

## Fuzzy linguistic hedges for the selection of manufacturing process for prosthetic sockets

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### CHRONICLE

### ABSTRACT

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In this paper, a comparison is presented between two prime methods of producing prosthetic sockets by using the fuzzy linguistic hedges approach on the qualitative feedback of Indian prosthetic users. Recent trends indicate that the Indian manufacturers have tried to adopt the newer technologies like reverse engineering (RE) approach to achieve the desired goals. However, the satisfaction of the user is of utmost importance for the unique and customized products for rehabilitation. In order to analyze the effectiveness of the manufacturing approaches, user case studies are taken, based on the linguistic feedbacks, and a comparative study is conducted. Thirteen users from four different manufacturing units are taken for study and sockets made by conventional as well as RE are experimented. Fuzzy membership functions are constructed using the linguistic hedges based on the user feedbacks. An analytical hierarchy process (AHP) is applied to arrive at a decision to select the manufacturing process for user satisfaction and manufacturing excellence.

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## 1. Introduction

The aim of custom fit prosthetics is to improve the quality of life of the user by providing optimized individual geometrical shape and requirements. In this context, the selection of the best fit method of fabrication of the prosthetic socket from the conventional method and the CAD/CAM/RE methodology is discussed in the present work. From the feedback of the prosthetic users and manufactures in qualitative linguistic terms and through a fuzzy QFD (quality function deployment) approach, the ranking of the parameters to reach the desired goal has been selected. On the basis of the weights of the needs, the ranking has shown that the most important feature for the prosthetic sockets is the manufacturing process followed by cost, material, smart technology, accuracy and surface finish. The objective of this paper is to apply a fuzzy multi-criterion approach for selection of fabrication technique using AHP based on user provided linguistic hedges. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh,

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1965). A substantive departure from the conventional fuzzy techniques of system analysis was proposed by using linguistic variables in place of numerical values (Zadeh, 1972). A linguistic variable is defined as a variable whose values are sentences in a natural or artificial language. Thus if *tall*, *not tall*, *very tall*, *very very tall*, etc. are values of *height*, then *height* is a linguistic variable. Similarly a linguistic hedge such as *very*, *more or less*, *much*, *essentially*, *slightly*, etc. may be viewed as an operator which acts on the fuzzy set representing the meaning of its operand. For example, in the case of the composite term *very tall man*, the operator *very* acts on the fuzzy meaning of the term *tall man* (Zadeh, 1972). Detailed works on linguistic variable and its applications to approximate reasoning are found in three published papers by Zadeh (1975). Further discussions on extended hedge algebras and their applications to fuzzy logic can be found in Nguyen and Wechler (1992), Lascio *et al* (1996) and Nguyen and Huynh (2002).

Some pioneering works on CAD/CAM applications to prosthetics can be found in Foort *et al* (1985), Klasson (1985) and Holden and Fernie (1986). Continuing this trend some recent applications can be found in Nepal *et al* (2011) where a case study in a North American orthopaedic industry was cited. Dangayach and Deshmukh (2005) presented a survey based analysis on status, scope and implementation criteria of advanced manufacturing technologies in 122 Indian firms from SME sector. A fuzzy AHP approach for the determination of importance of weights of customer requirements in QFD can prove to be quite beneficial. The data which is vague and unclear with consistent needs to be compared pair wise using a AHP technique. This approach was used to improve the imprecise rating of the customer requirements (Kwong and Bai, 2002). Ozden Bayazit (2005) presented an application of AHP tool in flexible manufacturing systems and Aktepe & Ersoz (2011) applied the fuzzy AHP to an industrial case study for the supplier selection process. The priority weights were calculated using the extent analysis method and the integral value calculation. Similarly Lee *et al* (2011) published a fuzzy AHP application for the selections in a green supply chain for a case study in Taiwan and Daim *et al* (2012) applied AHP for the selection of 3PL providers.

In section 2, the methodology for forming fuzzy linguistic hedges is presented. Application of the fuzzy linguistic hierarchy approach to various manufacturing case studies and the results are analyzed in § 3. The concluding remarks on the work is presented in § 4.

## 2. Formation of Fuzzy Linguistic Hedges

Four manufacturing units dealing with fabrication of prosthetic sockets have been surveyed for the purpose of comparing between CAD/CAM with conventional or manual method. The manufacturing excellence and user satisfaction are related to a number of factors or features which may be technical, design and process-based. The following parameters have been ranked or prioritized using a QFD (quality function deployment) tool on the feedbacks obtained from several manufacturers and users:

- Manufacturing process (*C1*)
- Material (*C2*)
- Smart Technology (*C3*)
- Cost Effectiveness (*C4*)
- Serviceability (*C5*)

In a fuzzy logic based system, sometimes the information is described linguistically. The linguistic hedge is an operator like a modifier used to modify the shape of membership functions. Linguistic hedge operators can be classified into three categories: concentration, dilation and contrast intensification. In this paper, only the concentrator and the dilators are used.

### 2.1 Concentrators and dilators

Applying a concentration operator to a fuzzy set results in the reduction of magnitude to the grade of membership. In contrast, the effect of dilation is opposite to that of concentration. The reinforcing modifiers provide a characterization which is stronger than the original one. Zadeh (1972) proposed the modifier *very* associated with the transformation  $tm(\mu)=\mu^2$ . More generally one can think of modifiers defined by transformations such that  $tm(\mu) \leq \mu$ , for any  $\mu \in [0, 1]$ . Other examples of these quantifiers include *extremely*, *positively* etc. Linguistic hedges with mathematical expressions proposed by Zadeh and others are shown in Table 1.

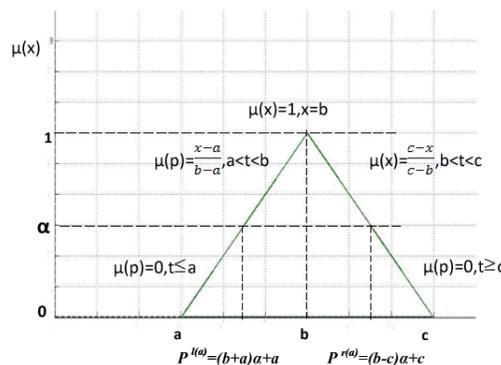
**Table 1**  
Definitions of linguistic hedges

Concentrators		Dilators	
Hedge	Operator definition	Hedge	Operator definition
Very F	F squared	More or less	Square root of F
Very Important F	F to power 1.5	Less Important	F to power 0.75
Extremely F	F to power 1.25	Minus F	F to power 0.3
Plus F	F to power 0.8	Insignificant	F to power 0.2
Very Very F	F to power 0.4		

The weakening modifiers provide a new characterization that is less strong than the original one. Zadeh introduced the modifier *more or less*, associated with the transformations  $tm(\mu) = \mu^{1/2}$ . Other weakening modifiers can be defined by choosing a transformation  $tm(\mu) \geq \mu$  for any  $\mu \in [0, 1]$ . Other examples of these quantifiers include *more or less*, *negatively* etc.

### 2.2 Development of membership Functions

There are several methods to get reasonable membership functions. Here is an illustration of one such method. Suppose one needs to model the notion of *high* performance with a fuzzy set as shown in Fig. 1, then the set U with a positive real numbers representing the totality of possible performance of the manufacturing units (as judged by the users).



**Fig. 1.** Membership function development for fuzzy linguistic hedges

Then by surveying a large number of users and finding out about the performance of the manufacturing process in a particular range, the proportion  $p$  of customers who thought that the performance ( $t$ ) of manufacturing unit lies between (5 and 20) is high, is approximately equal to  $p = (t - 5)/(20 - 5)$ . Of course everyone thought that the performance over 20 is high. The model of the fuzzy set “high performance” would be

$$\mu_p = \begin{cases} (t - 5)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$$

Similarly, for other criteria fuzzy sets are modelled.

### 3. Application to case studies

Taking into account the two main types of manufacturing process for the prosthetic sockets, the data from the four users are collected separately for each product type. Table 2 presents the compiled linguistic data from the users for the various sub-criteria of manufacturing process.

**Table 2**  
Linguistic data for different parameters of Manufacturing Process- MP (CI)

User No.	Shape Matching	Shape Retrieval	Shape management	Biomechanical Shape standard	Shape Sensing	Graphic Analysis	time elapsed	Custom Fit	Further Processing
1 (M)	2	1	1	1	1	1	1	2	2
2 (M)	1	1	2	1	2	1	2	1	1
3 (F)	2	2	2	3	3	1	2	3	2
4 (M)	2	2	3	2	1	1	1	3	1
Total	7	6	8	7	7	4	6	9	6
1 (M)	5	5	3	4	4	4	3	3	5
2 (M)	4	4	4	5	5	4	3	4	4
3 (F)	5	4	5	3	5	4	3	5	4
4 (M)	3	4	3	5	4	5	3	3	5
Total	17	17	15	17	18	17	12	15	18

Similar compiled data for other parameters are given in next section. The membership functions required for computation of performance score for each criterion with respect to alternatives are also developed.

#### 3.1 The proposed algorithm

The application of the linguistic fuzzy AHP comprises of the following steps:

##### Step 1 : Structuring the Problem as a Hierarchy

The problem can be structured into a hierarchy as shown in the Fig 2. On the top level, there is the overall goal i.e. selection of the type of manufacturing process. On the second level, there are the five criteria (C1 through C5) that contribute to the goal. On the third level, these five criteria are again decomposed into twenty five sub-criteria and on the bottom (fourth) level are the two alternative choices of manufacturing type that are evaluated in terms of the sub-criteria of the third level.

##### Step 2: Construction of Membership Function

Membership functions are constructed using section dilators as reference for each sub criterion represented in Table 3.

##### Step 3: Calculation of Performance Score

With the help of this membership function, the performance score is calculated for each sub criterion. For example, considering the value of manufacturing type 1 (manual) and manufacturing type 2 (RE) under sub-criterion performance,

The value of membership function for manufacturing type 1  $= (7-5) / 15 = 0.13$ ,  $5 \leq t \leq 20$ .

Similarly, for manufacturing type 2, the calculated value is  $(17 - 5) / 15 = 0.8$

Next all the scores are aggregated corresponding to its sub criterion (Table 3 and 4).

#### Step 4: Normalization of the Score

The decision matrix is normalized to obtain unique membership functions combined with other criteria to select the best manufacturing type. Here normalization is done by adding individual column elements and then dividing each column element by the sum.

#### Step 5: Dilation or centralization for each criterion

To determine the power of dilation or centralization for each criterion the various attributes with power of centralization or dilation is given Table 5. Using the power values from this table and applying it on alternative criterion, a set of values represented by a decision matrix is obtained. For example, for  $C_2$ , the relative weights are obtained by raising the column elements to power of 1.25 i.e.  $(0.5)^{1.25}=0.420448$ . In this way a decision matrix is obtained (Table 6) representing scores of all criteria with respect to each alternatives.

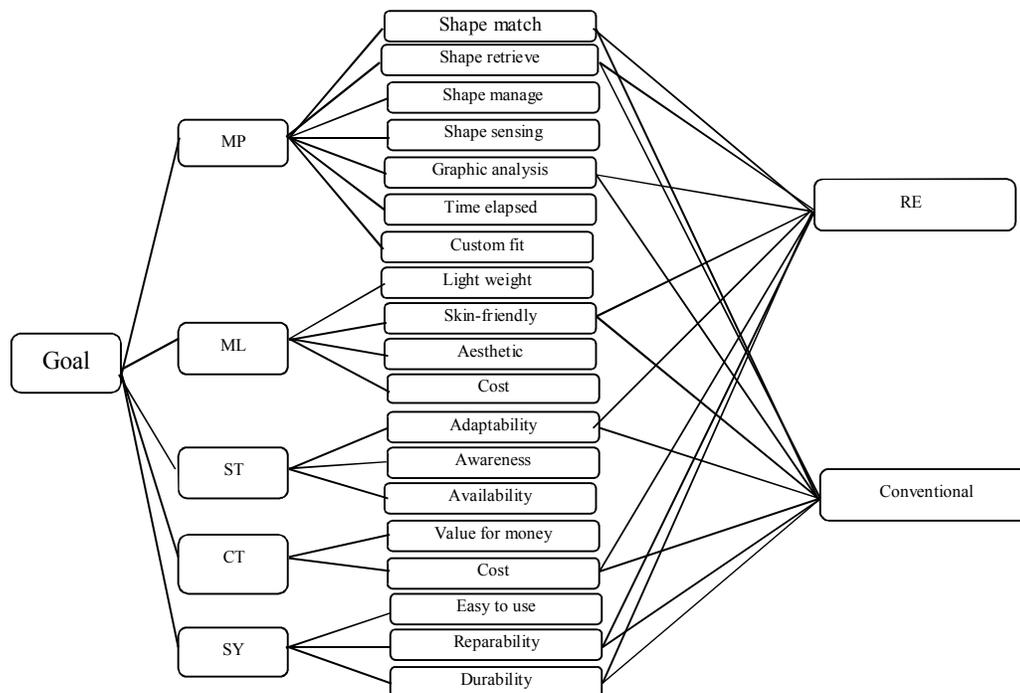
#### Step 6: Decision Making

To determine the best alternative solution, the max-min principle is applied. First by taking the minimum membership value over all of the criterion for different alternatives and then picking the alternative with maximum value. The ranking of best manufacturing type from Table 6 are found and the minimum value of all alternatives with respect to various criterion in the decision matrix are taken. For the given case study,

Minimum of manufacturing Type 1 (conventional) = 0.0169

Minimum of manufacturing type 2 (RE based) = 0.4099

Taking the maximum value between the two alternatives, manufacturing type 2, i.e. RE type is the best option.



**Fig. 2.** Hierarchical representation for the selection of manufacturing approach

**Table 3****Membership functions and scores for manufacturing process criteria**

Criteria	Manual (Score)	RE (Score)	Membership Function
Shape matching	7(0.13)	17(0.8)	$\mu_p = \begin{cases} (t-5)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Shape retrieval	6(0.06)	17(0.8)	$\mu_p = \begin{cases} (t-5)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Shape management	8(0.17)	15(0.75)	$\mu_p = \begin{cases} (t-6)/18, & 6 \leq t \leq 18 \\ 1, & 18 \leq t \end{cases}$
Biomechanical shape standard	7(0.08)	17(0.91)	$\mu_p = \begin{cases} (t-5)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Shape sensing	7(0.05)	18(0.63)	$\mu_p = \begin{cases} (t-6)/19, & 6 \leq t \leq 25 \\ 1, & 25 \leq t \end{cases}$
Graphic analysis	4(0.05)	17(0.82)	$\mu_p = \begin{cases} (t-3)/17, & 3 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Time elapsed	6(0.16)	12(0.67)	$\mu_p = \begin{cases} (t-5)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Custom fit	9(0.18)	15(0.72)	$\mu_p = \begin{cases} (t-7)/15, & 5 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Further processing	6(0.13)	18(0.88)	$\mu_p = \begin{cases} (t-4)/16, & 4 \leq t \leq 20 \\ 1, & 20 \leq t \end{cases}$
Total	1.01	6.88	

**Table 4****Linguistic data and scores for other parameters**

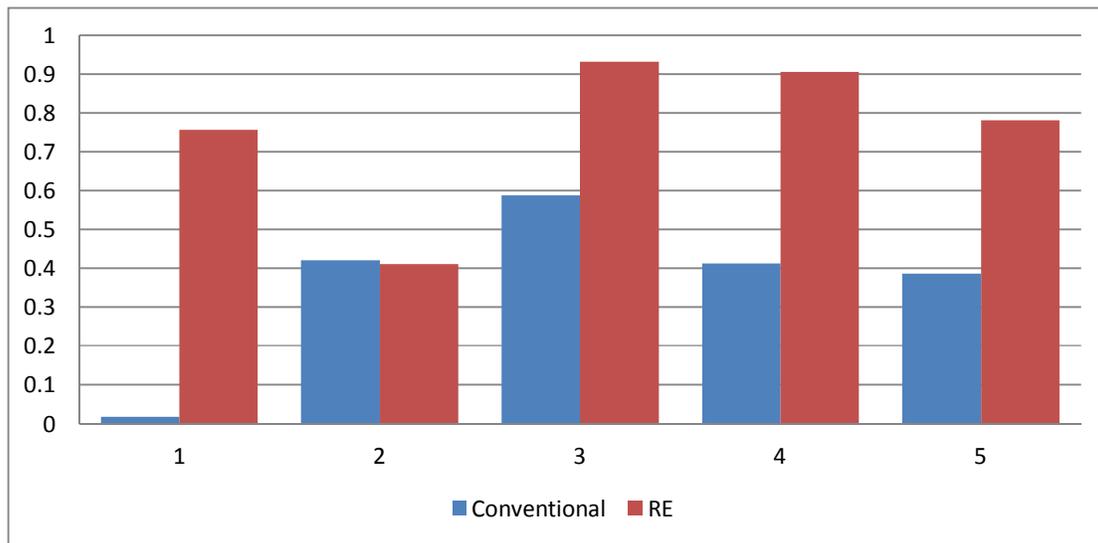
	Criteria	Manual	RE
Material	Environment friendly	11 (0.3)	15 (0.7)
	Light weight	12 (0.2)	18 (0.8)
	Easy replaceable	10 (0.72)	11 (0.33)
	Skin friendliness	11 (0.43)	11 (0.43)
	Aesthetic	12 (0.31)	17 (0.27)
	Cost	18 (0.75)	6 (0.15)
	Total	2.71	2.68
Cost Effectiveness	Easy replaceable	8 (0.14)	16 (0.71)
	Skin friendliness	14 (0.11)	18 (0.88)
	Aesthetic	13 (0.15)	16 (0.3)
Total	0.4	1.89	
Smart Technology	Awareness to smart	5 (0.12)	16 (0.81)
	Adaptability	7 (0.13)	16 (0.73)
	Cost	10 (0.13)	12 (0.25)
	Availability	11 (0.15)	18 (0.5)
Total	0.53	2.29	
Serviceability	Easy-to-use	13 (0.38)	15 (0.63)
	Reparability	7 (0.15)	15 (0.77)
	Durability	12 (0.25)	15 (0.63)
Total	0.78	2.03	

**Table 5****Various attributes with power of concentrator or dilators**

Criteria	Hedge	Power
$C_1$ : Manufacturing process	Very significant	2
$C_2$ : Material	Very important	1.25
$C_3$ : Smart Technology	Somewhat important	0.33
$C_4$ : Cost Effective	More or less important	0.5
$C_5$ : Serviceability	Negatively important	0.75

**Table 6**  
Final selection of best manufacturing process using max-min principle

Normalized Decision Matrix					
	C1	C2	C3	C4	C5
Conventional	1.01	2.71	0.53	0.4	0.78
RE	6.88	2.68	2.29	1.89	2.03
Concentrated/Dilated					
Conventional	0.13	0.5	0.2	0.17	0.28
RE	0.87	0.49	0.81	0.82	0.72
Decision Matrix					
Conventional	0.0169	0.420448	0.587949	0.412311	0.384918
RE	0.7569	0.409963	0.932825	0.905539	0.781627



**Fig 3.** Selection of best alternative using max-min principle

In Fig. 3 a graphical depiction of relative weights from the decision matrix is produced to clearly select the maximum value for the RE-based manufacturing approach which can be selected in order to achieve the manufacturing excellence leading user satisfaction.

#### 4. Conclusions

In this paper, a fuzzy linguistic hedges based AHP tool is implemented to analyse the responses from the users of the prosthetic sockets manufactured conventionally as well as using RE based approach. Four manufacturing units dealing with prosthetic sockets were taken for the study and users were identified for getting the responses on both the approaches of manufacturing. Five design parameters along with their rankings as obtained from a fuzzy QFD analysis earlier by the authors were used in the present work. A hierarchical problem is constructed to select the best alternative of manufacturing using the max-min principle of the linguistic hedges. Decision matrices are formed after normalization and using concentrators/dilators to the scoring weights. From the decision matrix the RE-based approach was selected as a better alternative which can satisfy the user requirements and achieve the manufacturing excellence. In present times the SMEs are enhancing their capabilities by procuring advanced technologies. During the survey, many rehabilitation units were found, those in the process of acquiring the RE-based technology to replace the manual or conventional method.

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