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A hybrid Tabu search-simulated annealing method to solve quadratic assignment problem

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A B S T R A C T

Quadratic assignment problem (QAP) has been considered as one of the most complicated problems. The problem is NP-Hard and the optimal solutions are not available for large-scale problems. This paper presents a hybrid method using tabu search and simulated annealing technique to solve QAP called TABUSA. Using some well-known problems from QAPLIB generated by Burkard et al. (1997) [Burkard, R. E., Karisch, S. E., & Rendl, F. (1997). QAPLIB–a quadratic assignment problem library. *Journal of Global Optimization*, *10*(4), 391-403.], two methods of TABUSA and TS are both coded on MATLAB and they are compared in terms of relative percentage deviation (RPD) for all instances. The performance of the proposed method is examined against Tabu search and the preliminary results indicate that the hybrid method is capable of solving real-world problems, efficiently.

1. Introduction

Koopmans and Beckman (1957) are believed to be the first who introduced the quadratic assignment problem (QAP) in the context of locating “indivisible economic activities”. The primary objective of QAP problem is to assign a set of facilities to a set of locations such that the total assignment cost is minimized. The assignment cost for a pair of facilities is considered as a function of the flow between the facilities and the distance between the locations of the facilities. Ahmed (2013) presented a new reformulation of the problem and developed a Lexisearch Algorithm (LSA) to obtain exact optimal solution to this problem. He performed a comparative study to show the efficiency of the algorithm against an existing algorithm for some medium sized instances from the QAP library, QAPLIB (Burkard et al., 1997; Burkard, 2013).

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Forghani and Mohammadi (2012) presented an integrated quadratic assignment and continuous facility layout problem. They obtained the arrangement of facilities within the departments through the QAP. They presented mathematical model as a mixed-integer programming (MIP) to minimize total material handling cost. In addition, they presented a heuristic method to solve the problem for large-scale problems and using several illustrative numerical examples, the performance of the model was examined. Tasgetiren et al. (2013) presented some metaheuristics to solve QAP problems. Tseng and Liang (2006) presented a hybrid metaheuristic for the quadratic assignment problem. Wang (2007) applied Tabu search to solve QAP problem.

2. The proposed study

2.1. Problem statement

In quadratic assignment problem, we are concerned with assignment of two facilities *i* and *j* in two possible places of *k* and *l*. Let x_{ik} be a binary variable, which is one if facility *i* is located in place *k* and zero, otherwise. In addition, Let x_{jl} be a binary variable, which is one if facility *j* is located in place *l* and zero, otherwise. Let c_{ijkl} be the cost of assigning location *i* in place *k* and location *j* in place *l*. Therefore, the proposed study considers the following,

$$(\text{QAP}) \min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} c_{ijkl}x_{ijkl} \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,\dots,n, \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,\dots,n, \tag{3}$$

$$x_{ij} = 0,1, i = 1,\dots,n, j = 1,\dots,n. \tag{4}$$

Eq. (1) states the objective function of the QAP problem and it minimizes sum of costs associated with facility assignment. Eq. (2) ensures that only any facility is assigned to only one place. Eq. (3) specifies that only each location is considered only for one place. Eq. (4) states that all variables are binay. The QAP problem is generally considered as an NP-Hard problem (Çela, 1998; Anstreicher, 2003) and cannot be solved using combinatorial optimization techniques. Therefore, an alternative solution is to use heuristic as well as meta-heuristics to tackle such problem although there are recently some studies concentrated on reformulation of QAP in an attempt to provide exact solution (Loiola et al., 2007).

2.2. Tabu search

During the past two decades, there have been various metaheuristics to solve QAP such as Tabu search, which was originally developed by Glover (1986, 1989, 1990). Neighborhood searches take a potential solution to a problem and verify its immediate local opportunities, which is, solutions that are similar except for one or two minor details to detect an improved solution. Local search techniques tend to become stuck in suboptimal regions or on plateaus where several solutions are equally fit. Tabu search takes advantage of the performance of these methods by using memory structures, which explain the visited solutions or user-provided sets of rules. If a potential solution has been already visited within a certain short-term period or if it has already violated a rule, it is marked as "tabu" (forbidden) so that the algorithm would not reconsider that possibility, repeatedly (Hertz et al., 1995). Hussin and Stutzle (2011) presented a high performing stochastic local search algorithms for the QAP and their performance in dependence to the instance structure and size. Misevičius (2003) proposed a modified simulated annealing algorithm for the QAP - M-SA-QAP. They examined their algorithm on a number of instances from the library of the QAP instances – QAPLIB and reported that the proposed algorithm seemed to be superior to earlier versions of the simulated annealing for the QAP. Saifullah Hussin and Stützle (2014) compared the performance of Tabu search vs. simulated annealing as a function of the size of quadratic assignment problem instances. They reported that the assertion whether one algorithm is better than the other could depend strongly on QAP instance size even if one focuses on instances with otherwise same characteristics. Wilhelm and Ward (1987) applied simulated annealing to solve QAP problem. Fig. 1 shows the structure of Tabu search method.

Step 1. Let S be the initial feasible solution and Z its objective function value; then, set S* = S, Z* = Z, max short-term memory (STM) = 5, and max iteration = 1,000; iter = 1. Best O value = O value.
Step 2. Random (i, j) = rand/Long-term memory (LTM) (i, j), (n1, n2) = the indices of maximum value in random.
Step 3. If there is none (n1, n2) in STM matrix, change n1 and n2 locations; otherwise, repeat step 2.
Step 4. Insert n1 and n2 in STM and release the last indices from STM (e.g., m1, m2); and LTM(m1, m2) = LTM(m1, m2) + 1.
Step 5. Calculate the objective function value (Z) of the new permutation.
Step 6. If Z ≤ Z*, then Z* = Z, S* = S, and iter = iter + 1.
Step 7. If iter ≤ max iteration, then repeat step 2; otherwise, print Z* and S*.

Fig. 1. Tabu search algorithm

2.3. Simulated annealing

Simulated annealing (SA) is a generic probabilistic metaheuristic for combinatorial optimization problem of locating a good approximation to the global optimum of a given function in a relatively large search space. The method is often implemented when the search space is discrete such as QAP problems. For certain problems, SA may be more efficient than exhaustive enumeration rather than the best possible solution. Paul (2010) reported that for a number of varied problem instances, SA could perform better for higher quality targets while TS performs better for lower quality targets. Fig. 2 shows details of the SA method.

```
s ← Generate Initial Solution( )
T ← T0
while termination conditions not met do
        s′ ← Pick At Random (N(s))
        if ( f ( s′ ) < f (s)) then
                s ← s′
        else
                Accept s′ as new solution with probability p (T, s′, s)
        end if

        Update(T)
end while
```

Fig. 2. The structure of SA method

The proposed study of this paper propose a hybrid of Tabu and SA method. Fig. 3 shows details of Pseudu code of the proposed study.

step1 :

initialize S as initial solution and z= evaluate objective fuction

step1. 3 : S*=S and Z*=Z ;

STM=5;// max short-term memory

maxiteration=1000; iter=1; best value O = O value

step2: randomize

step2.1: for i =1 to n do

for j =1 to n do

RANDOM(i,j)=rand/LTM;

step2.2 : (i,j) and(n1,n2)=index of (RANDOM $\in$ STM);

step3 : T=0;

for i=1 to size(STM,1) do

for j=1 to size(STM,2) do;

if((n1,n2)==STM(i,j))

T=1; repeat Step 2

if(T=0)

{

temp=n1;

n1=n2;

n2=temp

}

Step 4:

m1=size(STM,1) ;

m2=size(STM,2) ;

(n1,n2)=STM(m1,m2);

LTM(m1,m2)=LTM(m1,m2)+1;

step5: z=evaluate objective function;

step 6 :

if(z<=z*) ; z*=z

{

S*=S;

iter=iter+1

}

step7: if (iter<=max iteration) ; repeat step 2 ; else print z* and S*

Fig. 3. Hybrid Tabu-Simulated Annealing Pseudo-code

3. The results

In this section, we present details of the implementation of the proposed study (TABUSA) and compares the results with pure Tabu search method (TS). Using some well known problems from QAPLIB generated by Burkard et al. (1997), two methods of TABUSA and TS are both coded on MATLAB and they are compared with relative percentage deviation (RPD) for all instances. Table 1 shows details of our results.

Table 1
The summary of comparison of TABUSA versus TS

Name of instances	n	BKS	RPD of TS solutions	RPD of Hybrid Tabu-SA solutions
Nug 12	12	578	0.00	**0.00**
Nug 14	14	1014	0.39	**0.00**
Nug 15	15	1150	0.87	**0.00**
Nug 16a	16	1610	1.37	**0.00**
Nug 16b	16	1240	0.00	**0.00**
Nug 17	17	1732	0.69	**0.00**
Nug 18	18	1930	1.04	**0.00**
Nug 20	20	2570	1.56	**0.00**
Nug 25	25	3744	1.55	**0.38**
Bur26a	26	5426670	**0.09**	0.12
Bur26b	26	3817852	0.19	**0.00**
Bur26c	26	5426795	0.26	**0.02**
Bur26d	26	3821225	0.02	**0.01**
Bur26e	26	5386879	0.03	**0.01**
Bur26f	26	3782044	0.05	**0.00**
Bur26g	26	10117172	**0.01**	0.03
Bur26h	26	7098658	0.01	**0.00**
Tai25a	25	1167256	4.26	**2.74**
Tai30a	30	1818146	4.75	**2.71**
Tai40a	40	3139370	6.12	**3.82**
Tai50a	50	4938796	6.49	**4.12**
Average of RPDs			1.41	**0.66**

As we can observe from the results of Table 1, the proposed study of this paper is capable of finding optimal solutions in most cases and performs better than TS method. Fig. 4 shows details of error for two methods.

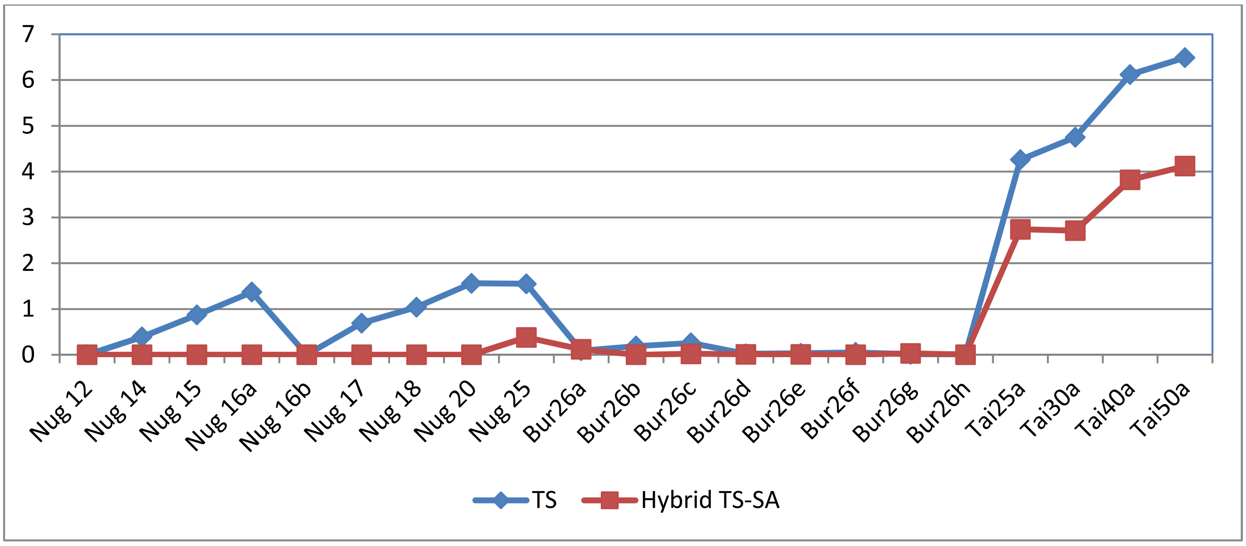

Fig. 4. Comparison of RPDs for TABUSA versus TS

4. Conclusion

In this paper, we have presented a hybrid method based on two well known methods, Tabu search and simulated annealing to solve QAP problem. The proposed model fo this paper has been implemented on some benchmarks and the results have confirmed that the new hybrid method could perform better than pure Tabu search method to solve QAP problems. However, we should cuatios about the findings since we may need to apply the proposed model of more instances to make more reliable conclusion.

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