may be fabricated and the fracture tests may be performed on them (cf. i). Note that the amount of the solid material used in the optimized and the conventional specimen can be the same and only the distribution and the thickness of the cells would be different. Ultimately, the fracture toughness of both fabricated specimens may be compared to each other and the effectiveness of the specimen with the optimized cell pattern can be explored. This is shown schematically in Fig. 2.

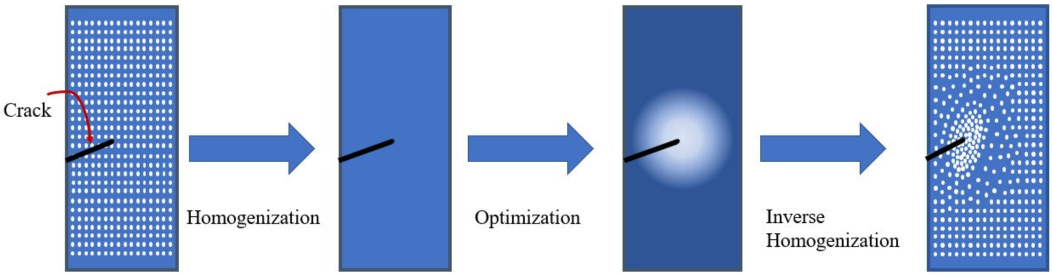

Fig. 2. Optimization of cellular structure for enhanced fracture toughness

3. Conclusions and Suggested Approaches to Address Future Research

The first step toward filling gaps in the field of fracture of cellular structures seems to be fabricating several cellular fracture specimens by using 3D printers and conducting mixed-mode fracture tests. The fabricated samples can have different printing orientations, cell geometry, cell pattern and cell distribution. The material used for the solid part of the cellular medium can be polymers exhibiting brittle behavior at room temperature (like acrylonitrile butadiene styrene also known as ABS). After performing numerous sets of fracture tests, the effect of each mentioned factor on the fracture behavior of the specimens can be individually examined and studied. This allows us to have an experimental vision of the fracture behavior of these samples which helps us to develop sophisticated fracture criteria by considering all observed phenomena. The fabrication of the samples can be started with the simplest cell configuration which is a uniform cell pattern. When the homogenization is done and the fracture models are developed, new sets of experiments can be performed to validate (and enhance, if needed) the proposed fracture model. Cellular topology optimization may be performed and an optimum cellular distribution for achieving the highest feasible fracture toughness may be obtained. Once the optimized cell configuration is known, another series of specimen fabrication can be performed, and new sets of fracture tests can be done. Note that the difference between the optimized specimen and the corresponding regular one can be the cell thickness and distribution throughout the sample, while the cell pattern, the mass of the solid part of the medium, and the printing orientation are the same. Various mixed mode loading conditions can be considered. The mode mixity in such specimens can be controlled by the two factors of the printing orientation and the angle of the crack in the specimen. Fig. 3 illustrates some examples for the specimens and their loading conditions together with the printing orientation.

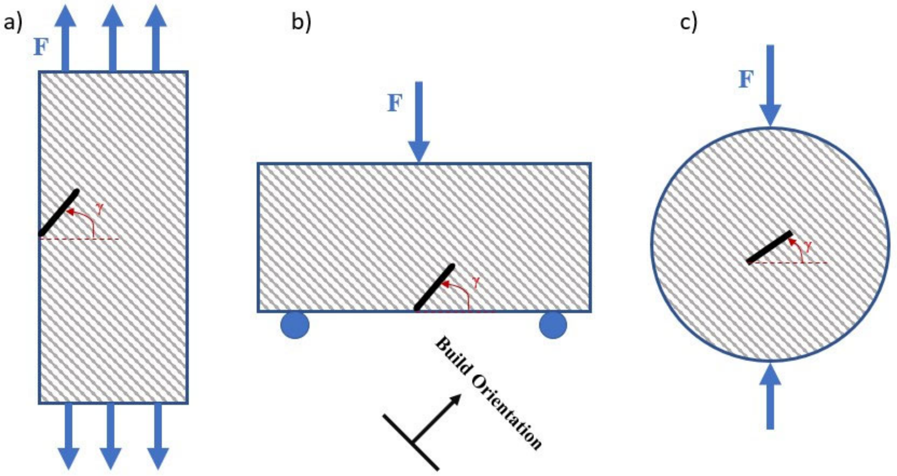

Fig. 3. Some examples for different fracture specimens with their support locations, loading conditions, and the build orientation.

Homogenization can be performed to determine the mechanical properties of the cellular structures. Fig. 4a, shows a cellular structure with uniform square pattern and its corresponding homogenized sample. In this figure, since the material and the voids in the cellular domain are evenly scattered, the equivalent homogenized material has the same density throughout its surface. However, when the cells are not uniformly distributed, like Fig. 4b, the density of the corresponding homogenized sample will change in accordance with the density of the original cellular structure. The inputs for obtaining the homogenized specimen are the mechanical properties of the solid material and the geometry of the cell where they can cause isotropic or anisotropic (cf. Fig. 4c) behavior in the material. In this figure, a and t are the geometric characteristics of the cell and E_i , $i = 1, 2$ are Young's Moduli corresponding to axis along and normal to the printing direction. Note that the inputs may vary based on the chosen homogenization approach.

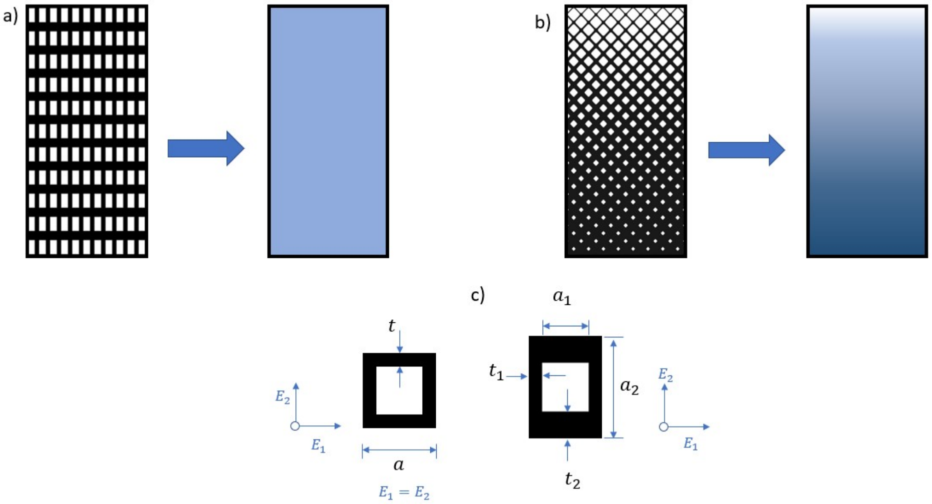

Fig. 4. a and b) Different cellular structures and their corresponding homogenized specimen. c) An example of isotropic and geometrically induced anisotropic cell.

There are many homogenization approaches including the beam theory, multi-scale homogenization method and so on but the machine learning methodology is found to be more interesting. The advantages of the homogenization method based on machine learning algorithms are abundant in comparison to other methods. For instance, in this approach, the computational cost is tremendously low, and it is not limited to cell topology or relative density.

The fracture behavior of the samples made of cellular materials and subjected to mixed-mode I/II loading conditions can be theoretically examined. For this purpose, suitable fracture criteria are needed to predict the fracture angle and the fracture toughness of the cracked specimens. To do so, the mechanical properties (material's constitutive parameters) obtained by homogenization can be utilized in the fracture model. Among many available fracture criteria, strain-based fracture criteria are found to be accurate in prediction of the mixed-mode fracture behavior of many engineering materials (Mirsayar, 2015, 2017,2018; Aliha et al., 2017; Mirsayar and Park, 2016b; Mirsayar et al., 2018a, b). According to a brittle fracture criterion, the onset of the fracture happens when at a certain distance in front of the crack tip, called critical distance ( ), the mentioned function reaches its critical value. Regardless of the loading conditions, it is considered to be constant and is widely recognized as the material property measured under pure mode I conditions. However, recent studies ((Mirsayar, 2021c; Mirsayar and Shahbazian, 2022)) have shown that such an assumption may not be accurate because the critical distance might considerably change when the fracture mode changes. To overcome this inaccuracy, the concept of effective critical distance (ECD) has been proposed recently (Mirsayar, 2021c) and the results achieved by it seem promising. Other fracture criteria include peridynamic (Mirsayar, 2022b), and phase-field approaches (Kuhn and Müller, 2010).

Obtaining an optimized medium with the maximum fracture toughness is an interesting future research in this field. Once the optimization for maximized fracture toughness is done, inevitably a functionally graded material (FGM) will be obtained. FGMs are advanced materials in which the material properties will change in a specific direction (Naebe and Shirvanimoghaddam, 2016). Similar to the artificial neural network that is used to emulate the human brain, the idea of FGMs is copied from nature to solve engineering problems with more efficient materials (Bohidar et al., 2014). Some of the natural FGMs are bones, bamboo trees and so on (Knoppers et al., 2005). Fig. 4b is a good example of a modeled FGM in which the density of the material and subsequently the mechanical properties of it gradually changes along a specific direction. Obviously, in this part, the developed fracture theory can be examined for this case too and the behavior of the crack can be investigated. The mentioned optimized spatially varying cell pattern can be obtained from the optimized homogenous FGM by the means of inverse homogenization. In this method, the cost function is to minimize the weight, the constraints are the given constitutive parameters, and the design variables are the topology of the material (Sigmund, 1994). Since, in this case the desired mechanical properties (constitutive parameters) of the material for enhanced fracture toughness is known through the FGM, the topology of the cellular medium can be acquired.

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