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Reliability analysis of two unit parallel repairable industrial system

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A B S T R A C T

The aim of this work is to present a reliability and profit analysis of a two-dissimilar parallel unit system under the assumption that operative unit cannot fail after post repair inspection and replacement and there is only one repair facility. Failure and repair times of each unit are assumed to be uncorrelated. Using regenerative point technique various reliability characteristics are obtained which are useful to system designers and industrial managers. Graphical behaviors of mean time to system failure (MTSF) and profit function have also been studied. In this paper, some important measures of reliability characteristics of a two non-identical unit standby system model with repair, inspection and post repair are obtained using regenerative point technique.

1. Introduction

Reliability is an important area that is receiving attention globally and it is vital for proper utilization and maintenance of any industrial system. It involves techniques for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. A two-unit system with one repairman has been one of the classical models in the literature of reliability theory. Several studies including Tuteja (1992) discussed the reliability and profit analysis of two single-unit models with three modes of the units normal (*N*), partial failure (*P*) and complete failure (*F*) and different repair policies of repairmen who appear and disappear randomly. This paper consists of two models: in model 1, the unit goes under repair the moment it fails partially, whereas in model 2 the unit goes for repair on complete failure. Pandey et al. (1995) explained the two non-identical unit systems with two types of repair, the internal and the external one. The external repair is called only when the internal staff failed to do the job. In the case of external repair, there was a provision of inspection, wherein if the repair is found unsatisfactory, it is sent for post repair. Hang (1997) investigated on non-destructive inspection,

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in this paper his proposed approach was illustrated by two examples in evaluating reliability with inspection information and in selecting an optimal inspection and maintenance schedule by minimizing the probability of time to failure before inspection and before the time at the end of remaining service life. Li et al. (1998) studied the availability characteristics and the reliability of a three-unidentical-unit repairable system with two various repair supports. Under some practical assumptions, they obtained the explicit expressions of the state probabilities of the system and then the explicit expressions of some performance measures of the system like steady-state availability, steady-state failure frequency steady-state renewal frequency etc. Tuteja et al. (1999) studied two server systems under the assumption that regular repairman is not always available with the system he/she appears disappears randomly. Rizwan et al. (2005) analyzed a system with the provision of multiple post repair inspection and accident during inspection. Two models were discussed: one was single unit operative system and the other was a two-unit cold standby system. In this paper, they assumed that after every repair, unit will go for further inspection to check whether it is in good mode or not. If not, it will again go for repair. This process is continued till the unit becomes operative.

Yadavalli et al. (2005) studied the steady-state availability of a two-component system in series and parallel subject to individual failures (I-failures) and common-cause shock (CCS) failures which was studied from a Bayesian viewpoint with different types of priors assumed for the unknown parameters in the system. Barron et al. (2006) studied an R out of N repairable system consisting of N independent components, operating where, at least, R components are functioning. The system fails whenever the number of good components decreases from R to $R - 1$. A failed component is sent to a repair facility having several repairmen, in this paper both cold and warm stand-by systems are considered. Parashar and Taneja (2007) examined such a system wherein two PLC were working in master-slave fashion. Initially, the master unit is operative, and the slave unit is in hot standby. The slave unit can also fail, but with a lower failure rate than the master unit. Taneja et al. (2007) discussed the Profit evaluation of 2-out-of-3 unit system for an ash handling plant where in situation of system failure did not arise. Yun-Shiow Chen et al. (2008) established an optimal inspection policies of reliability analysis for quantal-response product with Weibull lifetime components and considered a product consisting of m different components in series with lifetimes that follow Weibull distributions, and applied a competing failure model to examine the proposed series system for quantal-response products.

Goyal et al. (2010) performed a comparative study between two models for sulphated juice pump systems working seasonally and having different configurations. Papageorgiou and Kokolakis (2010) studied a parallel (2, n-2)-system where two units start their operations, simultaneously and each was replaced instantaneously on its failure by one of the (n-2) warm standbys. They assumed the availability of n non-identical, non-repairable units. The unit-lifetimes in full operational mode and in partial operational mode have general distribution functions Gi and H_i (i=1, …, n), respectively. The system reliability is evaluated by recursive relations. Sharma et al. (2010) presented the Availability Evaluation of an Agricultural Machine but without any inspection after repair. They assumed that repaired unit would work as well as new one after repair. Bhatti et al. (2011) discussed two identical unit cold standby systems with single repair man facility where concept of inspection policy has also been introduced for detecting the kind of failures (major or minor) before the failed unit get repaired by repairman. Narang et al. (2012) discussed the profit analysis of two unit standby oil delivering system with three types of failure complete failure, where normal to partial failure and partial to complete failure was also analyzed. Initially one unit is operative and the other is standby. In case of partial failure, repair of unit is accomplished by switching off the unit. When both units fail then for repairing, priority is given to partially failed unit over completely failed unit.

Kakkar et al. (2013) studied a two dissimilar parallel unit oil delivering system with preventive maintenance of both units with one repairman under the assumption that each unit is as good as new after the preventive maintenance and repair. Kumar et al. (2012) discussed the reliability modelling of a computer system with independent hardware and software failures subject to maximum operation and repair times. They discussed a reliability model in which a single server who visits the system immediately may do all the work of preventive maintenance (PM), repair and replacement of the units consisted by the system according to their failures. The unit undergoes for PM after a maximum operation time at normal mode. If repair of the component is not possible up to a pre-specific time (called Maximum Repair Time), the components are replaced by new one with some replacement time. However, only replacement of the software components by new one is made after failures. Gupta and Gupta (2013) discussed the one unit reliability model where they tried to minimize the cost incurred in component replacement of a system during post repair when system goes under all the process of repairing policy. Gupta et al. (2013) presented a profit analysis of a reliability model, which consists of one-unit system with post inspection, post repair, preventive maintenance and replacement of a particular unit. Bulama et al. (2013) discussed the stochastic modeling of a repairable warm standby system and explained a redundant system with warm standby units. They developed the explicit expressions for mean time to system failure (MTSF), steady-state availability, busy period and profit function by using Kolmogorov's forward equation method by assuming that after repair, unit is as good as the new one and there is no need for any further inspection. Gupta and Taneja (2014) investigated the reliability and profit analysis of a cement grinding system with failure in the nine important components namely; belt conveyor, bucket elevator, separator, roller press, diverting gate, process fan, cyclone, ball mill and fly ash system. Only one type of failure was considered for each of these components except diverting gate. Kakkar et al. (2014) discussed the probability analysis of a complex system working in a sugar mill with repair equipment failure and correlated life time. They presented a reliability analysis of a complex (SJP) system in a sugar mill with the assumption that repair equipment may also fail during the repair. They considered the analysis of a three-unit system with one big unit and two small identical units of a SJP System in a sugar mill. Failure and Repair times of each unit were assumed to be correlated.

All the above studies assumed that after repair each failed unit is "as good as the new". However, in real existing situations we observe that there are many sophisticated, expensive equipment where it is necessary to inspect a repaired equipment to check whether the repair executed is up to the desirable level or not. It has been observed before that the failed equipment may be sent for post-repair if repair is not found satisfactory during inspection. It has been also observed that when both units fail then there is no need of inspection after repair. Keeping the above real situation in view, we analyze a two-dissimilar-unit-system model in which a unit goes for repair, inspection and post repair as it fails whereas if the second unit also fails with another one then no unit will go for inspection after repair. The proposed approach has been applied to two unit parallel system in northern part of India. Keeping the above situation in view we analyze a two unit non identical parallel system introducing the concept of single post repair inspection and replacement.

2. System model description and assumptions

i) The system consists of two non-identical units (unit-1 and unit-2). Initially, system starts its operations from state S_0 in which unit-A and B both are operative.
ii) After the repair of a unit A or B is accomplished, it goes for inspection to decide whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operational, otherwise it is sent either for post repair or for replacement. The probability of having a perfect repair is fixed.
iii) Upon failure of both units, no unit goes for inspection after repair.

3. Notations

A_0 :	Unit A is in operative mode.
B_0:	Unit B is in operative mode.
A_f	Unit A is in failure mode.

B_f:	Unit B is in failure mode.
A_I:	Unit A is in inspection mode.
B_I:	Unit B is in inspection mode.
A_{f1}/B_{f1}:	Unit A and unit B are in failure mode before inspection.
A_{f2}/B_{f2} :	Unit A and unit B are in failure mode after inspection.
A_{fR}/B_{fR} :	Unit A and unit B are in replacement mode after inspection.
$\lambda_{i(i=1,2)}$:	Constant rate of failure of unit A and B, respectively,
$g_1(t)$:	Rate of repair of unit A and B, respectively,
$g_2(t)$:	Rate of repair of unit A and B, respectively,
a/x:	Probability of post repair after inspection of unit A and B,
b/y:	Probability of replacement of unit after inspection of A and B,
p/q:	Probability of unit is perfect after inspection of unit A and B,
$h_1(t))$:	Rate of inspection of the Unit A
$h_2(t)$:	Rate of inspection of the Unit B
$q_{ij}(.), Q_{ij}(.)$:	pdf & cdf of transition time from regenerative states S_i to S_j,
μ_i :	Mean sojourn time in state S_i,
$\oplus$:	Symbol of Laplace Convolution $A(t) \oplus B(t) = \int_0^t A(t-u)B(u)du$,
$\otimes$:	Symbol of laplace Stieltjes convolution $A(t) \otimes B(t) = \int_0^t A(t-u)dB(u)$.

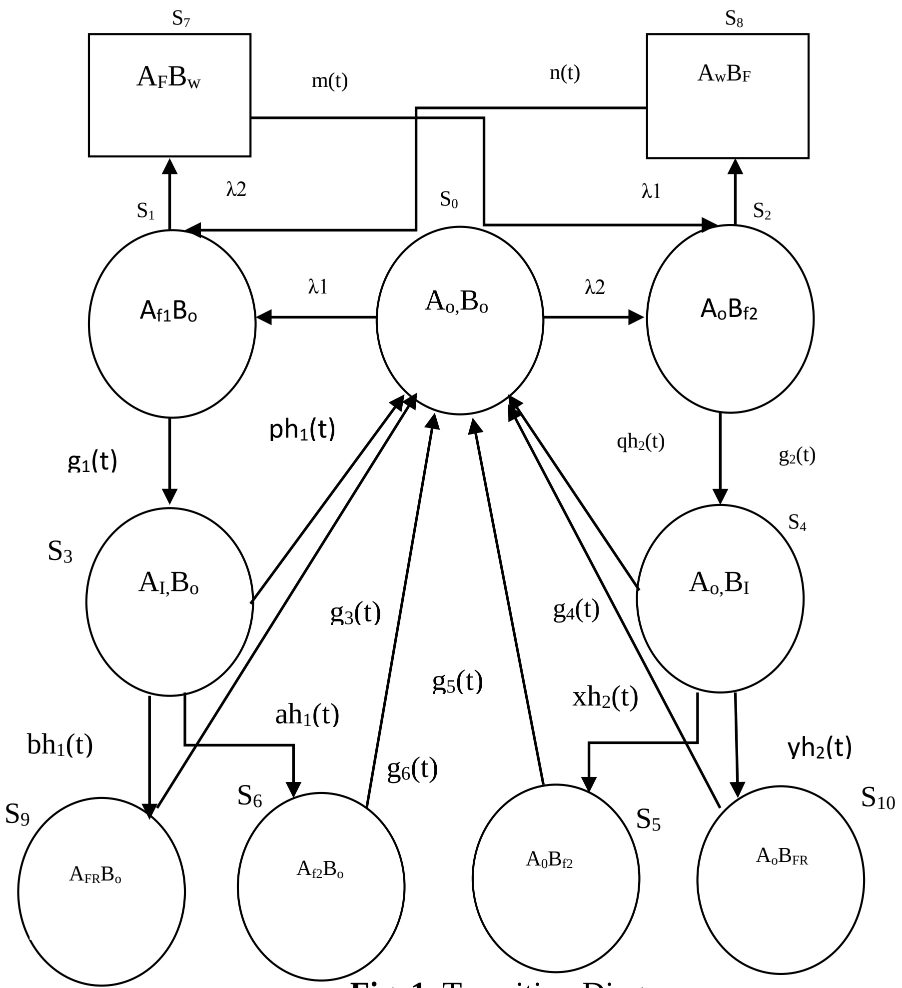

Fig. 1. Transition Diagram

3.1 Transition Probability and Sojourn Times

The steady state transition probability can be as follows

$$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \tag{1}$$

$$p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \tag{2}$$

$$p_{13} = g_1^*(\lambda_1 + \lambda_2), \tag{3}$$

$$p_{17} = (1 - g_1^*(\lambda_1 + \lambda_2)), \tag{4}$$

$$p_{24} = g_1^*(\lambda_1 + \lambda_2), \tag{5}$$

$$p_{28} = 1 - g_1^*(\lambda_1 + \lambda_2)), \tag{6}$$

$$p_{30} = \frac{p}{a+b+p}, \tag{7}$$

$$p_{39} = \frac{b}{a+b+p}, \tag{8}$$

$$p_{36} = \frac{a}{a+b+p}, \tag{9}$$

$$p_{40} = \frac{q}{x+y+q}, \tag{10}$$

$$p_{45} = \frac{x}{x+y+q}, \tag{11}$$

$$p_{410} = \frac{y}{x+y+q}. \tag{12}$$

Here we can see that

$$P_{01}+P_{02}=1, \tag{13}$$

$$P_{13}+P_{17}=1, \tag{14}$$

$$P_{20}+ P_{24}+P_{28}=1, \tag{15}$$

$$P_{30}+ P_{36}+P_{39}=1, \tag{16}$$

$$P_{40}+ P_{45}+P_{410}=1. \tag{17}$$

Mean sojourn times:

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2}, \tag{18}$$

$$\mu_1 = \frac{1 - g_1^*(\lambda_2)}{\lambda_2}, \tag{19}$$

$$\mu_2 = \frac{1 - g_1^*(\lambda_1)}{\lambda_1}, \tag{20}$$

$$\mu_3 = h_1^{*'}(0), \tag{21}$$

$$\mu_4 = h_2^{*'}(0). \tag{22}$$

4. Analysis of Characteristics

4.1 MTSF (Mean Time to System Failure)

T**o** determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

$$\phi_0(t) = Q_{01} \otimes \phi_1(t) + Q_{02} \otimes \phi_2(t), \tag{23}$$

$$\phi_1(t) = Q_{13} \otimes \phi_3(t) + Q_{17}(t), \tag{24}$$
$$\phi_2(t) = Q_{24} \otimes \phi_4(t) + Q_{28}(t), \tag{25}$$
$$\phi_3(t) = Q_{30} \otimes \phi_0(t) + Q_{36} \otimes \phi_6(t) + Q_{39} \otimes \phi_9(t), \tag{26}$$
$$\phi_4(t) = Q_{40} \otimes \phi_0(t) + Q_{45} \otimes \phi_5(t) + Q_{410} \otimes \phi_0(t), \tag{27}$$
$$\phi_5(t) = Q_{50} \otimes \phi_0(t), \tag{28}$$
$$\phi_6(t) = Q_{60} \otimes \phi_0(t), \tag{29}$$
$$\phi_9(t) = Q_{90} \otimes \phi_0(t), \tag{30}$$
$$\phi_{10}(t) = Q_{100} \otimes \phi_0(t). \tag{31}$$

Taking Laplace Stieltjes transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}, \tag{32}$$

where

$$N = \mu_0(1 - P_{14}P_{41}) + \mu_1(P_{01} + P_{02}P_{41}) + \mu_2(P_{01}P_{14} + P_{02}), \tag{33}$$

$$D = (1 - P_{14}P_{41}) - P_{01}P_{10} + P_{02}P_{41}P_{10}. \tag{34}$$

4.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant *t* given that the system entered regenerative state *i* at *t*=0. Using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations

$$A_0(t) = M_0(t) + q_{01} \oplus A_1(t) + q_{02} \oplus A_2(t), \tag{35}$$
$$A_1(t) = M_1(t) + q_{13} \oplus A_3(t) + q_{12.7} \oplus A_2(t), \tag{36}$$
$$A_2(t) = M_2(t) + q_{24} \oplus A_4(t) + q_{21.8} \oplus A_1(t), \tag{37}$$
$$A_3(t) = M_3(t) + q_{30} \oplus A_0(t) + q_{36} \oplus A_6(t) + q_{39} \oplus A_9(t), \tag{38}$$
$$A_4(t) = M_4(t) + q_{40} \oplus A_0(t) + q_{45} \oplus A_5(t) + q_{410} \oplus A_{10}(t), \tag{39}$$
$$A_5(t) = M_5(t) + q_{50} \oplus A_0(t), \tag{40}$$
$$A_6(t) = M_6(t) + q_{60} \oplus A_0(t), \tag{41}$$
$$A_9(t) = M_9(t) + q_{90} \oplus A_0(t), \tag{42}$$
$$A_{10}(t) = M_{10}(t) + q_{100} \oplus A_0(t). \tag{43}$$

Now, taking Laplace transform of these equations and solving them for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}. \tag{44}$$

The steady state availability is

$$A_0 = \lim_{s \to 0}(sA_0^*(s)) = \frac{N_1}{D_1}, \tag{45}$$

where

$$N_1 = \mu_0(1-P_{23})[P_{71}P_{18.7}(1-P_{56}P_{65}) + P_{78}P_{51}(P_{45.6}P_{14}+P_{15}) - P_{78}(1-P_{11.7}-P_{14}P_{41}) + P_{78}P_{65}P_{56}(1-P_{11.7}-P_{14}P_{41})] + (1-P_{23})P_{78}P_{01}[(\mu_1+\mu_4P_{14}) + P_{56}P_{65}(\mu_1+\mu_4P_{14})] \quad (46)$$

$$D_1 = \mu_0P_{78}P_{10}P_{20}P_{51} - \mu_1P_{78}P_{01}P_{20}P_{51} + \mu_2(P_{78}P_{51}(1-P_{14}P_{47.8}-P_{02}P_{10}) - P_{18.7}P_{51} - P_{78}P_{11.7}) + \mu_3P_{51}(1-P_{11.7}-P_{14}P_{41}-P_{01}P_{10}-P_{71}P_{14}P_{47.8})\mu_4(P_{01}P_{51}P_{14}P_{20}P_{78}P_{51}) + (\mu_6+\mu_5)[(P_{20}P_{01}P_{78}(P_{15}P_{45.6}+P_{51})] + \mu_7P_{01}[P_{20}(1-P_{11.7}) - P_{51}P_{20}(P_{14}+P_{18.7}) - P_{51}P_{14}(1-P_{47.8})] + \mu_8P_{02}P_{20}P_{51}[P_{78}(P_{10}-P_{14}P_{47.8}) + P_{18.7}] \quad (47)$$

4.3 Busy Period Analysis of the Repairman

Let $B_i(t)$ be the probability that the repairman is busy at instant *t*, given that the system is entered to regenerative state *i* at t=0. By probabilistic arguments, we have the following recursive relations for $B_i(t)$.

$$B_0(t) = q_{01} \oplus B_1(t) + q_{02} \oplus B_2(t), \quad (48)$$
$$B_1(t) = W_1(t) + q_{13} \oplus B_3(t) + q_{12.7} \oplus B_2(t), \quad (49)$$
$$B_2(t) = W_2(t) + q_{24} \oplus B_4(t) + q_{21.8} \oplus B_1(t), \quad (50)$$
$$B_3(t) = q_{30} \oplus B_0(t) + q_{36} \oplus B_6(t) + q_{39} \oplus B_9(t), \quad (51)$$
$$B_4(t) = q_{40} \oplus B_0(t) + q_{45} \oplus B_5(t) + q_{410} \oplus B_{10}(t), \quad (52)$$
$$B_5(t) = W_5(t) + q_{50} \oplus B_0(t), \quad (53)$$
$$B_6(t) = W_6(t) + q_{60} \oplus B_0(t), \quad (54)$$
$$B_9(t) = q_{90} \oplus B_0(t), \quad (55)$$
$$B_{10}(t) = q_{100} \oplus B_0(t). \quad (56)$$

Taking Laplace transform of the equations of busy period analysis and solving them for $B_0^*(s)$ yields

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)}. \quad (57)$$

In the steady state

$$B_0 = \lim_{s\to 0}(sB_0^*(s)) = \frac{N_2}{D_1} \quad (58)$$

where

$$N_2 = \mu_0(1-P_{23})[P_{71}P_{18.7}(1-P_{56}P_{65}) + P_{78}P_{51}(P_{45.6}P_{14}+P_{15}) - P_{78}(1-P_{11.7}-P_{14}P_{41}) + P_{78}P_{65}P_{56}(1-P_{11.7}-P_{14}P_{41})] \quad (59)$$

D_1 has already been specified.

4.4 Busy Period Analysis of the Repairman (Inspection time only)

Let $IB_i(t)$ be the probability that the repairman is doing his job, inspection time of repairing at instant t, given that the system is entered to regenerative state *i* at *t*=0. By probabilistic arguments we have the following recursive relations for $IB_i(t)$

$$IB_0(t) = q_{01} \oplus IB_1(t) + q_{02} \oplus IB_2(t), \quad (60)$$
$$IB_1(t) = q_{13} \oplus IB_3(t) + q_{12.7} \oplus IB_2(t), \quad (61)$$
$$IB_2(t) = q_{24} \oplus IB_4(t) + q_{21.8} \oplus IB_1(t), \quad (62)$$

$$IB_3(t) = W_3(t) + q_{30} \oplus IB_0(t) + q_{36} \oplus IB_6(t) + q_{39} \oplus IB_9(t), \tag{63}$$

$$IB_4(t) = W_4(t) + q_{40} \oplus IB_0(t) + q_{45} \oplus IB_5(t) + q_{410} \oplus IB_{10}(t), \tag{64}$$

$$IB_5(t) = q_{50} \oplus IB_0(t), \tag{65}$$

$$IB_6(t) = q_{60} \oplus IB_0(t), \tag{66}$$

$$IB_9(t) = q_{90} \oplus IB_0(t), \tag{67}$$

$$IB_{10}(t) = q_{100} \oplus IB_0(t). \tag{68}$$

Taking Laplace transform of the equations of busy period analysis and solving them for $IB_0^*(s)$ yields

$$IB_0^*(s) = \frac{N_3(s)}{D_1(s)}. \tag{69}$$

In the steady state

$$IB_0 = \lim_{s\to 0}(sIB_0^*(s)) = \frac{N_3}{D_1} \tag{70}$$

where

$$N_3 = \mu_0(1 - P_{23})P_{47.8} + \mu_2(P_{02} + P_{51}P_{18.7} + P_{02}P_{41}P_{48} + P_{02}P_{41}P_{48}). \tag{71}$$

D_1 has already been specified in Eq. (47).

4.5 Expected Number of Visits by the Repairman

We define $V_i(t)$ as the expected number of visits by the repairman in (0,t], given that the system initially starts from regenerative state S_i

By probabilistic arguments we have the following recursive relations for $V_i(t)$

$$V_0(t) = Q_{01} \otimes (1 + V_1(t)) + Q_{02} \otimes (1 + V_2(t)), \tag{72}$$

$$V_1(t) = Q_{13} \otimes V_3(t) + Q_{12.7} \otimes V_2(t), \tag{73}$$

$$V_2(t) = Q_{24} \otimes V_4(t) + Q_{21.8} \otimes V_1(t), \tag{74}$$

$$V_3(t) = Q_{30} \otimes V_0(t) + V_{36} \otimes V_6(t) + Q_{39} \otimes V_9(t), \tag{75}$$

$$V_4(t) = Q_{40} \otimes V_0(t) + Q_{45} \otimes V_5(t) + Q_{410} \otimes V_{10}(t), \tag{76}$$

$$V_5(t) = Q_{50} \otimes V_0(t), \tag{77}$$

$$V_6(t) = Q_{60} \otimes V_0(t), \tag{78}$$

$$V_9(t) = Q_{90} \otimes V_0(t), \tag{79}$$

$$V_{10}(t) = Q_{100} \otimes V_0(t). \tag{80}$$

Taking Laplace stieltjes transform of the equations of expected number of visits and solving them for $V_0^{**}(s)$ yields,

$$V_0^{**}(s) = \frac{N_4(s)}{D_1(s)}. \tag{81}$$

In steady state

$$V_0 = \lim_{s\to 0}(sV_0^*(s)) = \frac{N_4}{D_1}, \tag{82}$$

where

$$N_4 = \mu_0(1-P_{23}) + \mu_2(P_{02} + P_{51}P_{18.7} + P_{02}P_{41}P_{48}) + (\mu_4)P_{02}P_{48}P_{78} + \mu_8(1+P_{47.8}P_{02}) \tag{83}$$

D_1 has already been specified in Eq. (47).

4.6 Expected Number of Replacement

We define *R_i (t)* as the expected number of replacement in (0,t],given that the system initially starts from regenerative state S_i By probabilistic arguments we have the following recursive relations for *R_i (t)*,

$$R_0(t) = Q_{01} \otimes R_1(t) + Q_{02} \otimes R_2(t)), \tag{84}$$

$$R_1(t) = Q_{13} \otimes R_3(t) + Q_{12.7} \otimes R_2(t), \tag{85}$$

$$R_2(t) = Q_{24} \otimes R_4(t) + Q_{21.8} \otimes R_1(t), \tag{86}$$

$$R_3(t) = Q_{30} \otimes R_0(t) + R_{36} \otimes R_6(t) + Q_{39} \otimes (1 + R_9(t)), \tag{87}$$

$$R_4(t) = Q_{40} \otimes R_0(t) + Q_{45} \otimes R_5(t) + Q_{410} \otimes (1 + R_{10}(t)), \tag{88}$$

$$R_5(t) = Q_{50} \otimes R_0(t), \tag{89}$$

$$R_6(t) = Q_{60} \otimes R_0(t), \tag{90}$$

$$R_9(t) = Q_{90} \otimes R_0(t), \tag{91}$$

$$R_{10}(t) = Q_{100} \otimes R_0(t). \tag{92}$$

After Taking Laplace Stieltjes Transform of the equations of expected number of visits and solving them for $V_0^{**}(s)$, we get

$$R_0^{**}(s) = \frac{N_5(s)}{D_1(s)} \tag{93}$$

In steady state

$$R_0 = \lim_{s\to 0}(sR_0^*(s)) = \frac{N_5}{D_1} \tag{93}$$

where

$$N_5 = \mu_0(1-P_{23})P_{47.8} + \mu_2(P_{02} + P_{02}P_{41}P_{48}) + \mu_4 P_{02}P_{48}P_{78} + \mu_8 P_{47.8}P_{02} \tag{94}$$

D_1 has already been specified above in Eq. (47).

5. Profit analysis and conclusion

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2IB_0 - C_3V_0 - C_4R_0, \tag{96}$$

where

C_0 =Revenue/unit up time of the system,

C_1 =Cost/unit time for which repairman is busy,

C_2 =Cost/unit time for which repairman is busy in inspection,

C_3=Cost/visit of the repairman,

C_4 = Cost/unit replacement.

For the graphical interpretation, the following particular case is considered

$$g_1(t)=\theta_1 e^{-\theta_1 t},\ g_2(t)=\theta_2 e^{-\theta_2 t},\ h_2(t)=\phi_1 e^{-\phi_1 t},\ h_2(t)=\phi_2 e^{-\phi_2 t},$$

where θ_1 and θ_2 are the repair rate and ϕ_1 and ϕ_2 are the inspection rate of units A and B, respectively. For a more clear understanding of the system characteristics w.r.t. the various parameters involved, we trace the graphs for availability, MTSF and Profit Function in Fig. 2, Fig. 3 and Fig. 4, respectively w.r.t the failure rate parameters of unit A for three different values of p (probability of perfectness of unit after inspection) while the other parameters are kept constant as

a=0.05, b=0.01, x=0.02 ,y=0.004, $\beta_1 = 0.003, \beta_2 = 0.002, \lambda_1 = .02, \lambda_2 = 0.03, \theta_1 = 0.004, \theta_2 = 0.006$

$\phi_1 = 0.005, \phi_2 = 0.001, C_0 = 700, C_1 = 500, C_2 = 50, C_3 = 30, C_4 = 100$.

According to Fig. 2, availability decreases as the failure rate increases irrespective of other parameters. This bend also indicates that, for the same value of the failure rate, availability of the system is higher for higher values of “p”, so here we can see that the high value of “p” tends to increase the expected life time of the system. It can be interpreted from Fig.3 that as the failure rate is moving to the right hand side of the graph, MTSF goes down, this concludes that the reliability of the system also decreases with an increase in the failure rate. Also Fig. 4 reveals the variation in profit with respect to the failure rate and we can see that profit decreases as the failure rate increases. Also for the fixed value of failure rate, the profit is higher for higher value of “p”. It can be interpreted from Fig. 5 that with an increase in repair rate, profit increases. This concludes that reliability and profit of the system also increases as the repair rate increases. The observations drawn from the Fig. 4 is more interesting as for a specific value of p=0.7 (probability of perfectness of unit after inspection) the profit of the system is -ve if the failure rate is greater than 0.91, profit is nil (=0) if the failure rate is equal to 0.91 and profit is +ve if the failure rate is less than 0.91, so by doing this we can find the threshold limit for the failure rate, beyond this limit, profit will be no more. From Fig. 6 it is clear that as the revenue per unit up time of the system increases profit of the system moves up and for a fixed value of “C_0”(Revenue/unit up time of the system) we can observe that the profit is higher for higher value of “p” (probability of perfectness of unit after inspection). From the cut-off points of the revenue per unit up time, the cost for visiting the repairman can be fixed. Here we can conclude that cut off points for various failure rates and repair rates can be obtained which helps in deciding the optimum acceptable values of rates so that the system may be profitable. That is, the upper limit of the failure rate can be obtained so that the system can give the desirable profit.

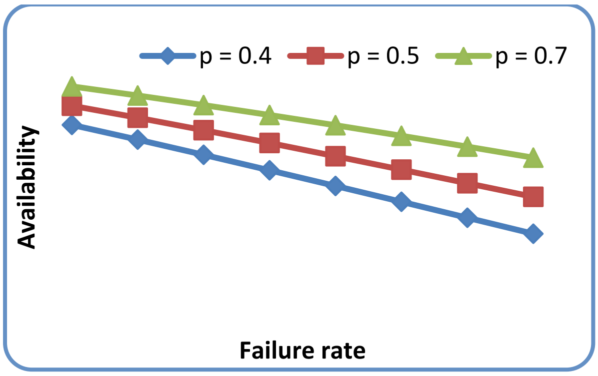

Fig. 2. Avalability vs Failure Rate

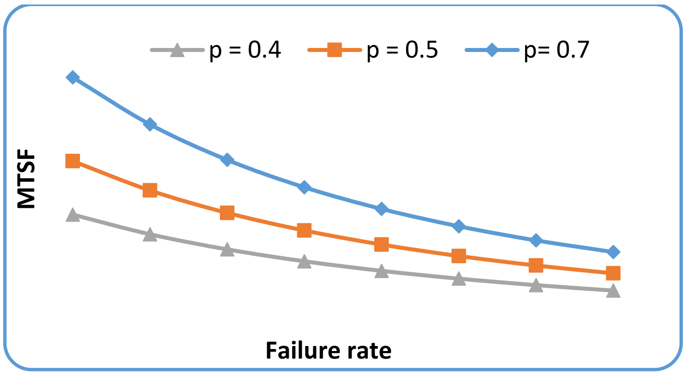

Fig. 3. MTSF vs Failure Rate

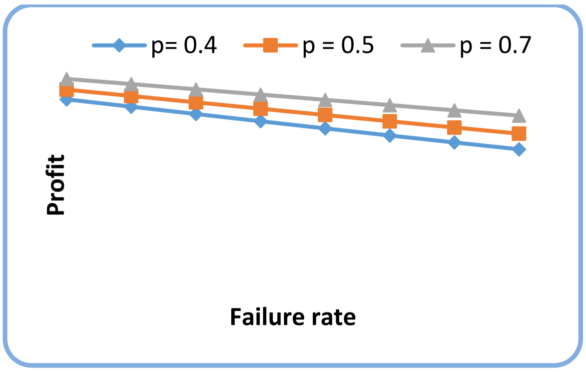

Fig. 4. Profit vs Failure Rate

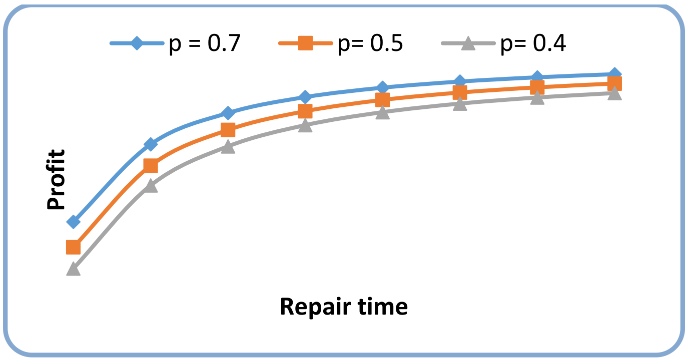

Fig. 5. Profit vs Repair Rate

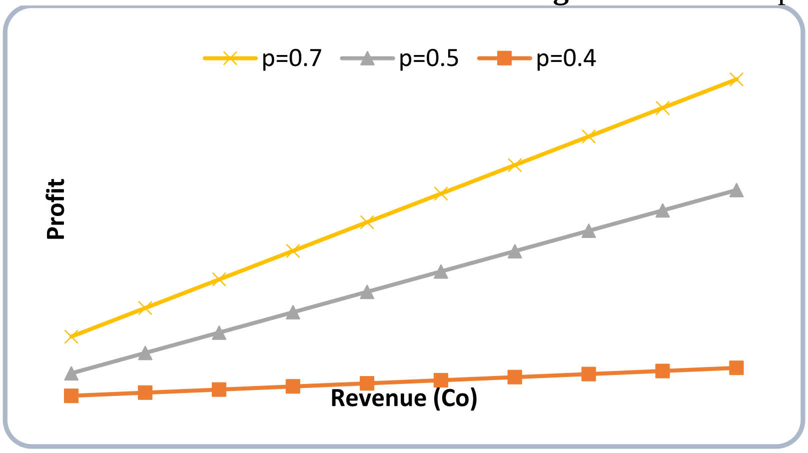

Fig. 6. Profit vs Revenue(Co)

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