Determination of the natural disaster insurance premiums by considering the mitigation fund reserve decisions: An application of collective risk model

Sukono\textsuperscript{a}, Kalfin\textsuperscript{b}, Riamana\textsuperscript{a}, Sudradjat Supian\textsuperscript{a}, Yuyun Hidayat\textsuperscript{c}, Jumadil Saputra\textsuperscript{d*} and Mustafa Mamate\textsuperscript{e}

\textsuperscript{a}Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia
\textsuperscript{b}Doctor Program of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia
\textsuperscript{c}Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia
\textsuperscript{d}Faculty of Business, Economics and Social Development, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia
\textsuperscript{e}Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Kuala Terengganu, Malaysia

\textbf{ABSTRACT}

In Indonesia, natural disasters cases have significantly increased from time to time and have the largest impact on economic losses. To avoid losses in the future due to natural disasters, the insurance company needs to estimate the risk and determine the rate of premium that would be charged to the policyholder. In conjunction with the present issue, this study seeks to determine the premium rate and estimate the size claim of insurance by considering the mitigation fund reserve decisions using The Collective Risk Model (CRM). The data was analyzed using the Poisson process with Weibull distribution to determine the natural disaster frequency and losses. The distribution of losses is estimated using Maximum Likelihood Estimation (MLE), and the magnitude of losses was estimated using the CRM. Also, the mean and variance estimators of the aggregate risk were used to estimate the premium charged. The results indicated that expectation and variance of the frequency of incident claims have the same value, i.e., 2562. Also, the loss claims follow the Weibull distribution with the expected value and variance of 5.81309×10\textsuperscript{10} and 2.5301×10\textsuperscript{22}, respectively. The mean and variance of the aggregate (collective) claims are 148,931,365,800,000 and 7.35×10\textsuperscript{25}, respectively. In conclusion, this study has successfully determined the efficient pure premium model through the Standard Deviation Principle (SDP). SDP provides a much cheaper premium than the Expected Value Principle with the same loading factor. The implications of the results of the premium determination are expected to be the basis for decision-making for insurance companies and the government in determining insurance policies for natural disaster mitigation.

\textbf{Keywords:}
Natural disasters
Mitigation Fund Reserve Decisions
Collective Risk Model
Insurance premium

1. Introduction

The potential for natural disasters in Indonesia is very high due to being located on the Pacific Ring of Fire (an area with a lot of tectonic activity) and the confluence of three large tectonic plates, namely the Indo-Australian Plate, the Eurasian Plate, and the Pacific plate. It makes Indonesia vulnerable to volcanic eruptions, earthquakes, and tsunamis (Martono, Satino, Nursalam, Efendi, & Bushy, 2019). In addition, climate change causes the potential for floods and landslides to continue to increase (Dewi & Istiadi, 2016). According to the National Disaster Management Agency (2017), the potential for natural disasters in Indonesia can be mapped based on each province, with Central Java, East Java, and West Java being...
the three provinces with the highest number of natural disasters. A study by Keerthiratne & Tol (2018) indicated that natural disasters have both direct and indirect impacts on the local community. The impacts caused by natural disasters are the emergence of fatalities, injuries, environmental damage, property losses, and psychological impacts (Benali & Feki, 2017). It is reinforced by Songwathana's (2018) opinion that natural disasters are very closely related and impact the socio-economic value of a country. For example, the tsunami disaster caused by the eruption of Anak Krakatau in the Sunda Strait in 2018 hit the coastal areas of Banten and Lampung. At least 426 people were killed and 7,202 injured, and 23 people were missing due to the incident. In addition, the disaster that occurred impacted the economic growth of the coastal areas of Banten and Lampung. Therefore, in minimizing the risk due to natural disasters, it is necessary to mitigate natural disasters.

Disaster mitigation is an effort to minimize the impact of a disaster or actions to reduce the risk caused by a disaster (El-Masri & Tipple, 2002). There are two mechanisms for mitigating natural disasters: pre- and post-disaster management (He & Zhuang, 2016; Kim & Marcouiller, 2018). Pre-disaster mitigation is a response effort carried out when a disaster does not occur by building various physical infrastructures and adopting a technological approach that can minimize risk. Some examples of pre-disaster mitigation are the construction of special canals to prevent flooding, tools for detecting volcanic activity, earthquake-resistant buildings, or the Early Warning System used to detect Tsunami waves (Kalflin et al., 2021). Meanwhile, post-disaster mitigation is a response effort carried out after a disaster in recovery, rehabilitation, and reconstruction (Banomyong, Beresford, & Pettit, 2009). Some examples of post-disaster mitigation are creating refugee posts, provision of health workers, and providing social assistance to disaster-affected communities. Judging from the events in the last few years in natural disaster management, the financial funds issued by the government are substantial. While the available sources of financial funds are limited, the disaster management process is not optimal (Kusumastuti, Husodo, Suardi, & Danarsari, 2014; Rindrasih, Witte, Spit, & Zoomers, 2019). It causes infrastructure development and economic recovery in disaster-affected areas to be not optimal and experience obstacles. Therefore, the Indonesian government needs to look for other alternatives in post-disaster management. One alternative that can be used is natural disaster insurance which several other countries have implemented (Pandigan, 2020; Saputra, Kusairi, & Sanusi, 2017b, 2017a).

Natural disaster insurance transfers risk from the government as a customer to an insurance company (Picard, 2008). There is also an agreement between the customer and the insurance company. The customer pays a premium agreed upon the loss events due to a natural disaster among these agreements. The insurance company must provide a guarantee against the risk of loss due to natural disasters or insurance risk (Aisah, Tampubolon, Napitupulu, Supian, & Sidi, 2018; Sukono, Parmikanti, Panggabean, Napitupulu, & Prabowo, 2020). Therefore, insurance companies must be able to manage and anticipate risks if there are many claims. Because if it is not expected, insurance companies will experience losses that can lead to bankruptcy or bankruptcy (Lesmana, Wulandari, Napitupulu, & Supian, 2018; Subartini et al., 2018). Thus, insurance companies need to be studied and considered in detail in determining premiums. In addition, in determining the premium, it is necessary to consider the customer's ability and not burden or harm the insurance company. In addition, in determining the premium for natural disaster insurance, it is necessary to consider several factors, namely the level of occurrence and loss of natural disasters and the level of economic growth in each region. Therefore, an efficient natural disaster insurance premium model is needed between customers and insurance companies (Hudson, 2018).

Previous researchers have developed several models of determining collective risk and disaster insurance premiums. For instance, Boratyńska (2008) used the collective risk model to determine the number of claims expected in insurance. Collective risk estimation is used to determine the minimum insurance premium. Besides that, the Bayesian method is used to describe losses that occur in the form of a random sample by combining prior data knowledge. A numerical simulation is conducted to determine the minimum insurance premium (McAneney, McAneney, Musulun, Walker, & Crompton, 2016; Porrini & Schwarze, 2014) to resolve the aggregate claim (collective risk) of the earthquake disaster case using Generalized Linear Models (GLM). The data was used in the form of earthquakes from 2000 to 2008 in Turkey. Maximum Likelihood Estimation (MLE) estimates the obtained model parameters. Zlateva & Velev (2016) analyzed the risk of natural disasters using the collective risk model. The collective risk is determined to determine the level of losses due to natural disasters.

In addition, it can also be used as a reference for local governments in providing efficient allocation of funds for natural disaster management. Kim and Marcouiller (2018) used a logistic distribution model with Contingent Valuation Methods (CVM) to estimate willingness to pay forest disaster insurance premiums. Furthermore, the Travel Cost Method (TCM) is used to measure the value of the subject's behavior in assuming. The data used is obtained from a survey of forest owners in Korea. Subartini et al. (2018) research used a Fuzzy Inference System (FIS) with the Tsukamoto method to determine the amount of flood insurance premiums in Bandung Regency. The data used is from losses due to floods that occurred in Bandung Regency. Besides that, Kalflin et al. (2020) used the Black Scholes Method in determining the amount of natural disaster insurance premiums in Indonesia. In determining natural disaster insurance premiums, consider the number of events and the magnitude of natural disaster losses. The data used consists of data on the incidence and losses of natural disasters in Indonesia.

Regarding previous elaborations, this study aims to determine the premium rate and estimate the size claim of insurance by considering the mitigation fund reserve decisions using the Collective Risk Model (CRM). The Poisson process with Weibull distribution is used to determine the natural disaster frequency and losses. The distribution of losses is estimated
using Maximum Likelihood Estimation (MLE), and the magnitude of losses was estimated using the CRM. Also, the mean
and variance estimators of the aggregate risk were used to estimate the premium charged.

2. Literature Review

2.1 Frequency of Natural Disasters

Lundberg (1903) first introduced the Poisson process as a model for many/frequency events processes. The Poisson process
determines the number of natural disasters occurring in a certain time interval. A stochastic process is called a counting
process if there are \( N(t) \), i.e. the number of natural disaster events that occur up to time \( t \), with \( t \geq 0 \). For example, the
counting process is the number of natural disasters in Indonesia at an interval of \( t \). The formulation and mathematical
modelling of the process of calculating \( \{N(t), t \geq 0\} \) from the number of natural disaster events. According to Alawiyah,
Johar, & Ruchjana (2021), there are reasonable assumptions that need to be fulfilled, namely \( N(t) \geq 0 \); \( N(t) \) is an integer;
If \( s < t \), then \( N(s) < N(t) \) and For \( s < t \), \( N(t) - N(s) \) states the number of natural disasters that occur in the time interval
\( (s,t] \)

Definition 1.

The process of calculating \( \{N(t), t \geq 0\} \) is called a Poisson process with parameter \( \lambda > 0 \) if it fulfils the following
assumptions:

At time \( t = 0 \), the number of events that occur is zero \( (N(0) = 0) \)
The process has independent increments and stationary increments.
If \( h \) is a short time interval, then\( = \lambda h + o(h) \).

\[
P(N(h) \geq 2) = o(h)
\]

On the basis of definition 1 for every \( t \geq 0 \) applies \( P_k(t) = P[N(t) = k|N(0) = 0] \) with \( (k = 0,1,2, \ldots) \) which is the
probability of occurrence in the time interval \([0, t)\). \( P_k(t) \) is the probability of a natural disaster occurring for the time value
\( t \). Based on the law of total probability, the number of probabilities is 1, which is formulated as follows:

\[
\sum_{k=0}^{\infty} P_k(t) = 1
\]  
(1)

Since the sum of the probabilities is 1 by applying the stationary of the Poisson process, we get

\[
P(N(s + t) - N(s) = k) = P[N(t) = k|N(0) = 0] = P_k(t)
\]  
(2)

For any \( t \geq 0 \) and \( s \geq 0 \).

The definition states a process to calculate \( \{N(t), t \geq 0\} \) for every \( s \geq 0 \) and \( k = 0,1,2,3, \ldots \) is called a Poisson process
with parameter \( \lambda > 0 \) having a probability for every \( k \) in the interval \( t \) formulated as following:

\[
P[N(t + s) - N(s) = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}
\]  
(3)

According to Alawiyah et al. (2021), if every \( s, t \geq 0 \), then \( (N(t + s) - N(s)) \sim \text{POI}(\lambda t) \), so the mean and variance of the
Poisson process on the number of natural disasters are formulated as follows:

\[
E(N(t)) = \lambda t
\]  
(4)

and

\[
\text{Var}(N(t)) = \lambda t
\]  
(5)

2.2 The Magnitude of Natural Disaster Losses

The Weibull distribution determines the magnitude of losses from natural disasters. The Weibull distribution has parameters
\( \lambda \) and \( k \), where the parameters \( \lambda \) and \( k \) are greater than 0. The cumulative distribution function of the Weibull distribution
is formulated as follows:

\[
F(x) = \begin{cases} 
1 - e^{-(\frac{x}{\lambda})^k} &; \ x \geq 0 \\
0 &; \ x < 0 
\end{cases}
\]  
(6)
where $\lambda > 0$ is a natural disaster loss parameter and $k > 0$ is a scale parameter (Ramaswamy & Anburajan, 2012). The probability density function of equation (6) is the derivative of the cumulative distribution function of the Weibull distribution. The probability density function of the Weibull distribution is formulated as follows:

$$f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$  \hspace{1cm} (7)$$

In Eq. (7), the determination of the expected value and variance of the Weibull distribution for natural disaster losses is formulated as follows (Qiao & Tsokos, 1994)

$$E(N(t)) = \lambda t$$  \hspace{1cm} (8)

$$Var(X) = \lambda^2 \left[ \Gamma \left( 1 + \frac{2}{k} \right) - \Gamma^2 \left( 1 + \frac{1}{k} \right) \right]$$  \hspace{1cm} (9)

2.3 Maximum Likelihood Estimator

According to Walpole, Myers, Myers, & Ye (1993), the Maximum Likelihood Estimator is one of the methods used to find the estimated value of a parameter. The joint density function of random variables $X_1, X_2, \ldots, X_n$ in $X_1, X_2, \ldots, X_n$ is $f(x_1, x_2, \ldots, x_n; \theta)$ which is called the likelihood function. For fixed $x_1, x_2, \ldots, x_n$, the likelihood function is a function of $\theta$ and is often denoted by $L(\theta)$, where $\theta^T = (\theta_1, \ldots, \theta_q)$. The likelihood function is another form of the combined probability distribution $X_1, X_2, \ldots, X_n$. If $X_1, X_2, \ldots, X_n$ represents a random sample with probability density $f(x; \theta)$, then $L(\theta) = \prod_{i=1}^{n} f(x_i; \theta)$.

It is often easier not to maximize $\theta$ directly but rather to Log-Likelihood, i.e. $\ln \theta$. It can be done because $\theta$ and $\ln \theta$ reach the global maximum on $\theta$. In addition, using $\ln L(\theta)$ will make it easier to find the global maximum on $\theta$. The natural logarithm function is an ascending monotone function, ensuring that $\theta$ has a global maximum. To obtain points that maximize $\theta$, it is necessary to do linearization. Linearization is carried out on the natural logarithm function of the likelihood function, which is done by differentiating to $\theta$. According to Hao, He, & Long (2018), to obtain the maximum likelihood estimator $(\hat{\theta})$ which is formulated as follows:

$$\frac{\partial \ln L(\theta)}{\partial \theta} = 0$$  \hspace{1cm} (10)

2.4 Collective risk model

Analyzing the collective risk that occurs considers the number of claims for natural disasters and the magnitude of the claims caused by natural disaster cases. Collective risk is one of the main problems in considering the claim's magnitude. The number of aggregate claims in insurance becomes a reference for the insurer in determining the number of reserves and insurance premiums. In the individual risk model or individual claims, if the $i$-th individual unit is viewed as a large unit of individual claims and is denoted $X_i$ by so in the individual risk model or individual claims, if the individual unit to $i$ is seen as a large unit of individual claims and is denoted by $X_i$ so that:

$$X = \{X_i\}, \quad i = 1, 2, 3, \ldots, N$$  \hspace{1cm} (11)

Where $X_i$ is assumed to be continuously distributed, a random variable with identical distribution independently (Sarabia, Gómez-Déniz, Prieto, & Jordá, 2016). The collective risk model can be used to assume a mixed distribution with aggregation claims which is the sum of $N$ individual claims. So that the total number of aggregations claims up to time $t$ can be formulated as follows:

$$S(t) = X_1 + X_2 + \cdots + X_{N(t)} = \sum_{i=1}^{N(t)} X_i$$  \hspace{1cm} (12)$$

where $N$ is a random variable that states many claims. $X_1, X_2, \ldots, X_{N(t)}$ is a random variable that states the size of individual claims which can be in the form of discrete and or continuous distributions that are identical and independent.

In general, insurance companies need to estimate the collective risk $S(t)$ because the collective risk estimator that has been analyzed becomes the company's reference in determining insurance reserves. The magnitude of the collective risk lies in the model solution used to determine the estimator of the expectation (mean) and variance of $S(t)$. The estimator of the amount of collective risk $S(t)$ that must be paid is shown in the size of the mean value, taking into account the number of
events $N(t)$ and the amount of loss $X(t)$. According to Aisah et al. (2018), mathematically, the expectation (mean) and variance of the collective risk $S(t)$ can be formulated as follows:

$$E(S(t)) = E[E(S(t) = x|N(t) = n)] = E(N(t))E(X(t))$$  \hspace{1cm} (13)

$$E \left( (S(t))^2 \right) = E \left[ E \left( (S(t))^2 | N(t) = n \right) \right] = \text{Var}(X(t))E(N(t)) + \left( E(X(t)) \right)^2 \left( E(N(t)) \right)^2 $$  \hspace{1cm} (14)

Furthermore, based on the information from equations (13) and (14), the Variance $\text{Var}(S(t))$ can be formulated as follows:

$$\text{Var}(S(t)) = E \left( (S(t))^2 \right) - \left( E(S(t)) \right)^2$$

$$= \text{Var}(X(t))E(N(t)) + \left( E(X(t)) \right)^2 \text{Var}(N(t)) + \left( E(N(t)) \right)^2 \text{Var}(X(t))$$  \hspace{1cm} (15)

2.5 **Premium Calculation Model**

Premium is a sum of money that must be paid by the insured (policyholder) at each predetermined time period as his obligation to the insurance company. Risk and premium are related because the premium is determined based on the risk mean. Therefore, any changes in risk must be considered in detail in the premium determination process. This study’s insurance premium calculation model considers the collective risk in many events and the magnitude of natural disaster losses. In addition, this model also considers the premium loading factor ($\alpha$) that occurs in cases of natural disasters. According to Đurić (2013), the model used in calculating insurance premiums is based on the following principles:

a) **The principle of expected value**

Premium calculation using the expected value principle based on the average aggregate risk (collective) or pure premium multiplied by the security loading (factor loading). Mathematically formulated as follows:

$$P_E(t) = (1 + \alpha)E(S(t))$$

with $\alpha > 0$.  \hspace{1cm} (16)

b) **Principle of Standard Deviation**

The premium calculation with this principle is based on the pure premium added up by the multiplication of the factor loading and the Standard Deviation of the collective risk. The standard deviation principle is widely used in non-life insurance because the proportional change in the premium calculation results is not too high. Mathematically formulated as follows:

$$P_{SD}(t) = E(S(t)) + \alpha \sqrt{\text{Var}(S(t))}$$

with $\alpha > 0$.  \hspace{1cm} (17)

3. **Materials and Methods**

Research methods are steps taken by researchers to collect data or information to be processed and analyzed scientifically. The data required is secondary data, which includes data on the frequency of natural disasters and large data on losses due to natural disasters in Indonesia. The required secondary data was obtained from the National Disaster Management Agency. The estimation of the distribution model of the frequency of natural disasters, which is assumed to follow the Poisson distribution model, refers to Eq. (3). The estimator of the distribution model is then used to determine the expected value and variance of the frequency of natural disasters, using Eq. (4) and Eq. (5), respectively. The estimation of the distribution model of the magnitude of losses due to natural disasters is assumed to follow the Weibull distribution model with the probability density function referring to Eq. (7). The estimator of the Weibull distribution model is then used to determine the expected value and variance of the amount of losses due to natural disasters, using Eq. (8) and Eq. (9). The estimation of the distribution model of the frequency of natural disasters in 2), and the estimation of the distribution model of the magnitude of the loss due to natural disasters 3) is carried out using the Maximum Likelihood Estimation (MLE) method regarding Eq. (10). Using the expectation and variance in the frequency of natural disasters 2), and the expectation and
variance in the magnitude of losses due to natural disasters), the collective risk expectation is determined by referring to Eq. (13) and the collective risk variance by referring to Eq. (15).

4. Results and Discussion

4.1 Descriptive Statistics Analysis

The data used in this study is from annual data in the form of incidents and losses from natural disasters in Indonesia. The data is obtained from the National Disaster Management Agency, used for 20 years (2000-2019). The data on cases of natural disasters is depicted in the form of a histogram using Microsoft Excel, given in Fig. 1.

![Fig. 1. Data on the Number of Natural Disasters in Indonesia](image)

Fig. 1 shows the cases of natural disasters in Indonesia continued to increase from 2000 to 2017. Furthermore, the number of natural disasters decreased from 2017 to 2019. In 2017, the number of natural disaster cases was 5,442, and it was the most natural disaster cases from 2000 to 2019. Furthermore, descriptive statistics on natural disaster cases are presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>138.00</td>
<td>5442.00</td>
<td>2562.0000</td>
<td>1587.41120</td>
</tr>
</tbody>
</table>

Table 1 displays the average number of natural disasters in Indonesia, which is 2,562 cases per year. Meanwhile, the standard deviation of data on natural disasters in Indonesia is 1,587.41120. In Table 1, it can also be seen that from the data on natural disasters that occurred in Indonesia from 2000 to 2019, the highest number of events was as many as 5,442 cases, while the smallest number of events (minimum) was 138 cases. Furthermore, descriptive statistics of natural disaster loss data are described in graphical form using the help of Microsoft Excel given in Fig. 2.

![Fig. 2. Graph of Large Data on Losses Due to Natural Disasters in Indonesia](image)
Fig. 2 captures the natural disaster losses in Indonesia that have increased or decreased (fluctuated) every year. The smallest natural disaster losses occurred in 2014, while the largest natural disaster losses occurred in 2019. Furthermore, descriptive statistics on natural disaster cases are presented in Table 2.

Table 2
Descriptive Statistics of Data on Natural Disaster Losses in Indonesia

<table>
<thead>
<tr>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2,567,641</td>
<td>159,920,522,794</td>
<td>42,066,867,692</td>
<td>47,804,880,752.85</td>
</tr>
</tbody>
</table>

Table 2 describes the data on losses from natural disasters in Indonesia from 2000 to 2019, for the highest number of losses amounted to IDR 159,920,522,794.00. Meanwhile, the smallest (minimum) loss due to natural disasters is IDR 2,567,641.00. Table 2 also shows that the average loss from natural disasters in Indonesia is IDR 42,066,867,692.00. Meanwhile, the standard deviation of data losses due to natural disasters in Indonesia is IDR 47,804,880,752.85.

4.2 Estimated Number of Natural Disasters

The model estimation of the number of confirmed cases of natural disasters is carried out using the Poisson process. The first step to determining the number of confirmed cases of natural disasters using the Poisson process is to determine the rate ($\lambda$) by assuming that the data on confirmed cases of natural disasters is valid with a time interval of $t = [0, 20]$.

$$\lambda = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{138 + 218 + \cdots + 1439}{20} = 2,562 \text{ cases/year}$$

Based on the analysis results, the rate ($\lambda$) of confirmed cases of natural disasters from January 2000 to December 2019 was 2,562 cases/year. Furthermore, from the value of $\lambda$, the expectation and variance of the natural disaster case data are determined using Eq. (4) and Eq. (5) with $t = 1$ year.

$$E(N(t)) = (2562)(1) = 2562$$
$$Var(N(t)) = (2562)(1) = 2562$$

Based on the calculation results, the expected number of claims for natural disaster cases in 1 year is 2562. Meanwhile, the variance of natural disaster cases in 1 year is 2562. Furthermore, the expected value and variance of the number of natural disasters are used in calculating the collective risk of natural disaster insurance.

4.3 Model for Estimating the Number of Natural Disasters Losses

Identification of the distribution model for the magnitude of claims for natural disasters is carried out using Easyfit 5.5 statistical software. The results of Goodness of Fit based on the ranking of the five best distributions of data on the size of claims for natural disasters can be seen in Table 3.

<table>
<thead>
<tr>
<th>No.</th>
<th>Distribution</th>
<th>Kolmogorov Smirnov</th>
<th>Anderson Darling</th>
<th>Chi-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Statistic</td>
<td>Rank</td>
<td>Statistic</td>
</tr>
<tr>
<td>1</td>
<td>Beta</td>
<td>0.13817</td>
<td>1</td>
<td>2.2233</td>
</tr>
<tr>
<td>2</td>
<td>Gamma (3P)</td>
<td>0.16386</td>
<td>2</td>
<td>2.2569</td>
</tr>
<tr>
<td>3</td>
<td>Log-Pearson 3</td>
<td>0.1725</td>
<td>3</td>
<td>0.65017</td>
</tr>
<tr>
<td>4</td>
<td>Weibull</td>
<td>0.1783</td>
<td>4</td>
<td>0.75079</td>
</tr>
<tr>
<td>5</td>
<td>Weibull (3P)</td>
<td>0.18543</td>
<td>5</td>
<td>2.4302</td>
</tr>
</tbody>
</table>

Table 3 shows the study of claims size for natural disasters using the Weibull distribution model. The Weibull distribution shows consistent results with ranks that are not too far away based on 3 conformity tests, namely the Kolmogorov Smirnov test at rank 4, Anderson Darling at rank 2 Chi-Squared at rank 4. In addition, the identification of the Weibull distribution model on the loss due to natural disasters is also carried out using Minitab 17 software. This stage is carried out to test the distribution suitability of the model that has been taken. The histogram of loss data due to natural disasters can be seen in Fig. 3.
Figure 3 shows the losses due to natural disasters in Indonesia using Minitab 17 software. A suitable distribution model is obtained for the amount of loss claims distributed by Weibull with parameters $\alpha = 0.31885$ and $\beta = 2.1760 \times 10^{10}$. For determining the distribution's suitability, the Anderson Darling test was carried out on the Weibull distribution with the help of Minitab 17 statistical software. Anderson Darling test results using Minitab 17 software are given in Figure 4. Using the Anderson Darling test given in Figure 4, the data on the number of claims for losses due to natural disasters follows the Weibull distribution model with an Anderson Darling (AD) test value of 1.44. In addition, a statistical summary of the Weibull distribution was determined using Minitab 17 software and obtained the results as given in Table 4.

Table 4
Summary of Weibull distribution statistics

<table>
<thead>
<tr>
<th>N</th>
<th>$E(\hat{X}(t))$</th>
<th>Std. Deviation</th>
<th>$Var(\hat{X}(t))$</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5.81309E+10</td>
<td>1.59063E+11</td>
<td>2.5301E+22</td>
<td>9,402,906,932</td>
</tr>
</tbody>
</table>

Fig. 3. Histogram of Data Losses Due to Natural Disasters in Indonesia

Fig. 4. Weibull Distribution Plot on Natural Disaster Loss Data
4.4 Collective Risk Model

Calculation of collective risk from data on cases of events and losses due to natural disasters is carried out to determine the premium predictions appropriate to be charged to the insured. The collective risk model considers two indicators, namely the number of events \(N(t)\) and the amount of loss \(X(t)\) from natural disasters that occurred in Indonesia. Calculation of expectations of collective risk in cases of natural disasters using equation (13) obtained the following results:

\[
E(S(t)) = E(N(t))E(X(t)) = (2,562) \times (58,130,900,000) = 148,931,365,800,000.00.
\]

While the calculation of the variance of the collective risk in the case of natural disasters using equation (15), the results obtained are as follows:

\[
Var(S(t)) = E(N(t))Var(X(t)) + Var(N(t))(E(X(t)))^2 = (2,562) \times (2.5301 \times 10^{22}) + (2,562) \times (58,130,900,000)^2 = 7.35 \times 10^{25}.
\]

The expected value and variance of the collective risk model in cases of natural disasters obtained previously can be used to predict insurance premiums to be paid by policyholders.

4.5 Insurance Premium Calculation

The insurance premium is the amount of money that must be paid by the insured (policyholder) to the insurance company. The amount of money paid by the insured is compensation for the risk of natural disasters that will occur to the insurance company. The premium amount depends on the collective risk in the form of the number of events and the magnitude of the loss from natural disasters. In addition, the amount of the premium must be greater than the amount of claim payments that will occur. If the specified premium is less than the amount of collective (aggregate) claim payments, the insurance company will suffer a loss. Therefore, it is necessary to multiply the cost factor (risk-free interest rate) in determining the premium. In this study, the cost factor \(\alpha\) is used from 1% to 10% in determining insurance premiums. The cost factor \(\alpha\) is in the 1% to 10% interval because insurance companies generally use a different cost factor but are in the 1%-10% interval. Using Eq. (16) and Eq. (17), the calculation of insurance premiums is given in Table 5.

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>Pure premium with the principle of expected value ((P_E))</th>
<th>Pure premium with standard deviation principle ((P_{SD}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>150,420,679,458,000.00</td>
<td>149,017,085,503,880.00</td>
</tr>
<tr>
<td>2%</td>
<td>151,909,993,116,000.00</td>
<td>149,102,805,207,759.00</td>
</tr>
<tr>
<td>3%</td>
<td>153,399,306,774,000.00</td>
<td>149,188,524,911,639.00</td>
</tr>
<tr>
<td>4%</td>
<td>154,888,620,432,000.00</td>
<td>149,274,244,615,519.00</td>
</tr>
<tr>
<td>5%</td>
<td>156,377,934,090,000.00</td>
<td>149,359,964,319,398.00</td>
</tr>
<tr>
<td>6%</td>
<td>157,867,247,748,000.00</td>
<td>149,445,684,023,278.00</td>
</tr>
<tr>
<td>7%</td>
<td>159,356,561,406,000.00</td>
<td>149,531,403,727,158.00</td>
</tr>
<tr>
<td>8%</td>
<td>160,845,875,064,000.00</td>
<td>149,617,123,431,037.00</td>
</tr>
<tr>
<td>9%</td>
<td>162,335,188,722,000.00</td>
<td>149,702,843,134,917.00</td>
</tr>
<tr>
<td>10%</td>
<td>163,824,502,380,000.00</td>
<td>149,788,562,838,797.00</td>
</tr>
</tbody>
</table>

Table 5 displays that the calculation of insurance premiums with the expected value principle has greater results than using the principle of standard deviation. Based on the value of the cost factor \(\alpha\), which continues to be increased from 1% to 10%, the premium value also increases based on the increase in the given cost factor. Based on Table 5, the smallest and largest insurance premiums with the expected value principle are IDR. 150,420,679,458,000.00 and IDR. 163,824,502,380,000.00. The smallest premium value occurs when the cost factor is 1%, while the largest premium occurs when the cost factor is 10%. Besides that, the smallest and largest insurance premiums for the standard deviation principle are IDR. 149,017,085,503,880.00 and IDR. 149,788,562,838,797.00, respectively. In addition, the smallest and largest premium values occur when the cost factor is 1% and 10%. Furthermore, the comparative analysis of the two principles of calculating insurance premiums based on the results in Table 5 can be seen in Fig. 5.
Fig. 5. Comparative analysis of premium calculation with the principle of expected value and standard deviation

Fig. 5 shows the determination of natural disaster insurance premiums with the principle of expected value, and the value continues to increase along with the increase in the given cost factor. The principle of standard deviation continues to increase along with the increase in the given cost factor. However, the delta (difference) of the premium based on the increase in the value of the cost factor from the expected value principle is very large compared to the standard deviation principle. In addition, based on Fig. 5, it can be seen that the principle of standard deviation has natural disaster insurance premiums that are cheaper or smaller than the principle of expected value at the same cost factor. In this case, the principle of standard deviation is very suitable to be used in the calculation of natural disaster insurance premiums. It is also reinforced by Đurić (2013) research, which states that the standard deviation principle is suitable for calculating non-life premiums. The standard deviation principle pays attention to the standard deviation of risks. Meanwhile, the principle of expected value does not pay attention to the standard deviation of the risk.

5. Conclusion

In conclusion, this study indicated that the expected value and variance using the Posson process in the number of natural disasters are 2,562 cases/year. As for the expectations and variances on the magnitude of natural disaster losses using the Weibull distribution. The expected value and variance of each loss claim amount of 5.81309×10^10 and 2.5301×10^22. Based on the results of data analysis of cases of events and losses due to natural disasters, the expected values and variance of collective risk are 148,931,365,800,000.00 and 7.35×10^25, respectively. In addition, the calculation process uses two principles, namely the expected value and the standard deviation. The premium value for these two principles continues to increase along with the increase in the given cost factor. However, when viewed from the proportional change in premium value, the standard deviation principle has the smallest change value compared to the expected value principle. Therefore, in this study, the standard deviation principle provides an efficient estimate of the premium value compared to the expected value principle. Where the estimated efficient premium value is IDR. 150,420,679,458,000.00 with a factor loading of 1%.

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References


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