Developing a multi stage mathematical production management model for auto industry

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ABSTRACT

In this paper, we establish an integrated mathematical production management model based on five parts consisted of finished goods production in production sites, inventory control of finished goods in production and distribution facilities, semi-finished goods production in producer’s facilities, inventory control of raw materials in production sites and inventory control of semi-finished products. The main goal of developed model is to minimize total cost of company in the mentioned parts. Finally, the model is validated with 7 real-world case studies from auto-industry in one of the largest Iranian car producer company, Iran Khodro co and the empirical results are illustrated at the end of the paper.

1. Introduction

The most important roles of a production management model are integrating and all needed stages of products producing process and executing them in an ordered and coordinated manner. In each stage in which some values are added to a manufactured product, consisted of supplying raw materials and intermediate components, manufacturing the finished products, packaging, transporting, warehousing, and logistics. Developing a mathematical production model because of its precision essence can be so applicable for industries. So mathematical modeling of a coordinated production management is still a largely undeveloped area of research. Sharma and Agrawal (2009) considered a multi-stages serial production system with the use of probabilistic demand situations for the end product. They considered the demand situations as binomial, exponential, lognormal and Poisson. Jung et al. (2008) considered a stock management for a multi-stage supply chain in the petrochemical, chemical and pharmaceutical industries, and they demonstrated the effectiveness of theirs safety stock management model on a realistically scaled polymer supply chain problem. The deterministic optimization problem of a profit-maximizing firm is studied by Jouini et al. (2007).
Fontes Dalila et al. (2007) addressed investment decisions in production systems by using real options. He tried to develop and implement stochastic dynamic programming models both for fixed and flexible capacity systems. Lejeune (2006) recommended a mathematical production management among a three-stage supply chain model, in which a sustainable inventory–production–distribution plan over a multi-period horizon is designed and he tried to solve it with a new mathematical method. Merdivenci Atici, and Uysal (2007) investigated the optimal production and inventory paths of HMMS type models on complex time domains. To solve problem they used a new mathematical theory with Time scale calculus name. Ouyang et al. (2008) presented an integrated inventory model with variable production and price-sensitive demand rates. They attempted to offer a best policy that aims at maximizing the joint total profit while the trade credit and freight rate are linked to the order quantity, simultaneously. A specification of strictly increasing and decreasing returns to scale in multi-output technologies was proposed by Boussemart et al. (2008), they established some necessary and sufficient conditions to characterize strictly increasing and decreasing returns to scale for a large class of technologies.

In this paper, we develop a mathematical production management model with decreasing costs main goal in following sections. (1) Raw materials holding costs, (2) semi-finished products production costs, (3) semi-finished products holding costs. (4) End products production costs. and (5) end products holding costs.

The remainder of the paper is organized as follows: In Section 2. We will introduce the notation used, explain the hypotheses of the model and formulate the integrated model and In Section 3. We will report the numerical results of the methodology, which is applied to a real-life industrial problem faced by a large production management in Iran khodro khorasan Razavi Company.

2. The Proposed model

2.1. Five production management components

In this paper, we develop a mathematical five-segment production management model presented in Fig. 1. The first stage, i.e., inventory control of Raw materials, semi-finished products and end products, is in charge of the procurement of raw materials and/or component, the second stage, i.e., production, represents the manufacturing and/or assembly of the finished and semi-finished goods. The customer demand and lead times, defined as the sum of the traveling, loading and unloading times, are known, and vary from period to period. There is no backlog; the demand, which cannot be satisfied from on-hand inventory is lost.

![Fig. 1. Five parts production management model](image)

The production management program, which is defined in this paper, follows below schemes,

- ✔ The ending inventory level required at each node and at each period in its three mentioned parts;
The production scheme, i.e., the quantity of semi-finished and finished products and raw materials to be produced or entered to each facility at each production facility and at each period;

2.2 Model formulation and notation

In the next subsections, we define the objective function, decision variables and parameters of developed model.

Decision variables

The decision variables involved in costs minimization of the production model are as follows,

- \( P_{it} \) Quantity of finished products, provided by facility \( i \) at time \( t \),
- \( b_{it} \) Quantity of raw materials provided by facility \( i \) at time \( t \),
- \( o_{it} \) Demand for semi-finished products at node \( i \) at time \( t \),
- \( s_{it} \) Products inventory level at facility \( i \) at the end of period \( t \),
- \( w_{it} \) Raw materials inventory level in facility \( i \) at the end of period \( t \),
- \( B_{it} \) Quantity of raw materials which are entered in facility \( i \) at time \( t \),
- \( o_{it} \) Semi-finished product inventory level at facility \( i \) and time \( t \).

2.2.1 Objective function

The objective function is defined for minimizing the holding costs (Raw materials, semi-finished products and finished product, parts 1,2 and 3 in objective function) and production costs (for finished and semi-finished product, parts 4 and 5 in objective function). The production costs vary with the production level at the facilities. The holding costs are proportion to the stock levels of products and raw materials at all nodes in production management. Generally, this objective function is designed for minimization of company’s costs in its mentioned five parts. The objective function is formulated as follows:

\[
\begin{align*}
\min & \sum_{i \in I} \sum_{t \in T} \left( h'_{it} \cdot (B_{it} + b_{it} + w_{it}) \right) + \sum_{i \in I} \sum_{t \in T} \left( h''_{it} \cdot (o_{it} + o_{it}) \right) \\
& + \sum_{i \in I} \sum_{t \in T} \left( h_{it} \cdot (P_{it} + s_{it}) \right) + \sum_{i \in I} \sum_{j \in J} \left( h_{jt} \cdot s_{jt} \right) \\
& + \sum_{i \in I} \sum_{t \in T} \left( o_{it} \cdot P_{it} \right) + \sum_{i \in I} \sum_{t \in T} o_{it} \cdot O_{it} \\
\end{align*}
\]

where \( I \) is the set of manufactures production facilities, \( J \) is the set of distributors, \( T \) is the set of time-periods in the developed model planning horizon, \( p_{it} \) is the unit production cost for finished products in facility \( i \) at time \( t \), \( o_{it} \) is the unit production cost for semi-finished product in facility \( i \) at time \( t \), \( h'_{it} \) is the unit holding costs for raw materials in facility \( i \) at time \( t \), \( h''_{it} \) is unit holding costs for semi-finished products in facility \( i \) at time \( t \), \( h_{it} \) is the unit holding cost at node \( k \) and time \( t \).
2.2.2. Constraints

Below, we list the constraints that must be satisfied over the planning horizon. The constraints should be defined to cover following parts:

- **Raw materials**
  - Sustainability, balance, Production, Storage,
- **Semi-finished products**
  - Balance, production, storage
- **Finished products**
  - Inventory, production, balance

- **raw materials constraints:**

Sustainability:

\[
\begin{align*}
    z_{jt} & \geq z_{j(t-1)}, & j \in J, t \in T \\
    s_{it} & \geq s_{i(t-1)}, & i \in I, t \in T \\
    w_{it} & \geq w_{i(t-1)}, & i \in I, t \in T
\end{align*}
\] (1) (2) (3)

In which index t presents the periods of planning horizon. A sustainable plan requires the inclusion of the constraints (1)–(3), which enforce that the end inventory levels of products and raw materials be at least equal to the before ones. The omission of this type of constraints would result in the construction of a myopic plan (Heyman. ET, all. 1984) that may be characterized by unacceptably low, possibly equal to 0, end inventory levels, and leave the production plan in an undesirable state for the future.

Flow balance of raw materials at supplier’s facilities:

\[
    w_i = w_{i(t-1)} + b_i - \sum_{v \in F \setminus \{i\}} \sum_{t \in T} q_{ivt} - f_i \times p_i - g_i \times o_i, & i \in I, t \in T
\] (4)

The raw materials are consumed at the suppliers’ and manufacturers’ facilities. All raw materials needed by the manufacturers’ facilities come from the suppliers’ facilities. Denoting by \( b_i \) the amount of raw materials available in time t, by \( q_{ivt} \) the amount of raw materials moved from the suppliers’ facility \( i \) to the manufacturers’ facility \( i' \) using \( v \) at \( t \), by \( p_i \) the production at the suppliers’ facility at \( t \), by \( f_i \) the amount of raw materials for a unit of production, by \( o_i \) the demand for the semi-finished product at \( t \), and by \( g_i \) the appropriate conversion coefficient, we obtain Eq. (4).

Flow balance of raw materials at manufacturers’ facilities:

\[
    w_i = w_{i(t-1)} + \sum_{v \in F \setminus \{i\}} \sum_{t \in T} q_{ivt} - f_i \times p_i - g_i \times o_i
\] (5)

Constraints (4), (5) enforce that current raw materials and product demands at facilities be met from current production and on-hand inventories.

Production capacity of raw materials:

\[
    0 \leq b_i \leq b_i^{\text{max}}, \quad i \in I, t \in T
\] (6)
Constraint (6) limits the production of raw materials and products below a specified maximum level.

Storage capacity of raw materials at facilities:

\[ 0 \leq w_{it} \leq w_{it}^{\text{max}} \quad , i \in I, t \in T \]  \hspace{1cm} (7)

Constraint (7) limits from above the inventories of raw materials and products. The non-negativity Restrictions ensure that no backlog occurs.

Lack of raw materials inventory level Prevention in facilities:

\[ B_{it}^f + w_{it} \geq f_{it} \times p_{it} + g_{it} + p_{it} \]  \hspace{1cm} (8)

where \( p_{it} \) is Company program for production of finished products at period \( t \) in facility \( i \), \( f_{it} \) is the conversion rate of raw materials into finished-product at node \( i \) and time \( t \), and \( g_{it} \) is the conversion rate of raw materials into semi-finished product at node \( i \) and time \( t \).

**Semi-finished products constraints:**

Flow balance of semi-finished products at manufacturer’s facilities:

\[ o_{it} = o_{it} - \sum_{\text{forall } j \in V} q_{ijt} - r_{it} \times p_{it} \]  \hspace{1cm} (9)

Constraint (9) enforce that the current semi-finished product and its demands in facilities be met from current production and on-hand inventory.

Production capacity of semi-finished products in manufacturers’ facilities

\[ 0 \leq o_{it} \leq o_{it}^{\text{max}} \]  \hspace{1cm} (10)

Storage capacity of raw materials at facilities:

\[ 0 \leq o_{it} \leq o_{it}^{\text{max}} \]  \hspace{1cm} (11)

Constraints (10) and (11) limit inventories and production of semi-finished products below a defined maximum level. Again, non-negativity Restrictions ensure that no backlog occurs.

Prevention of lack of semi-finished products inventory level in facilities:

\[ o_{it} + o_{it} \geq r_{it} \times p_{it} \]  \hspace{1cm} (12)

In which \( r_{it} \) , is conversion rate of semi-finished products into finished products at node \( i \) and time \( t \).

**End products constraints:**

Flow balance of finished- products at facilities:

\[ s_{it} = s_{it-1} + \sum_{\text{forall } j \in V} q_{ijt}, i \in I, t \in T \]  \hspace{1cm} (13)
Constraint (13) enforce that the current finished products and its demands in facilities or costumer node’s be met from current production and on-hand inventory.

Production capacity of finished-products:

$$0 \leq p_i \leq p_i^{\text{max}}, \quad i \in I, t \in T$$  \hspace{1cm} (14)

Storage capacity of finished-products at facilities:

$$0 \leq s_i \leq s_i^{\text{max}}, \quad i \in I, t \in T$$  \hspace{1cm} (15)

Constraints (14), (15) limit from above the productions and inventories of finished-products in manufacturers facilities. The non-negativity restrictions ensure that no backlog occurs.

Prevention of lack of semi-finished products inventory level in facilities:

$$p_i + s_i \geq p_{0i}$$  \hspace{1cm} (16)

Shipment indivisibility:

$$x_{ijt} \in Z^+, \quad i \in I, j \in J, v \in V, t \in T$$  \hspace{1cm} (17)

where $Z^+$ denotes the set of the positive integers;

Non-negativity:

$$z_{ijt}, P_{ijt}, s_{ijt}, w_{ijt}, b_{ijt}, q_{ijvt} \geq 0, \quad i \in I, j \in J, t \in T, v \in V$$  \hspace{1cm} (18)

Operationalization of the distribution scheme:

$$x_{ijt} = 0, \quad i \in I, v \in V$$  \hspace{1cm} (19)

Constraint (19) indicates the impossibility to deliver to a certain node $j$, at a given time $t$, this resulting, for example, from bad weather conditions or the closing of facilities for some time.

2.2.4. Notation (sets and parameters)

In this formulation, we define the following sets and indices so that using them in developed model:

$I = \{1, 2, \ldots, i, \ldots\}$ set of production facilities (manufacturers and suppliers),

$J = \{1, 2, \ldots, j, \ldots, c\}$ set of distributors,

$K = I + J = \{1, 2, \ldots, k, \ldots, i + c\}$ set of nodes (suppliers and manufacturers, and distributors),

$T = \{1, 2, \ldots, t, \ldots, s\}$ set of time periods in the planning horizon,

The parameters used in the formulation of the model are listed below:

$p_i$ \hspace{1cm} Unit production cost at facility $i$ and time $t$;
\( p_{it} \)  The company’s plan for producing goods in facility \( i \) and period \( t \);
\( h_{it} \)  Unit holding cost of finished-products at facility \( i \) and time \( t \);
\( O_{it} \)  Unit production cost of semi-finished products at facility \( i \) and time \( t \);
\( h_{it}' \)  Unit holding cost of raw materials at facility \( i \) and time \( t \);
\( h_{it}'' \)  Unit holding cost of semi-finished products at facility \( i \) and time \( t \);
\( r_{it} \)  Conversion rate of semi-finished product into finished-products at node \( i \) and time \( t \);
\( f_{it} \)  Conversion rate of raw materials into finished-products at node \( i \) and time \( t \);
\( g_{it} \)  Conversion rate of raw materials into semi-finished products at facility \( i \) and time \( t \);
\( O_{it}^{\text{max}} \)  Maximum production capacity for semi-finished products at facility \( i \) and time \( t \);
\( P_{i}^{\text{max}} \)  Maximum range of production capacity for finished-products at facility \( i \);
\( w_{i}^{\text{max}} \)  Maximum range of storage capacity for raw materials in facility \( i \);
\( o_{i}^{\text{max}} \)  Maximum range of storage capacity for semi-finished products in facility \( i \);
\( s_{i}^{\text{max}} \)  Maximum range of storage capacity for end products in facility \( i \).

3. The results

3.1. Test laboratory

As a case study for model proposed, we use the data provided by one of the largest Iranian companies. This company (Iran khodro khorasan Razavi co.) assembles three kinds of cars, Peugeot Pars, Peugeot 405Glx and Peugeot 405Gli. A one-year planning horizon with monthly time-periods is considered. In this research, we use Lindo (Standard Operational Research Optimization) software to reach the Numerical Results.

In the case study of Iran khodro co, we have tested our Model seven times. In every test, we have used the proposed model in formation of cars both single and combinatorial as follow,

Peugeot 405Glx;
Peugeot Pars;
Peugeot 405Gli ;
Peugeot Pars, Peugeot 405Glx and Peugeot 405 Gli.

The results of these tests are illustrated in Tables 1 to Table 7. Table 8 shows add up results. All tests results are classified in two categories (costs and savings) and five sections for each category based on five parts of developed model involve. Note that the all costs are denoted in terms of Tumans, which is equivalent to 10 Rials.

### Table 1

<table>
<thead>
<tr>
<th>Costs</th>
<th>Costs</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Real value(Tuman)</td>
<td>Predicted value</td>
</tr>
<tr>
<td>1-End product assembly costs</td>
<td>27,080,000</td>
<td>26,837,600</td>
</tr>
<tr>
<td>2-holding costs of End products</td>
<td>120,544,000</td>
<td>100,506,160</td>
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<tr>
<td>3-Semi-finished products assembly costs</td>
<td>8,720,000</td>
<td>8,123,040</td>
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<tr>
<td>4-Holding costs of raw material</td>
<td>138,007,600</td>
<td>129,312,400</td>
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<tr>
<td>5-Holding costs of semi-finished products</td>
<td>10,480,000</td>
<td>8,393,120</td>
</tr>
<tr>
<td>Table 2</td>
<td>Costs and Savings (PEUGEOT PARS)</td>
<td></td>
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<tr>
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</tr>
<tr>
<td>Costs</td>
<td>Real value (Tuman)</td>
<td>Predicted value</td>
</tr>
<tr>
<td>Part 1-End product assembly costs</td>
<td>40,620,000,000</td>
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<td>2-holding costs of End products</td>
<td>180,816,000</td>
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<tr>
<td>3-Semi-finished products assembly costs</td>
<td>13,080,000</td>
<td>12,164,560</td>
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<td>4-Holding costs of raw material</td>
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<td>190,218,600</td>
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<td>5-Holding costs of semi-finished products</td>
<td>15,720,000</td>
<td>12,680,000</td>
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<thead>
<tr>
<th>Table 3</th>
<th>Costs and Savings (PEUGEOT 405GLI)</th>
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<tr>
<td>Costs</td>
<td>Real value (Tuman)</td>
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<tr>
<td>Part 1-End product assembly costs</td>
<td>26,984,000,000</td>
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<tr>
<td>2-holding costs of End products</td>
<td>120,544,000</td>
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<tr>
<td>3-Semi-finished products assembly costs</td>
<td>8,526,000</td>
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<td>4-Holding costs of raw material</td>
<td>138,665,750</td>
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<td>5-Holding costs of semi-finished products</td>
<td>9,480,000</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Costs and Savings (Peugeot 405Glx and Peugeot Pars)</th>
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</thead>
<tbody>
<tr>
<td>Costs</td>
<td>Real value (Tuman)</td>
</tr>
<tr>
<td>Part 1-End product assembly costs</td>
<td>67,700,000,000</td>
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<tr>
<td>2-holding costs of End products</td>
<td>301,360,000</td>
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<tr>
<td>3-Semi-finished products assembly costs</td>
<td>21,800,000</td>
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<td>4-Holding costs of raw material</td>
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<td>5-Holding costs of semi-finished products</td>
<td>26,200,000</td>
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</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Costs and Savings (Peugeot 405Glxi and Peugeot Pars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>Real value (Tuman)</td>
</tr>
<tr>
<td>Part 1-End product assembly costs</td>
<td>67,605,680,000</td>
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<tr>
<td>2-holding costs of End products</td>
<td>281,980,700</td>
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<tr>
<td>3-Semi-finished products assembly costs</td>
<td>20,780,000</td>
</tr>
<tr>
<td>4-Holding costs of raw material</td>
<td>338,704,500</td>
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<td>5-Holding costs of semi-finished products</td>
<td>25,410,000</td>
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</table>

<table>
<thead>
<tr>
<th>Table 6</th>
<th>COSTS AND SAVINGS (PEUGEOT 405GLI AND PEUGEOT 405GLX)</th>
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</thead>
<tbody>
<tr>
<td>Costs</td>
<td>Real value (Tuman)</td>
</tr>
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<td>Part 1-End product assembly costs</td>
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<tr>
<td>2-holding costs of End products</td>
<td>210,004,700</td>
</tr>
<tr>
<td>3-Semi-finished products assembly costs</td>
<td>19,780,000</td>
</tr>
<tr>
<td>4-Holding costs of raw material</td>
<td>316,504,900</td>
</tr>
<tr>
<td>5-Holding costs of semi-finished products</td>
<td>21,432,400</td>
</tr>
</tbody>
</table>
Table 7
**COSTS AND SAVINGS (PEUGEOT 405GLI AND PEUGEOT 405GLX AND PEUGEOT PARS)**

<table>
<thead>
<tr>
<th>Part</th>
<th>Costs</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real value(Tuman)</td>
<td>Predicted value</td>
</tr>
<tr>
<td>1-End product assembly costs</td>
<td>94,684,000t</td>
<td>93,997,570t</td>
</tr>
<tr>
<td>2-holding costs of End products</td>
<td>421,904,000t</td>
<td>358,946,560t</td>
</tr>
<tr>
<td>3-Semi-finished products assembly costs</td>
<td>30,326,000t</td>
<td>29,128,640t</td>
</tr>
<tr>
<td>4-Holding costs of raw material</td>
<td>483,684,750t</td>
<td>458,944,900t</td>
</tr>
<tr>
<td>5-Holding costs of semi-finished products</td>
<td>35,680,000t</td>
<td>30,415,920t</td>
</tr>
</tbody>
</table>

For more clearness we add up our results in Table 8.

Table 8
**The summary of the result of testing the performance of the proposed model for seven products**

<table>
<thead>
<tr>
<th>Test number</th>
<th>Case(s)</th>
<th>Average savings</th>
<th>Average Percent of savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Peugeot 405Glx</td>
<td>273,826,880</td>
<td>11.31</td>
</tr>
<tr>
<td>2</td>
<td>Peugeot Pars</td>
<td>427,518,000</td>
<td>12.22</td>
</tr>
<tr>
<td>3</td>
<td>Peugeot 405Gli</td>
<td>253,861,530</td>
<td>11.69</td>
</tr>
<tr>
<td>4</td>
<td>Peugeot 405Glx&amp;Pars</td>
<td>618,017,200</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>Peugeot 405Gli&amp;Pars</td>
<td>585,247,500</td>
<td>10.13</td>
</tr>
<tr>
<td>6</td>
<td>Peugeot 405Glx&amp;Gli</td>
<td>472,740,300</td>
<td>9.85</td>
</tr>
<tr>
<td>7</td>
<td>Peugeot 405Glx,Gli&amp;Pars</td>
<td>780,588,730</td>
<td>9.69</td>
</tr>
</tbody>
</table>

In Table 8, we show the average value of each test and in Fig. 1 we try to show these values.

![Fig. 1. The average cost saving associated with each case study](image)

4. **Conclusion**

Production management and analysis involves determining a number of decisions such as: 1) the appropriate inventory levels for components and finished products, 2) safety production programming for all products. In a large enterprise with a production management involving hundreds of items, a detailed analysis becomes virtually impossible when each item is considered individually. The use of an integrated production management becomes necessary in this regard.

In this research, a comprehensive methodology has been presented a new model of production management under different restrictive assumptions. This model has parts which cover some famous issues in industrial engineering, such as inventory control of components, production planning of them and warehousing. We have formulated cost function that included all the costs of raw materials ordering, inventories and deliveries of finished and semi-finished products to our facilities. For
developing a mathematical model of production management in five parts we used a linear operational research planning, validated on a real-industrial life problem faced by a large Iranian production companies, the developed methodology proposed is very effective as it can be seen in table 7 which shows these savings in all parts of our model.

As a review, in this paper, first, we introduced the parts of our model, next we tried to test it in a large production case study and eventually, we showed numerical results of our study in Iran khodro khorasan razavi company. In the case presented above, there was reduction in the costs of all parts by using new proposed model.

References


