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The Equivalent Material Concept: Application to failure of O-notches

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ABSTRACT

The novel equivalent material concept, proposed originally by the author, was utilized together with the mean stress and the point stress failure concepts to predict the load-carrying capacity of O-notched ductile steel plates under pure tension. Unlike for V and U-notches, it was found that the point stress criterion combined with the equivalent material concept could estimate successfully the limited available experimental results reported in literature regarding four O-notched plates made of very ductile steel. By using the model, one may predict well the onset of tensile crack initiation in O-notched ductile components without requiring performing experiments or elastic-plastic analysis.

1. Introduction

Notches are employed in engineering components and structures because of their special design requirements. Various shapes of notches can be seen in structural elements that a large number of which are V, U and O-shaped. For example, one can remember the V and U-shaped threads in screws and O-shaped holes in riveted and bolted connections. Between all of the notch features, O-notches are probably the most widespread in mechanical engineering design. An O-notch is, in fact, a discontinuity which concentrates stresses around it and makes the notched component vulnerable to failure. It may also decrease dramatically the load-bearing capacity of the notched member depending upon the degree of stress concentration.

Depending on the material properties, different failure modes can be recognized in notched components, including O-notched ones, under static and monotonic loading conditions. For brittle materials, the yield area is not created around the notch during loading and hence, the notched

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component withstands elastically against the applied load till final fracture. In such conditions, the emanation of crack from the notch border consumes a large portion of the total fracture energy and the crack propagation is very less contributed in energy consumption. This is because the crack propagation is such a rapid and unstable phenomenon that the final fracture happens abruptly. Ductile materials, however, experience relatively large plastic deformations around the notch before crack initiation from the notch border. Both the crack emanation and growth consume significant amount of fracture energy during ductile rupture.

Although yielding is the most popular failure criterion in the design of ductile mechanical elements, there are many engineering applications for which the ultimate load that the ductile component can sustain is considered in the design. Fracture analysis of the mechanical elements containing stress concentrators like cracks and notches are usually performed by using the fracture mechanics. An important branch of the fracture mechanics namely the notch fracture mechanics (NFM) focuses on the failure of notched components. Because failure in notched brittle members is always catastrophic, most of the researchers have focused their investigations on the brittle fracture.

Brittle fracture has been frequently investigated in cracked domains by several researchers that a large number of which have been done by Ayatollahi and his co-researchers based on the generalized maximum tangential stress (GMTS) criterion (e.g. Ayatollahi et al. 2006, 2011a, Aliha & Ayatollahi 2008,2009; Ayatollahi & Aliha, 2008a,b, 2009a,b; Ayatollahi & Sistaninia, 2011; Saghafi et al., 2010). In notched domains, most of the researches have been focused on the brittle fracture of V and U-shaped notches. Various failure models can be found in open literature to estimate brittle fracture in V and U-notched elements that almost all of them have been developed based on the linear elastic fracture mechanics (LEFM). For example, one can see Sih and Ho (1991) that presented based on the critical energy density theory and those references on the basis of the local strain energy density and the cohesive zone concepts (see e.g. Ayatollahi et al., 2011b; Berto & Lazzarin, 2009; Ellyin & Kujawski, 1989; Glinka, 1985; Gomez & Elices, 2003a, 2003b, 2004; Gomez et al., 2000, 2008, 2009a, 2009b; Lazzarin et al., 2009; Lazzarin & Zambardi, 2001). Several papers have been published in recent years by Ayatollahi and Torabi for predicting the onset of sudden fracture in V and U-notched specimens under in-plane loading conditions. For instance, one can refer to Ayatollahi and Torabi (2010a, 2010b, 2010c, 2011a, 2011b) and Ayatollahi and Torabi (2009, 2010d) for V-notched and U-notched domains, respectively. Moreover, a combined experimental and theoretical investigation has been performed by Ayatollahi et al. (2011) on the brittle fracture of engineering components containing a sharp V-notch. As the author is aware, brittle fracture in engineering components or test specimens containing O-notches has not yet been investigated. However, fracture in O-notched ring-shape specimens containing pre-existing angled cracks has been studied by Aliha et al. (2008) using GMTS model.

Unlike brittle fracture, the number of researches in open literature dealing with fracture in ductile metallic materials containing cracks and notches under static and monotonic loading conditions is very limited. For instance, J-integral has been employed by Smith et al. (1998) to estimate brittle mode I, brittle-ductile mixed mode and ductile mode II fracture in the cracked specimens made of rotor steel. J-integral has also been evaluated under elastic-plastic conditions by Berto et al. (2007) as a governing parameter in fracture assessment of U and V-notched components made of ductile materials obeying a power-hardening law. Susmel and Taylor (2008a) predicted the load-bearing capacity for a type of ductile steel containing notches of different features (e.g. O-notches) by using the theory of critical distances (TCD) under pure tensile loading conditions. Their tested material has been a commercial cold-rolled carbon steel exhibiting very ductile behavior. The notched specimens tested by them showed large plastic deformations around the notches after fracture (Susmel & Taylor, 2008a). They predicted the maximum load that each notched specimen can sustain by performing linear elastic and elastic-plastic stress analyzes in conjunction with the use of the theory of critical distances (TCD) with a maximum discrepancy of about 15%. They have clearly stated in their paper (Susmel & Taylor, 2008a) that the good accuracy of TCD in the presence of large plastic deformations around notches is questionable. Although the experimental results reported in Susmel and Taylor (2008a) have been in a good agreement with the results of TCD, its application in engineering design together with linear elastic analysis cannot be prescribed from the viewpoint of fracture mechanics principles. A set of elastic-plastic analyzes have also been performed in Susmel and Taylor (2008a) accompanied by TCD to predict the load-bearing capacity of the O-notched specimens under pure tension which have finally resulted in satisfactory consistency with experimental results. Since elastic-plastic analyzes in the engineering design process are rather time-consuming and relatively complicated with respect to elastic ones, the author attempted to suggest a simple failure model to be conveniently used in predicting the crack emanation from O-notches in ductile materials under tension. In the author's most recent work (Torabi, 2012), the tensile load-bearing capacity of several V-notched specimens made of ductile commercial steel has been estimated by using the combined mean stress (MS) criterion and the equivalent material concept (EMC), i.e. the MS-EMC model. A very good agreement has been shown to exist between the experimental and theoretical results (Torabi, 2012). Recently, the MS-EMC model has been successfully employed by Torabi (2013) in a more applied work to predict the tensile load-bearing capacity of ductile steel bolts containing V-shaped threads. More recently, the ultimate bending strength of very ductile steel plates weakened by U-notches has been successfully evaluated by means of the MS-EMC criterion (Torabi, 2013b).

In this research, both the mean stress (MS) and the point stress (PS) failure concepts were utilized combined with EMC for predicting the tensile load-bearing capacity (*Note: this parameter is probably the most important item in mechanical design for monotonic loading conditions*) of a few O-notched specimens made of very ductile steel. The results showed that the PS-EMC model could be able to estimate the experimental results with a mean discrepancy of about 9% demonstrating the effectiveness of the model. While the MS-EMC has been shown in Torabi (2012, 2013) to be a very successful failure model for V-notched ductile components, it was found that this model with a discrepancy of 16% might not an appropriate failure model for ductile engineering elements containing O-shaped notches.

In the forthcoming sections, first, some experimental results reported in literature regarding tensile load-bearing capacity of O-notched ductile specimens are described and then, the MS-EMC and the PS-EMC models are explained and used to predict the experimental results. Finally, the theoretical and experimental results are compared and the accuracies of the failure models are evaluated.

2. Tensile test results on O-notched ductile specimens

Some experimental results have been reported in Susmel and Taylor (2008a) dealing with tensile tests on a few O-notched ductile specimens. The specimens have been four samples made of a type of ductile commercial cold-rolled low-carbon steel, namely En3B, and tested under uni-axial tension (Susmel and Taylor, 2008a). The mechanical properties of the ductile steel are presented in Table 1 (Susmel and Taylor, 2008a). The value of K_{Ic} has been determined experimentally by means of testing the C(T) specimens of 85 mm thick in accordance with the ASTM E399 (1990). Fig. 1 represents the tested O-notched specimens, schematically (Susmel and Taylor, 2008a).

Table 1. The mechanical properties of the En3B steel (Susmel and Taylor, 2008a).

σ_u (MPa)	σ_Y (MPa)	E (GPa)	K_{Ic} ($MPa\sqrt{m}$)	K (MPa)	n	σ_f	ε_f
638.5	606.2	197.4	97.4	882.7	0.06	851.8	0.56

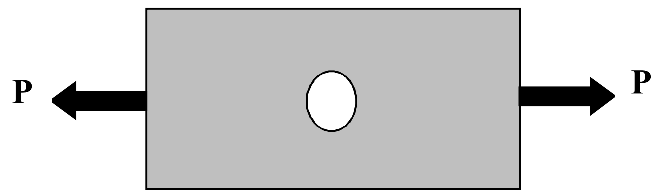

Fig. 1. The tested O-notched specimens (Susmel and Taylor, 2008a)

The length, width and thickness of the specimens have been 130, 25 and 6 mm, respectively (Susmel and Taylor, 2008a). The diameter of the central O-notches has been equal to 6 mm. Four specimens have been totally tested. The experimental values of the maximum loads that the specimens could sustain are presented in Table 2 (Susmel and Taylor, 2008a).

Table 2. The load-bearing capacity of the tested O-notched samples (Susmel and Taylor, 2008a)

Specimen	1	2	3	4	Average
Load-carrying capacity	83 (*kN*)	82.3 (*kN*)	84.6 (*kN*)	84.5 (*kN*)	83.6 (*kN*)

In the next section, the equivalent material concept (EMC) (Torabi, 2012) which equates a ductile material with a virtual brittle one from the view point of crack initiation is introduced.

3. The equivalent material concept (EMC)

Although the equivalent material concept (EMC) has been elaborated in Torabi (2012), the author wishes to describe it again in this section to make use of this manuscript more convenient for the readers. By using EMC, one can hypothetically consider in failure studies a virtual brittle material exhibiting linear elastic behavior instead of the ductile material with elastic-plastic behavior. Therefore, brittle fracture criteria may be employed to investigate the fracture phenomenon in ductile materials.

According to the EMC, the strain energy density (i.e. the area under the stress-strain curve in uni-axial tension) for the existing ductile material is assumed to be equal to that for a virtual brittle material having the same modulus of elasticity. The strain energy density (SED) is, in fact, the strain energy absorbed by a unit volume of material. For a ductile material with considerable plastic deformations and with exhibiting power-law strain-hardening relationship in the plastic zone, one can write

$$\sigma = K\varepsilon_P{}^n \tag{1}$$

In Eq. (1), σ and ε_p are the stress and the plastic strain, respectively. K and n denote also the strain-hardening coefficient and exponent. Fig. 2 reveals schematically a sample engineering tensile stress-strain curve for a typical ductile material. Some of the parameters presented in Table 1 are represented in Fig. 2.

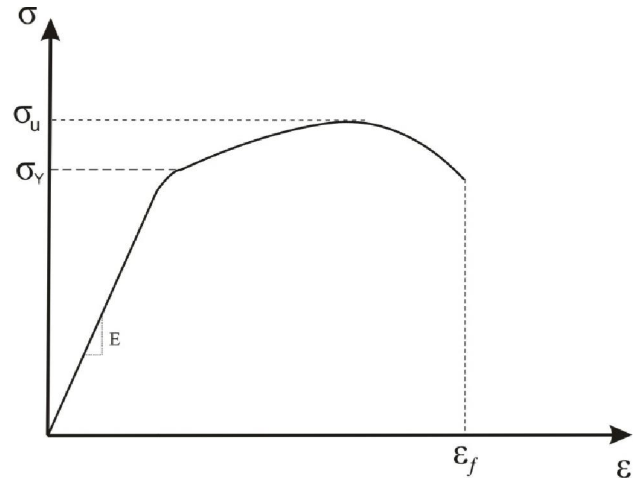

Fig. 2. A sample engineering tensile stress-strain curve for a typical ductile material

In Fig. 2, E, σ_Y, σ_u and ε_f denote the elastic modulus, the yield strength, the ultimate tensile strength and the engineering strain to rupture, respectively. The total SED can be written in a general form of elastic-plasticity as

$$(\mathrm{SED})_{\mathrm{tot.}} = (\mathrm{SED})_{\mathrm{e}} + (\mathrm{SED})_{\mathrm{p}} = \frac{1}{2}\sigma_{\mathrm{Y}}\varepsilon_{\mathrm{Y}} + \int_{\varepsilon^{p}{}_{\mathrm{Y}}}^{\varepsilon_{\mathrm{p}}} \sigma \, d\varepsilon_{\mathrm{p}} \tag{2}$$

Substituting $\varepsilon_Y = \dfrac{\sigma_Y}{E}$ and Eq.1 into Eq. 2 gives

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_{\mathrm{Y}}{}^{2}}{2\mathrm{E}} + \int_{\varepsilon^{p}{}_{\mathrm{Y}}}^{\varepsilon_{\mathrm{p}}} K\varepsilon_{P}{}^{n} d\varepsilon_{\mathrm{p}} \tag{3}$$

Thus

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_{\mathrm{Y}}{}^{2}}{2\mathrm{E}} + \frac{K}{n+1}(\varepsilon_{P}{}^{n+1} - \varepsilon^{p}{}_{\mathrm{Y}}{}^{n+1}) \tag{4}$$

If $\varepsilon^{p}{}_{\mathrm{Y}}$ is considered to be equal to 0.002 (corresponding to 0.2% offset yield strength), then

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_{\mathrm{Y}}{}^{2}}{2\mathrm{E}} + \frac{K}{n+1}(\varepsilon_{P}{}^{n+1} - (0.002)^{n+1}) \tag{5}$$

In order to calculate the total SED corresponding to the onset of crack initiation, one can replace ε_p in Eq.5 with $\varepsilon_{u,\mathrm{true}}$; i.e. the true strain at maximum load; which could be obtained by recording the length of the gage section of the standard tensile test specimen at maximum load (one can utilize simply $\varepsilon_{u,true} = \ln(l_u / l_0)$ where l_0 and l_u are the initial length and the length of the gage section at maximum load, respectively.).

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_{\mathrm{Y}}{}^{2}}{2\mathrm{E}} + \frac{K}{n+1}(\varepsilon_{u,true}{}^{n+1} - (0.002)^{n+1}) \tag{6}$$

The equivalent material considered in EMC is a virtual brittle material with the same values of the elastic modulus E and the plane-strain fracture toughness K_{Ic}, but unknown value of ultimate tensile strength. Fig. 3 shows schematically a sample uni-axial stress-strain curve for the virtual brittle material.

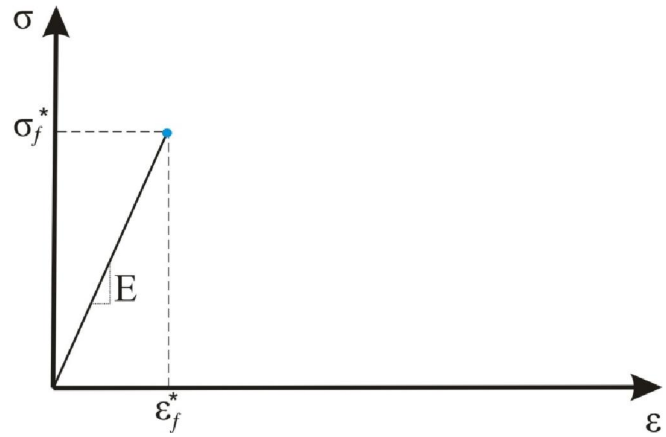

Fig. 3. A sample uni-axial stress-strain curve for the virtual brittle material

In Fig. 3, the parameter ε_f^* and σ_f^* are the strain at crack initiation (i.e. the final fracture due to the brittleness) and the ultimate tensile strength, respectively. The SED for this material at the onset of crack initiation is

$$(\text{SED})_{\text{EMC}} = \frac{\sigma_f^{*2}}{2\text{E}} \tag{7}$$

According to the requirements of EMC, SED values for both the real ductile and the virtual brittle materials must be equal. Therefore, Eqs. (6-7) are equal. Therefore,

$$\frac{\sigma_f^{*2}}{2\text{E}} = \frac{\sigma_{\text{Y}}^{2}}{2\text{E}} + \frac{K}{n+1}(\varepsilon_{u,true}{}^{n+1} - (0.002)^{n+1}) \tag{8}$$

Finally, σ_f^* can be obtained as

$$\sigma_f^* = \sqrt{\sigma_{\text{Y}}{}^2 + \frac{2EK}{n+1}(\varepsilon_{u,true}{}^{n+1} - (0.002)^{n+1})} \tag{9}$$

The parameter σ_f^* presented in Eq. 9 may be utilized together with K_{Ic} in any brittle fracture models to estimate the onset of crack initiation in notched ductile components.

In the two next sections, the mean stress (MS) and the point stress (PS) brittle fracture concepts are briefly explained. The combination of the EMC with the MS and the PS criteria is then utilized in forthcoming sections to predict the experimental results regarding tensile load-bearing capacity of O-notched ductile specimens.

4. The mean stress (MS) criterion

One of the most well-known brittle fracture criteria in notched domains under pure mode I loading conditions is the mean stress (MS) criterion previously utilized by several researchers. This fracture model has been successfully employed in the past by Strandberg (2002) for predicting brittle fracture in sharp V-notches and by Ayatollahi and Torabi (2010a, 2010b) in rounded-tip V-notches. According to the MS criterion, fracture takes place in any notched component when the mean value of the tensile stress over a specified critical distance ahead of the notch tip/border attains a critical value. The effective critical distance of MS model for V-notches has been presented in Ayatollahi and Torabi (2010a, 2010b) equal to

$$d_{c,V} = \frac{2}{\pi}(\frac{K_{Ic}}{\sigma_c})^2 \tag{10}$$

The parameter σ_c is the critical value of the tensile stress, which is usually assumed to be equal to σ_u for brittle materials (Ayatollahi & Torabi, 2010a, 2010b). Eq. (10) implies that the critical distance is a material property and it depends on none of the geometry of notched component and the type of loading (i.e. mode I, mode II or mixed mode I/II). Based on this assumption, the critical distance for O-notches was also considered to be equal to Eq. (10).

A simple MS-EMC procedure for estimating the tensile load-bearing capacity of the O-notched ductile elements can be presented as follows:

1. Determine K_{Ic} from experiment (Susmel and taylor, 2008a) and σ_f^* from Eq. (9).

2. Substitute K_{Ic} and σ_f^* (instead of σ_c) into Eq. (10) and compute the critical distance.

3. Create an appropriate finite element (FE) model for the notched component and apply the existing boundary conditions and a unit load to it. Note that the elements near the notch border must be fine enough to cover the high stress gradient.

4. Compute the mean value of linear elastic tensile stress for the unit load over the critical distance determined in the step 2 on the notch bisector line.

5. Increase the applied load till the mean stress value over the critical distance attains σ_f^*. The corresponding load is in-fact, the tensile load-bearing capacity of notched component.

5. The point stress (PS) criterion

Probably, the point stress (PS) criterion is the most ancient failure model in the context of brittle fracture of cracked bodies. This criterion with different names (e.g. the maximum tangential stress (MTS) criterion) has been frequently used by several researchers to predict the onset of brittle fracture in notched components under mode I (Ayatollahi & Torabi, 2010a), mode II and mixed mode I/II (Ayatollahi & Torabi, 2009, 2010c, 2010d, 2011a, 2011b) and also mixed mode I/III (Susmel and Taylor, 2008b). According to PS criterion, fracture occurs in notched component when the tensile stress at a certain critical distance ahead of the notch tip/border attains a critical value. The effective critical distance of PS model for V-notches has been reported in Ayatollahi and Torabi (2010a) equal to

$$r_{c,V} = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_c}\right)^2 \tag{11}$$

Eq. (11) implies obviously that the critical distance is also a material property. The critical distance of PS model for O-notches was considered to be equal to Eq. (11).

A PS-EMC procedure to predict the tensile load-bearing capacity of ductile components containing O-notches can be described as follows:

1. Determine K_{Ic} from experiment (Susmel and taylor, 2008a) and σ_f^* from Eq. (9).

2. Substitute K_{Ic} and σ_f^* (instead of σ_c) into Eq. (11) and compute the critical distance.

3. Create an appropriate finite element (FE) model for the notched component and apply the existing boundary conditions and a unit load to it. Note again that the elements near the notch border must be fine enough to cover the high stress gradient.

4. Calculate the linear elastic tensile stress for the unit load at the critical distance determined in the step 2 on the notch bisector line.

5. Increase the applied load till the stress value at the critical distance attains σ_f^*. The corresponding load is in-fact, the tensile load-bearing capacity of notched component.

In the next section, the theoretical results of the MS-EMC and the PS-EMC models are compared with the experimental results reported in Susmel and Taylor (2008a) and described in section 2.

6. Results

The theoretical result of the MS-EMC and the PS-EMC models in predicting the load-bearing capacity of the tested O-notched specimens are presented in Table 3 together with the average value of the experimental results. Also, presented in Table 3 is the discrepancy between the theoretical and the mean experimental results. As seen in Table 3, the discrepancy of the PS-EMC model is about 9% demonstrating the effectiveness of the model. However, the accuracy of the MS-EMC model seems to be unsatisfactory. This result suggests that unlike for V-shaped notches (Torabi, 2012, 2013), the MS-EMC model is not a suitable failure theory for O-notches.

Table 3. The theoretical results of the MS-EMC and the PS-EMC models together with the mean value of the experimental results with the discrepancies included

	Load-bearing capacity *(kN)*	Discrepancy (%)
PS-EMC	91	9
MS-EMC	97	16
Mean experimental	83.6	------

7. Discussion

The major finding of this research is perhaps the inefficiency of the MS-EMC criterion for O-notches. It has been previously demonstrated in the literature that the MS failure criterion is a very successful theory in predicting the onset of tensile fracture in brittle components containing V-shaped notches (see e.g. Ayatollahi and Torabi, 2010a, 2010b). It has also been shown in the author's most recent work (Torabi, 2012) that the MS criterion combined with the EMC (i.e. the MS-EMC model) is successful in estimating the tensile load-bearing capacity (i.e. the crack initiation from the notch tip) of V-notched specimens made of ductile steel. An unexpected result was found in this investigation from which the MS-EMC model was not successful for O-notched ductile components. This means that despite the mechanical behavior of material, the accuracy of each failure criterion depends basically on the notch feature and hence on the stress distribution around the notch border. Consequently, different notch shapes require different investigations.

Eqs. (10-11) clearly implies that the parameter K_{Ic} is necessary to compute the critical distances. This parameter is essential in not only the MS-EMC and the PS-EMC criteria but also in almost all of the criteria in brittle fracture context. If K_{Ic} is not valid for a ductile material, both the criteria will no longer be valid to be employed. As stated in Torabi (2012, 2013) and according to the ASTM E399 (1990), for metallic materials with yielding near the crack tip, the ratio of the maximum load to the load recorded at the end of the linear portion of the load-displacement curve during plane-strain fracture toughness test should not exceed 1.1. This suggests that if the values of σ_Y and σ_u are considerably different, the K_{Ic} test would not probably be valid. In such cases, stable crack growth is usually seen during the test and ductile rupture finally occurs. For this type of metallic materials, one cannot use K_{Ic} as a governing fracture parameter and must utilize instead the ductile fracture parameters such as J-integral, crack tip opening displacement (CTOD), crack tip opening angle (CTOA), R-curve etc. As a result, it seems that both the models described above could be used to study the mode I fracture in V-notched metallic domains only when the yield and the ultimate strengths of material are adequately close together (like En3B steel reported in Susmel and Taylor (2008a)). Another important parameter to successfully achieve the K_{Ic} parameter is the specimen thickness. According to ASTM E399 (1990), the thickness of C(T) specimen should be greater than the reference thickness $t_r = 2.5(K_{Ic}/\sigma_Y)^2$ in order to gain valid K_{Ic} (t_r for En3B steel is about 65 mm). Thus, for those ductile metallic materials having σ_Y and σ_u adequately close together, it is expected that selecting a specimen thickness greater than t_r will result in valid K_{Ic} value. Consequently, the strain to rupture value (i.e. ε_f) for such ductile materials is not an important parameter to identify the type of fracture either being stable ductile or unstable brittle fracture. As a conclusion from the above statements, it can be said that the MS and the PS criteria in conjunction with the equivalent material concept (i.e. MS-EMC and PS-EMC) can be utilized in both small-scale and large-scale yielding conditions.

As stated in ASTM E399 (1990), the plane-strain fracture toughness test may be valid for metallic materials if the specimen thickness is selected to become larger than $t_r = 2.5(K_{Ic}/\sigma_Y)^2$. The value of t_r for the ductile steel reported in Susmel and Taylor (2008a) (i.e. the En3B grade) is calculated to be equal to about 65 mm. Therefore, the thickness selected in Susmel and Taylor (2008a) has been equal to 85 mm for C(T) specimens. The thicknesses considered for the notched specimens in Susmel and Taylor (2008a) (also used in the present work) have been equal to 6 and 25 mm. Since both values are lower than 65 mm, the plane-strain fracture conditions are not completely satisfied and the K_{Ic} value should be modified to K_c values when applied to these notched specimens. As well-known, K_c depends basically on the specimen thickness. Because K_c values for En3B steel have not been reported in Susmel and Taylor (2008a) as a function of thickness, K_{Ic} has been utilized instead of K_c in both the MS-EMC and the PS-EMC models.

The TCD failure criterion presented in Susmel and Taylor (2008a) for fracture analysis of O-notched components made of the ductile steel En3B has been used based on the linear elastic stress analysis while the fracture phenomenon has been preceded by large amount of plastic deformations around the notch tip (Susmel and Taylor, 2008a). According to Susmel and Taylor (2008a), the successful predictions of linear elastic TCD are surprising and hence more studies are needed to justify the applicability of this criterion. In the author's opinion, although TCD results have been satisfactory, the justification of the capability of using a failure criterion developed fundamentally based on the linear elastic fracture mechanics (LEFM) in situations having large plastic deformations (like for EN3B steel) is very difficult. This major shortcoming of TCD does not exist for MS-EMC and PS-EMC criteria, because the ductile metallic material is initially simulated with a virtual brittle material with an ideal linear elastic behavior from beginning of loading up to final breakage. Beside linear elastic calculations, TCD has also been employed in Susmel and Taylor (2008a) accompanied by elastic-plastic analyzes for predicting the load-bearing capacity of the O-notched specimens and its good results have been revealed. However, it should be noted that the elastic-plastic analysis based on the finite element modeling is very time-consuming and relatively complicated in comparison with the linear elastic analysis. Therefore, linear elastic calculations are generally preferred in real engineering applications. Consequently, it can be stated from the viewpoint of engineering design that the accurate PS-EMC failure criterion is very easy and convenient to use, because the corresponding analysis is completely linear elastic. Despite accuracy, the major advantage of the PS-EMC criterion is that it provides a short and justifiable path to estimate the tensile load-bearing capacity of O-notched components having large plastic deformations around the notch border without requiring considering the fracture mechanism and performing elastic-plastic stress analysis. It is necessary to note that similar to TCD, the PS-EMC criterion can be utilized only to estimate the tensile load-bearing capacity of O-shaped notches in ductile materials and cannot identify the type of fracture and also cannot study the propagation of cracks initiated from the notch tip.

It should be reminded that the equivalent material concept simulates the ductile material with a virtual brittle one. The brittle fracture criteria (e.g. MS and PS) are applied to the virtual brittle material after this simulation. Since the mode I failure of O-notched specimens was considered in this research, the EMC was used together with the MS and PS criteria. It is expected that the EMC can be used accompanied by different failure criteria in the context of brittle fracture to estimate the load-bearing capacity of O-notched components made of ductile materials and loaded under mode I, Mode II, mixed mode I/II and also mode III contributed conditions. It is necessary to notice that although the very good accuracy of the PS-EMC criterion in estimating the tensile load-bearing capacity of a few O-notched steel samples was revealed, the applicability of this model should be examined for similar ductile metallic materials and also non-metallic ductile materials having valid K_{Ic} values.

8. Conclusions

The point stress (PS) criterion, proposed basically for predicting the mode I fracture toughness of cracked and notched components made of brittle materials, was successfully utilized combined with the equivalent material concept (EMC) in order to estimate the tensile load-bearing capacity of a few O-notched specimens made of a type of ductile steel (the accuracy was more than 90%). Unlike for V and U-notches, the MS-EMC criterion was unexpectedly found to be inefficient for O-notches. This suggests that despite the mechanical behavior of material, the accuracy of each failure criterion depends basically on the notch feature and hence on the stress distribution around the notch border.

The PS-EMC model is valid for those metallic materials having valid K_{Ic} values or at least valid K_c. Therefore, ductile rupture cannot be analyzed by this criterion. The PS-EMC criterion may be valid for ductile materials with both small and large-scale plastic deformations. This fact was demonstrated in this work for O-notched specimens with considerable yielding area around the notches. The applicability of the PS-EMC criterion in practical engineering design is very convenient since it does not require time-consuming elastic-plastic analysis.

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