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Ranking CCR-efficient units based on the indicator with limited resources

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A B S T R A C T

Data Envelopment Analysis (DEA) is one of the most popular techniques for measuring the relative efficiencies of a set of decision making units (DMUs), which use different inputs producing various outputs. Ranking of efficient DMUs is one of the most interesting DEA perspectives. However, there are cases where we see some limitations on available resources and the proposed model of this paper is associated with Indicator with Limited Sources (ILS), which affects ranking methods. The ILS exists as fixed amount in a community and the DMUs can own it with their abilities. When a DMU loses the same amount of the indicator, the rest of the DMUs are able to own some without even changing their capacities of other indicators and or vice versa. If a DMU looks for more of the same amount of the indicator, the rest of the DMUs have to supply it without even changing their capacity of other indicators. This paper develops a ranking method based on the ILS for the efficient DMUs, when there is changes either in inputs/ outputs ILS. The implementation of the proposed model is applied for a case study of banking system.

1. Introduction

Since the introduction of data envelopment analysis by Charnes et al. (1978, 1995), there have been tremendous efforts on developing different aspects of the original model. Jahanshahloo et al. (2005) is believed to be the first who introduced the idea of indicator with limited sources (ILS). There are many cases where the availability of some of the input or output indicators are relatively limited and the DMUs are able to own them with their abilities called as ILS. A good instance of ILS is where we attempt to measure the relative efficiencies of different banks located in a small community where there are limited number of customers. Obviously, there are some limited resources for banking deposit and there is a competition among various branches to absorb new customers.

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The number of people in medical care in small community is another instance of ILS since there are some limitations on customers and medication. In both mentioned examines, when a DMU losses some small amount of output, the rest of the DMUs are easily get more market share and improve their outputs. There are literally many evidences to believe that we have limited resources among various DMUs. Total amount of budget in a system is limited and it needs to be allocated among various branches based on their relative efficiencies. Another point is that if a DMU wants more of the same input, this amount must be supported by decreasing the input in the other DMUs. In this paper, we present an improved ILS ranking method for evaluating a set of efficient DMUs against variation in an input or output ILS.

This paper is organized as follows. We first review the related literature review on DEA method in section 2. Section 3 present a ranking method based on sensitivity analysis of the implementation of ILS for efficient DMUs. The proposed method of this paper is supported with an application, for ranking the branches of a famous Iranian bank in section 4. finally conclusion remarks are given at the end to summarize the contribution of the paper.

2. CCR-efficiency

Consider n DMUs, where the j-th DMU uses input vector $\mathrm{X}_{\mathrm{j}}^{\mathrm{T}} = (\mathrm{x}_{1\mathrm{j}}, \dots, \mathrm{x}_{\mathrm{mj}}) \in \mathbb{R}_{+}^{\mathrm{m}}$ and produces output vector $\mathrm{Y}_{\mathrm{j}}^{\mathrm{T}} = (\mathrm{y}_{1\mathrm{j}}, \dots, \mathrm{y}_{\mathrm{sj}}) \in \mathbb{R}_{+}^{\mathrm{s}}$, where $\mathrm{j} \in \{1, \dots, \mathrm{n}\}$.

In DEA literature, we construct a production technology, called Production Possibility Set (PPS), from the observed input-output vectors of the DMUs to study a particular case. An input-output vector (X, Y) model is determined in PPS when the output vector Y can be produced by the input vector X. To create the PPS, the following general assumptions need to be considered,

(A1) All actually observed input-output combinations (X_j, Y_j), $j = 1, \dots, n$, are in PPS.

(A2) The PPS is convex set, i.e. if $(\bar{X}, \bar{Y})$ and $(\hat{X}, \hat{Y})$ are in PPS then for any $0 \le \lambda \le 1$, (X_λ, Y_λ) is also in PPS, where $X_\lambda = \lambda \bar{X} + (1-\lambda)\hat{X}$ and $X_\lambda = \lambda \bar{Y} + (1-\lambda)\hat{Y}$.

(A3) Inputs are freely disposable, i.e. if $(\bar{X}, \bar{Y}))$ is in PPS then for any $X \ge \bar{X}$, $(X, \bar{Y})$ is also in PPS.

(A4) Outputs are freely disposable, i.e. if $(\bar{X}, \bar{Y}))$ is in PPS then for any $Y \le \bar{Y}$, $(\bar{X}, Y)$ is also in PPS.

We additionally assume that constant returns to scale (CRS) holds.

(A5) If $(\bar{X}, \bar{Y})$ is in PPS, then for any $0 \le \lambda$, $(\lambda\bar{X}, \lambda\bar{Y})$ is also in PPS.

On the basis of the observed input-output quantities and under the five assumptions, the PPS can be defined as follows:

$$\boldsymbol{T_C} = \left\{ (\boldsymbol{X}, \boldsymbol{Y}) |\ \boldsymbol{X} \ge \sum_{j=1}^{n} \boldsymbol{\lambda_j X_j}, \boldsymbol{Y} \le \sum_{j=1}^{n} \boldsymbol{\lambda_j Y_j}, \boldsymbol{\lambda_j} \ge \boldsymbol{0}, \boldsymbol{j} = \boldsymbol{1}, \dots, \boldsymbol{n} \right\}. \tag{1}$$

Here the subscript C indicates that the technology is characterized by CRS. The input-oriented linear programming problem formulation for the model proposed by Charnes et al. (CCR) (1978) model for evaluation of DMU_o, $j \in \{1, \dots, n\}$, (the multiplier side) is as follows:

$$\theta_o^* = \max \quad \sum_{r=1}^{s} u_r\, y_{ro}$$

subject to

$$\sum_{i=1}^{m} v_i\, x_{io} = 1, \tag{2}$$

$$\sum_{r=1}^{s} \boldsymbol{u_r\, y_{rj}} - \sum_{i=1}^{m} \boldsymbol{v_i\, x_{ij} \leq 0}, \qquad \boldsymbol{j = 1, \ldots\ , n},$$

$$v_i \geq \varepsilon, \qquad i = 1, \ldots\ , m,$$

$$u_r \geq \varepsilon, \qquad r = 1, \ldots\ , s$$

where ε is a non-Archimedean infinitesimal.

It is an easy task to show $0 < \theta_o^* \leq 1$ and DMU_o is efficient in the CCR model if $\theta_o^* = 1$. Otherwise, the DMU_o is inefficient (Charnes, 1978). Therefore, from Eq. (1), DMU_o is efficient, if there are $v_i \geq \varepsilon$, $i = 1, \ldots\ , m$ and $u_r \geq \varepsilon, r = 1, \ldots\ , s$, such that

$$\sum_{r=1}^{s} u_r\, y_{ro} - \sum_{i=1}^{m} v_i\, x_{io} = 0,$$

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0. \qquad j = 1, \ldots\ , n, j \neq o. \tag{3}$$

without loss of generality, we can verify that $\sum_{r=1}^{s} u_r - \sum_{i=1}^{m} v_i = 1$, because if v_i and u_r satisfy in Eq. (3), then $\bar{v}_i = t v_i$ and $\bar{u}_i = t u_i$ also satisfy, where

$$t = \frac{1}{\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i}.$$

3. Ranking efficient DMUs

One of the most important issues on most DEA problems appears when there is more than one efficient unit when DEA is applied (Mehrabian et al., 1999; Khaki et al., 2012). Matric and Savic (2001) developed a model to determine the relative efficiency among various units in an effort to discriminate the impacts of having more than one efficient unit. Alder et al. (2002) presented a review on varieties of DEA methods and divided all DEA methods into various categories. Sexton (1986) provided a ranking of various DMUs based on cross-efficiency. Torgersen et al. (1996) implemented a special DEA method where the slack variables in the original model were modified. They concluded that a DMU was highly ranked if it were chosen as a reference by other inefficient DMUs.

Fridman and Sinuany-Stern (1997) implemented statistical techniques for scaling the inputs and the outputs based on correlation analysis. They implemented multivariate statistical techniques such as canonical correlation analysis and discriminate analysis to rank all efficient and inefficient DMUs. One of the most popular methods for handling the difficulty arising in multiple efficiencies is a technique proposed by Anderson and Peterson (1993) and the method for ranking DMUs is called super-efficiency. Thrall (1996) pointed out that this technique may result in instability when some inputs are close to zero and proposed a modified one to overcome this issue. Tone (2002) provided a method to overcome this issue. In this section, we rank the efficient DMUs based on their stability ratios in decreasing order using the idea of ILS.

Suppose that DMU_o is an efficient DMU and the k-th output has a limited amount of sources. The changing amount of y_{kj} , ‘$j = 1, \dots, n$, is denoted by α_j, such that $0 \le \alpha_o \le y_{ko}$ and $0 \le \alpha_j \le p_j$, $j \neq o$, where p_j is a non-decreasing y_{kj} such that DMU_j is capable of producing $Y_j^{'} = (y_{1j}, \dots, y_{kj}^{'}, \dots, y_{sj})$ by the same inputs X_j, where

$$y_{ko}^{'} = y_{ko} - \alpha_o$$
$$y_{kj}^{'} = y_{kj} + \alpha_j, \ \ j \neq o$$

and $\sum_{j \neq o} \alpha_j \le \alpha_o$. $p_j \ge y_{ko}$ means that α_j can be increased without making any limitation.

Suppose that the attraction contribution of α_o by DMU_j is stated and is denoted by w_j where

$$W = (w_1, \dots, w_n), \ \ w_o = 0, w_j \ge 0, \sum_{j=1}^{n} w_j \le 1.$$

Therefore, we have $\alpha_j = w_j \alpha_o$. The stability interval of DMU_o is $[0, \alpha_o(W)]$, where $\alpha_o(W)$ has the most amount α_o such that DMU_o with input-output vector $(X_o, Y_o^{'})$ holds over as an efficient DMU among other DMUs with input-output vector $(X_j, Y_j^{'})$. From Eq. (3), we have,

$$\begin{aligned}
&\alpha_o(W) = \max \quad \alpha_o \\
&\text{subject to} \\
&\sum_{r=1}^{s} u_r\, y_{ro} - u_k\, \alpha_o - \sum_{i=1}^{m} v_i\, x_{io} = 0 \\
&\sum_{r=1}^{s} u_r\, y_{rj} + u_k\, w_j \alpha_o - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \quad j \neq o \\
&\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i \ = 1 \\
&0 \le \alpha_o \le y_{ko} \\
&w_j \alpha_o \le p_j \ \ j \neq o \\
&v_i \ge \varepsilon \ \ i = 1, \dots, m \\
&u_r \ge \varepsilon \ \ \ r = 1, \dots, s.
\end{aligned} \tag{4}$$

Eq. (4) is a nonlinear programming problem, which can be converted into linear form by $\bar{\alpha}_o = u_k\ \alpha_o$.

Now, suppose that the contribution of each DMU of $\alpha_o$is unknown. Hence, $\alpha_o(W)$ is a function of W. Let α_o^L and α_o^U be the lower and the upper bounds for $\alpha_o(W)$, respectively. Therefore $\alpha_o^L \le \alpha_o(W) \le \alpha_o^U$ holds for each W. Therefore, we have,

$$\alpha_o^L = \min\{\alpha_o(W) | W = (w_1, \dots, w_n), w_o = 0, w_j \ge 0, \sum_{j=1}^{n} w_j \le 1\} \tag{5}$$

$$\alpha_o^U = \max\{\alpha_o(W) | W = (w_1, \dots, w_n), w_o = 0, w_j \ge 0, \sum_{j=1}^{n} w_j \le 1\} \tag{6}$$

The amount of α_o^L is obtained by solving a $Min\ Max$ nonlinear programming problem. Jahanshahloo et al. (2008) provided a method for calculating α_o^L. It is obtained by solving $n-1$ linear programming problems.

Theorem 1. $\alpha_o^L = z^l$, where

$$z^l = \frac{1}{u_k^*} \min \quad z - \varepsilon\left(\sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^-\right) \qquad l = 1, \dots, n, l \neq o$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} y_{1j}\,\lambda_j + z - y_{ko}\,\beta - \sum_{j\neq o} p_j\,\mu_j - s_k^+ = 0 \\
&\sum_{j=1}^{n} y_{rj}\,\lambda_j + z - s_r^+ = 0, \qquad r \neq k \\
&\sum_{j=1}^{n} x_{ij}\,\lambda_j - z + s_i^- = 0, \qquad i = 1, \dots, m \\
&-\lambda_o + \beta + \lambda_l + \mu_l \geq 1 \\
&\lambda_j \geq 0 \\
&\mu_j \geq 0 \\
&\beta \geq 0 \\
&z, \lambda_o, \quad \text{free}
\end{aligned} \tag{7}$$

where u_k^* is optimal solution $u_k$in Eq. (4) for W^* such that W^* is optimal solution of Eq. (5) corresponds to α_o^L.

Proof. (See Jahanshahloo et al., 2008).

Based on Theorem 1, we have,

$$\alpha_o^L = z^q = \min\{z^k | k = 1, \dots, n, k \neq o\}.$$

The amount of α_o^U is computed by solving following problem,

$$\begin{aligned}
&\alpha_o^U = \max \quad \alpha_o \\
&\text{subject to} \\
&\sum_{r=1}^{s} u_r\,y_{ro} - u_k\,\alpha_o - \sum_{i=1}^{m} v_i\,x_{io} = 0 \\
&\sum_{r=1}^{s} u_r\,y_{rj} + u_k w_j \alpha_o - \sum_{i=1}^{m} v_i\,x_{ij} \leq 0, \qquad j \neq o \\
&\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1 \\
&0 \leq \alpha_o \leq y_{ko} \\
&w_j \alpha_o \leq p_j, \qquad j \neq o \\
&\textstyle\sum_{j\neq o} w_j = 1,\; w_o = 0, w_j \geq 0, j \neq o, \\
&v_i \geq \varepsilon, \qquad i = 1, \dots, m \\
&u_r \geq \varepsilon, \qquad r = 1, \dots, s.
\end{aligned} \tag{8}$$

Eqs. (8) represent a nonlinear programming problem. An approach for computing α_o^U is that we introduce some dummy DMUs located in PPS and allocate the amount $\alpha_o^U$to their k-th output. Now α_o^U can be obtained by solving following problem, which is a simple linear programming model by $\bar{\alpha}_o = u_k\,\alpha_o$.

$$\alpha_o^U = \max \quad \alpha_o$$
subject to
$$\sum_{r=1}^{s} u_r\, y_{ro} - u_k\, \alpha_o - \sum_{i=1}^{m} v_i\, x_{io} = 0$$
$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0 \qquad j \ne o \tag{9}$$
$$0 \le \alpha_o \le y_{ko}$$
$$v_i \ge \varepsilon \qquad i = 1, \dots, m,$$
$$u_r \ge \varepsilon \qquad r = 1, \dots, s.$$
The rank order of efficient DMUs are based on the following two criteria:

1. The *stability length*, α_o^L

2. The *agreement length*, $\alpha_o^U - \alpha_o^L$.

The stability length has a higher degree value respect to the agreement length since an efficient DMU could be inefficient when its ILS amount decreases more than α_o^L, otherwise it is definitely efficient. Therefore, we apply two weights M and m for α_o^L and $\alpha_o^U - \alpha_o^L$, respectively, such that m is sufficiently small compared with M. Therefore, we present the following measurement for ranking efficient DMUs based on their ILS,

$$r_o = M\alpha_o^L + \mathrm{m}(\alpha_o^{\mathrm{U}} - \alpha_o^L) \tag{10}$$

for the efficient DMU_o. It is clearly that the more r_o, the more rank order.

4. An application

We use a simple but sophisticated example from banking industry in Iran from Bagherzadeh Valami et al. (2012). It is always a tedious task to define inputs and outputs in banking industry and measure the relative efficiency levels. However, according to bank senior management and strategy consulting with experts following parameters were determined. In the current section, we implement the proposed method to rank 49 bank branches of a famous Iranian bank with three inputs and four outputs defined in Table 1. Details of our inputs of the DEA model are briefly explained as follows,

1. **Outstanding claims:** These indicators represent a percentage of the granted facilities to the customer where the debts are not collected on maturity date. (Criterion is based on a percentage of total foreign exchange and Rial facility is granted)
2. **Personnel costs:** This index shows total costs associated with personnel and facilities granted to employees to encourage and the scale is based on the million Rials.
3. **Administrative costs:** This index shows total operating costs, office supplies and the scale is based on the million Rials.

System outputs are also as follows,

1. **Total deposits:** This index shows that total amounts of deposit in each branch specified time intervals through the absorption of various deposits, including deposits loan current loan savings, short and long term investment of its customers shall collect in billion Rials.
2. **Total facility granted:** This index shows total amounts given to customers in each branch in terms of various facilities to its customers in billion Rials or foreign currency payment.
3. **Total customers:** This index indicates total numbers of customers in certain time intervals.
4. **Number of credit cards:** This index indicates the number of issued cards both debit card, credit card and gift and Ben in a specified period of time is.

Total amount for deposit in a community is a constant value and can be considered as ILS category. The implementation of the proposed model for this case study yields efficient units as follows,

$$E = \{DMU_j | j = 1,2,4,5,8,9,12,14,19,23,24,33,35,36,40,47\}.$$

The efficiency scores of DMUs are shown in Table 2 where the first output as an index of the ILS one is intended and $m = 1$, $M = 10$.

Table 1
Data for 49 bank branches

DMUs	Input1	Input2	Input3	Output1	Output2	Output3	Output4
1	100.39	415467.80	435093.30	128313.00	66186.00	2661355.00	21059.00
2	137.04	14136.80	3861.90	16911.00	16238.00	157437.00	425.00
3	147.92	10660.00	2348.80	4471.00	5839.80	117608.00	587.00
4	73.15	4251.00	1248.90	5565.90	2541.00	52544.00	280.00
5	21.94	5204.00	1419.80	6199.00	2402.00	74335.00	252.00
6	137.11	6772.30	1635.00	2435.50	3228.00	49222.00	233.00
7	40.24	5272.50	1431.20	2870.60	669.00	75697.00	391.00
8	11.10	3501.00	1509.70	4852.20	1084.40	35609.00	521.00
9	82.99	5183.90	3903.80	5799.40	5201.00	42794.00	412.00
10	41.00	3297.00	972.40	1818.50	493.00	27730.00	458.00
11	261.19	4018.80	1124.00	2031.00	356.00	30729.00	312.00
12	2.67	3123.90	1117.90	983.70	457.00	21282.00	88.00
13	19.32	2893.30	858.00	1329.90	1663.60	16964.00	122.00
14	1.05	2461.40	645.90	737.60	94.50	6312.00	300.00
15	1.19	2050.70	818.90	447.00	15.80	9297.00	97.00
16	0.50	2290.60	966.60	238.10	13.60	6478.00	31.00
17	0.50	2036.70	5189.50	275.10	0.80	4483.00	27.00
18	24.33	4351.90	1053.30	2577.20	331.20	13708.00	182.00
19	0.50	2454.30	678.30	468.80	90.40	16784.00	57.00
20	29.02	2024.00	720.50	1052.20	72.40	4307.00	43.00
21	0.50	2442.00	853.90	333.18	15.80	7397.00	22.00
22	0.50	1956.80	1580.10	478.10	25.78	6763.00	90.00
23	0.50	2523.80	2968.80	827.80	16.80	7516.00	59.00
24	0.50	2017.40	975.00	798.80	42.94	7010.00	269.00
25	57.21	4200.80	1405.30	2392.90	402.90	60583.00	582.00
26	1.89	2556.30	1022.10	598.30	84.00	15432.00	250.00
27	0.50	2246.60	2162.10	434.50	14.90	6369.00	160.00
28	34.75	3333.80	1307.90	726.80	214.60	30746.00	283.00
29	5.84	2269.10	1424.00	221.90	56.30	7575.00	95.00
30	26.64	2779.10	882.60	354.70	140.60	21508.00	121.00
31	33.28	2562.10	1148.30	397.00	133.40	13843.00	446.00
32	2.30	1880.00	1383.00	63.79	10.50	2476.00	30.00
33	46.26	11132.00	3146.00	7794.40	4008.00	198162.00	1274.00
34	113.84	4602.20	1521.40	1580.70	667.30	59439.00	338.00
35	8.77	2426.00	1113.20	352.40	150.00	16165.00	577.00
36	39.21	5128.80	1203.90	1750.80	1103.40	108084.00	120.00
37	4.23	2191.40	1206.00	110.40	33.80	4685.00	44.00
38	46.97	2850.00	1392.70	321.00	473.00	22694.00	563.00
39	3.30	2181.00	617.70	186.83	79.30	6076.00	287.00
40	4.84	3701.30	967.50	2205.10	240.90	29661.00	768.00
41	3.54	2244.10	830.10	196.60	144.00	12994.00	209.00
42	8.77	4689.00	1070.70	2947.30	701.00	39461.00	294.00
43	0.50	2321.50	1072.70	520.50	1.30	5016.00	36.00
44	16.50	4645.60	1247.00	1746.50	930.50	66144.00	400.00
45	19.89	3183.60	1372.00	439.80	365.00	26229.00	624.00
46	0.61	1397.70	2385.40	105.80	8.00	4301.00	16.00
47	96.97	4871.80	1283.50	4673.60	549.00	106176.00	314.00
48	80.10	6347.80	2433.40	1784.50	2972.60	29252.00	240.00
49	29.32	2456.10	833.90	349.00	255.40	13923.00	419.00

Table 2
The result ranking efficient DMUs

Efficient DMUs	α_o^L	DMU_q	α_o^U	$r_o = 9\alpha_o^L + \alpha_o^U$	Rank Order
DMU_1	3489.932	DMU_{41}	128350.0935	159759.4815	1
DMU_2	4.11E+03	DMU_{13}	16911.02551	53901.02551	3
DMU_4	2.55E+02	DMU_5	383.9446554	2678.944655	14
DMU_5	6.63E+02	DMU_2	1043.101031	7010.101031	10
DMU_8	1.11E+03	DMU_{40}	1731.382632	11721.38263	8
DMU_9	1.41E+03	DMU_{35}	3022.549414	15712.54941	7
DMU_{12}	4.59E+02	DMU_{19}	983.8132296	5114.81323	11
DMU_{14}	7.37E+02	DMU_{12}	737.0150943	7370.015094	9
DMU_{19}	4.68E+02	DMU_1	468.670948	4680.670948	12
DMU_{23}	2.18E+01	DMU_{24}	41.93542074	238.1354207	16
DMU_{24}	2.00E+02	DMU_{14}	395.5647448	2195.564745	15
DMU_{33}	7.79E+03	DMU_2	7794.374072	77904.37407	2
DMU_{35}	3.52E+02	DMU_1	352.4022317	3520.402232	13
DMU_{36}	1.75E+03	DMU_1	1750.775712	17500.77571	6
DMU_{40}	1.75E+03	DMU_{49}	2205.101132	17955.10113	5
DMU_{47}	1.68E+03	DMU_{36}	3449.602653	18569.60265	4

5. Conclusion

In this paper, we have presented an investigation on efficiency of banking industry where there are some limitations on resources. The proposed study of this paper has been used for a real-world case study of banking industry, which was active in some small cities with limited amount of bank deposit. The study has considered deposit as ILS factor and using the methods proposed in this paper, we have measured the relative efficiencies of different units. The main contribution of this paper is to provide a ranking method in based of an ILS. The approach is presented for single output ILS case and it is similarly expandable for single input with a limited source case. Finally, this approach is explained with an Application.

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